HEAT FLUX IN SURFACE SNOW AT MIZUHO STATION, ANTARCTICA: MONTHLY VALUES AND ERRORS

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***Abstract*:** Investigations are made on the determination of conductive snow heat flux at an Antarctic snow field, Mizuho Station, from the end of February to December 1979 during the first year of POLEX-South. Some preliminary analyses on the heat flux meter outputs raised questions on the data. As an alternative method, numerical differentiation and integration method is used to calculate snow heat flux from snow temperature. Errors accompanying the calculation are investigated, and it revealed that this method gives monthly heat flux with a relative error of about 10%. Snow temperature gradient in the surface layer is consistent with the present estimates. Estimated snow heat flux is compared with the results obtained at other sites in Antarctica.

1. Introduction

During the course of POLEX-South (1979–1982), Japanese Antarctic Research Expedition (JARE) carried out a micrometeorological observation at Mizuho Station (MAE *et al.*, 1981). One of the objects of the project is to determine heat budget on Antarctic snow surface. This paper aims at determination of snow heat flux, one of the heat budget components.

There have been many researches in which the snow heat flux is estimated, including DALRYMPLE *et al.* (1966) and WELLER and SCHWERDTFEGER (1977). But, there seem to be no investigations on the errors involved. Because the estimated values of snow heat flux are used for heat budget considerations, the error estimation is indispensable. In this paper, the most suitable method for present measurement is selected among various methods to estimate heat flux, and then the errors accompanying the method are investigated.

2. Selection of a Method for Estimation

The data compiled by WADA *et al.* (1981) provide hourly values of heat flux meter outputs at 0.1, 0.3 and 0.5 m, snow temperatures at 0.1, 0.3, 0.5, 1, 3 and 5 m in depth, and snow surface temperature from February 23 to December 31, 1979. Some questionable data are corrected with interpolation method using adjacent data. Figure 1 shows the daily mean values calculated from the data.

For the estimation of snow heat flux, there may be three methods applicable to this case:

1) Use an extrapolation technique on flux meter outputs to obtain flux at the surface.

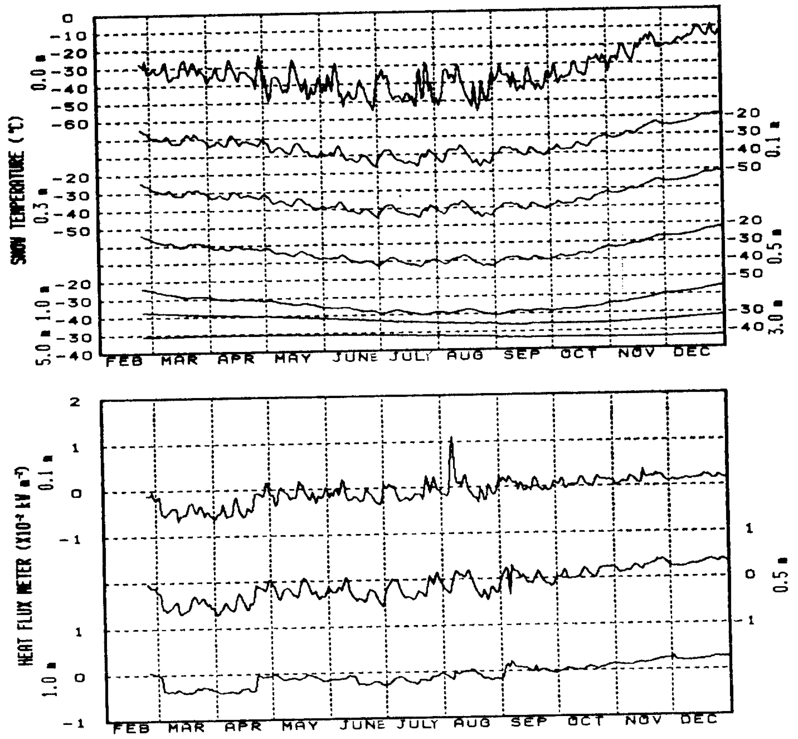

Fig. 1. Daily mean values of snow temperature and heat flux meter outputs at Mizuho Station from February 23 to December 31, 1979.

