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ATTEMPT TO DETECT COMBINATION TONES BETWEEN THE EARTH'S FREE OSCILLATION AND THE EARTH TIDE

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Abstract: When there are two oscillations in a nonlinear system, new oscillations called combination tones appear. We try to detect combination tones between the earth tides and the earth's free oscillations using the long-period seismograms of the Bolivian great deep earthquake on June 9, 1994 recorded by a broad band seismometer and a superconducting gravimeter installed at Syowa Station in Antarctica. The results suggest that combination tones between the free oscillations and the semidiurnal tide exist.

key words: nonlinear elasticity, the earth's free oscillation, the earth tide, combination tone

1. Introduction

The linear stress-strain relation (Hooke's law) has been successfully applied to analyses of seismic waves. Therefore, little attention has been paid to nonlinear elastic effects on seismic wave propagation in the earth, excepting strong ground motions on soft soil layers (*e.g.* DIMITRIU, 1988; BERESNEV *et al.*, 1995).

However, there is abundant evidence showing large nonlinearity of the elasticity of hard rock in the earth. Static and dynamic experiments in laboratories show that the nonlinear response in rock is large (*e.g.* BIRCH, 1960; JOHNSON and MCCALL, 1994; TENCATE *et al.*, 1996). On the other hand, precise field studies of the temporal variation of seismic wave velocities also indicate the existence of large nonlinearity in the crust; the tidal deformations in the crust cause changes in the P wave velocity of the order of 10^{-4} (*e.g.* DE FAZIO *et al.*, 1973). Moreover, AGNEW (1981) reported that some sort of nonlinearity is present in strain records of the earth tide, though it could be in the instrument or the ocean tides instead of the solid earth. These observations suggest the possibility that seismic waves are affected by the nonlinear rock elasticity.

One of the most prominent phenomena associated with oscillations of nonlinear media is the spontaneous generation of "combination tones" (LANDAU and LIFSHITZ, 1974). If two oscillations with different frequencies are present in a nonlinear medium, the nonlinearity will cause them to interact and generate new oscillations or combination tones.

If a large part of the earth is nonlinear, global oscillations such as earth tides and the earth's free oscillations may generate combination tones which can be detected as seismic signals. Of course, since the amplitudes of the combination tones, if any, are very small, high quality data must be analyzed to detect them. The aim of this study is to investigate the combination tones by using long period seismograms recorded at Syowa Station in Antarctica. The station is installed in a tectonically very stable area, so the quality of the data is expected to be good enough for the analysis.

2. Combination Tones

The generation of the combination tones in a nonlinear medium can be explained by a simple stress-strain relation. Following WIGLEY (1976), consider a strain $X(t)$

$$X(t) = A \sin \omega_0 + B \sin \omega_1, \qquad (1)$$

where ω_0 and ω_1 are the angular frequencies of the oscillation modes, and A and B are their amplitudes. For simplicity, the phase difference between two components and effects of attenuation are ignored here. Let $Y(t)$ be a stress in a nonlinear medium which is a function of the strain to second order.

$$Y(t) = \alpha X(t) + \beta X^2(t), \qquad (2)$$

where α and β are the linear and the second order elastic constants, respectively. Substituting (1) into (2), we obtain

$$Y(t) = \alpha \sin \omega_0 + \beta \sin \omega_1 - \frac{\beta A^2 \cos 2\omega_0}{2} - \frac{\beta B^2 \cos 2\omega_1}{2}$$

$$+ \beta AB \cos(\omega_0 - \omega_1) - \beta AB \cos(\omega_0 + \omega_1) + \frac{\beta(A^2 + B^2)}{2}, \qquad (3)$$

$Y(t)$ includes new oscillations with frequencies of $2\omega_0$, $2\omega_1$, $\omega_1+\omega_0$, and ω_1-ω_0, meaning combination tones. The frequencies of the combination tones are the sums or differences of the frequencies of the original oscillations. Furthermore, if we take account of higher order terms, more combination oscillations with frequencies such as $m\omega_0$ and $\omega_0+n\omega_1$ (m and n are integers), will be produced (Fig. 1).

