# A METHOD OF ION TEMPERATURE DETERMINATION WITH RETARDING POTENTIAL ANALYZER BY COMPUTER

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**Abstract:** We have been measuring the ion temperature by means of a retarding potential analyzer (RPA) aboard the sounding rocket whose velocity is comparable to the ion thermal velocity. In order to determine the ion temperature  $T_1$  from the RPA data, an optimum fitting procedure is used. It was believed that the deduction of  $T_1$  at low ram velocity by computer methods without assumption of ion mass number was impossible. This paper deals with an improved accurate fitting procedure to obtain  $T_1$  and mixed ion mass number. We advocate the usefulness of the improved error evaluation function. We also show the actual data analysis procedure.

#### 1. Introduction

Retarding potential analyzer (RPA) is one of the ion temperature  $(T_i)$  probes which has some grids for energy analysis. The RPA's have been used with space vehicles to measure the drifted plasma. By the use of the differences of mass and energy with the same drift velocity, ion mass concentration can be obtained. Ion temperature measurement using RPA has been attempted by many workers (Gringanz and Zelikman, 1957; Hanson and McKibbin, 1961; Hanson et al., 1970; Gordon, 1969; McClure et al., 1973; Benson et al., 1977).

The measurement of ion temperature by RPA on board satellites has been successful due to the large drift energy (1/3 eV/AMU). Since the RPA characteristic curves are usually so wide in energy, mass separation can be done easily. In the sounding rocket experiment, however, it is not easy to know the mass difference because the drift energy is very small. Although some RPA experiments aboard rockets had been done before satellites were used, ion temperature measurements were not available at altitudes less than 130 km. On the other hand, incoherent radars also provide reliable  $T_1$  in altitudes lower than 100 km (Benson et al., 1977). It may be likely that rocket RPAs are still potentially useful for ion temperature measurements compared with I. S. radar, since I. S. radar observation has poor time resolution.

The accurate measurement of  $T_1$  can be obtained by the RPA system with angle of attack measurements (MINAMI and TAKEYA, 1982; MINAMI et al., 1982) and by the improved data analysis method which is described in this paper.

It is shown that the ion mass ratio which was assumed in the past data reduction (Knudsen and Sharp, 1965) can be deduced from the RPA characteristics even when the drift velocities of ions are comparable to the thermal velocities of ions. This method

needs two planer RPAs (MINAMI and TAKEYA, 1982), one is used for usual RPA and the other is used for the measurement of the angle of attack.

## 2. Theory of the RPA Voltage Current Characteristics

Typical RPA with a grid G for energy analysis and an ion collector C is illustrated in Fig. 1. Other grids for dc potential supply are not shown. Theoretical planer RPA collector current  $I_c$  is expressed by Minami and Takeya (1982) as

$$I_{\rm e} = eN\eta \left\{ \frac{v_{\rm o}\cos\theta_{\rm a}}{2} (1 + {\rm erf}\,x) + \sqrt{\frac{KT_{\rm i}}{2\pi m_{\rm i}}} \right\} e^{-x^2},$$
 (1)

where

$$x = \sqrt{\frac{m}{2KT_1}} \left( v_0 \cos \theta_a - \sqrt{\frac{2e}{m}} V_R \sec^2 \theta_a \right), \tag{2}$$

 $V_{\rm R}$ : retarding voltage  $\theta_{\rm a}$ : angle of attack  $T_{\rm i}$ : ion temperature

 $m_i$ : ion mass

 $V_{\rm s}$ : plasma potential to the reference electrode of vehicle

 $N_o$ : plasma density  $v_o$ : velocity of vehicle.

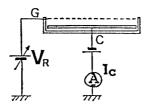


Fig. 1. A fundamental circuit of RPA.

And  $\eta$  which is a total transparency of the grids is calculated by comparing with other electron density probes.