2) Assume appropriate value for the thermal conductivity and calculate flux from temperature gradient at the surface using the flux-gradient relation,

$$S = -\lambda \frac{\partial T}{\partial z}, \tag{1}$$

where S is the snow heat flux, λ the thermal conductivity, T the temperature, and z is the depth.

3) Assume appropriate value for the snow heat capacity and calculate flux from the divergence theorem,

$$\rho c \frac{\partial T}{\partial t} = -\frac{\partial S}{\partial z}, \tag{2}$$

where ρ is the snow density, c the specific heat of ice, and t denotes the time.

As described in the report, the depth of instruments in snow changed as the snow accumulated. That is, the depth of the thermal flux meter first set at 0.1 m was about 0.4 m at the last. Moreover, the 0.1 m flux meter gives lower values than the other two flux meters in December, even though the snow temperature at 0.1 m and 0.5 m is still increasing, indicating that the measurement suffers from some drifts in electrical circuits. These situations make it difficult to use the extrapolation technique for estimating the surface value from the flux meters.

On the other hand, the snow surface temperature probe was kept at the surface though the depth of the other snow temperature probes changed with snow accumulation. This situation also makes it difficult to use the second method because the temperature gradient is the largest at the surface in most cases.

Unlike the other two, the last method suffers little from snow accumulation because it needs no extrapolations nor space derivatives of temperature. It contains a time derivative of the temperature which is easier to refine than the space derivative because there are more data on the time axis. It also contains a coefficient concerning heat capacity, but there are more reliable data available on this coefficient than the thermal conductivity.

One method to calculate heat flux through eq. (2) is done via calculating Fourier coefficients of temperature such as exploited by DALRYMPLE *et al.* (1966). But, this method needs complete one year data and cannot be used here because data from January 1 to February 22 are absent. Therefore, another method which uses numerical differentiation and integration is selected.

3. Snow Heat Flux Calculation Formula

Integration of eq. (2) from z_1 to z_2 ($z_2>z_1$) leads to

$$S_2-S_1=-\int_{z_1}^{z_2}\rho c\frac{\partial T}{\partial t}\mathrm{d}z\ . \tag{3}$$

Since there are no significant heat sources in the surface snow layer, the heat flux at an appropriate deep level must vanish to zero. Observations at the South Pole and Maudheim (DALRYMPLE *et al.*, 1966) and at Plateau (WELLER and SCHWERDTFEGER, 1977) show that temperature fluctuations at 10 m depth are as small as 0.5–1 K. Thus, assuming $T(z_2)=(\text{const.})$ and $S_2=0$ at $z_2=10$ m, an equation for calculating surface snow heat flux S_0 is obtained as

$$S_0=\int_{0}^{10\mathrm{m}}\rho c\frac{\partial T}{\partial t}\mathrm{d}z\ . \tag{4}$$

For the convenience of calculation, eq. (4) is rewritten using numerical differentiation and integration as

$$S_0(t)=\sum_{i=0}^{6}\frac{1}{2}\left\{\bar{\rho}_i c(T_{i,t})\frac{T_{i,t+\Delta t}-T_{i,t-\Delta t}}{2\Delta t}+\bar{\rho}_{i+1}c\,(T_{i+1,t})\frac{T_{i+1,t+\Delta t}-T_{i+1,t-\Delta t}}{2\Delta t}\right\}(z_{i+1}-z_i)\ , \tag{5}$$

where z_i (i varies from 0 to 7) is the depth of snow temperature observation, and the quantities with suffix i represent those at that depth. Time is represented by the second suffix t, and $2\Delta t$ is the time interval of flux estimation. The specific heat of ice c is 2.0 J g^{-1} K^{-1} at -26°C, and varies a little with temperature.

