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Symmetric generation

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Symmetric Generation

A Thesis

Presented to the

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of the Requirements for the Degree

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in

Mathematics

by

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Abstract

In this thesis we construct finite homomorphic images of infinite semi-direct products, $2^{*n} : N$, where 2^{*n} is a free product of n copies the cyclic group C_2 extended by N, a group of permutations on n letter. We construct several finite homomorphic images of the semi-direct products $2^{*4} : S_4$, $2^{*6} : L_2(5)$, and $3^{*3} : S_3$. In particular, we construct the finite groups $2^4 : S_4$, $2 \times S_5$, $PGL_2(7)$, $PGL_2(9)$, $3 \times PGL_2(9)$, A homomorphic image of $3^{*3} : S_3$, $L_2(11)$, and $M_{12} : 2$. The main result of the thesis is the construction of $M_{12} : 2$ as a homomorphic image of $2^{*4} : S_4$.

Acknowledgements

First of all, I would like to thank Dr. Zahid Hasan who taught me about group theory, the construction of finite groups, programming for magma, and the structure of several interesting groups. His patience seemed infinite. He spent many hours on my thesis, reviewing my work, and correcting errors. Without his help, my thesis could not have been complete. Next, I would like to thank my committee members, Dr. John Sarli and Dr. Gary Griffing for their time reviewing my work. Finally, I would like to thank my parents, for their unwavering support.

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Chapter 1

Introduction

In this thesis, we present a method for performing group theoretic operations on finite groups. Our method is, as will be demonstrated, more convenient to work with than the two existing methods; permutation or matrix representation. Many of the interesting groups have permutation representations on sets of inconveniently large size. Also; working with matrix representation for these groups are very time consuming. Although there exist group theory packages, such as MAGMA (see [Can05]) which enable us to efficiently perform group theoretic operations on permutations of very large size, the only means of communication of these elements is electronically. Our method applies to may finite groups including all finite non abelian simples groups. We will construct several groups, including the automorphism group, $M_{12} : 2$, of the Mathieu group M_{12}. These groups are constructed through a technique that is given in next section. This technique allows us to express elements of the constructed group in convenient short forms. Many groups have been constructed this way and their elements represented in short forms. For example, every element of the Janko group J, is represented in convenient short form instead of the usual 226 letter permutation representation, see [CH96] for reference.

1.1 Symmetric Generation of a Group

Let G be a group and

$$T = \{t_1, t_2, \ldots, t_n\} \subseteq G,$$

and let $N = N_G(T)$, the set normalizer in G of T.
If

(i) $G = \langle T \rangle$, and

(ii) N permutes T transitively, not necessarily faithfully.

Then G is a homomorphic image of the (infinite) *progenitor*

$$m^{*n} : N,$$

where m^{*n} represents a free product of n copies of the cyclic group C_m, m being the order of t_i, and N is a group of automorphisms of m^{*n} which permutes the n cyclic subgroups by conjugation. Thus, for $\pi \in N$, we have

$$t_i^{\pi} = \pi t_{(i)\pi},$$

N will simply act by conjugation as permutations of the n involutory symmetric generators. Now, since by the above, elements of N can be gathered on the left, every element of the progenitor can be represented as πw, where $\pi \in N$ and w is a word in the symmetric generators. Indeed this representation is unique provided w is simplified so that adjacent symmetric generators are distinct. Thus any additional relator by which we must factor the progenitor to obtain G must have the form

$$\pi w(t_1, t_2, .., t_n),$$

where $\pi \in N$ and w is a word in T. We will let i stand for the symmetric generator t_i and also let i to denote the coset Nt_i.

1.2 Manual Double Coset Enumeration

We have

$$N^i = C_N(t_i);$$

as the single point stabilizer in N and

$$N^{ij} = C_N(\langle t_i, t_j \rangle) \text{ etc},$$

as the two points stabilizer in N. The coset stabilizing subgroup of N, $N^{(w)}$, is given by,

$$N^{(w)} = \{\pi \in N : Nw\pi = Nw\},$$

for w a word in the symmetric generators. Then $N^w \leq N^{(w)}$, and the number of cosets in the double coset $[w] = NwN$ is given by $\mid N \mid / \mid N^{(w)} \mid$, since

$Nw\pi_1 \neq Nw\pi_2$

$\iff Nw\pi_1{\pi_2}^{-1} \neq Nw$

$\iff \pi_1{\pi_2}^{-1} \notin N^{(w)}$

$\iff N^{(w)}\pi_1{\pi_2}^{-1} \neq N^{(w)}$

$\iff N^{(w)}\pi_1 \neq N^{(w)}\pi_2$.

We find the index of N in G by performing a manual double coset enumeration of G over N. We need to find all double cosets $[w]$ and also find the number of single cosets in each of these. We know that the double coset enumeration is completed when the set of right cosets obtained is closed under right multiplication. We obtain the orbits of $N^{(w)}$ on the set of symmetric generators and identify, for each $[w]$, the double coset to which Nwt_i belongs for one symmetric generator t_i from each orbit.

Chapter 2

Construction of $2^4 : S_4$

We factor the progenitor $2^{*4} : S_4$ by the single relator, $[(0,1,2,3)t_0]^4$, and let,

$$G \cong \frac{2^{*4} : S_4}{[(0,1,2,3)t_0]^4}$$

a symmetric presentation of G is given by,

$$\langle x, y, t | x^4, y^2, (yx)^3, t^2, (t, y), (t^x, y), (xt)^4 \rangle,$$

where $N \cong S_4 = \langle x, y | x^4, y^2, xy^3 \rangle$, and the actions of x and y on the symmetric generator are given by $x \sim (0,1,2,3)$, $y \sim (2,3)$. Our relation is $[(0,1,2,3)t_0]^4 = 1$.

Now, let $(0,1,2,3) = \pi$ then $[(0,1,2,3)t_0]^4 = (\pi t_0)^4 = 1$.

$$\begin{aligned}
\pi t_0 \pi t_0 \pi t_0 \pi t_0 &= 1 \\
\Leftrightarrow \pi t_0 \pi t_0 \pi^2 \pi^{-1} t_0 \pi t_0 &= 1 \\
\Leftrightarrow \pi t_0 \pi^3 \pi^{-2} t_0 \pi^2 (t_0)^{\pi} t_0 &= 1 \\
\Leftrightarrow \pi t_0 \pi^3 (t_0)^{\pi^2} (t_0)^{\pi} t_0 &= 1 \\
\Leftrightarrow \pi^4 \pi^{-3} t_0 \pi^3 (t_0)^{\pi^2} (t_0)^{\pi} t_0 &= 1 \\
\Leftrightarrow \pi^4 (t_0)^{\pi^3} (t_0)^{\pi^2} (t_0)^{\pi} t_0 &= 1 \\
\Leftrightarrow t_3 t_2 t_1 t_0 &= 1 \\
\Leftrightarrow t_3 t_2 &= t_0 t_1
\end{aligned}$$

Manual double coset enumeration.

We note that; $NeN = \{Nen | n \in N\} = \{Nn | n \in N\} = \{N\}$. Thus NeN, denoted by $[\star]$, contains one single coset. N is transitive on $\{0,1,2,3\}$, so it has a single orbit $\{0,1,2,3\}$. We take a representative, say $\{0\}$ from the orbit, and find to which the double cosets Nt_0 belong? Clearly; $Nt_0 \in Nt_0N = \{Nt_0^n | n \in N\} = \{Nt_0, Nt_1, Nt_2, Nt_3\}$. Now consider the cosets stabilizer $N^{(0)}$ in N. The cosets stabilizer of Nt_0 is equal to the point stabilizer N^0, given by:

$$\begin{aligned} N^{(0)} &= N^0 \\ &= \langle (1,2,3),(1,2) \rangle \\ &= \{e,(1,2),(1,3),(2,3),(1,2,3),(2,1,3)\}, \end{aligned}$$

then the number of the single cosets in the double coset Nt_0N is at most: $\frac{|N|}{|N^{(0)}|} = \frac{24}{6} = 4$.

The orbits of $N^{(0)}$ on $\{0,1,2,3\}$ are $\{0\}$ and $\{1,2,3\}$. We take a representative, $\{0\}$ and say $\{1\}$, from each orbit, and find to which the double cosets Nt_0t_0 and Nt_0t_1 belong? However, $Nt_0t_0 = N \in [\star]$, then one symmetric generator goes back to the double coset NeN, and three symmetric generators go to the next double coset $Nt_0t_1 \in [01]$ (new).

Next, we consider the cosets stabilizer $N^{(01)}$ in N. Now, $N^{(01)} \geq N^{01} = \{e,(2,3)\}$, and we note that; $Nt_3t_2 = Nt_0t_1 \quad (32 \sim 01)$.

$Nt_3t_2^{(0,3)(1,2)} = Nt_0t_1^{(1,2)(0,3)} \Rightarrow Nt_2t_3 = Nt_0t_1 \Rightarrow (0,3)(1,2) \in N^{(01)}$.

$Nt_3t_2^{(0,1)} = Nt_0t_1^{(0,1)} \Rightarrow Nt_3t_2 = Nt_1t_0 \Rightarrow (0,1) \in N^{(01)}$.

Therefore; $N^{(01)} \geq \langle (2,3),(1,2)(0,3) \rangle = \{e,(0,1),(2,3),(1,3)(0,2),(0,3)(1,2),$ $(0,3,1,2),(0,2,1,3),(2,1,0,3)\}$, then the number of the single cosets in the double coset Nt_0t_1N is at most: $\frac{|N|}{|N^{(01)}|} = \frac{24}{8} = 3$ (with four names for each).

In order to find the other two distinct single cosets in Nt_0t_1N we find the right cosets of $N^{(01)}$ in N. They are $N^{(01)}, Nt_0t_1^{(0,1,3,2)}, Nt_0t_1^{(0,3,2,1)}$. We take a presentative from each of the cosets, we form the transversal T. $T = \{e,(0,3,2,1),(0,1,3,2)\}$. Conjugating the coset with four different names that is, $32 \sim 23 \sim 10 \sim 01$ by each of the elements in the set T, we get the other two distinct cosets in Nt_0t_1N with four names for each:

$31 \sim 13 \sim 02 \sim 20$.

$30 \sim 03 \sim 21 \sim 12$.

$N^{(01)}$ is transitive on $\{0,1,2,3\}$. Thus $N^{(01)}$ has a single orbits $\{0,1,2,3\}$. We take a representative, say $\{1\}$, from the orbit, and find to which the double cosets $Nt_0t_1t_0$ belong? However, $Nt_0t_1t_0 = Nt_0 \in [0]$. Therefore; four symmetric generators go back to the double coset Nt_0N.

Hence we must have complete the double coset enumerator since the set of right cosets of N in G is closed under right multiplication by $t_i s$. The double coset enumeration shows that the index of $N \cong S_4$ in G is at most: $\frac{|N|}{|N|} + \frac{|N|}{|N^{(0)}|} + \frac{|N|}{|N^{(01)}|} = 1 + 4 + 3 = 8$. $\Longrightarrow |G| \leq 8|N| = 8 \times 24 = 192$.

We now prove that $|G| = 192$. Determine the action of the symmetric generators x, y and t on 8 distinct cosets of N in G that we have found. Recall that $x \sim (0,1,2,3)$, $y \sim (2,3)$.

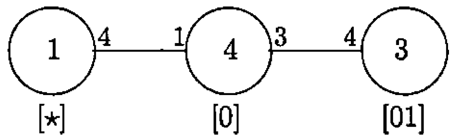

Figure 2.1: The Cayley Diagram of $2^4 : S_4$ over $2^{*4} : S_4$

Permutation Representation.

Coset	$x \sim (0,1,2,3)$	$y \sim (2,3)$	t_0
1 N	1 N	1 N	2 Nt_0
2 Nt_0	3 Nt_1	2 Nt_0	1 N
3 Nt_1	4 Nt_2	3 Nt_1	6 Nt_0t_1
4 Nt_2	5 Nt_3	5 Nt_3	7 Nt_0t_2
5 Nt_3	2 Nt_0	4 Nt_2	8 Nt_0t_3
6 Nt_0t_1	8 Nt_0t_3	6 Nt_0t_1	3 Nt_1
7 Nt_0t_2	7 Nt_0t_2	8 Nt_0t_3	4 Nt_2
8 Nt_0t_3	6 Nt_0t_1	7 Nt_0t_2	5 Nt_3

The action of N and the action of $t's$ is as follows: G×X⟶X defined by (g,x)⟶gx, and $f \longrightarrow S_x$, where f is homomorphism. In this problem; we have, $f : 2^{*4} : S_4 \longrightarrow S_x$, where $X = \{N, Nt_0, ..., Nt_0t_3\}$. Now we find,

$f(x) = (2,3,4,5)(6,8)$.

$f(y) = (4,5)(7,8)$.

$f(t_0) = (1,2)(3,6)(4,7)(5,8)$.

$f(t_1) = f({t_0}^x) = (ft_0)^{(f(x))} = (1,3)(4,8)(5,7)(2,6)$.

$f(t_2) = f({t_1}^x) = (ft_1)^{(f(x))}) = (1,4)(5,6)(2,7)(3,8)$.

$f(t_3) = f({t_2}^x) = (ft_2)^{(f(x))}) = (1,5)(2,8)(3,7)(4,6)$.

The Homomorphism Image of G.

$\bar{G} = f(2^{*4} : S_4) = \langle f(x), f(y), f(t_0), f(t_1), f(t_2), f(t_3)\rangle$.

However;

$\{f(t_1), f(t_2), f(t_3)\} \subseteq \langle f(x), f(y), f(t_0)\rangle$.

$\Longrightarrow \bar{G} = \langle f(x), f(y), f(t_0)\rangle = f(2^{*4} : S_4)$.

If the additional relation hold in $\langle f(x), f(t_0), f(y)\rangle$ then:

$$f\left(\frac{2^{*4} : S_4}{([(0,1,2,3)t_0])^4}\right) \cong \langle f(x), f(y), f(t_0)\rangle$$

The addition relation, namely $t_0t_1t_2t_3 = 1$ hold in $\langle f(x), f(y), f(t_0)\rangle$

if $f(t_0)f(t_1)f(t_2)f(t_3) = 1$.

Let $\mu = f(t_0)f(t_1)f(t_2)f(t_3)$=$(1,2)(3,6)(4,7)(5,8)(1,3)(4,8)(5,7)(2,6)(1,4)(5,6)(2,7)$ $(3,8)(1,5)(2,8)(3,7)(4,6)$=1.

Thus the relation holds in $\bar{G}$.

$\Longrightarrow \bar{G}$ is a homomorphism image of G.

Now, by the First Isomorphic Theorem $|G/Ker f| \cong \bar{G}$.

$\Longrightarrow |G| \geq |\bar{G}|$.

It is easily verified that $|\bar{G}| = |\langle f(x), f(y), f(t_0)\rangle| = 192$.

$\Longrightarrow |G| \geq 192$.

But we have see above that $|G| \leq 192$.

Hence; $|G| = 192$.

Chapter 3

Construction of $2 \times S_5$

We factor the progenitor $2^{*4} : S_4$ by the single relator, $[(0,1,2)t_0]^4$, and let,

$$G \cong \frac{2^{*4} : S_4}{[(0,1,2)t_0]^4}$$

a symmetric presentation of G is given by:

$$\langle x, y, t | x^4, y^2, (yx)^3, t^2, (t, y), (t^x, y), ((xy)^{yt})^4 \rangle,$$

where $N \cong S_4 = \langle x, y | x^4, y^2, xy^3 \rangle$, and the actions of x and y on the symmetric generator are given by $x \sim (0,1,2,3)$, $y \sim (2,3)$. Our relation is, $[(0,1,2)t_0]^4 = 1$. Now, let $(0,1,2) = \pi$ then $[(0,1,2)t_0]^4 = (\pi t_0)^4 = 1$.

$$\begin{aligned}
\pi t_0 \pi t_0 \pi t_0 \pi t_0 &= 1 \\
\Leftrightarrow \pi t_0 \pi t_0 \pi^2 \pi^{-1} t_0 \pi t_0 &= 1 \\
\Leftrightarrow \pi t_0 \pi^3 \pi^{-2} t_0 \pi^2 (t_0)^{\pi} t_0 &= 1 \\
\Leftrightarrow \pi t_0 \pi^3 (t_0)^{\pi^2} (t_0)^{\pi} t_0 &= 1 \\
\Leftrightarrow \pi^4 \pi^{-3} t_0 \pi^3 (t_0)^{\pi^2} (t_0)^{\pi} t_0 &= 1 \\
\Leftrightarrow \pi^4 (t_0)^{\pi^3} (t_0)^{\pi^2} (t_0)^{\pi} t_0 &= 1 \\
\Leftrightarrow (0,1,2) t_0 t_2 t_1 t_0 &= 1 \\
\Leftrightarrow (0,1,2) t_0 t_2 &= t_0 t_1
\end{aligned}$$

Manual double coset enumeration

We note that; $NeN = \{Nen | n \in N\} = \{Nn | n \in N\} = \{N\}$. Thus NeN, denoted by $[\star]$, contains one single coset. N is transitive on $\{0,1,2,3\}$, so it has a single orbit $\{0,1,2,3\}$. We take a representative, say $\{0\}$ from the orbit, and find to which the double cosets Nt_0 belong? Clearly; $Nt_0 \in Nt_0N = \{Nt_0^n | n \in N\} = \{Nt_0, Nt_1, Nt_2, Nt_3\}$. Now consider the cosets stabilizer $N^{(0)}$. The cosets stabilizer of Nt_0 is equal to the point stabilize N^0, given by:

$$\begin{aligned} N^{(0)} &= N^0 \\ &= \langle (1,2,3),(1,2) \rangle. \\ &= \{e,(1,2),(1,3),(2,3),(1,2,3),(2,1,3)\}, \end{aligned}$$

then the number of the single cosets in the double coset Nt_0N is at most: $\frac{|N|}{|N^{(0)}|} = \frac{24}{6} = 4$.

The orbit of $N^{(0)}$ on $\{0,1,2,3\}$ are $\{0\}$ and $\{1,2,3\}$. We take a representatives, $\{0\}$ and say $\{1\}$, from each orbit, and find to which the double cosets Nt_0t_0 and Nt_0t_1 belong? However, $Nt_0t_0 = N \in [\star]$, then one symmetric generator goes back to the double coset NeN, and three symmetric generators go to the next double coset $Nt_0t_1 \in [01]$ (new).

Next, we consider coset stabilizer $N^{(01)}$ in N. Now, $N^{(01)} \geq N^{01} = \{e,(23,)\}$, and we note that; $Nt_0t_1 = Nt_0t_2 \quad (01 \sim 02)$. $Nt_0t_1^{(1,2)} = Nt_0t_2 \Rightarrow (1,2) \in N^{(01)}$ but S_n can be generated by (12...n)(12). Therefore; $N^{(01)} = \langle (1,2,3),(1,2) \rangle = \{e,(1,2),(1,3),(2,3),(1,2,3),(2,1,3)\}$. Since; $(2,3) \in N^{(01)} \Rightarrow Nt_0t_2^{(2,3)} = Nt_0t_1^{(2,3)} \Rightarrow Nt_0t_3 = Nt_0t_1$. Thus $Nt_0t_1 = Nt_0t_2 = Nt_0t_3 \quad (01 \sim 02 \sim 03)$. Then the number of the single cosets in the double coset Nt_0t_1N is at most:$\frac{|N|}{|N^{(01)}|} = \frac{24}{6} = 4$ (with three names for each).

In order to find other three distinct single cosets in Nt_0t_1N we find the right cosets of $N^{(01)}$ in N. They are$N^{(01)}, Nt_0t_1^{(0,1)}, Nt_0t_1^{(0,2)}, Nt_0t_1^{(0,3)}$. We take a presentative from each of the cosets, we form the transversal T. $T = \{e,(0,1),(0,2),(0,3)\}$. Conjugating the coset with three different names that is; $01 \sim 02 \sim 03$ by each of the elements in the set T, we get other three distinct cosets in Nt_0t_1N with three names for each:

$10 \sim 12 \sim 13$.

$20 \sim 21 \sim 23$.

$30 \sim 31 \sim 32$.

The orbit of $N^{(01)}$ on $\{0,1,2,3\}$ are $\{0\}$ and $\{1,2,3\}$. We take a representatives, $\{0\}$ and say $\{1\}$, from each orbit, and find to which the double cosets $Nt_0t_1t_0$ and $Nt_0t_1t_1$ belong to? However, $Nt_0t_1t_1 = Nt_0 \in [0]$, then three symmetric generators go back to the double coset Nt_0N, and $Nt_0t_1t_0N$ is new double coset.

