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OPTIMUM MEAN IONOSPHERIC HEIGHT IN TOTAL ELECTRON CONTENT OBSERVATIONS

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Abstract: In order to obtain an accurate TEC (Total Electron Content) from the differential Doppler frequency measurements of two coherent signals transmitted from NNSS (Navy Navigation Satellite System) satellites received at a pair of ground stations, it is necessary to solve an integral constant which depends greatly on the assumed mean ionospheric height h_s.

We propose here a novel method which enables us to determine h_s in a reasonable way as follows.

By examining the rms sum of the calculated difference of TEC_v's ("composite difference" of the vertically corrected TEC) from six pairs of the four stations, Kokubunji, Sendai, Ebetsu, and Wakkanai, located in the northern part of Japan in the North-South direction, with various assumed h_s values, we select the combinations of neighboring nearest stations, Kokubunji-Sendai and Ebetsu-Wakkanai only, for which we obtain suitable h_s's with which the composite difference takes a single minimum for each pair of the stations. Then, we determine the latitudinal dependence of h_s by least-squares fit to a straight line for these locally deduced values. The mean ionospheric height h_s as a function of latitude thus derived should be optimum.

In practice, we examined the data set during the period from November 11 to 18, and from 21 to 25, 1994, throughout which magnetic activity remained low with K_p index in the range of 0_+~3_+, and we found 13 events to apply the present method effectively.

After showing a possible signature of medium-scale TID (Traveling Ionospheric Disturbance) observed at the time of $K_p=4_-$ on May 4, 1994, we give some comments on the observations of the medium-scale TID in the polar region.

1. Introduction

If we carry out the differential Doppler frequency measurement of TEC (Total Electron Content) at the ground by receiving two coherent signals transmitted from NNSS (Navy Navigation Satellite System) satellites with the "two-station method" which was first suggested by LEITINGER *et al.* (1975), we need to determine an unknown integral constant which must depend greatly on an assumed mean ionospheric height h_s.

LEITINGER *et al.* (1975) mentioned that the choice of 50 km above the height of maximum electron density is recommendable for h_s, and need not change the fixed height value. In another paper (LEITINGER *et al.*, 1984), they also stated that the height of 400 km is the best choice at least under quiet to moderately disturbed conditions of the ionosphere.

However, when we apply actually the "two-station method" to the TEC observations by Japanese network consisting of four stations from the South to North directions, Kokubunji (35.7°N, 139.5°E), Sendai (38.3°N, 140.9°E), Ebetsu (43.1°N, 141.5°E) and Wakkanai (45.4°N, 141.7°E), with a spatial distance (about 1100 km) similar to the case of LEITINGER *et al.* (1975), we have unfortunately been confronted with difficulty that the assumption of fixed 400 km causes discontinuous TEC's for different couples among the four stations. The degree of this discontinuity changed evidently with changing the values of h_s. This fact led us to an idea of "variable h_s".

In the present paper, after giving a brief outline of the principle of the TEC measurement we have conducted so far, we will propose a novel method that enables us to determine a reasonable optimum h_s value. Once h_s is determined as a function of latitude, an unknown constant must then be determined, and hence the absolute TEC can be finally calculated.

We will also discuss the TEC measurements from the viewpoint of applicability of the two-station method of the present paper to the observations of medium-scale TID (Traveling Ionospheric Disturbance) in high latitudes.

2. Principle of TEC Measurement

2.1. Differential Doppler measurement

The NNSS satellites transmit two coherent signals ($f_1=149.988$ and $f_2=399.968$ MHz) at the height of about 1100 km during their motion along circular polar orbit with the period of about 110 min. Doppler frequency shift of each signal is attributable to its propagation through the ionospheric medium. Thus we can observe the changing rate of TEC with time by the method of differential Doppler frequency. The wave signal is observable at a ground station for about 10 min on the average from horizon to horizon as the satellite passes over the earth. If the ionospheric condition does not vary so rapidly within such short-time interval, we can assume the TEC variation to be spatial. The available satellite data are restricted to those with elevation angles greater than 10 deg. so long as the propagation path is assumed to be straight. We assume also, as a first approximation, that the measured TEC is in the same longitude as that of NNSS satellite.