The snow density varies not only with depth but also with time, but it is almost impossible to incorporate the time variations in the calculation. It is also impractical to include the density fluctuations which are caused by the stratified structure. Therefore, some smoothing must be made on the actual density data, and this is denoted with $\bar{\rho}$ in eq. (5). MAENO and NARITA (1979) give comprehensive data of snow/ice density at Mizuho Station in 1970–1975. The snow density ($\bar{\rho}$) used here is obtained from their figure as

$$\bar{\rho}(z)=\begin{cases} 400+20\,z\ (\mathrm{kg\ m^{-3}}) & (z<8\ \mathrm{m}) \\ 510+8.2\,z & (z\geq 8\ \mathrm{m})\,, \end{cases} \tag{6}$$

which is the approximation of the scattered data.

4. Errors in Heat Flux Estimation

Since there are many assumptions and approximations in eq. (5) and observational data contain some errors, considerations are made on the errors in calculations.

4.1. From density estimation

The actual snow density shows considerable scatters from eq. (6), especially near the surface due to the stratified structure of snow. Therefore, it is necessary to consider the errors in the assumed density, which can be calculated from the original density data (NARITA and MAENO, 1978) as

$$\rho'=\left\{\frac{\sum(\rho_i-\bar{\rho})^2}{N(N-1)}\right\}^{1/2}, \tag{7}$$

where ρ_i is the individual value of snow density, and N is the number of samples. Table 1 lists the calculated error as a fraction of $\bar{\rho}$. Since only a few samples are available near the surface where the scatter is the largest, the error is the most significant in 0–0.1 m layer.

The effect of the error in density on the snow heat flux can be given as

$$\delta S_\rho=\int_0^{10\mathrm{m}}\rho' c\frac{\partial T}{\partial t}\mathrm{d}z\,. \tag{8}$$

Calculation of δS_ρ can be done converting the integral and differential into the expression similar to eq. (5).

Table 1. Errors in average density estimation.

Layer (cm)	Number of samples	Error (%)
0 - 10	3	30
10 - 30	3	10
30 - 50	3	3
50 - 100	10	3
100 - 300	35	2
300 - 500	48	1
500 -1000	150	0.5

4.2. From temperature resolution and time interval

Equation (5) leads to a serious error if the temperature difference $(T_{i,t+\Delta t}-T_{i,t-\Delta t})$ becomes comparable with the resolution of the analog-to-digital converter (0.1 K), as in the deep layer or when the time interval is small. This error is further amplified if the time interval $(2\Delta t)$ is small. The effect of the temperature resolution (δT) on the snow heat flux can be given as

$$\delta S_T=\int_0^{10\,\mathrm{m}} \rho c \frac{\delta T}{2\Delta t}\,\mathrm{d}z\ . \qquad (9)$$

Again, the integral is calculated with summation.

A method to make this error small is to average the temperature for some period. In this case, the error decreases as

$$\delta T=\frac{\delta T_0}{\sqrt{N}}\ , \qquad (10)$$

where δT_0 is the original resolution (0.1 K) and N is the number of data to be averaged. The temperature thus averaged has no information on shorter period variations. Therefore, it is useless to keep the time interval ($2\Delta t$) short. If $2\Delta t$ is lengthened, the error represented by eq. (9) can be made smaller.

4.3. From the assumption of zero heat flux depth

An assumption has been made that the snow heat flux at 10 m depth is zero. But, since there still remains temperature fluctuation of 0.5–1 K, it is probable that there is some amount of heat flux about the same order as that would exist in a homogeneous snow/ice. Equations (1) and (2) are solved with sinusoidal temperature wave as a boundary condition, and amplitude of heat flux wave is obtained. Then, the error of estimation of surface heat flux due to limiting the integral to 10 m is estimated as