Now, $Nt_0t_1t_0 \in [010]$ (new).

$$
\begin{aligned}
(0,1,2)(t_0t_2) &= (t_0t_1)\\
\Longrightarrow (0,1,2)(t_0t_2t_0) &= (t_0t_1t_0)\\
\text{Thus } Nt_0t_1t_0 &= Nt_0t_2t_0\\
&= Nt_0t_3t_0\\
&= Nt_0(3,0,2)t_3t_2\\
&= Nt_2t_3t_2\\
&= Nt_2t_1t_2\\
&= Nt_2t_0t_2\\
&= Nt_2(0,2,1)t_0t_1\\
&= Nt_1t_0t_1\\
&= Nt_1t_2t_1\\
&= Nt_1t_3t_1\\
&= Nt_0(3,2,0)t_2t_3\\
&= Nt_3t_3t_2\\
&= Nt_3t_0t_3\\
&= Nt_3t_1t_3.
\end{aligned}
$$

So,

$Nt_0t_1t_0$=$Nt_0t_2t_0$=$Nt_0t_3t_0$=$Nt_2t_3t_2$.

$Nt_2t_0t_2$=$Nt_1t_0t_1$=$Nt_1t_2t_1$=$Nt_1t_3t_1$.

$Nt_3t_1t_3$=$Nt_3t_2t_3$=$Nt_3t_0t_3$=$Nt_2t_1t_2$.

We consider the cosets stabilizer $N^{(010)}$ in N. $N^{(010)} \geq N^{010} = \{e,(2,3)\}$ and we already knew, $N^{(010)} \geq N^{(01)}$, which have six elements. Moreover; $(0,1,2,3) \in N^{(010)}$. Therefore; $N^{(010)} = \langle(0,1,2,3),(1,2)\rangle = 24 \Longrightarrow$ All elements in $S_4 \in N^{(010)}$. Then the number of the single cosets in the double coset $Nt_2t_3t_0N$ is at most: $\frac{|N|}{|N^{(010)}|} = \frac{24}{24} = 1$ (with 12 different names for each).

$N^{(010)}$ is transitive on $\{0,1,2,3\}$, so it has a single orbit $\{0,1,2,3\}$. We take a representative, say $\{0\}$ from the orbit, and find to which the double coset $Nt_0t_1t_0t_0$ belong?$Nt_0t_1t_0t_0 = Nt_0t_1 \in [01]$, then four symmetric generators go back to Nt_0t_1N.

Hence we must have complete the double coset enumerator since the set of right cosets of N in is closed under right multiplication by t_is. The double coset enumeration shows that the index of $N \cong S_4$ in G is at most: $\frac{|N|}{|N|} + \frac{|N|}{|N^{(0)}|} + \frac{|N|}{|N^{(01)}|} + \frac{|N|}{|N^{(010)}|} = 1+4+4+1 = 10$.

$\Longrightarrow | G | \leq 10|N| = 10 \times 24 = 240$.

We now prove that $|G| = 240$. Determine the action of the symmetric generators x, y and t on 10 distinct cosets of N in G that we have found. Recall that $x \sim (0,1,2,3)$, $y \sim (2,3)$.

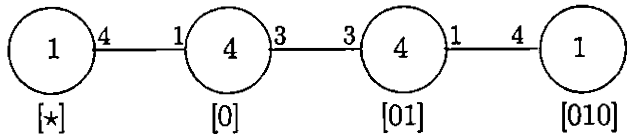

Figure 3.1: The Cayley Diagram of $2 \times S_5$ over $2^{*4} : S_4$

Permutation Representation.

Coset	$x \sim (0,1,2,3)$	$y \sim (2,3)$	t_0
1 N	1 N	1 N	2 Nt_0
2 Nt_0	3 Nt_1	2 Nt_0	1 N
3 Nt_1	4 Nt_2	3 Nt_1	7 Nt_1t_0
4 Nt_2	5 Nt_3	5 Nt_3	8 Nt_2t_0
5 Nt_3	2 Nt_0	4 Nt_2	9 Nt_3t_0
6 Nt_0t_1	7 Nt_1t_0	6 Nt_0t_1	10 $Nt_0t_1t_0$
7 Nt_1t_0	8 Nt_2t_0	7 Nt_1t_0	3 Nt_1
8 Nt_2t_0	9 Nt_3t_0	9 Nt_3t_0	4 Nt_2
9 Nt_3t_0	6 Nt_0t_1	8 Nt_2t_0	5 Nt_3
10 $Nt_0t_1t_0$	10 $Nt_0t_1t_0$	10 $Nt_0t_1t_0$	6 Nt_0t_1

The action of N and the action of $t's$ is as follows: G×X⟶X defined by (g,x)⟶gx, and $f \longrightarrow S_x$, where f is homomorphism. In this problem; we have, $f : 2^{*4} : S_4 \longrightarrow S_x$, where $X = \{N, Nt_0, ..., Nt_0t_1t_0\}$.

$f(x) = (2,3,4,5)(6,7,8,9)$.

$f(y) = (4,5)(8,9)$.

$f(t_0) = (1,2)(3,7)(4,8)(5,9)(6,10)$.

$f(t_1) = f(t_0{}^x) = (ft_0)^{(f(x))} = (1,3)(4,8)(5,9)(2,6)(7,10)$.

$f(t_2) = f(t_1{}^x) = (ft_1)^{(f(x))} = (1,4)(2,6)(3,7)(5,9)(4,8)$.

$f(t_3) = f(t_2{}^x) = (ft_2)^{(f(x))} = (1,5)(2,6)(3,7)(4,8)(9,10)$.

The Homomorphism Image Of G.

$\bar{G} = f(2^{*4} : S_4) = \langle f(x), f(y), f(t_0), f(t_1), f(t_2), f(t_3)\rangle$.

However;

$\{f(t_1), f(t_2), f(t_3)\} \subseteq \langle f(x), f(y), f(t_0)\rangle$.

$\Longrightarrow \bar{G} = f(2^{*4} : S_4) = \langle f(x), f(y), f(t_0)\rangle$.

If the additional relation hold in $\langle f(x), f(y), f(t_0)\rangle$ then

$$f\left(\frac{2^{*4}:S_4}{[(0,1,2)t_0]^4}\right) \cong \langle f(x), f(y), f(t_0)\rangle.$$

The addition relation, namely $t_0t_1t_2t_0 = (0,1,2)$ holds in $\langle f(x), f(y), f(t_0)\rangle$.

if $t_0t_1t_2t_0$ acts as $f(t_0) \to f(t_1) \to f(t_2)$.

Let $\mu = f(t_0)f(t_1)f(t_2)f(t_0) = (2,3,4)(6,7,8)$.

Then $f(t_0)^\mu = f(t_1)$, $f(t_1)^\mu = f(t_2)$ and $f(t_2)^\mu = f(t_0)$.

Thus the relation holds in $\bar{G}$.

$\Longrightarrow \bar{G}$ is a homomorphism image of G.

Now, by the First Isomorphic Theorem $|G/Kerf| \cong \bar{G}$.

$\Longrightarrow |G| \geq |\bar{G}|$.

It is easily verified that $|\bar{G}| = |\langle f(x), f(y), f(t_0)\rangle| = 240$.

$\Longrightarrow |G| \geq 240$.

But we have seen that $|G| \leq 240$.

Hence; $|G| = 240$.

Chapter 4

Construction of $PGL_2(7)$

We factor the progenitor $2^{*4} : S_4$ by the single relator, $[(2,3)(t \cdot t^x)^2]$, and let

$$G \cong \frac{2^{*4} : S_4}{[(23)(t \cdot t^x)^2]}$$

a symmetric presentation of G is given by:

$$\langle x, y, t | x^4, y^2, (yx)^3, t^2, (t, y), (t^x, y), y(t \cdot t^x)^2 \rangle,$$

where $N \cong S_4 = \langle x, y | x^4, y^2, xy^3 \rangle$, and the actions of x and y on the symmetric generator are given by $x \sim (0,1,2,3)$, $y \sim (2,3)$. Our relation is $(2,3)(t \cdot t^x)^2$.

Now, let t is equivalent with t_0, then $t^x = t_0^x \Longrightarrow t_0^{(0,1,2,3)} = t_1$.

$$\begin{aligned} (2,3)(t \cdot t^x)^2 &= 1 \\ \Leftrightarrow (2,3)(t_0 t_1)^2 &= 1 \\ \Leftrightarrow (2,3)(t_0 t_1 t_0 t_1) &= 1 \\ \Leftrightarrow (2,3) t_0 t_1 &= t_1 t_0 \\ \Leftrightarrow N t_0 t_1 &= N t_1 t_0 \end{aligned}$$

Manual double coset enumeration.

We note that; $NeN = \{Nen | n \in N\} = \{Nn | n \in N\} = \{N\}$. Thus NeN, denoted by $[\star]$, contains one single coset. N is transitive on $\{0,1,2,3\}$, so it has a single orbit $\{0,1,2,3\}$. We take a representative, say $\{0\}$ from the orbit, and find to which the double cosets Nt_0 belong? Clearly; $Nt_0 \in Nt_0N = \{Nt_0^n | n \in N\} = \{Nt_0, Nt_1, Nt_2, Nt_3\}$.

Now consider the cosets stabilizer $N^{(0)}$. The cosets stabilizer of Nt_0 is equal to the point stabilize N^0, given by:

$$\begin{aligned} N^{(0)} &= N^0 \\ &= \langle (1,2,3),(1,2)\rangle. \\ &= \{e,(1,2),(1,3),(2,3),(1,2,3),(2,1,3)\}, \end{aligned}$$

then the number of the single cosets in double coset Nt_0N: $\frac{|N|}{|N^{(0)}|} = \frac{24}{6} = 4$.

The orbits of $N^{(0)}$ on $\{0,1,2,3\}$ are $\{0\}$ and $\{1,2,3\}$. We take a representative, $\{0\}$ and say $\{1\}$ from each orbit, and find to which the double cosets Nt_0t_0 and Nt_0t_1 belong? However, $Nt_0t_0 = N \in [\star]$, then one symmetric generator goes back to the double coset NeN, and three symmetric generators go to the next double coset $Nt_0t_1 \in [01]$ (new).

Next, we consider the cosets stabilizer $N^{(01)}$ in N. Now, $N^{01} = \{e,(2,3)\}$, and we note that; $Nt_0t_1 = Nt_1t_0$. $Nt_0t_1^{(0,1)} = Nt_1t_0 \Rightarrow (0,1) \in N^{(01)}$. $Nt_0t_1^{(0,1)(2,3)} = Nt_1t_0 \Rightarrow (0,1)(2,3) \in N^{(01)}$. Therefore; $N^{(01)} = \langle (0,1),(2,3)\rangle = \{e,(0,1),(2,3),(0,1)(2,3)\}$, then the number of the single cosets in double coset Nt_0t_1N is at most: $\frac{|N|}{|N^{(01)}|} = \frac{24}{4} = 6$ (with two names for each).

In order to find other five distinct single cosets in Nt_0t_1N we find the right cosets of $N^{(01)}$ in N. They are $N^{(0,1)}, Nt_0t_1^{(0,2)}, Nt_0t_1^{(0,3)}, Nt_0t_1^{(1,2)}, Nt_0t_1^{(1,3)}, Nt_0t_1^{(0,3)(1,2)}$. We take a presentative from each of the cosets, we form the transversal T. T= $\{e,(0,2),(0,3),(1,2),(1,3),(0,2)(1,3)\}$. Conjugating the coset with two different names; that is, $01 \sim 10$ by each of the elements in the set T, we get other five distinct cosets in Nt_0t_1N with two names each:

$21 \sim 12$.

$31 \sim 13$.

$02 \sim 20$.

$03 \sim 30$.

$23 \sim 32$.

The orbits of $N^{(01)}$ on $\{0,1,2,3\}$ are $\{0,1\}$ and $\{2,3\}$. We take a representative, say $\{0\}$ and $\{2\}$, from each orbit, and find to which the double cosets $Nt_0t_1t_1$ and $Nt_0t_1t_2$ belong? However; $Nt_0t_1t_1 = Nt_0 \in [0]$, then two symmetric generators go back to the double coset Nt_0N. So, $Nt_0t_1t_2N$ is new double coset.

Now, $Nt_0t_1t_2 \in [012]$ (new).

$$\begin{aligned}
(2,3)(t_0t_1) &= (t_1t_0)\\
\Longrightarrow (2,3)(t_0t_1t_2) &= (t_1t_0t_2)\\
\text{Thus } Nt_0t_1t_2 &= Nt_1t_0t_2\\
&= Nt_1(1,3)t_2t_0\\
&= Nt_3t_2t_0\\
&= Nt_2t_3t_0\\
&= Nt_2(1,2)t_0t_3\\
&= Nt_1t_0t_3\\
&= Nt_0t_1t_3\\
&= Nt_0(0,2)t_3t_1\\
&= Nt_2t_3t_1\\
&= Nt_3t_2t_1.
\end{aligned}$$

So,

$Nt_0t_1t_2$=$Nt_1t_0t_2$=$Nt_3t_2t_0$=$Nt_2t_3t_0$.

$Nt_1t_0t_3$=$Nt_0t_3t_1$=$Nt_2t_3t_1$=$Nt_3t_2t_1$.

We consider the cosets stabilizer $N^{(012)}$. $N^{012} = \{e\}$ and we know that $\{(0,1),(2,3),(0,2,3),(0,3,1,2),(0,1)(2,3),(1,3)(0,2)\} \in N^{(012)}$. Therefore; $N^{(012)} = \langle(0,1)(2,3),(1,3)(0,2)\rangle = \{e,(0,1),(2,3),(0,2,3),(0,3,2),(0,3,1,2),(0,1)(2,3),(1,3)(0,2)\}$, then the number of the single cosets in the double coset $Nt_2t_3t_0N$ is at most: $\frac{|N|}{|N^{(010)}|} = \frac{24}{8} = 3$ (with eight names for each).

In order to find other two distinct single cosets with eight names for each in $Nt_0t_1t_2N$ we find the right cosets of $N^{(012)}$ in N. They are $N^{(012)}, Nt_0t^1t_2^{(0,2)}, Nt_0t_1t_2^{(0,3)}$. We take a a presentative from each of the cosets, we form the transversal T.

T= $\{e,(0,2),(0,3)\}$. Conjugating the coset with eight different names; that is, $012 \sim 102 \sim 320 \sim 230 \sim 103 \sim 013 \sim 231 \sim 321$ by each of the elements in the set T, we get other two distinct cosets in Nt_0t_1N with eight names for each:

$210 \sim 120 \sim 302 \sim 032 \sim 123 \sim 213 \sim 031 \sim 301$.

$312 \sim 132 \sim 023 \sim 203 \sim 130 \sim 310 \sim 201 \sim 021$.

$N^{(012)}$ is transitive on $\{0,1,2,3\}$, so it has a single orbit $\{0,1,2,3\}$. We take a representative, say $\{2\}$ from the orbit, and find to which the double coset $Nt_0t_1t_2t_2$ belong? $Nt_0t_1t_2t_2 = Nt_0t_1 \in [01]$. Thus four symmetric generators go back to Nt_0t_1N.

Hence we must have complete the double coset enumerator since the set of right cosets of N in is closed under right multiplication by t_is. The double coset enumeration shows that the index of $N \cong S_4$ in G is at most: $\frac{|N|}{|N|}+\frac{|N|}{|N^{(0)}|}+\frac{|N|}{|N^{(01)}|}+\frac{|N|}{|N^{(2)}|} = 1+4+6+3 = 14$.

$\Longrightarrow |\, G \,| \leq 14|N| = 14 \times 24 = 336.$

We now prove that $|G| = 336$. Determine the action of the symmetric generators x, y and t on 14 distinct cosets of N in G that we have found. Recall that $x \sim (0,1,2,3)$, $y \sim (2,3)$.

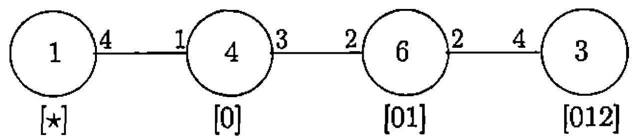

Figure 4.1: The Cayley Diagram of $PGL_2(7)$ over $2^{*4} : S_4$

Permutation Representation.

$Coset$	$x \sim (0,1,2,3)$	$y \sim (2,3)$	t_0
1 N	1 N	1 N	2 Nt_0
2 Nt_0	3 Nt_1	2 Nt_0	1 N
3 Nt_1	4 Nt_2	3 Nt_1	6 Nt_0t_1
4 Nt_2	5 Nt_3	5 Nt_3	7 Nt_0t_2
5 Nt_3	2 Nt_0	4 Nt_2	8 Nt_0t_3
6 Nt_0t_1	9 Nt_1t_2	6 Nt_0t_1	3 Nt_1
7 Nt_0t_2	10 Nt_1t_3	8 Nt_0t_3	4 Nt_2
8 Nt_0t_3	6 Nt_0t_1	7 Nt_0t_2	5 Nt_3
9 Nt_1t_2	11 Nt_2t_3	10 Nt_1t_3	13 $Nt_2t_1t_0$
10 Nt_1t_3	7 Nt_0t_2	9 Nt_1t_2	14 $Nt_3t_1t_2$
11 Nt_2t_3	8 Nt_0t_3	11 Nt_2t_3	12 $Nt_0t_1t_2$
12 $Nt_0t_1t_2$	13 $Nt_2t_1t_0$	12 $Nt_0t_1t_2$	11 Nt_2t_3
13 $Nt_2t_1t_0$	12 $Nt_0t_1t_2$	14 $Nt_3t_1t_2$	9 Nt_1t_2
14 $Nt_3t_1t_2$	14 $Nt_3t_1t_2$	13 $Nt_2t_1t_0$	10 Nt_1t_3

The action of N and the action of $t's$ is as follows: G×X⟶X defined by (g,x)⟶gx, and $f \longrightarrow S_x$, where f is homomorphism. In this problem; we have, $f : 2^{*4} : S_4 \longrightarrow S_x$, where $X = \{N, Nt_0, ..., Nt_3t_1t_2\}$. Now we find,

$f(x) = (2,3,4,5)(6,9,11,8)(7,10)(12,13)$.

$f(y) = (4,5)(7,8)(9,10)(13,14)$.

$f(t_0) = (1,2)(3,6)(4,7)(5,8)(9,13)(10,14)(11,12)$.

$f(t_1) = f(t_0{}^x) = (f(t_0))^{(f(x))} = (1,3)(4,9)(5,10)(2,6)(11,12)(7,14)(8,13)$.

$f(t_2) = f(t_1{}^x) = (f(t_1))^{(f(x))} = (1,4)(5,11)(2,7)(3,9)(13,8)(10,14)(6,12)$.

$f(t_3) = f(t_2{}^x) = (f(t_2))^{(f(x))} = (1,5)(2,8)(3,10)(4,11)(6,12)(7,14)(9,13)$.

The Homomorphism Image of G.

$\bar{G} = f(2^{*4} : S_4) = \langle f(x), f(y), f(t_0), f(t_1), f(t_2), f(t_3)\rangle$.

However; $\{f(t_1), f(t_2), f(t_3)\} \subseteq \langle f(x), f(y), f(t_0)\rangle$.

$\Longrightarrow \bar{G} = \langle f(x), f(y), f(t_0)\rangle = f(2^{*4} : S_4)$.

If the additional relation hold in $\langle f(x), f(t_0), f(y)\rangle$ then:

$$f\left(\frac{2^{*4} : S_4}{([t \cdot t^x])^2}\right) \cong \langle f(x), f(y), f(t_0)\rangle$$

The addition relation; namely, $t_0t_1t_0t_1 = (2,3)$ hold in $\langle f(x), f(y), f(t_0)\rangle$ if $t_0t_1t_0t_1$ acts as $f(t_2) \rightarrow f(t_3)$ and $f(t_3) \rightarrow f(t_2)$.

Let $\mu = f(t_0)f(t_1)f(t_0)f(t_1)$=(4,5)(7,8)(9,10)(13,14).

$\Longrightarrow \quad f(t_2)^\mu = (1,5)(2,8)(3,10)(4,11)(6,12)(7,14)(9,13) = f(t_3)$.

$\Longrightarrow \quad f(t_3)^\mu = (1,4)(5,11)(2,7)(3,9)(13,8)(10,14)(6,12) = f(t_2)$.

Thus the relation holds in $\bar{G}$.

$\Longrightarrow \bar{G}$ is a homomorphism image of G.

Now, by the First Isomorphic Theorem $|G/Kerf| \cong \bar{G}$.

$\Longrightarrow |G| \geq |\bar{G}|$.

It is easily verified that $|\bar{G}| = |\langle f(x), f(y), f(t_0)\rangle| = 336$.

$\Longrightarrow |G| \geq 336$.

But we have see above that $|G| \leq 336$.

Hence; $|G| = 336$.