For the two coherent receiving signals of f_1 and f_2 ($f_1/f_2=3/8$ for NNSS), the differential Doppler frequency $\mathrm{d}\Psi/\mathrm{d}t$ is given as

$$\frac{\mathrm{d}\Psi}{\mathrm{d}t}=\Delta f_1-\frac{f_1}{f_2}\Delta f_2, \qquad (1)$$

where Δf_1 and Δf_2 denote observed Doppler frequency shifts for each frequency (DAVIES, 1990). Integrating both sides of eq. (1) with time t under the condition of both f_1 and f_2 being much greater than plasma frequency in the region of interest, we find the expression of the phase parameter Ψ as follows:

$$\Psi + \Phi_0 = C_D \mathrm{TEC}, \tag{2}$$

where TEC is defined as the line integral of electron density along a radio ray path s as follows,

$$\mathrm{TEC} = \int_s N \mathrm{d}s, \tag{3}$$

and

$$C_D = \frac{e^2}{8\pi^2 m \varepsilon_0 c} \frac{f_2^2 - f_1^2}{f_1 f_2^2} = 7.70 \times 10^{-16} [\mathrm{m}^2] \text{ (for NNSS)}, \tag{4}$$

where e denotes the elementary electric charge, m the electron mass, ε_0 the dielectric constant of vacuum, and c the light speed, respectively.

The value of Ψ itself can be calculated by the numerical integration of eq. (1) assuming an appropriate integration time interval, which is taken as 4.6 s, *i.e.*, data sampling time of the present study. Then from eq. (2) we can deduce TEC, provided that the integral constant Φ_0 is determined.

2.2. Two-station method

Let us choose arbitrarily two ground stations, A and B, among the four stations, and apply eq. (2) to these combined stations. Here we assume an appropriate h_s at which two slant lines between the satellite and each station intersect at a point P. As the satellite moves, a number of such crossing points P_i ($i = 1,2,3,\cdots, l$) with the spatial interval corresponding to sampling time may be taken on the same h_s. Then, for these points, eq. (2) can be written in the following form:

$$\Psi_{\mathrm{AP}i} + \Phi_{0\mathrm{A}} = C_D \frac{\mathrm{TEC}_{v\mathrm{AP}_i}}{\cos \chi_{\mathrm{AP}_i}}, \tag{5}$$

$$\Psi_{\mathrm{BP}i} + \Phi_{0\mathrm{B}} = C_D \frac{\mathrm{TEC}_{v\mathrm{BP}_i}}{\cos \chi_{\mathrm{BP}_i}}, \tag{6}$$

$$(i = 1,2,3\cdots, l).$$

Here, the vertically corrected $\mathrm{TEC}_v = \mathrm{TEC} \cos \chi$, where χ stands for the zenith angle of satellite at the point P, and subscripts A, B, and P_i refer respectively to the two

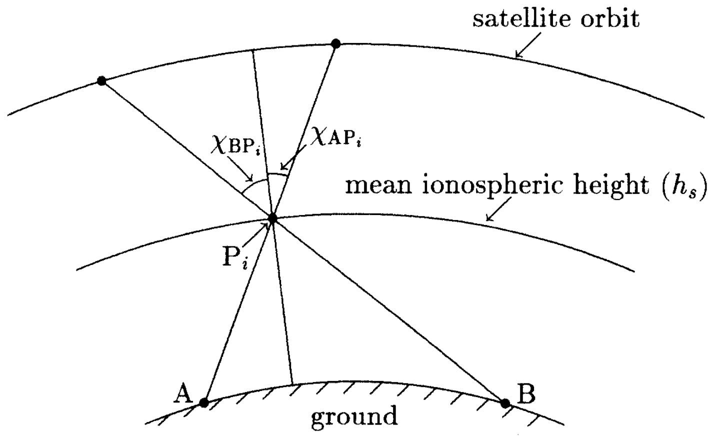

Fig. 1. Geometrical configuration of "two-station method".

ground stations and the ionospheric crossing point of the radio signals from the satellite (see Fig. 1). When the sampling time is 4.6 s, the number of data sample per station, l, amounts to 158–164.