In this study we investigate the case in which ω_0 is the frequency of an earth tide component and ω_1 is that of a mode of the earth's free oscillations. The earth is subjected to sinusoidal tidal forcing from the moon and the sun, the largest forcing coming from the tidal component M_2 whose period is 12.4 hours. By the interaction between the tidal

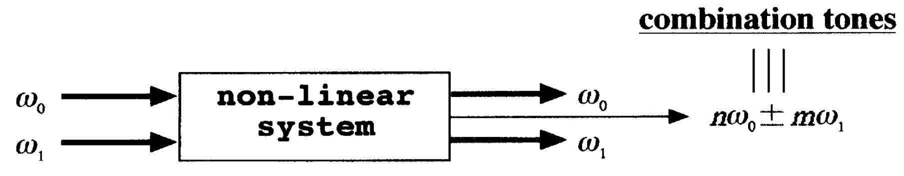

Fig. 1. Conceptual map of combination tones. ω_0 and ω_1 represent the frequencies of input signals. n and m are integers.

oscillations and the free oscillations excited by an earthquake, the nonlinearity will cause energy to appear at discrete frequencies around those of the free oscillations which are predicted from the linear theory. The frequency intervals between the free oscillations and the combination tones are integral multiples of the frequencies of the earth tides.

3. Data and Analysis

The data used are seismograms of the Bolivian great earthquake (Mw=8.2, Depth=600 km, June 9, 1994) recorded at Syowa Station in Antarctica. Two kinds of digital records are analyzed. One is the vertical component of the LP output of a broad band seismograph (STS-1) and the other is the MODE output of a superconducting gravimeter (SCG). The characteristics of STS seismographs at Syowa Station are de-