Error function x is expressed as

$$\operatorname{erf} x = \int_{x}^{\infty} \exp(-y^{2}) dy.$$
 (3)

If two species of ions exist, ion mass ratio P is decided for NO+ and O+

$$P = \frac{N_{30}}{\text{total ion density } (N_{16} + N_{30})}, \tag{4}$$

where  $N_{30}$  and  $N_{16}$  are the partial ion density of NO<sup>+</sup> and O<sup>+</sup> respectively. The numerical calculations of eq. (1) are shown below.

In Fig. 2 examples of theoretical RPA characteristics for various values of  $T_1$  and three cases of P (0, 0.5, 1.0) are shown. Figure 3 shows theoretical RPA characteristics for various values of  $\theta_a$  and three cases of P. Figure 4 shows examples of theoretical RPA characteristics for various values of  $\theta_a$  and three cases of P.

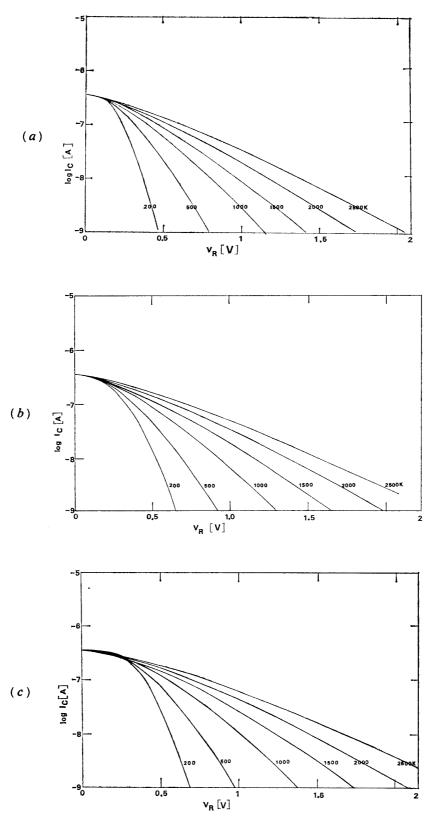
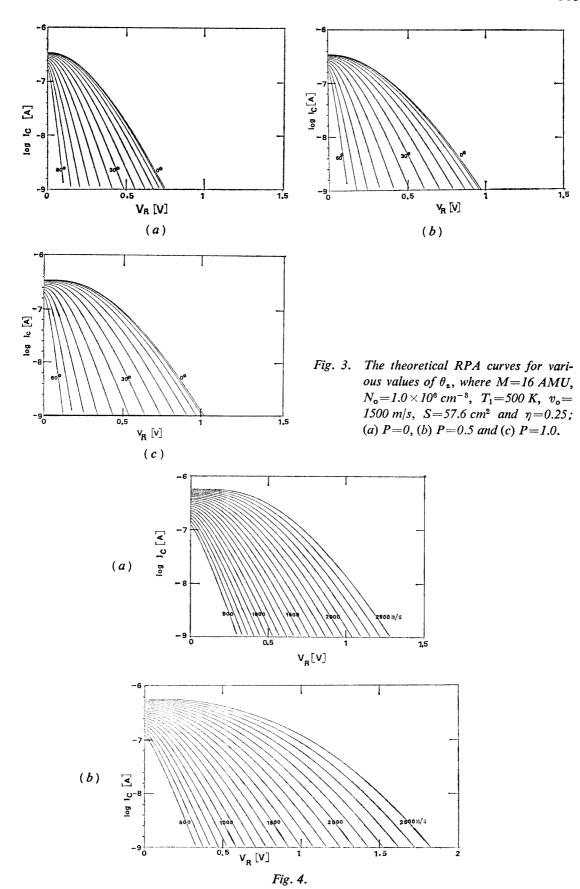


Fig. 2. The examples of theoretical RPA curves for various values of  $T_1$ ; (a) P=0, (b) P=0.5 and (c) P=1.0, where M=16 AMU,  $N_0=1.0\times10^8$  cm<sup>-8</sup>,  $\theta_a=10^\circ$ ,  $v_o=1500$  m/s, S=57.6 cm<sup>2</sup> and  $\eta=0.25$ .