Then, we will be able to determine the unknown constants, Φ_{0A} and Φ_{0B}, by least-squares minimization of the following quantity;

$$\sum_{i=1}^{l} (\mathrm{TEC}_{vAP_i}-\mathrm{TEC}_{vBP_i})^2=\frac{1}{C_D^2}\sum_{i=1}^{l}\{(\Psi_{AP_i}+\Phi_{0A})\cos\chi_{AP_i}-(\Psi_{BP_i}+\Phi_{0B})\cos\chi_{BP_i}\}^2. \quad (7)$$

Setting the differentiations of eq. (7) with Φ_{0A} and Φ_{0B} equal to zero, we solve them as simultaneous algebraic equations with respect to the variables, Φ_{0A} and Φ_{0B} (LEITINGER *et al.*, 1975).

3. Determination of Optimum Mean Ionospheric Height

We will first examine the dependence on h_s of "composite difference", σ, that is defined as follows:

$$\sigma=\sqrt{\sum_{i=1}^{l}\frac{(\mathrm{TEC}_{vAP_i}-\mathrm{TEC}_{vBP_i})^2}{l}}\,. \quad (8)$$

In order to make our explanation clearly understandable, we will now take up a data sample around 1410 JST, November 11, 1994. For the data set of six paired stations, Kokubunji-Sendai, Kokubunji-Ebetsu, Kokubunji-Wakkanai, Sendai-Ebetsu, Sendai-Wakkanai, and Ebetsu-Wakkanai, we calculated σ as a function of h_s in the range from 200 to 500 km. The results are shown in Fig. 2 for short-distance pairs, and Fig. 3 for long-distance pairs. We can see from the figures that the h_s-dependence

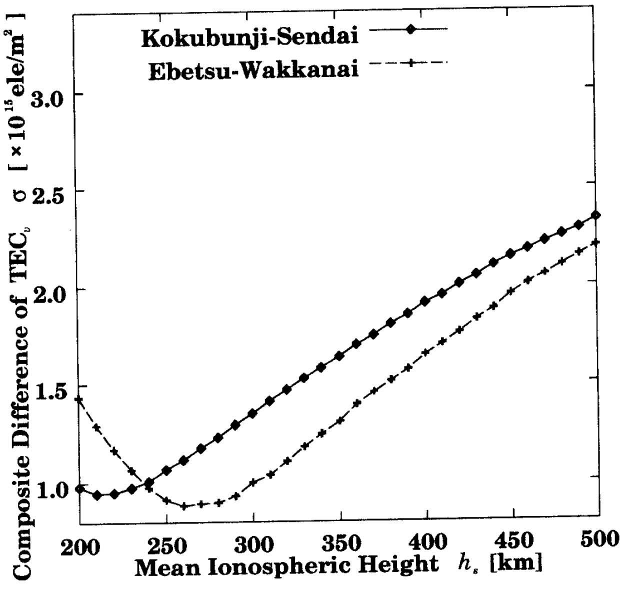

Fig. 2. Dependence of the "composite difference" of TEC$_v$ observed at the two combined ground stations on assumed mean ionospheric height in the case of pair data set from Kokubunji-Sendai, and Ebetsu-Wakkanai.

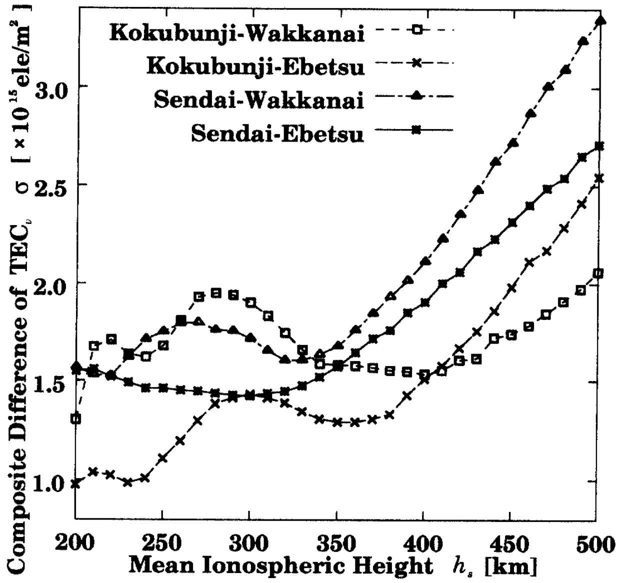

Fig. 3. Dependence of the "composite difference" of TEC_v *observed at the two combined ground stations on assumed mean ionospheric height in the case of pair data set from Kokubunji-Wakkanai, Kokubunji-Ebetsu, Sendai-Wakkanai, and Sendai-Ebetsu.*