Chapter 5

Construction of $3 \times PGL_2(9)$

We factor the progenitor $2^{*6} : L_2(5)$ by the single relator, $(0, \infty)(1, 4)(t_2t_3)^2$, and let,

$$G \cong \frac{2^{*6} : L_2(5)}{[(\infty, 0)(1, 4) = (t_2t_3)^2]}.$$

a symmetric presentation of G is given by:

$$\langle x, y, t | x^5, y^3, (xy)^2, t^2(t, x), (t^{(yx^2)}, xy), (t^{y^{(x^3 t y^{x^2})}})^2 \rangle,$$

where $N \cong L_2(5) = \langle x, y | x^5, y^3, (xy)^2 \rangle$, and the actions of x and y on the symmetric generator are given by $x \sim (0, 1, 2, 3, 4)$, $y \sim (\infty, 0, 1)(2, 4, 3)$. Our relation is $(0, \infty)(1, 4)(t_2t_3)^2 = 1$

$$\begin{aligned} (\infty, 0)(1, 4) &= (t_2t_3)^2 \\ \Leftrightarrow (\infty, 0)(1, 4) &= t_2t_3t_2t_3 \\ \Leftrightarrow (\infty, 0)(1, 4)t_3t_2 &= t_2t_3 \\ \Leftrightarrow (\infty, 0)(1, 4)t_2t_3 &= t_3t_2 \end{aligned}$$

Manual double coset enumeration

We note that; $NeN = \{Nen | n \in N\} = \{Nn | n \in N\} = \{N\}$. Thus NeN, denoted by $[\star]$, contains one single coset. N is transitive on $\{0, 1, 2, 3, 4, \infty\}$, so it has a single orbit $\{0, 1, 2, 3, 4, \infty\}$. We take a representative, say $\{2\}$ from the orbit, and find to which the double coset Nt_2 belong? Clearly; $Nt_2 \in Nt_2N = \{Nt_2^n | n \in N\}$ $= \{Nt_0, Nt_1, Nt_2, Nt_3, Nt_4, Nt_\infty\}$. Now consider the cosets stabilizer $N^{(2)}$. The cosets stabilizer of Nt_2 is equal to the point stabilizer N^2, given by, $N^{(2)} = N^2 = \langle(3,0,4,1,\infty),(0,4)(1,3)\rangle = \{e,(1,0)(3,\infty),(1,4)(0,\infty),(3,0)(4,\infty),(1,3)(4,0),(1,\infty)(3,4),(1,4,0,3,\infty),(1,\infty,3,0,4),(1,3,4,\infty,0),(1,0,\infty,4,3)\}$, then the number of the single cosets in the double coset Nt_2N is at most: $\frac{|N|}{|N^{(2)}|} = \frac{60}{10} = 6$.

The orbits of $N^{(2)}$ on $\{0,1,2,3,4,\infty\}$ are $\{2\}$ and $\{0,1,3,4,\infty\}$. We take a representative, $\{2\}$ and say $\{3\}$, from each orbit, and find to which the double cosets Nt_2t_2 and Nt_2t_3 belong? However, $Nt_2t_2 = N \in [\star]$, then one symmetric generator goes back to the double coset NeN, and five symmetric generators go to the next double coset $Nt_2t_3 \in [23]$ (new).

Next, we consider the cosets stabilizer $N^{(23)}$. $N^{(23)} \geq N^{23} = \{e,(1,4)(0,\infty)\}$, and we note that; $Nt_2t_3 = Nt_3t_2$. $Nt_2t_3^{(1,4)(2,3)} = Nt_3t_2 \Rightarrow (1,4)(2,3) \in Nt_2t_3$. $Nt_2t_3^{(0,\infty)(2,3)} = Nt_3t_2 \Rightarrow (0,\infty)(2,3) \in Nt_2t_3$. Therefore; $N^{(23)} = \langle(0,\infty)(2,3),(1,4)(2,3)\rangle = \{e,(1,4)(0,\infty),(1,4)(2,3),(2,3)(0,\infty)\}$, then the number of the single cosets in the double coset Nt_2t_3N is at most: $\frac{|N|}{|N^{(23)}|} = \frac{60}{4} = 15$ (with two names for each).

In order to find the other fourteen distinct single cosets with two different names for each in Nt_2t_3N, we find the right cosets of $N^{(23)}$ in N. They are $Nt_2t_3(1,3)(4,0)$, $Nt_2t_3(1,2)(0,3)$, $Nt_2t_3(\infty,4)(0,3)$, $Nt_2t_3(1,0)(\infty,3)$, $Nt_2t_3(1,\infty,4,2,3)$, $Nt_2t_3(2,0)(4,3)$, $Nt_2t_3(1,\infty,0)(2,4,3)$, $Nt_2t_3(2,\infty)(4,0)$, $Nt_2t_3(1,0)(2,4)$, $Nt_2t_3(1,\infty)(2,0)$, $Nt_2t_3(1,4,3,\infty,2)$, $Nt_2t_3(2,4)(\infty,3)$, $Nt_2t_3(1,\infty,3)(2,4,0)$, $Nt_2t_3(1,2,0)(3,\infty,4)$. Taking are presentative from each of the cosets, we form the transversal T. $T = \{e,(1,3)(4,0),(1,\infty,0)(2,4,3),(3,0)(4,\infty),(1,0)(3,\infty),(1,\infty,4,2,3),(1,\infty,3)(2,4,0),(1,2)(3,0),(1,4,3,\infty,2),(1,0)(2,4),(1,\infty)(2,0),(2,\infty)(4,0),(2,0)(3,4),(2,4)(3,\infty),(1,2,0)(3,\infty,4)\}$. Conjugating the coset with two different names; that is, $23 \sim 32$ by each of the elements in the set T, we get the other fourteen distinct cosets in Nt_2t_3N with two names for each:

$21 \sim 12$ $(\infty,4)(3,0)$.
$24 \sim 42$ $(\infty,3)(1,0)$.
$20 \sim 02$ $(\infty,1)(3,4)$.
$2\infty \sim \infty 2$ $(0,4)(3,1)$.
$31 \sim 13$ $(\infty,2)(4,0)$.
$03 \sim 30$ $(\infty,4)(1,2)$.
$34 \sim 43$ $(\infty,1)(2,0)$.
$3\infty \sim \infty 3$ $(2,4)(1,0)$.

$40 \sim 04$ $(\infty,2)(3,1)$.

$4\infty \sim \infty 4$ $(1,2)(3,0)$.

$0\infty \sim \infty 0$ $(1,4)(3,2)$.

$41 \sim 14$ $(\infty,0)(3,2)$.

$10 \sim 01$ $(\infty,3)(2,4)$.

$\infty 1 \sim 1\infty$ $(3,4)(2,0)$.

The orbits of $N^{(23)}$ on $\{0,1,2,3,4,\infty\}$ are $\{0,\infty\}$, $\{1,4\}$ and $\{2,3\}$. We take a representative, say $\{0\}$, $\{1\}$ and $\{3\}$ from each orbit,and find to which the double cosets $Nt_2t_3t_0$, $Nt_2t_3t_1$ and $Nt_2t_3t_3$ belong? However, $Nt_2t_3t_3 = Nt_2 \in [2]$, then two symmetric generators go back to the double coset Nt_2N, and $Nt_2t_3t_0N$, $Nt_2t_3t_1N$ are new double cosets.

Now, $Nt_2t_3t_0 \in [230]$ (new).

$$\begin{aligned}
(\infty,0)(1,4)t_2t_3 &= t_3t_2\\
\Longrightarrow (\infty,0)(1,4)(t_2t_3t_0) &= t_3t_2t_0\\
\text{Thus } Nt_2t_3t_0 &= Nt_3t_2t_0\\
&= Nt_3(3,4)(1,\infty)t_0t_2\\
&= Nt_4t_0t_2\\
&= Nt_0t_4t_2\\
&= Nt_0(0,1)(3,\infty)t_2t_4\\
&= Nt_1t_2t_4\\
&= Nt_2t_1t_4\\
&= Nt_2(\infty,0)(3,2)t_4t_1\\
&= Nt_3t_4t_1\\
&= Nt_4t_3t_1\\
&= Nt_4(4,0)(\infty,2)t_1t_3\\
&= Nt_0t_1t_3\\
&= Nt_1t_0t_3.
\end{aligned}$$

So,

$Nt_2t_3t_0 = Nt_3t_2t_0 = Nt_4t_0t_2 = Nt_0t_4t_2 = Nt_1t_2t_4 = Nt_3t_4t_1 = Nt_4t_3t_1 = Nt_2t_1t_4 = Nt_0t_1t_3 = Nt_1t_0t_3$.

Now, we consider the cosets stabilizer $N^{(230)}$. $N^{230} = \{e\}$ and $Nt_2t_3t_0^{(0,1,2,3,4)} = Nt_3t_4t_1 \Rightarrow (0,1,2,3,4) \in N^{(230)}$. $Nt_2t_3t_0^{(0,2)(3,4)} = Nt_0t_4t_2 \Rightarrow (0,2)(3,4) \in N^{(230)}$. Therefore; $N^{(230)} = \langle (0,1,2,3,4),(0,2)(3,4)\rangle = \{e,(1,4)(2,3),(0,1,2,3,4),(0,2,4,1,3),(0,3,1,4,2),(0,4,3,2,1),(2,0)(4,3),(1,2)(3,0),(1,0)(2,4),(1,3)(4,0)\} \cong D_{10}$. Then the number of the single cosets in the double coset $Nt_2t_3t_0N$ is at most: $\frac{|N|}{|N^{(230)}|} = \frac{60}{10} = 6$ (with ten different names for each).

In order to find the other five distinct cosets with ten different names for each in $Nt_2t_3t_0N$, we find the right distinct cosets of $N^{(230)}$ in N. They are $N^{(230)}(1,4)(0,\infty)$, $N^{(230)}(1,0)(3,\infty)$, $N^{(230)}(4,0)(2,\infty)$, $N^{(230)}(3,0)(4,\infty)$, $N^{(230)}(2,0)(1,\infty)$. We take a representative from each of the cosets, we form the transversal T. T= $\{(1,4)(0,\infty),(1,0)(3,\infty),(4,0)(2,\infty),(3,0)(4,\infty),(2,0)(1,\infty)\}$. Conjugating the coset with ten different names; that is, **230** ~ 320 ~ 402 ~ 024 ~ 124 ~ 214 ~ 341 ~ 431 ~ 013 ~ 103 by each of the elements in the set T, we get the other five distinct cosets in $Nt_2t_3t_0N$ with ten different names for each:

1∞2 ~ 23∞ ~ 32∞ ~ ∞12 ~ 421 ~ 241 ~ 314 ~ 134 ~ ∞43 ~ 4∞3.

2∞1 ~ ∞21 ~ 412 ~ 142 ~ 01∞ ~ 204 ~ ∞40 ~ 4∞0 ~ 10∞ ~ 024.

∞10 ~ ∞34 ~ 3∞4 ~ 04∞ ~ 40∞ ~ 143 ~ 301 ~ 031 ~ 413 ~ 1∞0.

203 ~ 023 ~ ∞32 ~ 3∞2 ~ 130 ~ 21∞ ~ 0∞1 ~ ∞01 ~ 310 ~ 12∞.

032 ~ 302 ~ 420 ~ 240 ~ ∞23 ~ 0∞4 ~ 34∞ ~ 43∞ ~ 2∞3 ~ ∞04.

The orbits of $N^{(230)}$ on $\{0,1,2,3,4,\infty\}$ are $\{0,1,2,3,4\}$ and $\{\infty\}$. We take a representative, $\{\infty\}$ and say $\{0\}$ from each orbit, and find to which the double cosets $Nt_2t_3t_0t_0$ and $Nt_2t_3t_0t_\infty$ belong? $Nt_2t_3t_0t_0 = Nt_2t_3 \in [23]$, then five symmetric generators go back to Nt_2t_3N and $Nt_2t_3t_0t_\infty N$ is a new double coset.

Now, $Nt_2t_3t_0t_\infty \in [230\infty]$ (new).

$$\begin{aligned}
Nt_2t_3t_0t_\infty &= Nt_2t_3(1,4)(2,3)t_\infty t_0 \\
&= Nt_3t_2t_\infty t_0 \\
&= Nt_3(3,1)(0,4)t_\infty t_2t_0 \\
&= Nt_1t_\infty t_2t_0 \\
&= Nt_1t_\infty(\infty,1)(3,4)t_0t_2 \\
&= Nt_\infty t_1t_0t_2 \\
&= Nt_\infty(\infty,3)(2,4)t_0t_1t_2 \\
&= Nt_3t_0t_1t_2 \\
&= Nt_3t_0(\infty,4)(0,3)t_2t_1 \\
&= Nt_0t_3t_2t_1 \\
&= Nt_0(0,\infty)(1,4)t_2t_3t_1 \\
&= Nt_\infty t_2t_3t_1 \\
&= Nt_\infty t_2(\infty,2)(0,4)t_1t_3 \\
&= Nt_2t_\infty t_1t_3
\end{aligned}$$

$= Nt_\infty t_2 t_1 t_3$

$= Nt_\infty(\infty,4)(0,3)t_1 t_2 t_3$

$= Nt_4 t_1 t_2 t_3$

$= Nt_1 t_4 t_2 t_3$

$= Nt_1(\infty,3)(0,1)t_2 t_4 t_3$

$= Nt_0 t_2 t_4 t_3$

$= Nt_0 t_2(\infty,1)(0,2)t_3 t_4$

$= Nt_2 t_0 t_3 t_4$

Then, $Nt_2t_3t_0t_\infty$ has six different names,

$Nt_2t_3t_0t_\infty = Nt_1t_\infty t_2t_0 = Nt_\infty t_1t_0t_2 = Nt_0t_3t_2t_1 = Nt_2t_\infty t_1t_3 = Nt_2t_0t_3t_4$.

Now recall,

230 $\sim 320 \sim 402 \sim 024 \sim 124 \sim 214 \sim 341 \sim 431 \sim 013 \sim 103$.

1∞2 $\sim 23\infty \sim 32\infty \sim \infty12 \sim 421 \sim 241 \sim 314 \sim 134 \sim \infty43 \sim 4\infty3$.

∞10 $\sim \infty34 \sim 3\infty4 \sim 04\infty \sim 40\infty \sim 143 \sim 301 \sim 031 \sim 413 \sim 1\infty0$.

032 $\sim 302 \sim 420 \sim 240 \sim \infty23 \sim 0\infty4 \sim 34\infty \sim 43\infty \sim 2\infty3 \sim \infty04$.

2∞1 $\sim \infty21 \sim 412 \sim 142 \sim 01\infty \sim 204 \sim \infty40 \sim 4\infty0 \sim 10\infty \sim 024$.

203 $\sim 023 \sim \infty32 \sim 3\infty2 \sim 130 \sim 21\infty \sim 0\infty1 \sim \infty01 \sim 310 \sim 12\infty$.

Thus $Nt_2t_3t_0t_\infty$ has sixty different names:

230∞ $\sim 320\infty \sim 402\infty \sim 024\infty \sim 124\infty \sim 214\infty \sim 341\infty \sim 431\infty \sim 013\infty \sim 103\infty$
$\sim$ **1∞20** $\sim 23\infty0 \sim 32\infty0 \sim \infty120 \sim 4210 \sim 2410 \sim 3140 \sim 1340 \sim \infty430 \sim 4\infty30$
$\sim$ **∞102** $\sim \infty342 \sim 3\infty42 \sim 04\infty2 \sim 40\infty2 \sim 1432 \sim 3012 \sim 0312 \sim 4132 \sim 1\infty02$
$\sim$ **0321** $\sim 3021 \sim 4201 \sim 2401 \sim \infty231 \sim 0\infty41 \sim 34\infty1 \sim 43\infty1 \sim 2\infty31 \sim \infty041$
$\sim$ **2∞13** $\sim \infty213 \sim 4123 \sim 1423 \sim 01\infty3 \sim 2043 \sim \infty403 \sim 4\infty03 \sim 10\infty3 \sim 0243$
$\sim$ **2034** $\sim 0234 \sim \infty324 \sim 3\infty24 \sim 1304 \sim 21\infty4 \sim 0\infty14 \sim \infty014 \sim 3104 \sim 12\infty4$.

Now, we consider the cosets stabilizer for $N^{(230\infty)}$. $N^{230\infty} = \{e\}$ and

$Nt_2t_3t_0t_\infty^{(0,1,2,3,4)} = Nt_3t_4t_1t_\infty \Rightarrow (0,1,2,3,4) \in N^{(230\infty)}$.

$Nt_2t_3t_0t_\infty^{(\infty,0,1)(2,4,3)} = Nt_4t_2t_1t_0 \Rightarrow (\infty,0,1)(2,4,3) \in N^{(230\infty)}$.

Therefore; $N^{(230\infty)} = \langle(0,1,2,3,4),(\infty,0,1)(2,4,3)\rangle = N \cong L_2(5) = \langle x,y|x^5,y^3,(xy)^2\rangle$.

Then the number of the single cosets in the double coset $Nt_2t_3t_0t_\infty N$ is at most: $\frac{|N|}{|N^{(230\infty)}|} = \frac{60}{60} = 1$ (with sixty different names for each).

$N^{(230\infty)}$ is transitive on $\{0,1,2,3,4,\infty\}$, so $N^{(230\infty)}$ has a single orbit $\{0,1,2,3,4,\infty\}$. We take the representative namely $\{\infty\}$ from the orbit, we find the double coset to which $Nt_2t_3t_0t_\infty t_\infty$ belong? $Nt_2t_3t_0t_\infty t_\infty = Nt_2t_3t_0 \in [230]$. Therefore; six symmetric generators go back to $Nt_2t_3t_0N$.

Now, $Nt_2t_3t_1 \in [231]$ (new) .

$$\begin{aligned}
(\infty,0)(1,4)(t_2t_3) &= t_3t_2\\
\Longrightarrow (\infty,0)(1,4)(t_2t_3t_1) &= t_3t_2t_1\\
\text{Thus } Nt_2t_3t_1 &= Nt_3t_2t_1\\
&= Nt_3(4,\infty)(0,3)t_1t_2\\
&= Nt_0t_1t_2\\
&= Nt_1t_0t_2\\
&= Nt_1(3,4)(1,\infty)t_2t_0\\
&= Nt_\infty t_2t_0\\
&= Nt_2t_\infty t_0\\
&= Nt_2(1,4)(3,2)t_0t_\infty\\
&= Nt_3t_0t_\infty\\
&= Nt_0t_3t_\infty\\
&= Nt_0(1,0)(4,2)t_\infty t_3\\
&= Nt_1t_\infty t_3\\
&= Nt_\infty t_1t_3
\end{aligned}$$

So,

$Nt_2t_3t_1 = Nt_3t_2t_1 = Nt_0t_1t_2 = Nt_1t_0t_2 = Nt_1t_\infty t_3 = Nt_\infty t_1t_3 = Nt_3t_0t_\infty = Nt_0t_3t_\infty = Nt_\infty t_2t_0 = Nt_2t_\infty t_0$.

Now, we consider the cosets stabilizer $N^{(231)}$. $N^{231} = \{e\}$ and $Nt_2t_3t_1^{(0,\infty,3,1,2)} = Nt_0t_1t_2 \Rightarrow (0,\infty,3,1,2) \in N^{(231)}$. $Nt_2t_3t_1^{(1,2)(0,3)} = Nt_1t_0t_2 \Rightarrow (1,2)(0,3) \in N^{(231)}$. Therefore; $N^{(231)} = \langle(0,\infty,3,1,2),(1,2)(3,0)\rangle = \{e,(1,2)(3,0),(1,3)(2,\infty),(2,3)(0,\infty),(1,0)(\infty,3),(1,\infty)(2,0),(1,2,0,\infty,3),(1,3,\infty,0,2),(1,\infty,2,0,3),(1,0,3,2,\infty)\} \cong D_{10}$. Then the number of the single cosets in double coset $Nt_2t_3t_1N$ is at most: $\frac{|N|}{|N^{(231)}|} = \frac{60}{10} = 6$ (with ten different names for each).