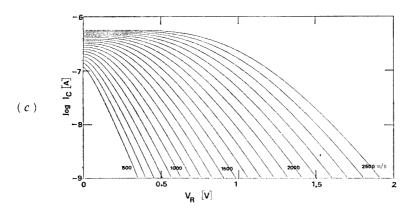


Fig. 4. The theoretical RPA curves for various values of velocity  $v_o$ , where total density  $N_o = 1.0 \times 10^8$ ,  $T_i = 500$  K,  $\theta_a = 10^\circ$ , S = 57.6 cm<sup>2</sup> and  $\eta = 0.25$ ; (a) P = 0, (b) P = 0.5 and (c) P = 1.0.

cal RPA characteristics for various values of rocket velocity  $v_o$  and three cases of P. These figures show that the difference of curves to the parameter P is not so clear as in the case when the velocity  $v_o$  is very large.

Although the current reduction curves in Fig. 4 are very similar when  $v_o$  is small, the gradients of the curves near the saturation region differ very much from each other. They contain the information about ion mass.

## 3. Data Analysis

Parameters are determined as follows:

- (1) Vehicle velocity  $v_0$  is decided from radar tracking data.
- (2) The angle of attack  $\theta_a$  between the normal direction of grid and the velocity vector of the vehicle is decided as referenced in a previous paper (MINAMI and TAKEYA, 1982).
- (3) Plasma density  $N_o$  is decided from the ion saturation current. Ion saturation current  $I_{is}$  does not depend on  $T_i$ ,  $V_s$ ,  $m_i$  as shown in Fig. 2. It depends only on  $N_o$  and  $\theta_a$ .
- (4)  $T_1$ ,  $m_1$  and  $V_s$  are derived using the optimum fitting procedure. By using the ion current data of RPA which is A-D converted, the plasma parameters  $T_1$ ,  $m_1$ , P,  $V_s$  and the error evaluation function G are obtained by computer analysis. The experimental current data  $J(V_1)$ 's whose total sampling number is n are compared with the theoretical current  $I(V_1)$ 's which correspond to all sampling points  $V_1$  of  $V_R$ . The characteristic error evaluation function G

$$G = \sum_{i=1}^{n} \{ \log J(V_i) - \log I(V_i) \}^2,$$

is used, which emphasizes the gradient of the curve. As the above theoretical curves show, information of ion temperature is contained not in the absolute current but in the gradient of the curve. We choose the same weight for every current.

There are the following three calculation loops for optimum fitting:

- (1) The calculation loop of temperature  $T_i$  gives G under the assumed  $V_s$ ,  $m_i$ , P.
- (2) The calculation loop of potential  $V_s$  gives G under the assumed  $m_i$  and P.

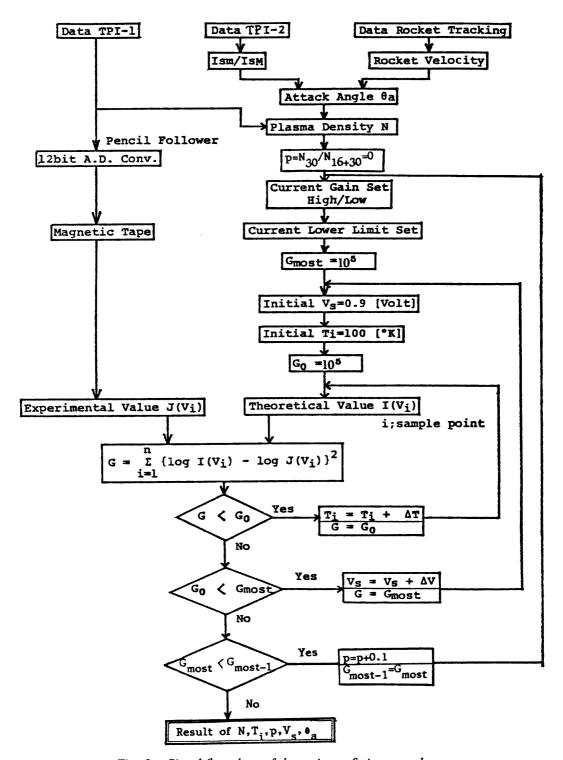


Fig. 5. Signal flow chart of the optimum fitting procedure.