of σ shows a simple form for the neighboring pairs, Kokubunji-Sendai and Ebetsu-Wakkanai, with a single minimum, while the dependences become more complex as the mutual distance between stations increases. Although the Sendai-Ebetsu pair shows also a single minimum in Fig. 3, we notice some overlapping (unstable) fluctuations in the higher values of h_s.

We have easily confirmed that if we choose $h_s=210$ km at which the composite difference takes a single minimum for the short-distance pair of Kokubunji-Sendai, the latitudinal variations of TEC_v calculated individually for each station coincide well one another. The same is true for another short-distance pair of Ebetsu-Wakkanai for $h_s=260$ km where the composite difference takes a single minimum. This implies that we can reasonably select from a coupled neighboring nearest stations an optimum h_s at which the composite difference takes a single minimum, say $h_s=210$ km for Kokubunji-Sendai pair, and 260 km for Ebetsu-Wakkanai one.

Adopting these values of h_s, we applied the two-station method to calculate the variation of TEC_v vs latitude as shown in Fig. 4 for Kokubunji-Sendai and Ebetsu-Wakkanai pairs. We see in this figure that a clear discontinuity, DC offset, appears between the two-paired curves, which will be caused from our assumption of the two different values of constant h_s for each pair of the stations.

In order to reduce this discrepancy as much as possible, we now propose an idea that at first for the coupled neighboring stations we estimate provisionally optimum h_s's as described above, and then as a second step determine the latitudinal dependence of h_s by least-squares fit to a straight line for these locally deduced values. For example, the resultant relation for the present problem is expressed as

$$h_s=4.05\times\varphi+67.7, \qquad (9)$$

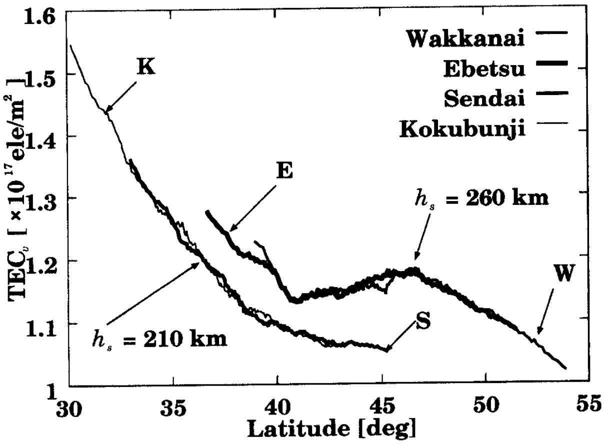

Fig. 4. *Latitudinal variation of TEC$_v$ obtained by applying "two-station method" to the pair data set from Kokubunji-Sendai, and Ebetsu-Wakkanai, assuming the mean ionospheric height of 210 and 260 km, respectively.*

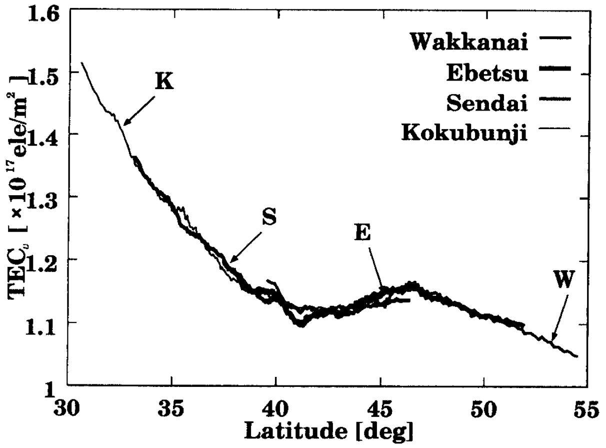

Fig. 5. *Latitudinal variation of TEC$_v$ obtained by applying "two-station method" to the pair data set from Kokubunji-Sendai, and Ebetsu-Wakkanai, assuming the mean ionospheric height with consideration on its latitudinal variation.*

where φ denotes the latitude in degree and h_s is given in km. As a final step, adopting the values of h_s that satisfy eq. (9), we calculated again the TEC$_v$'s for Kokubunji-Sendai pair and Ebetsu-Wakkanai one, and show the results in Fig. 5. It is obvious from the figure that the gap among the results from four stations has almost disappeared. Needless to say, eq. (9) is not a general relation, but the one to be obtained for each event.