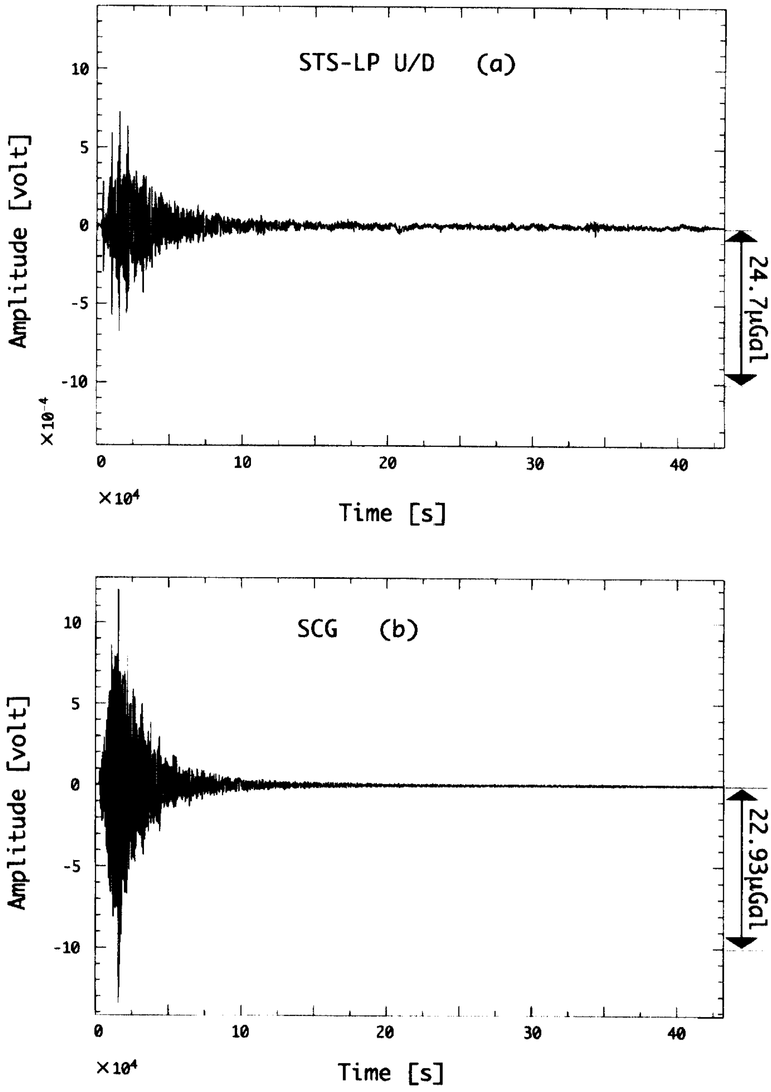

Fig. 2. Low-pass filtered observed records (5 days). The horizontal axis is time passed from 0000 (UT) on 1994/06/09. Gravity conversion values are also indicated. (a) STS record. (b) SCG record.

scribed in detail by Kanao and Kaminuma (1993) and those of SCG are given in Sato *et al.* (1995).

Although the LP output of STS-1 is originally designed to have a flat response for periods longer than 20 s, the response is not sufficient for periods longer than 12 hours (Kanao and Kaminuma, 1994). Since we seek combination tones in a frequency band of the earth's free oscillations, a high-pass filter will be adopted as follows.

The sampling intervals of the original data are 3 and 2 s for the STS and the SCG records, respectively. We use the data for five days after the earthquake. STS records are contaminated by high drift rates induced by the instrumental temperature changes (Kanao and Kaminuma, 1993), step-wise shifts caused by manual zero point adjustments of the instrument, and short durations of missing data. The fraction of the total of these durations without data points to a original data length of less than 1%. We use straight line

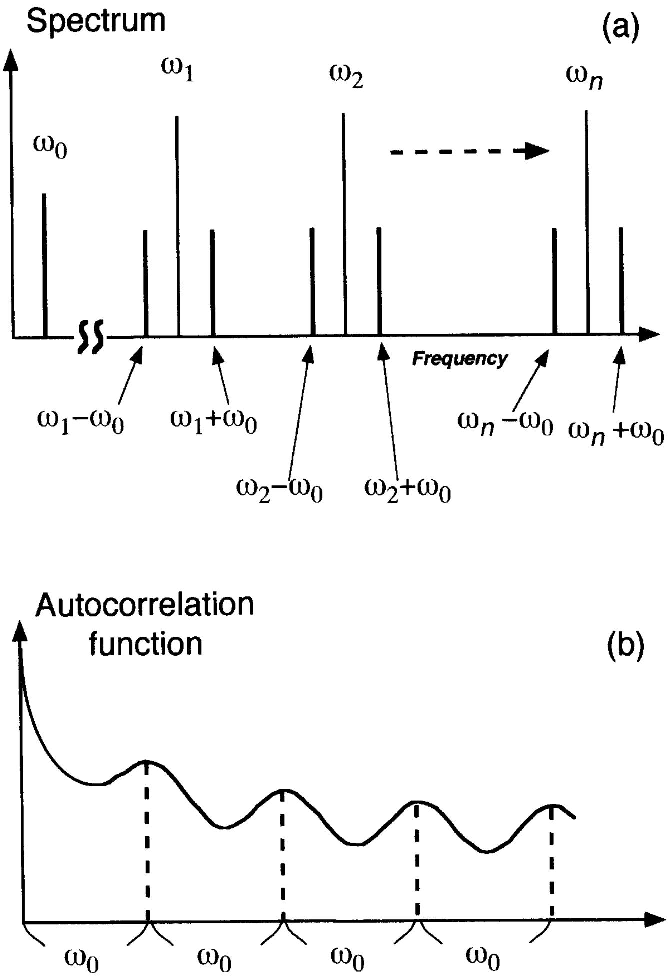

Fig. 3. Schematic map of detection of combination tones between the earth oscillations and the earth tides. n is an integer. ω_0 *is a frequency of the earth tide.* $\omega_n (n \geqq 1)$ *is each frequency of the earth oscillation. (a) Spectral structure. Higher order combination tones (*$\omega_n \pm m\omega_0$*, m is a natural number larger than 1) are not drawn here. (b) Expected autocorrelation function from (a).*

interpolation for the parts of data missing. The step-wise shifts of the records are corrected by adjusting the base lines of records. The data are resampled with an interval of 30 s and then a high-pass filter with a cut-off frequency of 1 mHz applied. The seismograms after these corrections are depicted in Fig. 2.