(3) The calculation loop of ion mass gives G while varying ion mass ratio P. Through the above three calculation loops the most probable parameters of  $T_1$ ,  $V_s$ ,  $m_1$  and P are decided. In Fig. 5 the signal flow chart of numerical parameter calculation is illustrated.

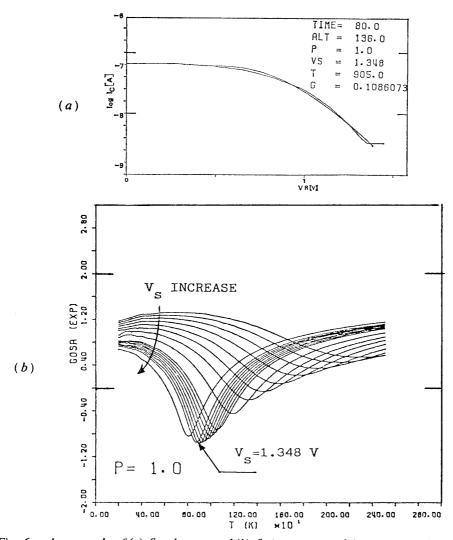
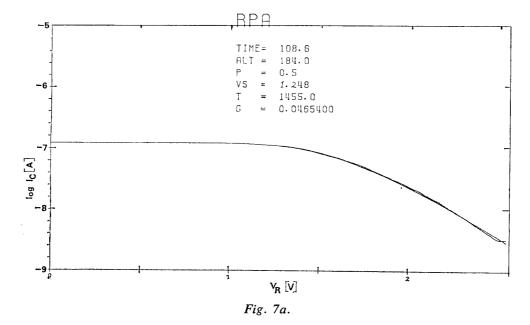


Fig. 6. An example of (a) fitted curve and (b) fitting process of G versus  $T_i$  (altitude 136 km).



## 4. An Example of Actual Data Analysis

This improved data analysis method was applied to the data obtained by K-9M-67 sounding rocket experiment. One RPA characteristic curve was obtained in about 0.3 s with the energy resolution of 0.007 eV.

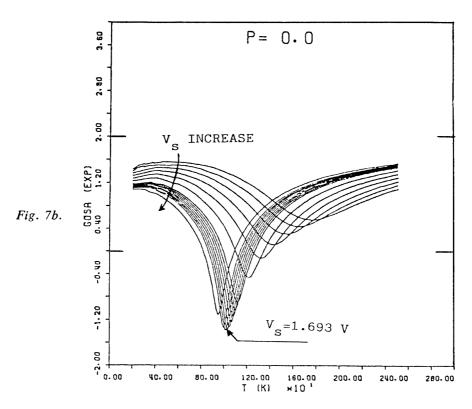
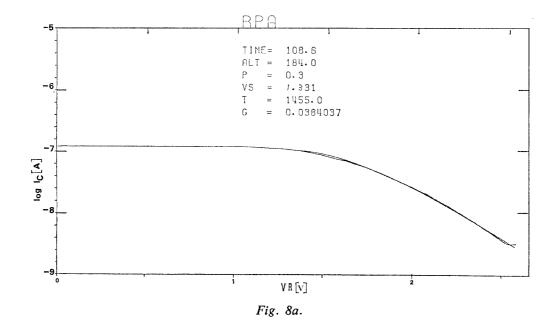
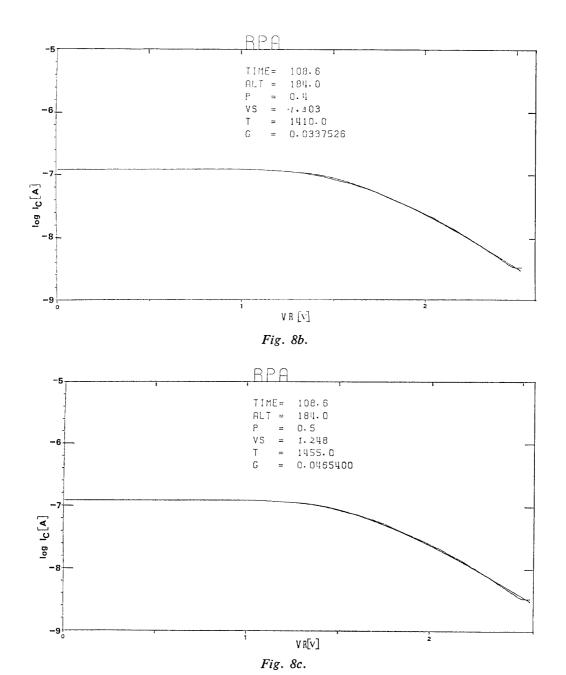