4. Discussions

We attempted, in practice, to examine the data set of 65 in number during the period from November 11 to 18, and from 21 to 25, 1994, throughout which magnetic activity was low with K_p index of 0_+~3_+. We have then found 13 events to which the two-station method with variable h_s for the combinations of Kokubunji-Sendai and Ebetsu-Wakkanai was effectively applied. From remaining 52 events, we had 10 events with no such effective combinations, 22 events with only one effective couple, 11 and 9 events with 2 and 3 effective couples, respectively (except Kokubunji-Sendai and Ebetsu-Wakkanai pairs).

At nearly the same time as the above successful 13 events, the height of the maximum electron density over Kokubunji station, h_m [km], can be calculated using transmission factor, $M(3000)F2$, which was deduced from vertical sounding data at Kokubunji as follows (SHIMAZAKI, 1955):

$$h_m = \frac{1490}{M(3000)F2} - 176. \tag{10}$$

From the comparison of h_s with h_m for Kokubunji station as shown in Fig. 6, we can clearly see that h_s not only varies in proportion to h_m, *viz.*, the correlation coefficient amounts to 0.82, but also becomes higher than h_m, as the value of h_s increases. Specifically, the least-squares fit to a straight line gives an expression as

$$h_s = 1.76 \times h_m - 174. \tag{11}$$

Since the optimum h_s may *a priori* be thought as a boundary that divides equally the contribution of TEC_V (LEITINGER *et al.*, 1975), eq. (11) implies that the TEC of the topside ionosphere becomes larger than that of the bottomside one as the value of h_m increases. It can be expected also that if K_p index increases, the resultant increase in scale height of the topside ionosphere will make TEC there increase to result in the increase of h_s. We hope these statements would be confirmed in the near future from the physical viewpoint by theoretical consideration as well as observation.

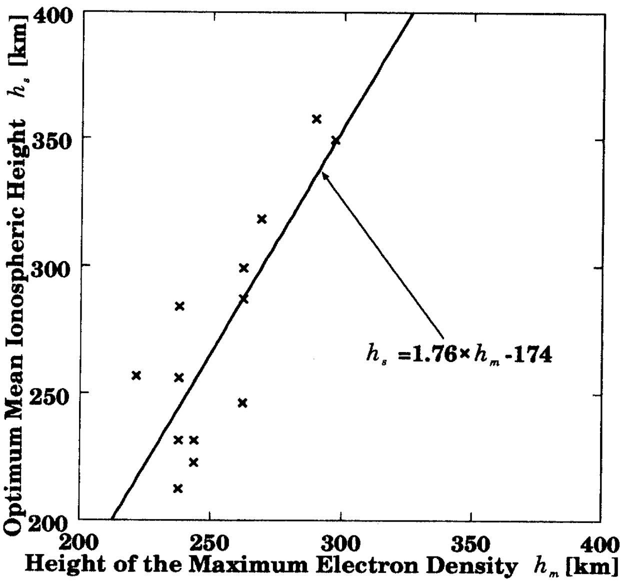

Fig. 6. Comparison between the optimum mean ionospheric height and the height of the maximum electron density at Kokubunji.

Apart from the data set for the period of low magnetic activity, we present also an interesting example of TEC$_v$ vs latitude obtained at the time of moderate magnetic activity (K_p=4$_-$) on May 4, 1994 in Fig. 7, which illustrates two distributions of TEC$_v$ vs latitude around 0142 and 0213 JST, respectively, obtained by applying the new method of the present paper. Now, paying our attention to two neighboring humps, *i.e.*, one is negative and the other is positive around 45° latitude in (a) (43° in (b)), we can recognize that these humps are not formed in the TEC$_v$'s from Kokubunji and Sendai, but built up in those of Wakkanai and Ebetsu only. This may be attributable to a large-scale density structure of the ionosphere slanted to the low latitude over the northern part of Japan (EVANS *et al.*, 1983). If we observe such structure from the higher-latitude side, the latitudinal variation of TEC$_v$ may become wavy, whereas from the lower-latitude side such pattern would not be detected because of a smoothing effect.