We calculate the spectra of the seismic records. To investigate the temporal change, the spectra are calculated for three time windows with a duration of three days, each window overlapping previous one for two days. We used a routine fast Fourier transform after applying the hanning window.

As the amplitudes of the combination tones should be very small, the direct observation of the combination tones on the spectra is difficult. Therefore, we calculate the autocorrelation functions of the spectra as a function of the frequency difference. As shown in Fig. 3a, we can assume that a spectral peak of each mode of the earth's free oscillations is accompanied by peaks of combination tones that keep equidistant intervals from the mode; *i.e.* the k-th mode with the frequency ω_k is accompanied by the

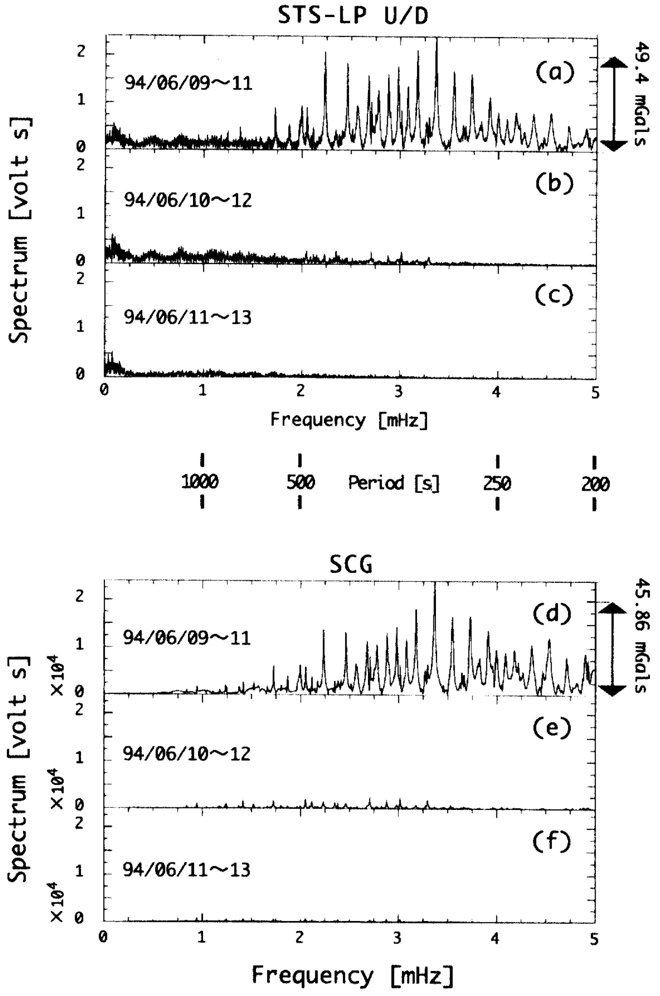

Fig. 4. FFT amplitude spectra of a data length of three days with a mutual time lag of two days. (a) and (d) give good correspondence to modes of the earth oscillation. The conditions of damping are also clear.

combination tones with the frequencies of $\omega_k+m\omega_0$ and $\omega_k-m\omega_0$ (m is a natural number, but Fig. 3a neglects the terms of $m>1$). If such a structure with equidistant peaks exists in the spectra, we can observe a sinusoidal oscillation of the autocorrelation function whose period is ω_0 (Fig. 3b).

4. Results

Figure 4 shows the obtained amplitude spectra. Many spectral peaks are recognized for both STS and SCG, in particular on the top ((a),(d)) parts which depict the spectra for the first three days. They represent the modes of the free oscillations. It is also clearly shown that the amplitude of the free oscillations decays with time; the large peaks disappear in the middle ((b),(e)) and bottom ((c),(f)) parts of Fig. 4.