Fig. 7. An example of (a) fitted curve and (b) fitting process of G versus  $T_i$  (altitude 229 km).



As the signal flow chart shows, at first, step width of  $T_1$  and  $V_8$  loops are 50 K and about 60 mV respectively, finally  $T_1$  and  $V_8$  are decided with accuracy of 5 K and 7 mV respectively. In Fig. 6 the optimum fitted curves and experimental curve are illustrated. Figure 6b shows the fitting process of G vs.  $T_1$  for various values under fixed  $V_8$ . Ion temperature  $T_1$  is obtained from the minimum value of G. At this curve (altitude is 136 km) ion mass is assumed as NO<sup>+</sup> only from the well-known results of mass spectrometry (Holmes *et al.*, 1965; Salah, 1974). In Fig. 7 the optimum fitted curves and the process of fitting are drawn in the case of the measured altitude of 227 km.

The optimum fitting procedure including ion mass ratio loop must be done in an altitude range of 150 to 220 km. Figure 8 is an example of the fitting process for three



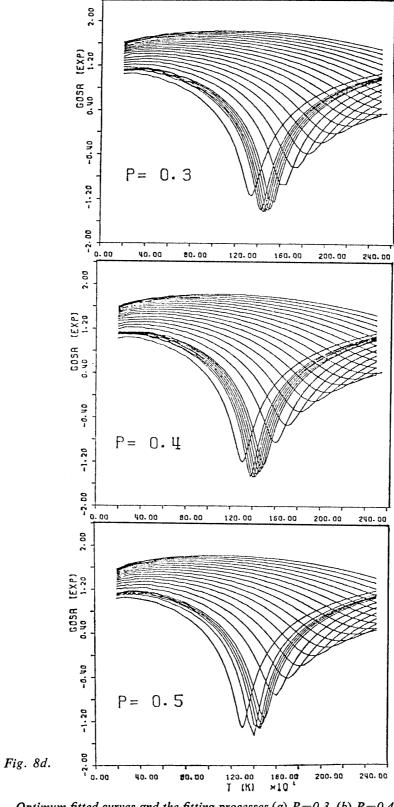


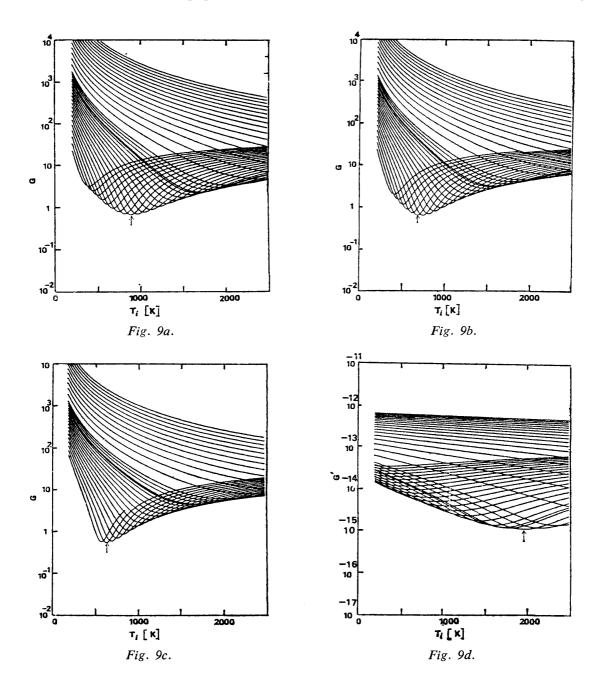
Fig. 8. Optimum fitted curves and the fitting processes (a) P=0.3, (b) P=0.4, (c) P=0.5 and (d) fitting processes for various values of P.

values of P(0.3, 0.4, 0.5) at an altitude of 184 km. In Fig. 8d the fitting curves for the above three values are drawn. The comparison of each minimum G which is illustrated in the figures shows that the most probable parameter of P is 0.4.

These data are compared with the past experimental profiles of  $T_i$  and  $m_i$ . As a result, they are consistent with the well known geomagnetic data (MINAMI and TAKEYA, 1982).

## 5. Discussion

Because of an accurate RPA measurement system and the improved data analysis method described in this paper, accurate data of  $T_1$  and  $m_1$  by RPA aboard sounding



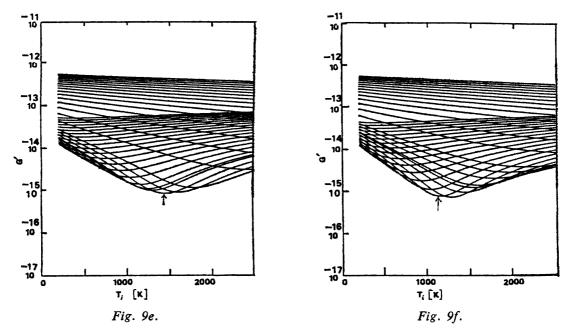


Fig. 9. The example of the results of comparison between G and G' for assumed values of P. The parameters of these curves are the assumed potential  $V_{\rm s}$ .