In order to find the other five distinct cosets with ten different names for each in $Nt_2t_3t_0N$, we find the right distinct cosets of $N^{(231)}$ in N. They are are listed below: $Nt_2t_3t_1(1,4)(2,3)$, $Nt_2t_3t_1(3,0)(4,\infty)$, $Nt_2t_3t_1(4,0)(2,\infty)$, $Nt_2t_3t_1(1,0)(4,\infty)$, $Nt_2t_3t_1(2,0)(3,4)$. We take a representative from of the cosets, we form the transversal T. T= $\{(1,4)(2,3),(3,0)(4,\infty),(4,0)(2,\infty),(1,0)(4,\infty),(2,0)(3,4)\}$. Conjugating the coset with ten different names for each; that is, **231** $\sim 321 \sim 012 \sim 1\infty3 \sim \infty13 \sim 30\infty \sim 03\infty \sim \infty20 \sim 2\infty0 \sim 102$ by each of the elements in the set T, we get the other five distinct cosets in $Nt_2t_3t_1N$ with ten different names for each:

324 $\sim 234 \sim 20\infty \sim 02\infty \sim 3\infty0 \sim \infty30 \sim 4\infty2 \sim \infty42 \sim 043 \sim 403$

132 $\sim 201 \sim 021 \sim 243 \sim 423 \sim 034 \sim 304 \sim 312 \sim 410 \sim 140$

24∞ $\sim 041 \sim 401 \sim 0\infty 2 \sim \infty 02 \sim 42\infty \sim 120 \sim 210 \sim \infty 14 \sim 1\infty 4$

213 $\sim \infty 31 \sim 3\infty 1 \sim \infty 24 \sim 2\infty 4 \sim 342 \sim 432 \sim 14\infty \sim 41\infty \sim 123$

31∞ $\sim 430 \sim 340 \sim 4\infty 1 \sim \infty 41 \sim 13\infty \sim 014 \sim 104 \sim \infty 03 \sim 0\infty 3$.

The orbits of $N^{(231)}$ on $\{0,1,2,3,4,\infty\}$ are $\{0,1,2,3,\infty\}$ and $\{4\}$. We take a representative, $\{4\}$ and say $\{1\}$ from each orbit, and find to which the double cosets $Nt_2t_3t_1t_1$ and $Nt_2t_3t_1t_4$ belong? $Nt_2t_3t_1t_1 = Nt_2t_3 \in [23]$, then five symmetric generators go back to Nt_2t_3N.

Now, $Nt_2t_3t_1t_4 \in [2314]$ (new).

$Nt_2t_3t_1t_4 = Nt_2t_3(\infty,0)(2,3)t_4t_1$
$= Nt_3t_2t_4t_1$
$= Nt_3(0,1)(\infty,3)t_4t_2t_1$
$= Nt_\infty t_4t_2t_1$
$= Nt_\infty t_4(\infty,4)(0,3)t_1t_2$
$= Nt_4t_\infty t_1t_2$
$= Nt_4(0,2)(3,4)t_1t_\infty t_2$
$= Nt_3t_1t_\infty t_2$
$= Nt_3t_1(0,4)(1,3)t_2t_\infty$
$= Nt_1t_3t_2t_\infty$
$= Nt_1(0,\infty)(1,4)t_2t_3t_\infty$
$= Nt_4t_2t_3t_\infty$
$= Nt_4t_2(2,4)(1,0)t_\infty t_3$
$= Nt_2t_4t_\infty t_3$
$= Nt_4t_2t_\infty t_3$
$= Nt_4(0,4)(1,3)t_\infty t_2t_3$
$= Nt_0t_\infty t_2t_3$
$= Nt_\infty t_0t_2t_3$
$= Nt_\infty(1,\infty)(3,4)t_2t_0t_3$
$= Nt_1t_2t_0t_3$
$= Nt_1t_2(4,\infty)(1,2)t_3t_0$
$= Nt_2t_1t_3t_0$

Then $Nt_2t_3t_1t_4$ has six different names,
$Nt_2t_3t_1t_4 = Nt_3t_2t_4t_1 = Nt_1t_3t_2t_\infty = Nt_2t_4t_\infty t_3 = Nt_2t_1t_3t_0 = Nt_3t_1t_\infty t_2$.

Now recall,

231 $\sim 321 \sim 012 \sim 1\infty 3 \sim \infty 13 \sim 30\infty \sim 03\infty \sim \infty 20 \sim 2\infty 0 \sim 102$

324 $\sim 234 \sim 20\infty \sim 02\infty \sim 3\infty 0 \sim \infty 30 \sim 4\infty 2 \sim \infty 42 \sim 043 \sim 403$

132 $\sim 201 \sim 021 \sim 243 \sim 423 \sim 034 \sim 304 \sim 312 \sim 410 \sim 140$

24∞ $\sim 041 \sim 401 \sim 0\infty 2 \sim \infty 02 \sim 42\infty \sim 120 \sim 210 \sim \infty 14 \sim 1\infty 4$

213 $\sim \infty 31 \sim 3\infty 1 \sim \infty 24 \sim 2\infty 4 \sim 342 \sim 432 \sim 14\infty \sim 41\infty \sim 123$

31∞ $\sim 430 \sim 340 \sim 4\infty 1 \sim \infty 41 \sim 13\infty \sim 014 \sim 104 \sim \infty 03 \sim 0\infty 3$

Thus $Nt_2t_3t_1t_4$ has sixty different names:

2314 $\sim 3214 \sim 0124 \sim 1\infty 34 \sim \infty 134 \sim 30\infty 4 \sim 03\infty 4 \sim \infty 204 \sim 2\infty 04 \sim 1024$
$\sim$ **3241** $\sim 2341 \sim 20\infty 1 \sim 02\infty 1 \sim 3\infty 01 \sim \infty 301 \sim 4\infty 21 \sim \infty 421 \sim 043 \sim 4031$
$\sim$ **132∞** $\sim 201\infty \sim 021\infty \sim 243\infty \sim 423\infty \sim 034\infty \sim 304\infty \sim 312\infty \sim 410\infty \sim 140\infty$
$\sim$ **24∞3** $\sim 0413 \sim 4013 \sim 0\infty 23 \sim \infty 023 \sim 42\infty 3 \sim 1203 \sim 2103 \sim \infty 143 \sim 1\infty 43$
$\sim$ **2130** $\sim \infty 310 \sim 3\infty 10 \sim \infty 240 \sim 2\infty 40 \sim 3420 \sim 4320 \sim 14\infty 0 \sim 41\infty 0 \sim 1230$
$\sim$ **31∞2** $\sim 4302 \sim 3402 \sim 4\infty 12 \sim \infty 412 \sim 13\infty 2 \sim 0142 \sim 1042 \sim \infty 032 \sim 0\infty 32$.

Now, we consider the cosets stabilizer for $N^{(2314)}$. $N^{2314} = \{e\}$ and $Nt_2t_3t_1t_4^{(0,1,2,3,4)} = Nt_3t_4t_2t_0 = Nt_3t_4(\infty,1)(3,4)t_0t_2 = Nt_4t_3t_0t_2 \Rightarrow (0,1,2,3,4) \in N^{(2314)}$. $Nt_2t_3t_1t_4^{(\infty,0,1)(2,4,3)} = Nt_4t_2t_\infty t_3 = Nt_4t_2(0,1)(2,4)t_3t_\infty = Nt_2t_4t_3t_\infty \Rightarrow (\infty,0,1)(2,4,3) \in N^{(2314)}$. Therefore; $N^{(2314)} = \langle (0,1,2,3,4),(\infty,0,1)(2,4,3)\rangle = N \cong L_2(5) = \langle x,y|x^5,y^3,(xy)^2\rangle$. Then the number of the single cosets in the double coset $Nt_2t_3t_1t_4N$ is at most: $\frac{|N|}{|N^{(2314)}|} = \frac{60}{60} = 1$ (with sixty different names for each).

$N^{(2314)}$ is transitive on $\{0,1,2,3,4,\infty\}$, then $N^{(2314)}$ has a single orbit on $\{0,1,2,3,4,\infty\}$. We take a representative; namely $\{4\}$ from the orbit, we find the double coset to which $Nt_2t_3t_1t_4t_4$ belong to? $Nt_2t_3t_1t_4t_4 = Nt_2t_3t_1 \in [231]$, then six symmetric generators go back to $Nt_2t_3t_1N$.

Hence; we must have complete the double coset enumerator since the set of right coset of N is closed under right multiplication by $t_i s$. The double coset enumeration shows that the index of $N \cong L_2(5)$ in G is at most: $\frac{|N|}{|N|} + \frac{|N|}{|N^{(2)}|} + \frac{|N|}{|N^{(23)}|} + \frac{|N|}{|N^{(230)}|} + \frac{|N|}{|N^{(231)}|} + \frac{|N|}{|N^{(230\infty)}|} + \frac{|N|}{|N^{(2314)}|} = 1 + 6 + 15 + 6 + 6 + 1 + 1 = 36$.
$\Longrightarrow |G| \leq 36|N| = 36 \times 60 = 2160$.

We now prove that $|G| = 2160$. Determine the action of the symmetric generator x, y and t on 36 distinct cosets of N in G that we have found. Recall that $x \sim (0,1,2,3,4)$, $y \sim (0,1,\infty)(2,4,3)$.

Permutation Representation

Coset	$x \sim (01234)$	$y \sim (01\infty)(243)$	t_0
1 N	1 N	1 N	2 Nt_0
2 Nt_0	3 Nt_1	3 Nt_1	1 N
3 Nt_1	4 Nt_2	7 Nt_∞	8 Nt_1t_0
4 Nt_2	5 Nt_3	6 Nt_4	13 Nt_2t_0
5 Nt_3	6 Nt_4	4 Nt_2	17 Nt_3t_0
6 Nt_4	2 Nt_0	5 Nt_3	20 Nt_4t_0
7 Nt_∞	7 Nt_∞	2 Nt_0	22 $Nt_\infty t_0$
8 Nt_1t_0	9 Nt_1t_2	12 Nt_1t_∞	3 Nt_1
9 Nt_1t_2	14 Nt_2t_3	21 Nt_4t_∞	32 $Nt_2t_1t_0$
10 Nt_1t_3	15 Nt_2t_4	16 Nt_2t_∞	26 $Nt_2t_0t_3$
11 Nt_1t_4	13 Nt_2t_0	19 Nt_3t_∞	31 $Nt_2t_0t_1$
12 Nt_1t_∞	16 Nt_2t_∞	22 $Nt_\infty t_0$	28 $Nt_3t_0t_1$
13 Nt_2t_0	10 Nt_1t_3	11 Nt_1t_4	4 Nt_2
14 Nt_2t_3	18 Nt_3t_4	15 Nt_2t_4	23 $Nt_2t_3t_0$
15 Nt_2t_4	17 Nt_3t_0	18 Nt_3t_4	27 $Nt_3t_0t_2$
16 Nt_2t_∞	19 Nt_3t_∞	20 Nt_4t_0	29 $Nt_2t_3t_1$
17 Nt_3t_0	11 Nt_1t_4	9 Nt_1t_2	5 Nt_3
18 Nt_3t_4	20 Nt_4t_0	14 Nt_2t_3	34 $Nt_1t_0t_4$
19 Nt_3t_∞	21 Nt_4t_∞	13 Nt_2t_0	30 $Nt_2t_3t_4$
20 Nt_4t_0	8 Nt_1t_0	10 Nt_1t_3	6 Nt_4
21 Nt_4t_∞	22 $Nt_\infty t_0$	17 Nt_3t_0	25 $Nt_2t_0t_4$
22 $Nt_\infty t_0$	12 Nt_1t_∞	8 Nt_1t_0	7 Nt_∞
23 $Nt_2t_3t_0$	23 $Nt_2t_3t_0$	24 $Nt_2t_4t_1$	14 Nt_2t_3
24 $Nt_2t_4t_1$	27 $Nt_3t_0t_2$	27 $Nt_3t_0t_2$	35 $Nt_2t_3t_0t_\infty$
25 $Nt_2t_0t_4$	26 $Nt_2t_0t_3$	28 $Nt_3t_0t_1$	21 Nt_4t_∞
26 $Nt_2t_0t_3$	24 $Nt_2t_4t_1$	25 $Nt_2t_0t_4$	10 Nt_1t_3
27 $Nt_3t_0t_2$	28 $Nt_3t_0t_1$	23 $Nt_2t_3t_0$	15 Nt_2t_4
28 $Nt_3t_0t_1$	25 $Nt_2t_0t_4$	26 $Nt_2t_0t_3$	12 Nt_1t_∞
29 $Nt_2t_3t_1$	33 $Nt_2t_1t_3$	32 $Nt_2t_1t_0$	16 Nt_2t_∞
30 $Nt_2t_3t_4$	34 $Nt_1t_0t_4$	31 $Nt_2t_0t_1$	19 Nt_3t_∞
31 $Nt_2t_0t_1$	31 $Nt_2t_0t_1$	33 $Nt_2t_1t_3$	11 Nt_1t_4
32 $Nt_2t_1t_0$	29 $Nt_2t_3t_1$	34 $Nt_1t_0t_4$	9 Nt_1t_2
33 $Nt_2t_1t_3$	30 $Nt_2t_3t_4$	30 $Nt_2t_3t_4$	36 $Nt_2t_3t_1t_4$
34 $Nt_1t_0t_4$	32 $Nt_2t_1t_0$	29 $Nt_2t_3t_1$	18 Nt_3t_4
35 $Nt_2t_3t_0t_\infty$	35 $Nt_2t_3t_0t_\infty$	35 $Nt_2t_3t_0t_\infty$	24 $Nt_2t_4t_1$
36 $Nt_2t_3t_1t_4$	36 $Nt_2t_3t_1t_4$	36 $Nt_2t_3t_1t_4$	33 $Nt_2t_1t_3$

The action of N and the action of $t's$ is as follows: G×X⟶X defined by (g,x) ⟶gx, and $f \longrightarrow S_x$, where f is homomorphism. In this problem; we have, $f : 2^{*6} : L_2(5) \longrightarrow S_x$, where $X = \{N, Nt_0, ..., Nt_2t_3t_1t_4\}$. Now, we find

$f(x) = (2,3,4,5,6)(8,9,14,18,20)(10,15,17,11,13)(12,16,19,21,22)$ $(24,27,28,25,26)$

(30, 34, 32, 29, 33).

$f(y) = (2,3,7)(4,6,5)(8,12,22)(9,21,17)(10,16,20)(11,19,13)(14,15,18)\ (23,24,27)$ $(25,28,26)(29,32,34)(30,31,33)$.

$f(t_0) = (1,2)(3,8)(4,13)(5,17)(6,20)(7,22)(9,32)(10,26)(11,31)(12,28)(14,23)\ (15,27)$ $(16,29)(18,34)(19,30)(21,25)(24,35)(33,36)$.

$f(t_1) = f(t_0{}^x) = (f(t_0))^{f(x)} = (1,3)(4,9)(5,10)(6,11)(2,8)(7,12)(14,29)\ (15,24)(13,31)$ $(16,25)(18,23)(17,28)(19,33)(20,32)(21,34)(22,26)(27,35)(30,36)$.

$f(t_2) = f(t_1{}^x) = (f(t_1))^{f(x)} = (1,4)(5,14)(6,15)(2,13)(3,9)(7,16)(18,33)\ (17,27)(10,31)$ $(19,26)(20,23)(15,31)$

$(11,25)(21,30)(8,29)(22,32)(12,24)(28,35)(34,36)$.

$f(t_3) = f(t_2{}^x) = (f(t_2))^{f(x)} = (1,5)(2,17)(3,10)(4,14)(6,18)(7,19)(8,23)\ (9,33)(11,28)$ $(12,29)(13,26)(16,27)(20,30)(21,24)(22,34)(25,35)(32,36)$.

$f(t_4) = f(t_3{}^x) = (f(t_3))^{f(x)} = (1,6)(2,20)(3,11)(4,15)(5,18)(7,21)(8,34)\ (9,23)(10,24)$ $(12,32)(13,25)(14,30)(16,33)(17,31)(19,28)(22,27)(26,35)(29,36)$.

$f(t_\infty) = f(t_1{}^y) = (f(t_1))^{f(y)} = (1,7)(2,22)(3,12)(4,16)(5,19)(6,21)\ (8,25)(9,26)(10,34)$ $(11,33)(13,30)(14,24)(23,35)(31,36)(15,32)(17,29)(18,27)(20,28)$

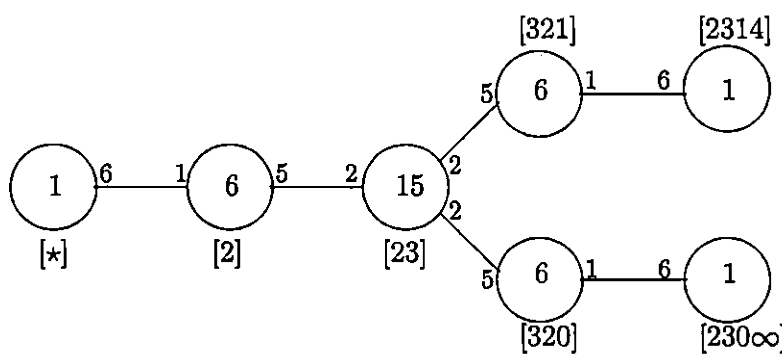

Figure 5.1: The Cayley Diagram of $3 \times PGL_2(9)$ over $2^{*6} : L_2(5)$

The Homomorphism Image Of G

$\bar{G} = f(2^{*6} : L_2(5)) = \langle f(x), f(y), f(t_0), f(t_1), f(t_2), f(t_3), f(t_4), f(t_\infty)\rangle$.

However;

$\{f(t_1), f(t_2), f(t_3), f(t_4), f(t_\infty)\} \subseteq \langle f(x), f(y), f(t_0))\rangle$.

Then $\bar{G} = f(2^{*6} : L_2(5)) = \langle f(x), f(y), f(t_0) \rangle$.

If the additional relation hold in $\langle f(x), f(y), f(t_0), f(t_\infty)\rangle$ then:

$$f\left(\frac{2^{*6} : L_2(5)}{[(t_2t_3)^2 = (\infty,0)(1,4)]}\right) = \langle f(x), f(y), f(t_0)\rangle.$$

The addition relation, namely $t_2t_3t_2t_3 = (\infty,0)(1,4)$ holds in $\langle f(x), f(y), f(t_0), f(t_\infty)\rangle$ if $f(t_2)f(t_3)f(t_2)f(t_3)$ acts as $f(t_\infty) \rightarrow f(t_0)$ and $f(t_1) \rightarrow f(t_4)$.

Let $\mu = f(t_2)f(t_3)f(t_2)f(t_3)$ $=(2,7)(3,6)(8,21)(9,15)(10,18)(12,20)(13,16)$ $(17,19)(23,24)(26,27)(29,30)(31,33)$.

Then $(f(t_\infty))^\mu = f(t_0)$ and $(f(t_0))^\mu = f(t_\infty)$.

$(f(t_1))^\mu = f(t_4)$ and $(f(t_4))^\mu = f(t_1)$.

Thus the relation holds in $\bar{G} = \langle f(x), f(y), f(t_0), f(t_\infty)\rangle$.

$\Longrightarrow \bar{G}$ is a homomorphism image of G.

Now, by the First Isomorphic Theorem $|G/Kerf| \cong \bar{G}$.

$\Rightarrow |G| \geq |\bar{G}|$.

It is easily verified that, $|\bar{G}| = |\langle f(x), f(y), f(t_0)\rangle| = 2160$.

$\Longrightarrow |G| \geq 2160$.

But we have seen above that $|G| \leq 2160$.

Hence; $|G| = 2160$.

Now, $3 \times PGL_2(9)$ has a unique normal subgroup of order 3, generated by; (1,35,36)(2,34,31) (3,33,29)(4,27,30)(5,26,22)(6,20,25)(7,19,32)(8,21,17)(9,23,28)(10,13,16) (11,12,24) (14,15,18) whose symmetric representative form is $[(1,3,\infty)(2,0,4)t_2t_3t_0t_\infty]$. In the next chapter, we will factor $3 \times PGL_2(9)$ by the unique normal subgroup of order 3 given above to obtained $PGL_2(9)$.

Chapter 6

Construction of $PGL_2(9)$

We consider the progenitor $2^{*6} : L_5$by the single relator, $(1,3,\infty)(2,0,4)t_2t_3t_0t_\infty$, and let

$$G \cong \frac{2^{*6} : L_2(5)}{[t_2t_3t_0t_\infty = (1,\infty,3)(2,4,0)]}$$

A symmetric presentation of G is given by:

$$\langle x, y, t | x^5, y^3, (xy)^2, t^2, (t, x^2y^{-1}), (t, yx^{-1}yx^2y^{-1}), (tt^x)^2, (xy), (tt^x t^{x^3} t^{(x^4y)}), (x^2y^2xy^2)\rangle.$$

where $N \cong L_2(5) = \langle x, y | x^5, y^3, (xy)^2\rangle$, and the actions of x and y on the symmetric generator are given by x $\sim (0,1,2,3,4)$, y $\sim (\infty,0,1)(2,4,3)$. Our relation is,

$$\begin{aligned} t_2t_3t_0t_\infty &= (1,\infty,3)(2,4,0) \\ \Leftrightarrow t_2t_3 &= (1,\infty,3)(2,4,0)t_\infty t_0. \end{aligned}$$

Manual double coset enumeration

We note that $NeN = \{Nen | n \in N\} = \{Nn | n \in N\} = \{N\}$.
Thus NeN, denoted by $[\star]$, contains one single coset. N is transitive on $\{0,1,2,3,4,\infty\}$, so it has a single orbit $\{0,1,2,3,4,\infty\}$. We take a representative, say $\{2\}$ from the orbit, and find to which the double coset Nt_2 belong? Clearly; $Nt_2 \in Nt_2N = \{Nt_2^n | n \in N\} = \{Nt_0, Nt_1, Nt_2, Nt_3, Nt_4, Nt_\infty\}$.