Figure 5 shows the autocorrelation functions. It is clearly observed that there is a sinusoidal variation on the autocorrelation function of the second term of STS data (see

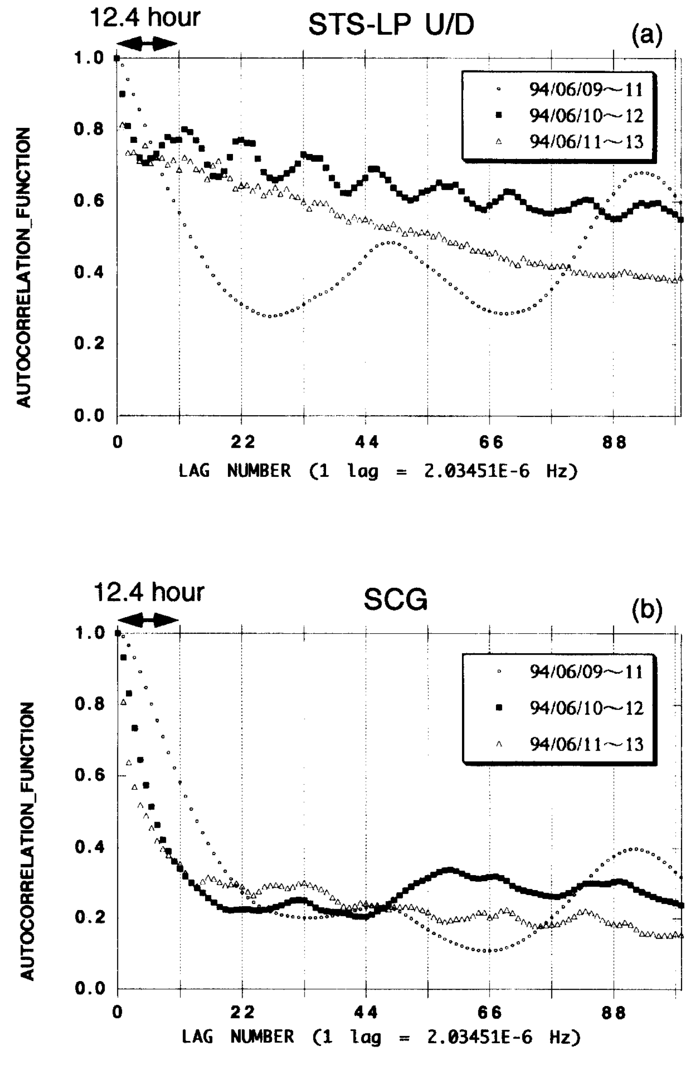

Fig. 5. Autocorrelation function of Fig. 4. The grid interval of lateral axes is about 12.4 hours (=11 lag numbers).

solid squares in the top part of Fig. 5). The period of the sinusoidal oscillation is about 12 hours. However, no large sinusoidal variation is recognized in the other terms of STS data or any terms of SCG data.

5. Discussion and Conclusions

Using digital records of STS and SCG of the Bolivian great deep earthquake, we try to detect combination tones between the earth's free oscillations and the earth tide. The existence of the sinusoidal oscillation of the autocorrelation function, whose period coincides with that of most strong semidiurnal tides, suggests a possibility that we have detected a combination tone.

However, several questions may remain. They include (1) why combination tones are not observed in the first term when the amplitudes of the free oscillations are large, and (2) why they are not registered on SCG records.

The first problem is not an unexpected result considering the growth of combination tones. The combination tones grow by energy transfer from the earth tide and the free oscillations. This means that the combination tones appear with some time lag from the excitation of the free oscillations. The combination tones also decay with time soon or later. Such temporal variations of the amplitudes of the combination tones may be responsible for their observation only in the second terms.

The second problem is serious for the contention that combination tones. This may suggest that the observed combination tone is caused by the nonlinearity of the STS system instead of the earth's response, or by the quality of the STS records lower than the SCG one. Taking a conservative view, therefore, we would be justified in concluding that the combination tones are detected if we can rule out contributions from the nonlinearity of the instruments.

Although detecting the combination tones requires further accumulation of more seismological data with theoretical studies of the nonlinear elasticity, we believe that observation of the combination tones will shed new light on the oscillation of the earth.

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