rocket could be obtained. We used the improved evaluation function G which is effective in the slow drifting plasma.

We discuss the difference between this function G and the function G' of usual minimum square method. G' is expressed as

$$G' = \sum_{i=1}^{n} \{J(V_i) - I(V_i)\}^2$$
,

(Moore and Zeigler, 1960; Moss and Hyman, 1968; Patterson, 1969). Figure 9 shows the example of the result of comparison between G and G' for three assumed values of P. These results are obtained from the data at an altitude of 136 km. It is obvious that the minimum error by G' does not show the true temperature  $T_1$ . The ion saturation current curve sometimes shows the tilted distortion as a result of sheath expansion (Minami and Takeya, 1982). As function G' puts weight around the ion saturation region, data analysis using G' is much influenced by the ion saturation current. Therefore, it is thought that  $T_1$  obtained by G' contains the error due to distortion of ion saturation current. If the distortion in the ion saturation region exists, the difference of G and G' might be larger. On the other hand, RPA aboard satellites can escape such distortion at the saturation region because of comparably large input ion energy with the sheath potential  $V_8$  (Hanson et al., 1970).

### 6. Conclusion

It was shown that an accurate deduction of  $T_1$  and  $m_1$  could be done by the use of the improved data analysis method from the RPA curve even in the slow drifting plasma (in sounding rocket experiment). Although the problem about this method is a long calculation time, it may be not so serious when more powerful computers become avail-

able.

This result motivates us to use RPA in the polar region to get the accurate measurement of the  $T_i$  profile which is useful to discuss thermodynamic behavior of the atmosphere by sounding rocket.

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