Now consider the cosets stabilizer of $N^{(2)}$. The cosets stabilizer of Nt_2 is equal to the point stabilizer N^2, given by: $N^{(2)} = N^2 = \langle (3,0,4,1,\infty), (0,4)(1,3)\rangle = \{e, (1,0)(3,\infty), (1,4)(0,\infty), (3,0)(4,\infty), (1,3)(4,0), (1,\infty)(3,4), (1,4,0,3,\infty), (1,\infty,3,0,4),$

$(1,3,4,\infty,0),(1,0,\infty,4,3)\}$, then the number of the single cosets in the double coset Nt_2N is at most: $\frac{|N|}{|N^{(2)}|} = \frac{60}{10} = 6$.

The orbits of $N^{(2)}$ on $\{0,1,2,3,4,\infty\}$ are $\{2\}$ and $\{0,1,3,4,\infty\}$. We take a representative, $\{2\}$ and say $\{3\}$, from each orbit, and find to which the double cosets Nt_2t_2 and Nt_2t_3 belong? However, $Nt_2t_2 = N \in [\star]$, then one symmetric generator goes back to the double coset NeN, and five symmetric generators go to the next double coset $Nt_2t_3 \in [23]$.

Next, we consider the cosets stabilizer $N^{(23)}$. The cosets stabilizer of Nt_2t_3 is given by: $N^{(23)} \geq N^(23) = \{e,(1,4)(0,\infty)\}$, and we note that;
$Nt_2t_3 = Nt_\infty t_0$. $Nt_2t_3^{(1,4)(0,\infty)} = Nt_\infty t_0^{(1,4)(0,\infty)} \Longrightarrow Nt_2t_3{=}Nt_0t_\infty$.
$Nt_2t_3^{(2,3)(0,\infty)} = Nt_\infty t_0^{(2,3)(0,\infty)} \Longrightarrow Nt_3t_2 = Nt_0t_\infty$ and $(2,3)(0,\infty) \in N^{(23)}$.
$Nt_2t_3^{(1,4)(0,\infty)} = Nt_\infty t_0^{(1,4)(0,\infty)} \Longrightarrow Nt_2t_3 = Nt_0t_\infty$ and$(1,4)(0,\infty) \in N^{(23)}$.
$Nt_2t_3^{(1,0,2)(3,4,\infty)} = Nt_\infty t_0^{(1,0,2)(3,4,\infty)} \Longrightarrow Nt_1t_4 = Nt_3t_2$ and $(1,0,2)(3,4,\infty) \in N^{(23)}$.
$Nt_2t_3^{(1,\infty 3)(2,4,0)} = Nt_\infty t_0^{(1,\infty,3)(2,4,0)} \Longrightarrow Nt_4t_1{=}Nt_3t_2$ and $(1,\infty,3)(2,4,0) \in N^{(23)}$.
So,
$Nt_2t_3{=}Nt_3t_2{=}Nt_\infty t_0{=}Nt_0t_\infty{=}Nt_4t_1{=}Nt_1t_4$.
Therefore;
$N^{(23)} = \langle(0,\infty)(1,4),(1,\infty,2)(3,4,0)\rangle{=}\ \{e,(1,4)(0,\infty),(1,\infty,2)(3,4,0),(1,\infty,3)(2,4,0),$ $(1,2,\infty)(3,0,4),(1,2,0)(3,\infty,4),(1,0,2)(3,4,\infty),(1,3,\infty)(2,0,4),(1,3,0)(2,\infty,4),$ $(2,3)(0,\infty),(1,0,3)(2,4,\infty),(1,4)(2,3)\}$. Then the number of single cosets in the double coset Nt_2t_3N is at most: $\frac{|N|}{|N^{(23)}|} = \frac{60}{12} = 5$. (with six names for each)

In order to find the other four distinct cosets with six names for each in Nt_2t_3N we find the right distinct cosets of $N^{(23)}$ in N. They are listed below: $Nt_2t_3(1,0)(3,\infty)$, $Nt_2t_3(1,\infty,0)(2,4,3)$, $Nt_2t_3(\infty,4)(0,3)$, $Nt_2t_3(1,3)(4,0)$. We take a representatives from of the cosets, we form the transversal T. T$=\{e,(1,0)(3,\infty),(1,\infty,0)(2,4,3),(3,0)(4,\infty),$ $(1,3)(4,0)\}$. Conjugating the coset with six different names; that is, $23 \sim 32 \sim 14 \sim 41 \sim 0\infty \sim \infty 0$ by each of the elements in the set T, we get the other four distinct cosets in Nt_2t_3N with six names for each:

$2\infty \sim \infty 2 \sim 31 \sim 13 \sim 04 \sim 40$

$42 \sim 24 \sim 01 \sim 10 \sim 36\infty \sim 6\infty 3$

$34 \sim 43 \sim 1\infty \sim 6\infty 1 \sim 20 \sim 02$

$12 \sim 21 \sim \infty 4 \sim 4\infty \sim 03 \sim 30$

Now, $N^{(23)}$ is transitive on $\{0,1,2,3,4,\infty\}$, so it has a single orbit $\{0,1,2,3,4,\infty\}$. We take a representative; namely, $\{3\}$ from the orbit, we note that $Nt_2t_3t_3 = Nt_2 \in [2]$. Therefore, six symmetric generators go back to the double coset Nt_2N.

Hence we must have complete the double coset enumerator since the set of right coset of N is closed under right multiplication by t_is. The double coset enumeration shows that the index of $N \cong L_2(5)$ in G is at most: $\frac{|N|}{|N|} + \frac{|N|}{|N^{(2)}|} + \frac{|N|}{|N^{(23)}|} = 1 + 6 + 5 = 12$
$\Longrightarrow | G | \leq 12|N| = 12 \times 60 = 720$

We now prove that $|G| = 720$. Determine the action of the symmetric generator x, y and t on 12 distinct cosets of N in G that we have found. Recall that $x \sim (0,1,2,3,4)$, $y \sim (0,1,\infty)(2,4,3)$.

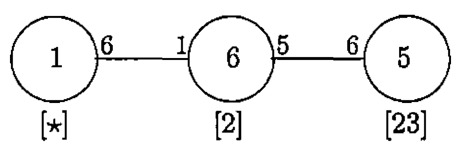

Figure 6.1: The Cayley Diagram of $PGL_2(9)$ over $2^{*6} : L_2(5)$

Permutation Representation

Coset	$x \sim (0,1,2,3,4)$	$y \sim (0,1,\infty)(2,4,3)$	t_0
1 N	1 N	1 N	2 Nt_0
2 Nt_0	3 Nt_1	3 Nt_1	1 N
3 Nt_1	4 Nt_2	7 Nt_∞	10 Nt_2t_4
4 Nt_2	5 Nt_3	6 Nt_4	11 Nt_2t_0
5 Nt_3	6 Nt_4	4 Nt_2	12 Nt_2t_1
6 Nt_4	2 Nt_0	5 Nt_3	9 Nt_2t_∞
7 Nt_∞	7 Nt_∞	2 Nt_0	8 Nt_2t_3
8 Nt_2t_3	11 Nt_3t_4	10 Nt_2t_4	7 Nt_∞
9 Nt_2t_∞	10 Nt_3t_∞	9 Nt_4t_∞	6 Nt_4
10 Nt_2t_4	12 Nt_3t_0	11 Nt_2t_0	3 Nt_1
11 Nt_2t_0	9 Nt_3t_1	8 Nt_2t_3	4 Nt_2
12 Nt_2t_1	8 Nt_2t_3	12 Nt_2t_1	5 Nt_3

The action of N and the action of $t's$ is as follows: G×X⟶X defined by (g,x)⟶gx, and $f \longrightarrow S_x$, where f is homomorphism. In this problem; we have, $f : 2^{*6} : L_2(5) \longrightarrow S_x$, where $X = \{N, Nt_0, ..., Nt_2t_1\}$.
Now we find,

$f(x) = (3,4,5,6,2)(8,11,9,10,12)$.

$f(y) = (2,3,7)(4,6,5)(8,10,11)$.

$f(t_0) = (1,2)(3,10)(4,11)(5,12)(6,9)(7,8)$.

$f(t_1) = f(t_0{}^x) = (f(t_0))^{f(x)} = (1,3)(4,12)(5,9)(6,8)(2,10)(7,11)$.

$f(t_2) = f(t_1{}^x) = (f(t_1))^{f(x)} = (1,4)(5,8)(6,10)(2,11)(3,12)(7,9)$.

$f(t_3) = f(t_2{}^x) = (f(t_2))^{f(x)} = (1,5)(6,11)(2,12)(3,9)(4,8)(7,10)$.

$f(t_4) = f(t_3{}^x) = (f(t_3))^{f(x)} = (1,6)(2,9)(3,8)(4,10)(5,11)(7,12)$.

$f(t_\infty) = f(t_1{}^y) = (f(t_1))^{f(y)} = (1,7)(6,12)(4,9)(5,10)(3,11)(2,8)$.

The Homomorphism Image Of G

$\bar{G} = f(2^{*6} : L_2(5)) = \langle f(x), f(y), f(t_0), f(t_1), f(t_2), f(t_3), f(t_4), f(t_\infty)\rangle$.

However;

$\{f(t_1), f(t_2), f(t_3), f(t_4), f(t_\infty)\} \subseteq \langle f(x), f(y), f(t_0))\rangle$.

Then $\bar{G} = f(2^{*6} : L_2(5)) = \langle f(x), f(y), f(t_0))\rangle$.

If the additional relation hold in $\langle f(x), f(y), f(t_0), f(t_\infty)\rangle$ then:

$$f\left(\frac{2^{*6} : L_2(5)}{[(t_2t_3t_0t_\infty = (1,\infty,3)(2,4,0)]}\right) = \langle f(x), f(y), f(t_0)\rangle.$$

The addition relation, namely $t_2t_3t_0t_\infty = (1,\infty,3)(2,4,0)$ holds in $\langle f(x), f(y), f(t_0), f(t_\infty)\rangle$ if $f(t_2)f(t_3)f(t_0)f(t_\infty)$ acts as $f(t_1) \to f(t_\infty) \to f(t_3)$ and $f(t_2) \to f(t_4) \to f(t_0)$.

Let $\mu = f(t_2)f(t_3)f(t_0)f(t_\infty) = (2,4,6)(3,7,5)(9,11,10)$

$(17,19)(23,24)(26,27)(29,30)(31,33)$.

Then $(f(t_\infty))^\mu = f(t_3)$, $(f(t_3))^\mu = f(t_1)$ and $(f(t_1))^\mu = f(t_\infty)$.

$(f(t_2))^\mu = f(t_4)$, $(f(t_4))^\mu = f(t_0)$ and $(f(t_0))^\mu = f(t_2)$.

Thus; the relation holds in $\bar{G}$.

$\Longrightarrow \bar{G}$ is a homomorphism image of G.

Now, by the First Isomorphic Theorem $|G/Kerf| \cong \bar{G}$.

$\Rightarrow |G| \geq |\bar{G}|$.

It is easily verified that $|\bar{G}| = |\langle f(x), f(y), f(t_0))\rangle| = 720$.

$\Longrightarrow |G| \geq 720$.

But we have seen above that $|G| \leq 720$.

Hence; $|G| = 720$.

Now, we need to find the order of $PGL_2(9)$.

$GL_2(9)$ is the group of 2x2 matrices with nonzero determinant and whose entries are symmetric generators of a field of order 9, F_9, and $PGL_2(9)$ is the projective linear group that is obtained from the general linear group $GL_2(9)$ by factoring its center.

$SL_2(9)$ is a normal subgroup of $GL_2(9)$ with determinant equal to 1. $L_2(9)$ [Artin] or $PSL_2(9)$ [Dickson] is the projective special linear group that is obtained from the special linear group $SL_2(9)$ by factoring its center.

Construction of of $PSL_2(9)$. Dickson gives the following definition of $PSL_2(9)$: $PSL_2(9)=\{\frac{ax+b}{cx+d}|$ ad-bc = 1 or a square, a,b,c,d $\in F_9\}$

We first construct a field of order 9, we know that $Z_3[x]/\langle x^2+1\rangle$ is a field of order 9 since x^2+1 is an irreducible polynomial over Z_3, then
$F_9 = Z_3[x]/\langle x^2+1\rangle = \{0,1,2,x,x+1,x+2,2x,2x+1,2x+2\}$.
Now,

$$\begin{aligned} x^2+1+\langle x^2+1\rangle &= \langle x^2+1\rangle \\ \Rightarrow x^2+\langle x^2+1\rangle &= -1+\langle x^2+1\rangle \\ \Rightarrow x^2 &= -1 \\ \Rightarrow x &= \pm i \end{aligned}$$

Since $x=\pm i$ and the leading coefficients of the polynomial in $Z_3=\{0,1,2=-1$ mod $(3)\}$ then

$$\begin{aligned} 2x+2+\langle x^2+1\rangle &= \text{2i+2+ } \langle x^2+1\rangle \\ &= -i-1+\langle x^2+1\rangle, \\ 2x+1+\langle x^2+1\rangle &= \text{2i+1 } +\langle x^2+1\rangle \\ &= -i+1+\langle x^2+1\rangle, \\ x+2+\langle x^2+1\rangle &= \text{i+2+ } \langle x^2+1\rangle \\ &= i-1+\langle x^2+1\rangle, \\ x+1+\langle x^2+1\rangle &= \text{i+1+ } \langle x^2+1\rangle \\ &= i+1+\langle x^2+1\rangle. \end{aligned}$$

Now, we know that $PSL_2(9)$ has action on 9+1 letters. We label the symmetric generators of F_9 as follows:

∞,	0,	1,	-1,	i,	-i,	1+i,	1-i,	-1+i,	-1-i
10,	9,	1,	2,	3,	4,	5,	6,	7	8

It has been shown by Conway that $L_2(9)=\langle \alpha : x\mapsto x+1, \beta : x\mapsto kx, \gamma : x\mapsto \frac{-1}{x}\rangle$, where k is a square whose powers give all nonzero squares. In this problem, the set squares in F_9 is $\{1,-1,i,-i\}$, powers of i or -i produce the set.

We compute α, β, γ;

$\alpha : x\mapsto x+1 : (\infty)(0,1,2)(i,i+1,i-1)(-i,1-i,-1-i) = (10)(0,1,2)(3,5,7)(4,6,8)$.

$\beta : x \mapsto kx = ix : (\infty)(0)(1, i, -1, -i)(i{+}1, i{-}1, -i{-}1, -i{+}1) = (1,3,2,4)(5,7,8,6)(9)(10)$.
$\gamma : x \mapsto \frac{-1}{x} : (0,\infty)(1,-1)(i)(-i)(i+1,-i+1)(i-1,-i-1) = (10,9)(3)(4)(5,6)(7,8)$.
Thus; $PSL_2(9) = \langle \alpha, \beta, \gamma \rangle \Longrightarrow |PSL_2(9)| = 360$.

Second, we know that $PGL_2(9) \cong PSL_2(9) : 2$. In order to find $PGL_2(9) \cong PSL_2(9) : 2$. We define δ an automorphism of $PSL_2(9)$ such that $\{\delta : x \mapsto \frac{ax+b}{cx+d} |$ ad-bc $\neq 1$, ad-bc $\neq 0$, a,b,c,d $\in F_9\}$ and order of $(\delta) = 2$. We have $\delta : x \mapsto \frac{x+i}{x-1}$: $(1,\infty)(-1,i-1)(i,-i-1)(0,-i)(i+1,-i-1) = (1,10)(2,7)(3,6)(4,9)(5,8)$.
Therefore; $PGL_2(9) = \langle PSL_2(9), \delta \rangle = \langle \alpha, \beta, \gamma, \delta \rangle \Longrightarrow |PGL_2(9)| = 720$.

Next, we need to show that $G = \frac{2^{*6}:L_2(5)}{[t_2t_3t_0t_\infty=(1,\infty,3)(2,4,0)]}$ is isomorphic to $PGL_2(9)$.

$$G = \frac{2^{*6}:L_2(5)}{[t_2t_3t_0t_\infty=(1,\infty,3)(2,4,0)]} \cong PGL_2(9)$$

$L_2(5) = \langle x, y \rangle$, where $x \sim (0,1,2,3,4)$, $y \sim (\infty,0,1)(2,4,3)$. We label the symmetric generators of F_9 as follows;

∞,	0,	1,	-1,	i,	-i,	1+i,	1-i,	-1+i,	-1-i
10,	9,	1,	2,	3,	4,	5,	6,	7	8

We construct a homomorphism ϕ from the progenitor $2^{*6} : L_2(5)$ to $PGL_2(9)$ be defining
$\phi(x) = (\frac{(1+i)\eta+i}{\eta+1}) = (1,-1{+}i,1{-}i,-i,-1{-}i)(-1,\infty,1{+}i,0,i) = (1,7,6,4,8)(2,10,5,9,3)$
$\phi(y) = (\frac{(i-1)\eta-1-i}{(i+1)\eta+1}) = (1,i{+}1,-1{+}i)(i,-i{+}1,\infty)(-i,0,-1{-}i) = (1,5,7)(3,6,10)(4,9,8)$
Since the order of $\phi(x), \phi(y)$, and $\phi(x)\phi(y)$ are 5, 3 and 2, respectively,
$N = \langle \phi(x), \phi(x) \rangle \cong L_2(5)$. We now let
$\phi(t_\infty) = (\frac{\eta-1}{i\eta-1}) = (1,0)(-i,\infty)(-1{+}i,i)(1{+}i,-1{-}i)(-1,1{-}i) = (1,9)(2,6)(3,7)(4,10)(5,8)$
It is readily verified that $PGL_2(9) \cong \langle \phi(x), \phi(y), \phi(t_\infty) \rangle$
We now show that ϕ preserves the operation of $2^{*6} : L_2(5)$.
We find that $|\phi(t_\infty)^N| = 6$ and
$\phi(t) = \phi(t_\infty) = (1,9)(2,6)(3,7)(4,10)(5,8)$
$\phi(t_0) = \phi(t_\infty^y) = (5,8)(2,10)(6,1)(3,9)(7,4)$
$\phi(t_1) = \phi(t_\infty^y) = (1,9)(5,10)(4,7)(2,3)(6,8)$
$\phi(t_2) = \phi(t_1^x) = (3,7)(5,9)(8,6)(2,10)(1,4)$
$\phi(t_3) = \phi(t_2^x) = (2,6)(3,9)(1,4)(5,10)(7,8)$
$\phi(t_4) = \phi(t_3^x) = (4,10)(2,3)(7,8)(5,9)(1,6)$
and the N permutes the six images of t_∞, by conjugation, as the group $L_2(5)$ given by

$\phi(x) : (\phi(t_0), \phi(t_1), \phi(t_2), \phi(t_3), \phi(t_4), \phi(t_\infty)$ and
$\phi(y) : (\phi(t_\infty), \phi(t_0), \phi(t_1), \phi(t_2), \phi(t_4), \phi(t_3))$. Thus $\phi(2^{*6} : L_2(5)) = PGL_2(9)$.

Now the addition relation given by $(0,4,2)(3,\infty,1)t_2t_3t_0t_\infty = 1 \Leftrightarrow t_2t_3t_0t_\infty = (1,\infty,3)(2,4,0)$ is satisfied in $PGL_2(9)$, because $\phi(t_2)\phi(t_3)\phi(t_0)\phi(t_\infty) = (1,5,2)(3,4,8)(6,9,10)$ acts as $(\phi(t_1), \phi(t_\infty), \phi(t_3), \phi(t_2), \phi(t_0), \phi(t_4))$, by conjugation, on the images of the six symmetric generators. This shows that $PGL_2(9)$ is an image of G. Thus $|G| \geq |PGL_2(9)|$; but $|G| \leq 720 = PGL_2(9)$; and so the equality holds and $G \cong PGL_2(9)$.