$$\delta S_{10}=2\lambda A_0\sqrt{\omega/2K}\ , \qquad (11)$$

where A_0 is the amplitude of temperature fluctuation at 10 m, ω is the radian frequency of temperature fluctuation taken as $2\times 10^{-7}\,(\mathrm{s}^{-1})$ which corresponds to the annual wave. With $K=(\lambda/\rho c)=10^{-6}\,\mathrm{m}^2\,\mathrm{s}^{-1}$, $A_0=0.5$ K, $\lambda=0.8\,\mathrm{J\,m^{-1}\,K^{-1}\,s^{-1}}$, which are the values typical at 10 m depth (WELLER and SCHWERDTFEGER, 1977), eq. (11) gives an approximate value of $\delta S_{10}=2\times 10^{-1}\,\mathrm{W\,m^{-2}}$.

4.4. From numerical integration

The errors in calculation of integral can be estimated with assumptions of homogeneous snow and sinusoidal wave at the interface. In this case, the temperature profile can be obtained theoretically as

$$T(z,t)=A_0\exp\left[-\frac{z}{l}\right]\cos\left(\omega t-\frac{z}{l}\right), \qquad (12)$$

where $l=\sqrt{2K/\omega}$ and ω is the radian frequency of the sinusoidal wave.

Integration with eq. (5) is to approximate the profile with a straight line. Therefore, the error in the numerical integration becomes significant if the thickness of the layer between two temperature probes becomes larger than the characteristic length (l). Table 2 lists estimated values of l for the layers of interest and for the period of temperature waves. Thermal diffusivity (K) is estimated from YAMADA and HASEMI (1974) and DALRYMPLE *et al.* (1966).

Table 2 indicates that estimation of daily variation of heat flux suffers seriously from the integration error. On the other hand, calculation of longer period, such as monthly, heat flux is relatively accurate, because it contains only the annual wave.

Table 2. Characteristic length for temperature wave.

Layer (cm)	K ($\times 10^{-7}$ m^2 s^{-1})	l (cm) Daily	Annual
0 - 10	1.0	5.2	100
10 - 30	1.5	6.4	123
30 - 50	2.0	7.4	141
50 - 100	3.0	9.1	174
100 - 300	7.0	13.9	265
300 - 500	8.5	15.3	292
500 -1000	9.3	16.0	305

4.5. Errors: summary

Snow heat flux S_0 and its errors δS_ρ and δS_T are calculated throughout the data period varying the time interval of flux determination ($2\Delta t$) and averaging the temperature over the interval. Then the root mean square values are calculated for δS_ρ and δS_T, and divided by the root mean square value of S_0, thus give relative errors which are shown in Fig. 2 as a function of the time interval. As indicated in Subsection 4.2, $\delta S_T/S_0$ decreases rapidly as the time interval increases. The relative error from density estimation is largest at shortest interval because heat, in short period, can penetrate only in shallow layer where the density estimation is erroneous.

The error from an assumption of zero heat flux depth at 10 m is constant as noted in Subsection 4.3. But, as also shown in Fig. 2, its relative significance becomes larger if the interval of flux estimation becomes longer because relatively large fluctuations of shorter period are smoothed away by averaging.

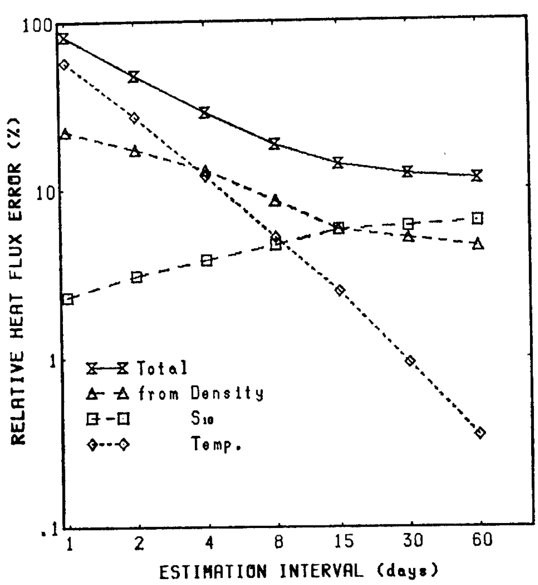

Fig. 2. Errors in snow heat flux estimation as a function of interval of estimation. Temperature is averaged over a period which is equal to the interval.