In order to determine the background component of TEC$_v$, we fitted two cubic

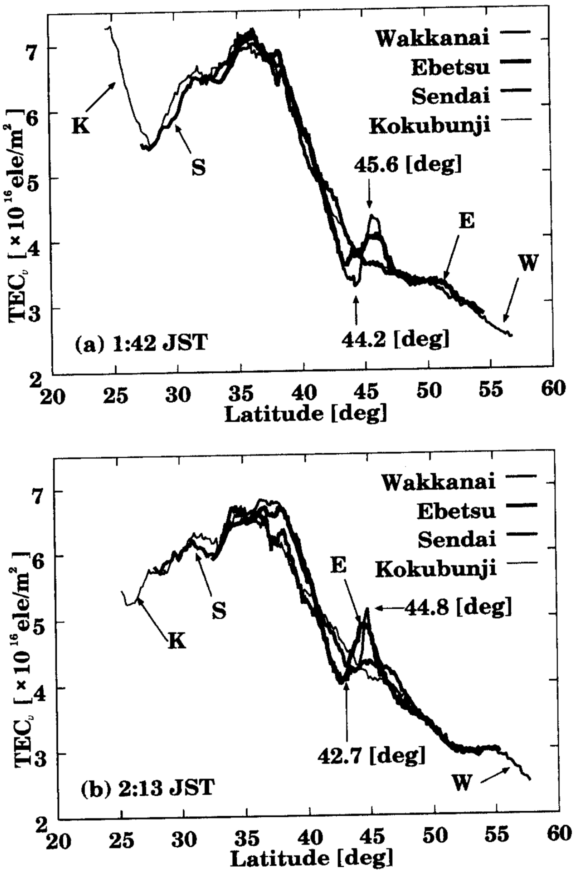

Fig. 7. Latitudinal variations of TEC$_v$ on May 4, 1994: (a) for 0142 JST, and (b) for 0213 JST.

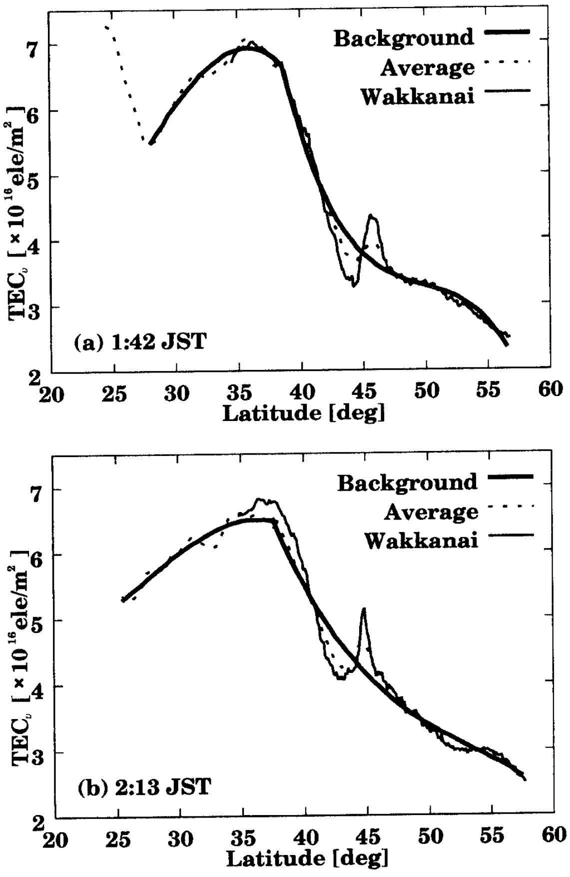

Fig. 8. The background component of TEC$_v$, and the deviations from it as a function of latitude, which are based on the result of Fig. 7: (a) for 0142 JST, and (b) for 0213 JST, May 4, 1994.