Chapter 7

A Homomorphic Image of $3^{*3} : S_3$

We consider the progenitor:

$$3^{*3} : S_3 \cong \langle\, x, y, t | x^3 = y^2 = (yx)^2 = t^3 = [t, y] \rangle$$

where $x \sim (0, 1, 2)$, $y \sim (1, 2)$, $t \sim t_0$. The following relation may used for the purpose of manual double coset enumeration:

1. $[(0, 1, 2)t_0]^9 = 1$.
2. $[(0, 1)t_0]^6 = 1$.
3. $[t_0 t_1^{-1}]^3 = 1$.
4. $[(0, 1, 2)t_0 t_1^{-1}]^3 = 1$.
5. $[(0, 1)t_0 t_2]^6 = 1$.
6. $[(0, 1)t_0 t_2^{-1}]^2 = 1$.

We now perform the manual double coset enumeration of $G \cong \frac{3^{*3}:S_3}{[(t_0 t_2^{-1})]^2}$.
We use relation 2 and relation 6 to find the manual double coset enumeration.
Expanding relation 2: $[(0, 1)t_0]^6 = 1$.

$$\begin{aligned} [t_1 t_0 t_1 t_0 t_1 t_0] &= 1 \\ t_0 t_1 t_0 &= t_1^{-1} t_0^{-1} t_1^{-1} \\ N t_0 t_1 t_0 &= N t_1^{-1} t_0^{-1} t_1^{-1} \end{aligned}$$

Expanding relation 6: $[(0,1)t_0t_2^{-1}]^2 = 1$.

$$\begin{aligned}
[(0,1)t_0t_2^{-1}]^2 &= 1\\
(0,1)t_0t_2^{-1}(0,1)t_0t_2^{-1} &= 1\\
(t_0t_2^{-1})^{(0,1)}t_0t_2^{-1} &= 1\\
t_1t_2^{-1}t_0t_2^{-1} &= 1\\
t_1t_2^{-1} &= t_2t_0^{-1}\\
Nt_1t_2^{-1} &= Nt_2t_0^{-1}\\
Nt_2^{-1}t_0 &= Nt_0^{-1}t_2
\end{aligned}$$

We start with the double coset representative word of length zero, $NeN = N$; denoted [*]. Next, we consider the double cosets with word length one. N is transitive on $T = \{t_0, t_1, t_2\} = \{0, 1, 3\}$, and their inverses $\overline{T} = \{t_0^{-1}, t_2^{-1}, t_2^{-1}\} = \{\overline{0}, \overline{1}, \overline{2}\}$. Thus; $Nt_0N = \{Nt_0^n | n \in N\} = \{Nt_0, Nt_1, Nt_2\} = \{N0, N1, N2\}$ denoted [0], and its inverse, $Nt_0^{-1}N = \{N(t_0^{-1})^n | n \in N\} = \{Nt_0^{-1}, Nt_1^{-1}, Nt_2^{-1}\} = \{N\overline{0}, N\overline{1}, N\overline{2}\}$ denoted $[\overline{0}]$. Now, we determine for [0] and $[\overline{0}]$ to which the double coset Nt_0t_i and $Nt_0^{-1}t_i$ belong, for one t_i from each orbit of $N^0 = N^{\overline{0}}$ on T and $\overline{T}$. Since; $N^0 = \{n \in N | t_0^n = t_0\} = \langle e, (1,2)\rangle$, thus; $N^0 = N^{\overline{0}}$ has orbits $\{0\}$, $\{1, 2\}$ on T, and $\{\overline{0}\}$, $\{\overline{1}, \overline{2}\}$ on $\overline{T}$.

Next, we need to consider the double cosets [00], [01], $[0\overline{0}]$, $[0\overline{1}]$, $[\overline{0}\overline{0}]$, $[\overline{0}\overline{1}]$, $[\overline{0}0]$, $[\overline{0}1]$. $00 = \overline{0} \Longrightarrow [00] = [\overline{0}]$, so the generator t_0 takes [0] go back to single coset in $[\overline{0}]$.
$\overline{0}\overline{0} = 0 \Longrightarrow [\overline{0}\overline{0}] = [0]$, so the generator t_0^{-1} takes $[\overline{0}]$ go back to single coset in [0].
$0\overline{0} = e = \overline{0}0 \Longrightarrow [0\overline{0}] = [*] = [\overline{0}0]$, so the generators t_0^{-1} and t_0 take [0] and $[\overline{0}]$ to a single coset in [*].

Recall relation 6:
$Nt_1t_2^{-1} = Nt_2t_0^{-1}$ $(1\overline{2} \sim 2\overline{0})$.
$Nt_1t_2^{-1\,(0,1,2)} = Nt_2t_0^{-1\,(0,1,2)}$ $(2\overline{0} \sim 0\overline{1})$.
$Nt_1t_2^{-1\,(0,2,1)} = Nt_2t_0^{-1\,(0,2,1)}$ $(1\overline{2} \sim 2\overline{0})$.
Thus, $Nt_1t_2^{-1} = Nt_2t_0^{-1} = Nt_0t_1^{-1}$ $(1\overline{2} \sim 2\overline{0} \sim 0\overline{1})$.
Then, $\overline{Nt_1t_2^{-1}} = \overline{Nt_2t_0^{-1}} = \overline{Nt_0t_1^{-1}}$ $(\overline{1\overline{2}} \sim \overline{2\overline{0}} \sim \overline{0\overline{1}})$ will be:
$Nt_2t_1^{-1} = Nt_0t_2^{-1} = Nt_1t_0^{-1}$ $(\overline{1}0 \sim \overline{0}2 \sim \overline{2}1)$.

$[01] = Nt_0t_1N = \{N01, N02, N10, N12, N20, N21\}$, and the coset stabilizer of N01 is given by $N^{(01)} = N^{01} = \{e\}$. Then, $N^{(01)}$ has orbits $\{0\}$, $\{1\}$, $\{2\}$, $\{\overline{0}\}$, $\{\overline{1}\}, \{\overline{2}\}$. Hence; the number of single coset in the double coset [01] is at most 6. Taking a representative from each orbit, we note,

$[010]$: *new*

$[011] = Nt_0t_{\bar{1}}^{-1} \in [0\bar{1}]$

$[012]$: *new*

$[01\bar{0}] = Nt_0^{-1}t_2^{-1} \in [\overline{01}]$

$[01\bar{1}] = Nt_0 \in [0]$

$[01\bar{2}] = Nt_{\bar{1}}^{-1}t_2^{-1} \in [\overline{01}]$

Therefore; t_1 takes [01] go back to $[0\bar{1}]$, $t_{\bar{1}}^{-1}$ takes [01] go back to [0], t_0^{-1}, t_2^{-1} take [01] go back to $[\overline{01}]$ and [010], [012] are two new double cosets.

$[\overline{01}] = Nt_0^{-1}t_1^{-1}N = \{N\overline{01}, N\overline{02}, N\overline{10}, N\overline{12}, N\overline{20}, N\overline{21}\}$, and the cosets stabilizer of $N\overline{01}$ given by $N^{(\overline{01})} = N^{\overline{01}} = \{e\}$, thus; $N^{(\overline{01})}$ has orbits $\{0\}$, $\{1\}$, $\{2\}$, $\{\bar{0}\}$, $\{\bar{1}\}, \{\bar{2}\}$. Hence; the number of single coset in the double coset $[\overline{01}]$ is at most 6. Taking a representative from each orbit, we note,

$[\overline{01}0] = Nt_0t_2 \in [01]$

$[\overline{01}1] = Nt_0^{-1} \in [\bar{0}]$

$[\overline{01}2] = Nt_0t_1 \in [01]$

$[\overline{010}]$: *new*

$[\overline{011}] = Nt_0^{-1}t_1 \in [\bar{0}1]$

$[\overline{012}]$: *new*

Therefore, t_0, t_2 take $[\overline{01}]$ go back to [01], $t_{\bar{1}}^{-1}$ takes $[\overline{01}]$ go back to $[\bar{0}1]$, t_1 takes $[\overline{01}]$ go back to $[\bar{0}]$ and $[\overline{010}]$, $[\overline{012}]$ are two new double cosets.

$[0\bar{1}] = N0\bar{1}N = N1\bar{2}N = N2\bar{0}N.$

$N^{(0\bar{1})} \geq N^{0\bar{1}} = \{n \in N|(t_0t_{\bar{1}}^{-1})^n = t_0t_{\bar{1}}^{-1}\} = \{e\}$; thus $N^{(\overline{01})}$ has orbits $\{0\}$, $\{1\}$, $\{2\}$, $\{\bar{0}\}$, $\{\bar{1}\}$, $\{\bar{2}\}$, by the relation 6: $\{(0,1,2);(0,2,1)\} \in N0\bar{1}N$. Hence, the number of the single cosets in the double coset $[0\bar{1}]$ is at most 2 with 3 different names for each. In order to find another distinct cosets in $[0\bar{1}] = N0\bar{1}N = N1\bar{2}N = N2\bar{0}N$, we conjugate the coset with three different names that is already know to us by (0, 1) then we get $[1\bar{0}] = N1\bar{0}N = N2\bar{1}N = N0\bar{2}N$. Taking a representative from each orbit, we note,

$[0\bar{1}0] = Nt_2 \in [0]$

$[0\bar{1}1] = Nt_0 \in [0]$

$[0\bar{1}2] = Nt_1 \in [0]$

$[0\overline{10}] = Nt_2t_1 \in [01]$

$[0\overline{1}\overline{1}] = Nt_0t_1 \in [01]$

$[0\overline{1}\overline{2}] = Nt_1t_2 \in [01]$

Therefore, t_0, t_1, t_2, take $[0\overline{1}]$ go back to $[0]$ and t_0^{-1}, t_1^{-1}, t_2^{-1} take $[0\overline{1}]$ go back to $[01]$.

$[\overline{0}1] = N\overline{2}0N = N\overline{1}2N = N\overline{0}1N.$

$N^{(\overline{0}1)} \geq N^{\overline{0}1} = n \in N|(t_0^{-1}t_1)^n = t_0^{-1}t_1 = \{e\}$; thus $N^{\overline{(01)}}$ has orbits $\{0\}$, $\{1\}$, $\{2\}$, $\{\overline{0}\}$, $\{\overline{1}\}$, $\{\overline{2}\}$, by the relation 6: $\{(0,1,2);(0,2,1)\} \in N\overline{0}1N$. Hence, the numbers of single coset in double coset $[\overline{0}1]$ is at most 2 with 3 different names each. In order to find another distinct cosets in $[\overline{0}1] = N\overline{2}0N = N\overline{1}2N = N\overline{0}1N$, we conjugate the coset with three different names that is already know to us by (0,1) then we get $[\overline{1}0] = N\overline{0}2N = N\overline{2}1N = N\overline{1}0N$. Taking a representative from each orbit, we note

$[\overline{0}10] = Nt_2^{-1}t_0^{-1} \in [\overline{01}]$

$[\overline{0}11] = Nt_0^{-1}t_1^{-1} \in [\overline{01}]$

$[\overline{0}12] = Nt_1^{-1}t_2^{-1} \in [\overline{01}]$

$[\overline{0}1\overline{0}] = Nt_2^{-1} \in [\overline{0}]$

$[\overline{0}1\overline{1}] = Nt_0^{-1} \in [\overline{0}]$

$[\overline{0}1\overline{2}] = Nt_1^{-1} \in [\overline{0}]$

Therefore, t_0, t_1, t_2, take $[0\overline{1}]$ go back to t_0^{-1}, t_1^{-1}, t_2^{-1} take $[0\overline{1}]$ go back to $[\overline{01}]$.

Now, we consider the double cosets $[010]$; $[012]$; $[\overline{010}]$; and $[\overline{012}]$.

Consider $[010]$, the coset stabilizer of $[010]$ is $\{e\}$, apply relation 6;

$[010] = [0\overline{11}0] = [2\overline{00}2] = [202]$

$[010] = [0\overline{11}0] = [1\overline{22}1] = [121]$

Hence, $[010] = [202] = [121]$ which implied $\{(0,1,2);(0,2,1)\} \in N010N$.

Recall relation 2: $Nt_0t_1t_0 = Nt_1^{-1}t_0^{-1}t_1^{-1} = [010] = [\overline{101}]$

Conjugating $[010] = [\overline{101}]$ by $\{e;(0,1,2);(0,2,1)\}$

$\Rightarrow [010] = [202] = [121] = [\overline{101}] = [\overline{212}] = [\overline{020}]$.

Hence, the numbers of single coset in double coset $[010]$ is at most 2 with 6 different names each. In order to find another distinct cosets in $[010] = [202] = [121] = [\overline{101}] = [\overline{212}] = [\overline{020}]$, conjugating the coset with six different names that is already know to us by (0,1) then that implied $[101] = [212] = [020] = [\overline{010}] = [\overline{202}] = [\overline{121}]$. Thus; $[\overline{010}] \subseteq [010]$.

The orbits of $[010]$ are $\{0\}$, $\{1\}$, $\{2\}$, $\{\overline{0}\}$, $\{\overline{1}\}, \{\overline{2}\}$. Taking a representative from each orbit, we note,

$[0100] = Nt_0^{-1}t_2^{-1} \in [\overline{01}]$
$[0101] = Nt_1^{-1}t_0^{-1} \in [\overline{01}]$
$[0102] = Nt_2^{-1}t_1^{-1} \in [\overline{01}]$
$[010\bar{0}] = Nt_0t_1 \in [01]$
$[010\bar{1}] = Nt_1t_2 \in [01]$
$[010\bar{2}] = Nt_2t_0 \in [01]$
Therefore, t_0, t_1, t_2 take [010] go back to $[\overline{01}]$ and t_0^{-1}, t_1^{-1}, t_2^{-1}take [010] go back to [01].

Consider [012], the coset stabilizer of [012] is {e}, apply relation 6;
$[012] = [0\bar{1}\bar{1}2] = [1\bar{2}\bar{2}0] = [120]$
$[012] = [0\bar{1}\bar{1}2] = [2\bar{0}\bar{0}1] = [201]$
Hence, $[012] = [120] = [201]$ which implied $\{(0,1,2);(0,2,1)\} \in N012N$.
By relation 6: $Nt_0t_1t_2 = Nt_0^{-1}t_1^{-1}t_2^{-1} = [012] = [\overline{012}]$
Conjugating $[012] = [\overline{012}]$ by $\{e;(0,1,2);(0,2,1)\}$
$\Rightarrow [012] = [120] = [201] = [\overline{012}] = [\overline{120}] = [\overline{201}]$.

Hence, the numbers of single coset in double coset [012] is at most 2 with 6 different names each. In order to find another distinct cosets in $[012] = [120] = [201] = [\overline{012}] = [\overline{120}] = [\overline{201}]$, conjugating the coset with six different names that is already know to us by (0,1) then that implied $[102] = [021] = [210] = [\overline{102}] = [\overline{021}] = [\overline{210}]$.
Thus; $[\overline{012}] \subseteq [012]$.
The orbits of [010] are $\{0\}$, $\{1\}$, $\{2\}$, $\{\bar{0}\}$, $\{\bar{1}\}$, $\{\bar{2}\}$
Taking representative from each orbit:
$[0120] = Nt_2^{-1}t_1^{-1} \in [\overline{01}]$
$[0121] = Nt_0^{-1}t_2^{-1} \in [\overline{01}]$
$[0122] = Nt_1^{-1}t_0^{-1} \in [\overline{01}]$
$[012\bar{0}] = Nt_1t_2 \in [01]$
$[012\bar{1}] = Nt_2t_0 \in [01]$
$[012\bar{2}] = Nt_0t_1 \in [01]$
Therefore, t_0, t_1, t_2 take [010] go back to $[\overline{01}]$ and t_0^{-1}, t_1^{-1}, t_2^{-1} take [010] go back to [01].

Our double coset enumeration must be complete since the set of right coset is close under right multiplication by the symmetric generators. Double coset enumeration shows that the index of $N \cong S_3$ in G is at most: $\frac{|N|}{|N|}+\frac{|N|}{|N^{(0)}|}+\frac{|N|}{|N^{(\bar{0})}|}+\frac{|N|}{|N^{(01)}|}+\frac{|N|}{|N^{(\overline{01})}|}+\frac{|N|}{|N^{(0\bar{1})}|}+\frac{|N|}{|N^{(\bar{0}1)}|}+\frac{|N|}{|N^{(010)}|}+\frac{|N|}{|N^{(012)}|} = 1+3+3+6+6+2+2+2+2$.

$\Longrightarrow |G| \leq 27|N| = 27 \times 6 = 162.$

We now want to prove $|G|$=162. Determine the action of the symmetric generator x, y and t on 27 distinct cosets of N in G that we have found. Recall $x \sim (0,1,2)$, $y \sim (1,2)$ and$t \sim t_0$.

Permutation Representation

Coset	$x \sim (0,1,2)$	$y \sim (1,2)$	t_0
1 N	1 N	1 N	2 Nt_0
2 Nt_0	3 Nt_1	2 Nt_0	5 Nt_0^{-1}
3 Nt_1	4 Nt_2	4 Nt_2	10 Nt_1t_0
4 Nt_2	2 Nt_0	3 Nt_1	12 Nt_2t_0
5 Nt_0^{-1}	6 Nt_1^{-1}	5 Nt_0^{-1}	1 N
6 Nt_1^{-1}	7 Nt_2^{-1}	7 Nt_2^{-1}	23 $Nt_0^{-1}t_2$
7 Nt_2^{-1}	5 Nt_0^{-1}	6 Nt_1^{-1}	22 $Nt_0^{-1}t_1$
8 Nt_0t_1	11 Nt_1t_2	9 Nt_0t_2	24 $Nt_0t_1t_0$
9 Nt_0t_2	10 Nt_1t_0	8 Nt_0t_1	25 $Nt_1t_0t_1$
10 Nt_1t_0	13 Nt_2t_1	12 Nt_2t_0	21 $Nt_0t_2^{-1}$
11 Nt_1t_2	12 Nt_2t_0	13 Nt_2t_1	26 $Nt_0t_1t_2$
12 Nt_2t_0	8 Nt_0t_1	10 Nt_1t_0	20 $Nt_0t_1^{-1}$
13 Nt_2t_1	9 Nt_0t_2	11 Nt_1t_2	27 $Nt_0t_2t_1$
14 $Nt_0^{-1}t_1^{-1}$	17 $Nt_1^{-1}t_2^{-1}$	15 $Nt_0^{-1}t_2^{-1}$	9 Nt_0t_2
15 $Nt_0^{-1}t_2^{-1}$	16 $Nt_1^{-1}t_0^{-1}$	14 $Nt_0^{-1}t_1^{-1}$	8 Nt_0t_1
16 $Nt_1^{-1}t_0^{-1}$	19 $Nt_2^{-1}t_1^{-1}$	18 $Nt_2^{-1}t_0^{-1}$	6 Nt_1^{-1}
17 $Nt_1^{-1}t_2^{-1}$	18 $Nt_2^{-1}t_0^{-1}$	19 $Nt_2^{-1}t_1^{-1}$	11 Nt_1t_2
18 $Nt_2^{-1}t_0^{-1}$	14 $Nt_0^{-1}t_1^{-1}$	16 $Nt_1^{-1}t_0^{-1}$	7 Nt_2^{-1}
19 $Nt_2^{-1}t_1^{-1}$	15 $Nt_0^{-1}t_2^{-1}$	17 $Nt_1^{-1}t_2^{-1}$	13 Nt_2t_1
20 $Nt_0t_1^{-1}$	20 $Nt_0t_1^{-1}$	21 $Nt_0t_2^{-1}$	4 Nt_2
21 $Nt_0t_2^{-1}$	21 $Nt_0t_2^{-1}$	20 $Nt_0t_1^{-1}$	3 Nt_1
22 $Nt_0^{-1}t_1$	22 $Nt_0^{-1}t_1$	23 $Nt_0^{-1}t_2$	18 $Nt_2^{-1}t_0^{-1}$
23 $Nt_0^{-1}t_2$	23 $Nt_0^{-1}t_2$	22 $Nt_0^{-1}t_1$	16 $Nt_1^{-1}t_0^{-1}$
24 $Nt_0t_1t_0$	24 $Nt_0t_1t_0$	25 $Nt_1t_0t_1$	15 $Nt_0^{-1}t_2^{-1}$
25 $Nt_1t_0t_1$	25 $Nt_1t_0t_1$	24 $Nt_0t_1t_0$	14 $Nt_0^{-1}t_1^{-1}$
26 $Nt_0t_1t_2$	26 $Nt_0t_1t_2$	27 $Nt_0t_2t_1$	17 $Nt_1^{-1}t_2^{-1}$
27 $Nt_0t_2t_1$	27 $Nt_0t_2t_1$	26 $Nt_0t_1t_2$	19 $Nt_2^{-1}t_1^{-1}$

The action of N and the action of $t's$ is as follows: G×X⟶X defined by (g,x)⟶gx, and $f \longrightarrow S_x$, where f is homomorphism. In this problem; we have,

$f : 3^{*3} : S_3 \longrightarrow S_x$, where $X = \{N, Nt_0, ..., Nt_0t_2t_1\}$.