The total error of δS_ρ, δS_T and δS_{10} is shown in Fig. 2 as a solid line. If the interval is taken as one day, calculated results contain no more than errors. The total error decreases as the time interval increases and reaches a limit about 10% at time interval of 30 days. Although the error which accompanies numerical integration cannot be shown here, it increases the total error at short intervals and changes little at long intervals. In order to obtain meaningful results, only the monthly heat flux is considered hereafter. It is not impossible to estimate shorter period flux, but it needs considerable amount of calculations to correct errors.

5. Results and Discussions

In Fig. 3 are the tautochrones showing monthly average temperature versus depth. Note that the snow accumulation is represented as a surface rise. The area between two profiles multiplied by the heat capacity represents the heat flux.

As a check of the calculation, estimated heat flux is compared with heat flux meter outputs and with surface layer temperature gradient in Fig. 4. The temperature gradient is converted into flux, using an average value of thermal conductivity ($0.16\ \mathrm{J\ m^{-1}\ s^{-1}\ K^{-1}}$) calculated from the present estimation of heat flux and the temperature gradient itself. In other words, the heat flux from gradient shown in Fig. 4 is fitted to that from divergence theorem. But, it should be noted that the thermal conductivity assumed here is within a range of experimental scatters (MELLOR, 1964).

As shown in Fig. 4, the scatters in the flux meter outputs are larger than those in the gradient. Since time and space derivatives of temperature are independent of each other, errors in temperature measurement would cause inconsistent results. This again raises doubts on the flux meter measurements.

Figure 5 shows calculated snow heat flux (S_0) together with its confidence limit and net radiation heat flux (R) (YAMANOUCHI *et al.*, 1981). Although individual

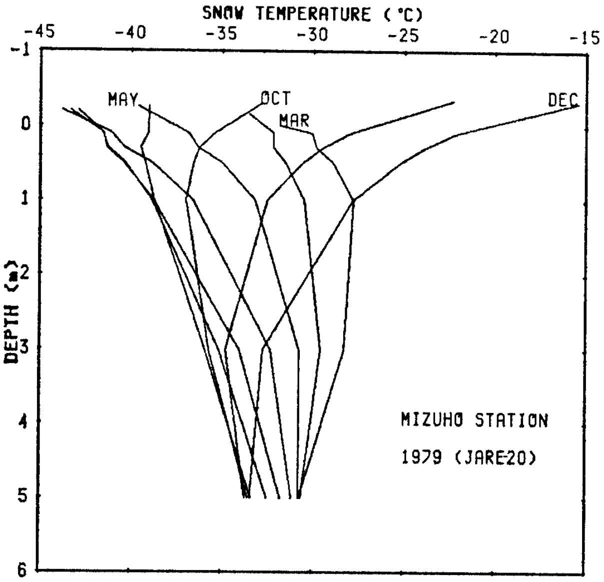

Fig. 3. Monthly average snow temperature versus depth at Mizuho Station from March to December 1979.

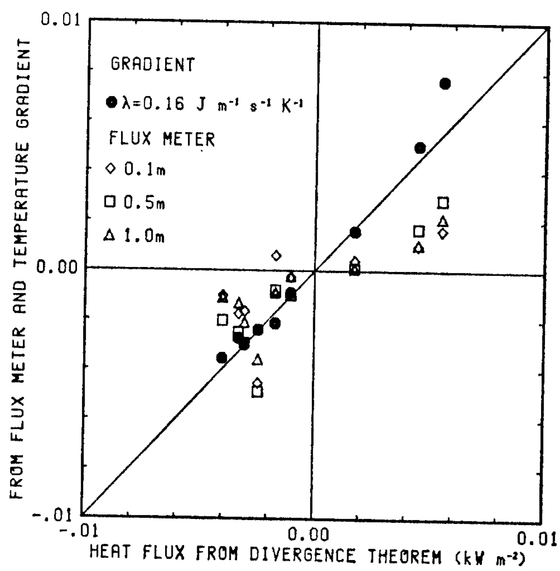

Fig. 4. Comparison of monthly heat flux estimated from divergence theorem (present estimates) with those from heat flux meters and temperature gradient.