equations, in least-squares sense, to each curve of the same TEC_v variations of Fig. 7, and then exhibit the results by thick solid lines in Fig. 8. The dotted line and the thin solid one indicate the average of the TEC_v's and the TEC_v from Wakkanai of Fig. 7, respectively. The deviation from the background may signify the exsistence of either a TID or a non-propagating ionospheric irregularity. We notice that, for each of the above humped structure in Fig. 8, its fractional amplitude is estimated to be 0.14–0.22 with the horizontal width of 240–420 km, and change together their positions in a time interval of 31 min. Then, there is a great possibility that we have found a signature of medium-scale TID which propagates with an apparent southward speed of 51–96 m/s with a horizontal wavelength of 660–840 km, because these values are consistent with those of the medium-scale TID (*e.g.*, FRANCIS, 1975).

There are a couple of papers on the observations of the medium-scale TID's by means of the NNSS satellites, *i.e.*, EVANS *et al.* (1983) and OGAWA *et al.* (1987), for middle and high latitudes, respectively. A common point in their papars is that they observed relative TEC, in other word, they took up the differential Doppler frequency $d\Psi/dt$. They identified *a priori* the TID's when they found oscillations on the trace of $d\Psi/dt$ observed at a single ground station. In contrast to their study we can calculate the absolute TEC, so that we will be more sure to detect the medium-scale TID in the way as described above.

OGAWA *et al.* (1987) reported that their statistical results of the medium-scale TID's obtained at Syowa Station (69.0°S, 39.6°E), Antarctica, were nearly consistent with the TID observations made by EVANS *et al.* (1983) at middle latitudes by using similar technique. Therefore, it will be expected that a large number of the medium-scale TID's can be detected by a TEC receiver network set up in the polar region, where several kinds of auroral activities, *i.e.*, temporal variations of auroral electrojet, particle precipitation and supersonic movement of auroral arcs, are believed to be possible origin of the atmospheric gravity waves, in addition to the waves of the lower atmospheric origins, (*e.g.*, HUNSUCKER, 1982). If we want to put the new idea in the future program of JARE, two additional stations are needed in addition to Syowa and Dome Fuji (77.3°S, 39.7°E; 920 km apart from Syowa) to constitute a network of TEC measurement.

We also note here one weak point in the application of the present method: *i.e.*, it is likely that the TID observation frequency may become less in comparison with that of OGAWA *et al.* (1987), because we will not be able to encounter so much flight chance of the NNSS satellites over the receiving stations on the ground in a nearly straight line stretching along the North-South direction.

Anyway, we hope our novel method of the determination of the absolute TEC will be useful to some extent for the observation of medium-scale TID in high latitudes as well as at middle latitudes.

5. Summary

As for TEC observations, we applied "two-station method" to the data set of the differential Doppler frequency from four stations, Kokubunji, Sendai, Ebetsu, and Wakkanai, which are spanning the northern part of Japan with about 1100 km length in the North-South direction.

Firstly, we examined the rms sum of the calculated difference of TEC_v's ("composite difference") from six pairs of the four stations assuming a number of mean ionospheric height h_s in the range of 200 to 500 km. Then, in order to avoid the effect from local structure of the ionosphere as much as possible, we select the combinations of neighboring nearest stations, Kokubunji-Sendai and Ebetsu-Wakkanai only, for which we obtained suitable h_s's at which the composite difference takes a single minimum for each pair of the stations.

As a next step, we estimated the latitudinal dependence of h_s by least-squares fit to a straight line for these locally deduced height values. Finally, regarding the height vs latitude thus derived as optimum h_s, we applied again the two-station method to obtain reliable values of the absolute TEC_v's.

Practically, we examined the data set during the period from November 11 to 18, and from 21 to 25, 1994, throughout which magnetic activity was low with K_p index of $0_+ \sim 3_+$ and the present method was effectively applied to 13 events.

A statistical estimate indicated that the optimum h_s thus obtained not only varies in proportion to the height of the maximum electron density h_m at Kokubunji, but also becomes higher than the latter.

We also showed a possible signature of medium-scale TID from the latitudinal variations of TEC_v observed at the time of magnetic activity with $K_p = 4_-$ on May 4, 1994.

Having the above result of the TID in mind, we gave also some comments on the observations of medium-scale TID in high latitudes.

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