$f(x) = (2,3,4)(5,6,7)(8,11,12)(9,10,13)(14,17,18)(15,16,19)$.

$f(y) = (3,4)(6,7)(8,9)(10,12)(11,13)(14,15)(16,18)(17,19)(20,21)(22,23)(24,25)(26,27)$.

$f(t_0) = (1,2,5)(3,10,21)(4,12,20)(6,23,16)(7,22,18)(9,25,14)(11,26,17)(8,24,15)(13,27,19)$.

$f(t_1) = f(t_0{}^x) = (f(t_0)^{(}f(x)) = (1,3,6)(2,8,20)(4,13,21)(5,22,14)(7,23,19)(9,27,15)$
$(10,25,17)(11,24,16)(12,26,18)$.
$f(t_2) = f(t_1{}^x) = (f(t_1)^{(}f(x)) = (1,4,7)(2,9,21)(3,11,20)(5,23,15)(6,22,17)(8,26,14)$
$(10,27,16)(12,24,19)(13,25,18)$.
Verify relation 1: $t_2t_1t_0t_2t_1t_0t_2t_1t_0 = 1$
Verify relation 2: $t_1t_0t_1t_0t_1t_0 = 1$
Verify relation 3: $t_0t_1^{-1}t_0t_1^{-1}t_0t_1^{-1} = 1$
Verify relation 4: $t_0t_1^{-1}t_0t_1^{-1}t_0t_1^{-1} = 1$
Verify relation 5: $t_1t_2t_0t_2t_1t_2t_0t_2t_1t_2t_0t_2 = 1$
Verify relation 6: $t_0t_2^{-1}t_0t_2^{-1} = 1$

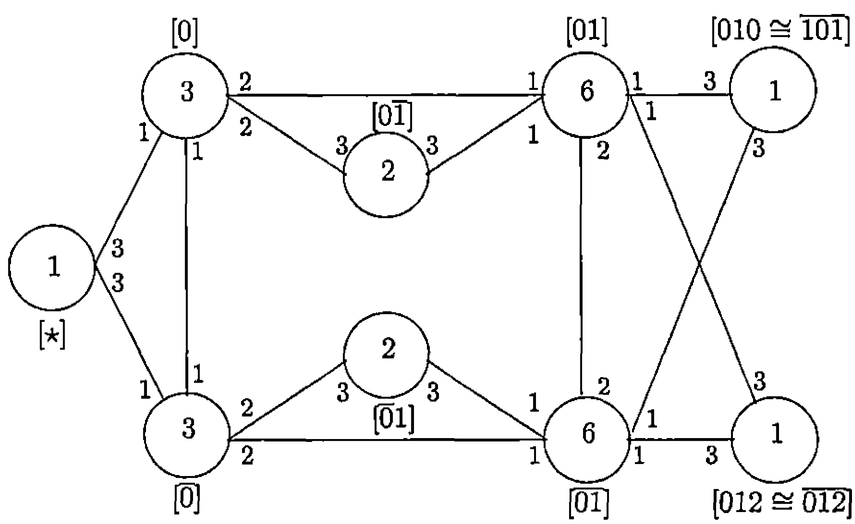

Figure 7.1: The Cayley Diagram of A homomorphic image of $3^{*3} : S_3$

Now, we want to show that $| G |= 162$.

The Homomorphism Image Of G

$\bar{G} = f(3^{*3} : S_3) = \langle f(x), f(y), f(t_0), f(t_1), f(t_2)\rangle$.
However;
$\{f(t_1), f(t_2)\} \in \{f(x), f(y), f(t_0)\}$.
$\Longrightarrow \bar{G} = f(3^{*3} : S_3) = \langle f(x), f(y), f(t_0)\rangle$.

If the addition relation hold in $\langle f(x), f(y), f(t_0)\rangle$ then:

$$f\left(\frac{3^{*3} : S_3}{[(t_0 t_2^{-1})]^2 = 1}\right) = \langle f(x), f(y), f(t_0)\rangle.$$

The addition relation, namely $f(t_1)f(t_2^{-1})f(t_0)f(t_2^{-1})$ must equal to an identity

Let $\mu = f(t_1)f(t_2^{-1})f(t_0)f(t_2^{-1}) = 1$.

Thus the relation holds in $\bar{G}$.

$\Longrightarrow \bar{G}$ is a homomorphism image of G.

Now, by the First Isomorphic Theorem $|G/Kerf| \cong \bar{G}$.

$\Longrightarrow |G| \geq |\bar{G}|$.

It is easily verified that $|\bar{G}| = |\langle f(x), f(y), f(t_0)\rangle| = 162$.

$\Longrightarrow |G| \geq 162$.

But we have seen that $|G| \leq 162$.

Hence; $|G| = 162$.

Chapter 8

Construction of $L_2(11)$

We consider the progenitor:

$$3^{*3} : S_3 \cong \langle\, x, y, t | x^3 = y^2 = (yx)^2 = t^3 = [t, y] \rangle$$

where $x \sim (0, 1, 2)$, $y \sim (1, 2)$, $t \sim t_0$
The following relation may used for the purpose of manual double coset enumeration:

1. $[(0, 1, 2)t_0]^6 = 1$

2. $[(0, 1)t_0]^5 = 1$

3. $[t_0 t_1^{-1}]^5 = 1$

4. $[(0, 1, 2)t_0 t_1^{-1}]^5 = 1$

5. $[(0, 1)t_0 t_2]^5 = 1$

6. $[(0, 1)t_0 t_2^{-1}]^3 = 1$

We now perform the manual double coset enumeration of

$$G \cong \frac{3^{*3} : S_3}{[(0, 1)t_0]^5 = 1}.$$

We use the relation 1, 2, and 6 to find the manual double coset enumeration.
Expanding relation 1: $[(0,1,2)t_0]^6 = 1$

$$\begin{aligned} [(0,1,2)t_0]^6 &= 1 \\ t_2t_1t_0t_2t_1t_0 &= 1 \\ t_2t_1t_0 &= t_0^{-1}t_1^{-1}t_2^{-1} \\ Nt_2t_1t_0 &= Nt_0^{-1}t_1^{-1}t_2^{-1} \end{aligned}$$

Conjugating $Nt_2t_1t_0 = Nt_0^{-1}t_1^{-1}t_2^{-1}$ with all elements in S_3 we get:
$(t_2t_1t_0)^{(e)} = (t_0^{-1}t_1^{-1}t_2^{-1})^{(e)} \Longrightarrow t_2t_1t_0 = t_0^{-1}t_1^{-1}t_2^{-1} \Longrightarrow Nt_2t_1t_0 = Nt_0^{-1}t_1^{-1}t_2^{-1}$
$(t_2t_1t_0)^{(0,1)} = (t_0^{-1}t_1^{-1}t_2^{-1})^{(0,1)} \Longrightarrow t_2t_0t_1 = t_1^{-1}t_0^{-1}t_2^{-1} \Longrightarrow Nt_2t_0t_1 = Nt_1^{-1}t_0^{-1}t_2^{-1}$
$(t_2t_1t_0)^{(0,2)} = (t_0^{-1}t_1^{-1}t_2^{-1})^{(0,2)} \Longrightarrow t_0t_1t_2 = t_2^{-1}t_1^{-1}t_0^{-1} \Longrightarrow Nt_0t_1t_2 = Nt_2^{-1}t_1^{-1}t_0^{-1}$
$(t_2t_1t_0)^{(1,2)} = (t_0^{-1}t_1^{-1}t_2^{-1})^{(1,2)} \Longrightarrow t_1t_2t_0 = t_0^{-1}t_2^{-1}t_1^{-1} \Longrightarrow Nt_1t_2t_0 = Nt_0^{-1}t_2^{-1}t_1^{-1}$
$(t_2t_1t_0)^{(0,1,2)} = (t_0^{-1}t_1^{-1}t_2^{-1})^{(0,1,2)} \Longrightarrow t_0t_2t_1 = t_1^{-1}t_2^{-1}t_0^{-1} \Longrightarrow Nt_0t_2t_1 = Nt_1^{-1}t_2^{-1}t_0^{-1}$
$(t_2t_1t_0)^{(0,2,1)} = (t_0^{-1}t_1^{-1}t_2^{-1})^{(0,2,1)} \Longrightarrow t_1t_0t_2 = t_2^{-1}t_0^{-1}t_1^{-1} \Longrightarrow Nt_1t_0t_2 = Nt_2^{-1}t_0^{-1}t_1^{-1}$
Expanding relation 2: $[(0,1)t_0]^5 = 1$

$$\begin{aligned} [(0,1)t_0t_1t_0t_1t_0] &= 1 \\ (0,1)t_0t_1 &= t_0^{-1}t_1^{-1}t_0^{-1} \\ Nt_0t_1 &= Nt_0^{-1}t_1^{-1}t_0^{-1} \end{aligned}$$

Conjugating $Nt_0t_1 = Nt_0^{-1}t_1^{-1}t_0^{-1}$ with all elements in S_3 we get:
$((0,1)t_0t_1)^{(e)} = (t_0^{-1}t_1^{-1}t_0^{-1})^{(e)} \Longrightarrow (0,1)t_0t_1 = t_0^{-1}t_1^{-1}t_0^{-1} \Longrightarrow Nt_0t_1 = Nt_0^{-1}t_1^{-1}t_0^{-1}$
$((0,1)t_0t_1)^{(0,1)} = (t_0^{-1}t_1^{-1}t_0^{-1})^{(0,1)} \Longrightarrow (0,1)t_1t_0 = t_1^{-1}t_0^{-1}t_1^{-1} \Longrightarrow Nt_1t_0 = Nt_1^{-1}t_0^{-1}t_1^{-1}$
$((0,1)t_0t_1)^{(0,2)} = (t_0^{-1}t_1^{-1}t_0^{-1})^{(0,2)} \Longrightarrow (1,2)t_2t_1 = t_2^{-1}t_1^{-1}t_2^{-1} \Longrightarrow Nt_2t_1 = Nt_2^{-1}t_1^{-1}t_2^{-1}$
$((0,1)t_0t_1)^{(1,2)} = (t_0^{-1}t_1^{-1}t_0^{-1})^{(1,2)} \Longrightarrow (0,2)t_0t_2 = t_0^{-1}t_2^{-1}t_0^{-1} \Longrightarrow Nt_0t_2 = Nt_0^{-1}t_2^{-1}t_0^{-1}$
$((0,1)t_0t_1)^{(0,1,2)} = (t_0^{-1}t_1^{-1}t_0^{-1})^{(0,1,2)} \Longrightarrow (1,2)t_1t_2 = t_1^{-1}t_2^{-1}t_1^{-1} \Longrightarrow Nt_1t_2 = Nt_1^{-1}t_2^{-1}t_1^{-1}$
$((0,1)t_0t_1)^{(0,2,1)} = (t_0^{-1}t_1^{-1}t_0^{-1})^{(0,2,1)} \Longrightarrow (0,2)t_2t_0 = t_2^{-1}t_0^{-1}t_2^{-1} \Longrightarrow Nt_2t_0 = Nt_2^{-1}t_0^{-1}t_2^{-1}$
Now;
$(0,1)\overline{t_0t_1} = \overline{t_0^{-1}t_1^{-1}t_0^{-1}} \Longrightarrow Nt_0^{-1}t_1^{-1} = Nt_0t_1t_0$
$(0,1)\overline{t_1t_0} = \overline{t_1^{-1}t_0^{-1}t_1^{-1}} \Longrightarrow Nt_1^{-1}t_0^{-1} = Nt_1t_0t_1$
$(1,2)\overline{t_2t_1} = \overline{t_2^{-1}t_1^{-1}t_2^{-1}} \Longrightarrow Nt_2^{-1}t_1^{-1} = Nt_2t_1t_2$
$(0,2)\overline{t_0t_2} = \overline{t_0^{-1}t_2^{-1}t_0^{-1}} \Longrightarrow Nt_0^{-1}t_2^{-1} = Nt_0t_2t_0$
$(1,2)\overline{t_1t_2} = \overline{t_1^{-1}t_2^{-1}t_1^{-1}} \Longrightarrow Nt_1^{-1}t_2^{-1} = Nt_1t_2t_1$

$(0,2)\overline{t_2t_0} = \overline{t_2^{-1}t_0^{-1}t_2^{-1}} \Longrightarrow Nt_2^{-1}t_0^{-1} = Nt_2t_0t_2$

Expanding relation 6: $[(0,1)t_0t_2^{-1}]^3 = 1$

$$\begin{aligned} [(0,1)t_0t_2^{-1}t_0t_2^{-1}t_0t_2^{-1}] &= 1 \\ (0,1)t_0t_2^{-1}t_1 &= t_2t_0^{-1}t_2 \\ Nt_0t_2^{-1}t_1 &= Nt_2t_0^{-1}t_2 \end{aligned}$$

Conjugating $Nt_0t_2^{-1}t_1 = Nt_2t_0^{-1}t_2$ with all elements in S_3 we get:

$((0,1)t_0t_2^{-1}t_1)^{(e)} = (t_2t_0^{-1}t_2)^{(e)} \Longrightarrow (0,1)t_0t_2^{-1}t_1 = t_2t_0^{-1}t_2 \Longrightarrow Nt_0t_2^{-1}t_1 = Nt_2t_0^{-1}t_2$

$((0,1)t_0t_2^{-1}t_1)^{(0,1)} = (t_2t_0^{-1}t_2)^{(0,1)} \Longrightarrow (0,1)t_1t_2^{-1}t_1 = t_2t_1^{-1}t_2 \Longrightarrow Nt_1t_2^{-1}t_0 = Nt_2t_1^{-1}t_2$

$((0,1)t_0t_2^{-1}t_1)^{(0,2)} = (t_2t_0^{-1}t_2)^{(0,2)} \Longrightarrow (1,2)t_2t_0^{-1}t_1 = t_0t_2^{-1}t_0 \Longrightarrow Nt_2t_0^{-1}t_1 = Nt_0t_2^{-1}t_0$

$((0,1)t_0t_2^{-1}t_1)^{(1,2)} = (t_2t_0^{-1}t_2)^{(1,2)} \Longrightarrow (0,2)t_0t_1^{-1}t_2 = t_1t_0^{-1}t_1 \Longrightarrow Nt_0t_1^{-1}t_2 = Nt_1t_0^{-1}t_1$

$((0,1)t_0t_2^{-1}t_1)^{(0,1,2)} = (t_2t_0^{-1}t_2)^{(0,1,2)} \Longrightarrow (1,2)t_1t_0^{-1}t_2 = t_0t_1^{-1}t_0 \Longrightarrow Nt_1t_0^{-1}t_2 = Nt_0t_1^{-1}t_0$

$((0,1)t_0t_2^{-1}t_1)^{(0,2,1)} = (t_2t_0^{-1}t_2)^{(0,2,1)} \Longrightarrow (0,2)t_2t_1^{-1}t_0 = t_1t_2^{-1}t_1 \Longrightarrow Nt_2t_1^{-1}t_0 = Nt_1t_2^{-1}t_1$

Now;

$(0,1)\overline{t_0t_2^{-1}t_1} = \overline{t_2t_0^{-1}t_2} \Longrightarrow (0,1)t_0^{-1}t_2t_1^{-1} = t_2^{-1}t_0t_2^{-1} \Longrightarrow Nt_0^{-1}t_2t_1^{-1} = Nt_2^{-1}t_0t_2^{-1}$

$(0,1)\overline{t_1t_2^{-1}t_0} = \overline{t_2t_1^{-1}t_2} \Longrightarrow (0,1)t_1^{-1}t_2t_0^{-1} = t_2^{-1}t_1t_2^{-1} \Longrightarrow Nt_1^{-1}t_2t_0^{-1} = Nt_2^{-1}t_1t_2^{-1}$

$(1,2)\overline{t_2t_0^{-1}t_1} = \overline{t_0t_2^{-1}t_0} \Longrightarrow (1,2)t_2^{-1}t_0t_1^{-1} = t_0^{-1}t_2t_0^{-1} \Longrightarrow Nt_2^{-1}t_0t_1^{-1} = Nt_0^{-1}t_2t_0^{-1}$

$(0,2)\overline{t_0t_1^{-1}t_2} = \overline{t_1t_0^{-1}t_1} \Longrightarrow (0,2)t_0^{-1}t_1t_2^{-1} = t_1^{-1}t_0t_1^{-1} \Longrightarrow Nt_0^{-1}t_2t_1^{-1} = Nt_2^{-1}t_0t_2^{-1}$

$(1,2)\overline{t_1t_0^{-1}t_2} = \overline{t_0t_1^{-1}t_0} \Longrightarrow (1,2)t_1^{-1}t_0t_2^{-1} = t_0^{-1}t_1t_0^{-1} \Longrightarrow Nt_1^{-1}t_0t_2^{-1} = Nt_0^{-1}t_1t_0^{-1}$

$(0,2)\overline{t_2t_1^{-1}t_0} = \overline{t_1t_2^{-1}t_1} \Longrightarrow (0,2)t_2^{-1}t_1t_0^{-1} = t_1^{-1}t_2t_1^{-1} \Longrightarrow Nt_2^{-1}t_1t_0^{-1} = Nt_1^{-1}t_2t_1^{-1}$

We start with the double coset representative word of length zero, $NeN = N$; denoted [*]. Next we consider the double cosets with word length one. N is transitive on $T = \{t_0, t_1, t_2\} = \{0, 1, 3\}$ and therefore on their inverse $\overline{T} = \{t_0^{-1}, t_2^{-1}, t_2^{-1}\} = \{\overline{0}, \overline{1}, \overline{2}\}$, thus; $Nt_0N = \{Nt_0^n | n \in N\} = \{Nt_0, Nt_1, Nt_2\} = \{N0, N1, N2\}$ denoted [0] and $Nt_0^{-1}N = \{N(t_0^{-1})^n | n \in N\} = \{Nt_0^{-1}, Nt_1^{-1}, Nt_2^{-1}\} = \{N\overline{0}, N\overline{1}, N\overline{2}\}$ denoted $[\overline{0}]$.

Now we determine for [0] and $[\overline{0}]$ to which coset Nt_0t_i and $Nt_0^{-1}t_i$ belong for one t_i from each orbit of $N^0 = N^{\overline{0}}$ on T and $\overline{T}$. Since $N^0 = \{n \in N | t_0^n = t_0\} = \langle e, (1,2)\rangle$, which implies; $N^0 = N^{\overline{0}}$ has orbits {0}, {1,2} on T , and $\{\overline{0}\}$, $\{\overline{1}, \overline{2}\}$ on $\overline{T}$.

Next, we need to consider the double cosets [00], [01], $[0\overline{0}]$, $[0\overline{1}]$, $[\overline{0}\overline{0}]$, $[\overline{0}\overline{1}]$, $[\overline{0}0]$, $[\overline{0}1]$.

$00 = \overline{0} \Longrightarrow [00] = [\overline{0}]$, so the generator t_0 takes [0] back to single coset in $[\overline{0}]$.

$\overline{00} = 0 \Longrightarrow [\overline{00}] = [0]$, so the generator $t_{\overline{0}}^{-1}$ takes $[\overline{0}]$ back to single coset in $[0]$.
$0\overline{0} = e = \overline{0}0 \Longrightarrow [0\overline{0}] = [*] = [\overline{0}0]$, so the generators $t_{\overline{0}}^{-1}$ and t_0 takes $[0]$ and $[\overline{0}]$ to a single coset in $[*]$.

$[01] = N01N = \{N01, N02, N10, N12, N20, N21\}$, and coset stabilizer of $N01$ given by $N^{(01)} = N^{01} = \{e\}$, thus; $N^{(01)}$ has orbits $\{0\}$, $\{1\}$, $\{2\}$, $\{\overline{0}\}$, $\{\overline{1}\}$, $\{\overline{2}\}$. Hence, the number of the single cosets in the double coset $[01]$ is at most 6.
Taking representative from each orbit:
$[010] = (0,1)\overline{01} \in [\overline{01}]$
$[011] = [0\overline{1}]$
$[012] = [\overline{210}]$
$[01\overline{0}] = (0,1)\overline{01}0 \in [\overline{01}0]$
$[01\overline{1}] = [0]$
$[01\overline{2}] = [01\overline{2}]$
Therefore, $\{0\}$ takes $[01]$ go back to $[\overline{01}]$, $\{1\}$ takes $[01]$ go back to $[0\overline{1}]$, $\{2\}$ takes $[01]$ go to $[\overline{210}]$, $\{\overline{0}\}$ takes $[01]$ go to $[\overline{01}0]$, $\{\overline{1}\}$ takes $[01]$ go back to $[0]$, $\{\overline{2}\}$ takes $[01]$ go to $[01\overline{2}]$.