estimates of other components of heat balance cannot be shown here, the difference $(R-S_0)$ must be the sum of sensible and latent heat flux into the air. The net radiation is negative (cooling the surface) almost always in this period, whereas the snow heat flux changes its sign in September and becomes positive from October. Absolute value of snow heat flux is smaller than that of the net radiation from March to October, which indicates the radiation is balanced with the sensible and latent heat flux into the air. In November and October, when the absolute value of R becomes small, cooling of the surface by the underlying snow is almost balanced with warming by the air.

Although the data for January and February are absent, the tautochrones in Fig. 3 show an asymmetric pattern with smaller deviation on the left side. Similar pattern is found at the South Pole (DALRYMPLE *et al.*, 1966) and is an indication of 'coreless winter'. In this case, the asymmetric pattern is less evident than at the South Pole.

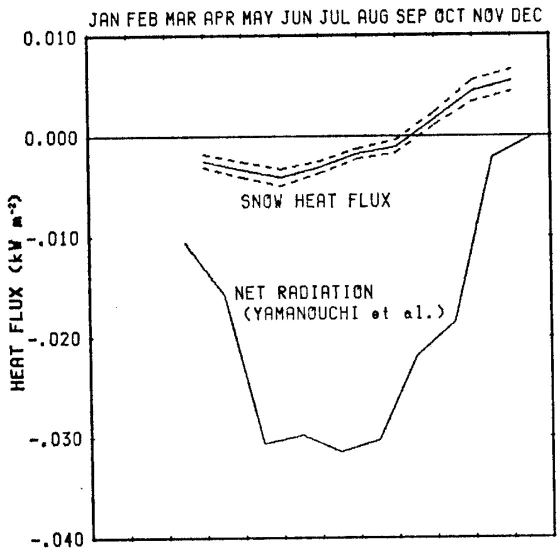

Fig. 5. Calculated snow heat flux with estimated error and net radiative heat flux (YAMANOUCHI et al., 1981) at Mizuho Station (March to December, 1979).

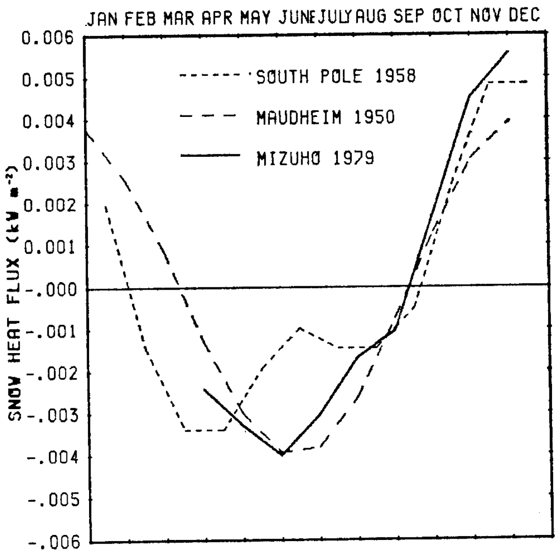

Fig. 6. Seasonal variation of snow heat flux at different sites in Antarctica: South Pole (90°S, 2800 m), Maudheim (71°03′S, 10°56′W, 37 m), Mizuho (70°42′S, 44°20′E, 2230 m).

Figure 6 compares the present results with others from different sites in Antarctica. Heat flux minima appear in March and April at the South Pole, between May and June at Mizuho, and in June at Maudheim, respectively. These appearances of flux minima at different locations indicate that 'coreless winter' is the most significant at the South Pole, deep inland, and becomes less significant with decreasing distance to the coast.

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