$[\overline{01}] = N\overline{01}N = \{N\overline{01}, N\overline{02}, N\overline{10}, N\overline{12}, N\overline{20}, N\overline{21}\}$, and coset stabilizer of $N\overline{01}$ given by $N^{\overline{(01)}} = N^{\overline{01}} = \{e\}$, thus; $N^{\overline{(01)}}$ has orbits $\{0\}$, $\{1\}$, $\{2\}$, $\{\overline{0}\}$, $\{\overline{1}\}$, $\{\overline{2}\}$. Hence, the number of the single cosets in the double coset $[\overline{01}]$ is at most 6.
Taking representative from each orbit:
$[\overline{01}0] = (0,1)01\overline{0} \in [01\overline{0}]$
$[\overline{01}1] = [\overline{0}]$
$[\overline{01}2] = [\overline{01}2]$
$[\overline{010}] = (0,1)01 \in [01]$
$[\overline{011}] = [\overline{0}1]$
$[\overline{012}] = [210]$
Therefore, $\{0\}$takes $[\overline{01}]$ go back to $[01\overline{0}]$, $\{1\}$ takes $[\overline{01}]$ go back to $[\overline{0}]$, $\{2\}$ takes $[\overline{01}]$ go to $[\overline{01}2]$, $\{\overline{0}\}$ takes $[\overline{01}]$ go back to $[01]$, $\{\overline{1}\}$ takes $[\overline{01}]$ go back to $[\overline{0}1]$, $\{\overline{2}\}$ takes $[\overline{01}]$ go back to $[210]$.

$[0\bar{1}] = N0\bar{1}N = \{N0\bar{1}, N0\bar{2}, N1\bar{0}, N1\bar{2}, N2\bar{0}, N2\bar{1}\}$, and coset stabilizer of $N0\bar{1}$ given by $N^{(0\bar{1})} = N^{0\bar{1}} = \{e\}$, thus; $N^{(0\bar{1})}$ has orbits $\{0\}$, $\{1\}$, $\{2\}$, $\{\bar{0}\}$, $\{\bar{1}\}, \{\bar{2}\}$. Hence, the number of the single cosets in the double coset $[0\bar{1}]$ is at most 6.

Taking representative from each orbit:

$[0\bar{1}0] = (1,2)1\bar{0}2 \in [1\bar{0}2]$

$[0\bar{1}1] = [0]$

$[0\bar{1}2] = (0,2)1\bar{0}1 \in [1\bar{0}1]$

$[0\bar{1}\bar{0}] = (0,1)\bar{1}01 \in [\bar{1}01]$

$[0\bar{1}\bar{1}] = [0\bar{1}]$

$[0\bar{1}\bar{2}] = [0\bar{1}\bar{2}]$

Therefore, $\{0\}$ takes $[0\bar{1}]$ go to $[1\bar{0}2]$, $\{1\}$ takes $[0\bar{1}]$ go back to $[0]$, $\{2\}$ takes $[0\bar{1}]$ go to $[\bar{1}01]$,$\{\bar{0}\}$ takes $[0\bar{1}]$ go to $[\bar{1}01]$, $\{\bar{1}\}$ takes $[0\bar{1}]$ go back to $[01]$, $\{\bar{2}\}$ takes $[0\bar{1}]$ go to $[0\bar{1}\bar{2}]$.

$[\bar{0}1] = N\bar{0}1N = \{N\bar{0}1, N\bar{0}2, N\bar{1}0, N\bar{1}2, N\bar{2}0, N\bar{2}1\}$, and coset stabilizer of $N\bar{0}1$ given by $N^{(\bar{0}1)} = N^{\bar{0}1} = \{e\}$, thus; $N^{(\bar{0}1)}$ has orbits $\{0\}$, $\{1\}$, $\{2\}$, $\{\bar{0}\}$, $\{\bar{1}\}, \{\bar{2}\}$. Hence, the number of the single cosets in the double coset $[\bar{0}1]$ is at most 6.

Taking representative from each orbit:

$[\bar{0}10] = (0,1)10\bar{1} \in [10\bar{1}]$

$[\bar{0}11] = [\bar{0}\bar{1}]$

$[\bar{0}12] = [\bar{0}12]$

$[\bar{0}1\bar{0}] = (1,2)\bar{1}0\bar{2} \in [\bar{1}0\bar{2}]$

$[\bar{0}1\bar{1}] = [\bar{0}]$

$[\bar{0}1\bar{2}] = (0,1)\bar{1}0\bar{1} \in [\bar{1}0\bar{1}]$

Therefore, $\{0\}$ takes $[\bar{0}1]$ go to $[10\bar{1}]$, $\{1\}$ takes $[\bar{0}1]$ go back to $[\bar{0}\bar{1}]$, $\{2\}$ takes $[\bar{0}1]$ go to $[\bar{0}12]$,$\{\bar{0}\}$ takes $[\bar{0}1]$ go to $[\bar{1}0\bar{2}]$, $\{\bar{1}\}$ takes $[\bar{0}1]$ go back to $[\bar{0}]$, $\{\bar{2}\}$ takes $[\bar{0}1]$ go to $[\bar{1}0\bar{1}]$.

Next, we considering the double coset with a words length 3.

$[012] = N012N = \{N012, N102, N210, N021, N120, N210\}$, and coset stabilizer of $N012$ given by $N^{(012)} = N^{012} = \{e\}$, thus; $N^{(012)}$ has orbits $\{0\}$, $\{1\}$, $\{2\}$, $\{\bar{0}\}$, $\{\bar{1}\}, \{\bar{2}\}$. Hence, the number of the single cosets in the double coset $[012]$ is at most 6.

Taking representative from each orbit:

$[0120] = [\bar{0}\bar{1}]$
$[0121] = [0\bar{1}2]$
$[0122] = [01\bar{2}]$
$[012\bar{0}] = [\bar{0}\bar{1}2]$
$[012\bar{1}] = [\bar{0}12]$
$[012\bar{2}] = [01]$
Therefore, $\{0\}$ takes $[012]$ go back to $[\bar{0}\bar{1}]$, $\{1\}$ takes $[012]$ go to $[0\bar{1}\bar{2}]$, $\{2\}$ takes $[012]$ go to $[01\bar{2}]$, $\{\bar{0}\}$ takes $[012]$ go to $[\bar{0}\bar{1}2]$, $\{\bar{1}\}$ takes $[012]$ go to $[\bar{0}12]$, $\{\bar{2}\}$ takes $[01\bar{2}]$ go to $[01]$.

$[01\bar{2}] = N01\bar{2}N = \{N01\bar{2}, N10\bar{2}, N21\bar{0}, N02\bar{1}, N12\bar{0}, N20\bar{1}\}$, and coset stabilizer of $N01\bar{2}$ given by $N^{(01)\bar{2}} = N^{01\bar{2}} = \{e\}$, thus; $N^{(01)\bar{2}}$ has orbits $\{0\}$, $\{1\}$, $\{2\}$, $\{\bar{0}\}$, $\{\bar{1}\}$, $\{\bar{2}\}$. Hence, the number of the single cosets in the double coset $[01\bar{2}]$ is at most 6.
Taking representative from each orbit:
$[01\bar{2}0] = [01\bar{2}0]$
$[01\bar{2}1] = [\bar{0}\bar{1}2]$
$[01\bar{2}2] = [01]$
$[01\bar{2}\bar{0}] = [01\bar{2}\bar{0}]$
$[01\bar{2}\bar{1}] = [1\bar{0}\bar{1}]$
$[01\bar{2}\bar{2}] = [012]$
Therefore, $\{0\}$ takes $[01\bar{2}]$ go back to $[01\bar{2}0]$, $\{1\}$ takes $[01\bar{2}]$ go back to $[\bar{0}\bar{1}2]$, $\{2\}$ takes $[01\bar{2}]$ go back to $[01]$, $\{\bar{0}\}$ takes $[01\bar{2}]$ go to $[01\bar{2}\bar{0}]$, $\{\bar{1}\}$ takes $[01\bar{2}]$ go to $[1\bar{0}\bar{1}]$, $\{\bar{2}\}$ takes $[01\bar{2}]$ go to $[012]$.

$[\bar{0}1] = N\bar{0}1N = \{N\bar{0}1, N\bar{0}2, N\bar{1}0, N\bar{1}2, N\bar{2}0, N\bar{2}1\}$, and coset stabilizer of $N\bar{0}1$ given by $N^{(\bar{0}1)} = N^{\bar{0}1} = \{e\}$, thus; $N^{(\bar{0}1)}$ has orbits $\{0\}$, $\{1\}$, $\{2\}$, $\{\bar{0}\}$, $\{\bar{1}\}$, $\{\bar{2}\}$. Hence, the number of the single cosets in the double coset $[\bar{0}1]$ is at most 6.
Taking representative from each orbit:
$[\bar{0}10] = (0,1)1\bar{0}\bar{1} \in [1\bar{0}\bar{1}]$
$[\bar{0}11] = [\bar{0}\bar{1}]$
$[\bar{0}12] = [\bar{0}12]$
$[\bar{0}1\bar{0}] = (1,2)\bar{1}0\bar{2} \in [\bar{1}0\bar{2}]$
$[\bar{0}1\bar{1}] = [0]$

$[\bar{0}1\bar{2}] = (0,1)\bar{1}0\bar{1} \in [\bar{1}0\bar{1}]$

Therefore, $\{0\}$ takes $[\bar{0}1]$ go to $[1\bar{0}\bar{1}]$, $\{1\}$ takes $[\bar{0}1]$ go back to $[\bar{0}\bar{1}]$, $\{2\}$ takes $[\bar{0}1]$ go to $[\bar{0}12]$,$\{\bar{0}\}$ takes $[\bar{0}1]$ go to $[\bar{1}0\bar{2}]$, $\{\bar{1}\}$ takes $[\bar{0}1]$ go back to $[\bar{0}]$, $\{\bar{2}\}$ takes $[\bar{0}1]$ go to $[\bar{1}0\bar{1}]$.

Next, we considering the double coset with a words length 3.

$[012] = N012N = \{N012, N102, N210, N021, N120, N210\}$, and coset stabilizer of $N012$ given by $N^{(012)} = N^{012} = \{e\}$, thus; $N^{(012)}$ has orbits $\{0\}$, $\{1\}$, $\{2\}$, $\{\bar{0}\}$, $\{\bar{1}\}$, $\{\bar{2}\}$. Hence, the number of the single cosets in the double coset $[012]$ is at most 6.

Taking representative from each orbit:

$[0120] = [\bar{0}\bar{1}]$

$[0121] = [0\bar{1}2]$

$[0122] = [01\bar{2}]$

$[012\bar{0}] = [\bar{0}\bar{1}2]$

$[012\bar{1}] = [\bar{0}12]$

$[012\bar{2}] = [01]$

Therefore, $\{0\}$ takes $[012]$ go back to $[\bar{0}\bar{1}]$, $\{1\}$ takes $[012]$ go to $[0\bar{1}\bar{2}]$, $\{2\}$ takes $[012]$ go to $[01\bar{2}]$, $\{\bar{0}\}$ takes $[012]$ go to $[\bar{0}\bar{1}2]$, $\{\bar{1}\}$ takes $[012]$ go to $[\bar{0}12]$, $\{\bar{2}\}$ takes $[01\bar{2}]$ go to $[01]$.

$[01\bar{2}] = N01\bar{2}N = \{N01\bar{2}, N10\bar{2}, N21\bar{0}, N02\bar{1}, N12\bar{0}, N20\bar{1}\}$, and coset stabilizer of $N01\bar{2}$ given by $N^{(01)\bar{2}} = N^{01\bar{2}} = \{e\}$, thus; $N^{(01)\bar{2}}$ has orbits $\{0\}$, $\{1\}$, $\{2\}$, $\{\bar{0}\}$, $\{\bar{1}\}$, $\{\bar{2}\}$. Hence, the number of the single cosets in the double coset $[01\bar{2}]$ is at most 6.

Taking representative from each orbit:

$[01\bar{2}0] = [01\bar{2}0]$

$[01\bar{2}1] = [\bar{0}\bar{1}2]$

$[01\bar{2}2] = [01]$

$[01\bar{2}\bar{0}] = [01\bar{2}\bar{0}]$

$[01\bar{2}\bar{1}] = [1\bar{0}\bar{1}]$

$[01\bar{2}\bar{2}] = [012]$

Therefore, $\{0\}$ takes $[01\bar{2}]$ go back to $[01\bar{2}0]$, $\{1\}$ takes $[01\bar{2}]$ go back to $[\bar{0}\bar{1}2]$, $\{2\}$ takes $[01\bar{2}]$ go back to $[01]$, $\{\bar{0}\}$ takes $[01\bar{2}]$ go to $[01\bar{2}\bar{0}]$, $\{\bar{1}\}$ takes $[01\bar{2}]$ go to $[1\bar{0}\bar{1}]$, $\{\bar{2}\}$ takes $[01\bar{2}]$ go to $[012]$.

$[\bar{0}\bar{1}2] = N\bar{0}\bar{1}2N = \{N\bar{0}\bar{1}2, N\bar{1}\bar{0}2, N\bar{2}\bar{1}0, N\bar{0}\bar{2}1, N\bar{1}\bar{2}0, N\bar{2}\bar{0}1\}$, and coset stabilizer of $N\bar{0}\bar{1}2$ given by $N^{\overline{(01)}2} = N^{\bar{0}\bar{1}2} = \{e\}$, thus; $N^{\overline{(01)}2}$ has orbits $\{0\}$, $\{1\}$, $\{2\}$, $\{\bar{0}\}$, $\{\bar{1}\}$, $\{\bar{2}\}$. Hence, the number of the single cosets in the double coset $[\bar{0}\bar{1}2]$ is at most 6.

Taking representative from each orbit:

$[\bar{0}\bar{1}20] = [\bar{0}\bar{1}20]$

$[\bar{0}\bar{1}21] = [0\bar{1}\bar{0}]$

$[\bar{0}\bar{1}22] = [\bar{0}\bar{1}2]$

$[\bar{0}\bar{1}2\bar{0}] = [\bar{0}\bar{1}2\bar{0}]$

$[\bar{0}\bar{1}2\bar{1}] = [01\bar{2}]$

$[\bar{0}\bar{1}2\bar{2}] = [\bar{0}\bar{1}]$

Therefore, $\{0\}$ takes $[\bar{0}\bar{1}2]$ go back to $[\bar{0}\bar{1}20]$, $\{1\}$ takes $[\bar{0}\bar{1}2]$ go to $[0\bar{1}\bar{0}]$, $\{2\}$ takes $[\bar{0}\bar{1}2]$ go to $[\bar{0}\bar{1}2]$, $\{\bar{0}\}$ takes $[\bar{0}\bar{1}2]$ go to $[\bar{0}\bar{1}2\bar{0}]$, $\{\bar{1}\}$ takes $[\bar{0}\bar{1}2]$ go to $[01\bar{2}]$, $\{\bar{2}\}$ takes $[\bar{0}\bar{1}2]$ go back to $[\bar{0}\bar{1}]$

$[0\bar{1}\bar{0}] = N0\bar{1}\bar{0}N = \{N0\bar{1}\bar{0}, N1\bar{0}\bar{1}, N2\bar{1}\bar{2}, N0\bar{2}\bar{0}, N1\bar{2}\bar{1}, N2\bar{0}\bar{2}\}$, and coset stabilizer of $N0\bar{1}\bar{0}$ given by $N^{(0\bar{1}\bar{0})} = N^{0\bar{0}\bar{1}} = \{e\}$, thus; $N^{(0\bar{1}\bar{0})}$ has orbits $\{0\}$, $\{1\}$, $\{2\}$, $\{\bar{0}\}$, $\{\bar{1}\}$, $\{\bar{2}\}$. Hence, the number of the single cosets in the double coset $[0\bar{1}\bar{0}]$ is at most 6.

Taking representative from each orbit:

$[0\bar{1}\bar{0}0] = [0\bar{1}]$

$[0\bar{1}\bar{0}1] = (0,1)\bar{1}0\bar{1} \in [\bar{0}1\bar{0}]$

$[0\bar{1}\bar{0}2] = [0\bar{1}0]$

$[0\bar{1}\bar{0}\bar{0}] = (0,2)12\bar{0} \in [01\bar{2}]$

$[0\bar{1}\bar{0}\bar{1}] = [\bar{0}1]$

$[0\bar{1}\bar{0}\bar{2}] = [\bar{0}\bar{1}2]$

Therefore, $\{0\}$ takes $[0\bar{1}\bar{0}]$ go to $[0\bar{1}]$, $\{1\}$ takes $[0\bar{1}\bar{0}]$ go to $[\bar{0}1\bar{0}]$, $\{2\}$ takes $[0\bar{1}\bar{0}]$ go to $[01\bar{2}]$,$\{\bar{0}\}$ takes $[0\bar{1}\bar{0}]$ go to $[0\bar{1}0]$, $\{\bar{1}\}$ takes $[0\bar{1}\bar{0}]$ go back to $[\bar{0}1]$, $\{\bar{2}\}$ takes $[0\bar{1}\bar{0}]$ go to $[\bar{0}\bar{1}2]$.

$[\bar{0}1\bar{0}] = N\bar{0}1\bar{0}N = \{N\bar{0}1\bar{0}, N\bar{1}0\bar{1}, N\bar{0}2\bar{0}, N\bar{1}2\bar{1}, N\bar{2}1\bar{2}, N\bar{2}0\bar{2}\}$, and coset stabilizer of $N\bar{0}1\bar{0}$ given by $N^{(\bar{0}1\bar{0})} = N^{\bar{0}1\bar{0}} = \{e\}$, thus; $N^{(\bar{0}1\bar{0})}$ has orbits $\{0\}$, $\{1\}$, $\{2\}$, $\{\bar{0}\}$, $\{\bar{1}\}$, $\{\bar{2}\}$. Hence, the number of the single cosets in the double coset $[\bar{0}1\bar{0}]$ is at most 6.

Taking representative from each orbit:

$[\bar{0}1\bar{0}0] = [\bar{0}1]$

$[\bar{0}1\bar{0}1] = [\bar{0}1\bar{0}1]$

$[\bar{0}1\bar{0}2] = [\bar{0}1]$

$[\bar{0}1\bar{0}\bar{0}] = [\bar{0}10]$

$[\bar{0}1\bar{0}\bar{1}] = [\bar{0}1\bar{0}\bar{1}]$

$[\bar{0}1\bar{0}\bar{2}] = [\bar{0}\bar{1}2]$

Therefore, $\{0\}$ takes $[\bar{0}1\bar{0}\bar{1}]$ go back to $[\bar{0}1]$, $\{1\}$ takes $[\bar{0}1\bar{0}\bar{1}]$ go to $[\bar{0}1\bar{0}1]$, $\{2\}$ takes $[\bar{0}1\bar{0}1]$ go back to $[\bar{0}1]$,$\{\bar{0}\}$ takes $[\bar{0}1\bar{0}1]$ go to $[\bar{0}10]$, $\{\bar{1}\}$ takes $[\bar{0}1\bar{0}1]$ go to $[\bar{0}1\bar{0}\bar{1}]$, $\{\bar{2}\}$ takes $[\bar{0}1\bar{0}1]$ go to $[\bar{0}12]$.

$[0\bar{1}0] = N0\bar{1}0N = \{N0\bar{1}0, N0\bar{2}0, N1\bar{0}1, N1\bar{2}1, N2\bar{0}2, N2\bar{1}2\}$, and coset stabilizer of $N0\bar{1}0$ given by $N^{(0\bar{1}0)} = N^{0\bar{1}0} = \{e\}$, thus; $N^{(0\bar{1}0)}$ has orbits $\{0\}$, $\{1\}$, $\{2\}$, $\{\bar{0}\}$, $\{\bar{1}\}$, $\{\bar{2}\}$. Hence, the number of the single cosets in the double coset $[0\bar{1}0]$ is at most 6.

Taking representative from each orbit:

$[0\bar{1}00] = [0\bar{1}\bar{0}]$

$[0\bar{1}01] = [0\bar{1}01]$

$[0\bar{1}02] = (1,2)1\overline{02} \in [0\bar{1}\bar{2}]$

$[0\bar{1}0\bar{0}] = [0\bar{1}]$

$[0\bar{1}0\bar{1}] = [0\bar{1}0\bar{1}]$

$[0\bar{1}0\bar{2}] = [1\bar{0}]$

Therefore, $\{0\}$ takes $[0\bar{1}0]$ go to $[0\bar{1}\bar{0}]$, $\{1\}$ takes $[0\bar{1}0]$ go to $[0\bar{1}01]$, $\{2\}$ takes $[0\bar{1}0]$ go to $[0\bar{1}\bar{2}]$,$\{\bar{0}\}$ takes $[0\bar{1}0]$ go back to $[0\bar{1}]$, $\{\bar{1}\}$ takes $[0\bar{1}0]$ go to $[0\bar{1}0\bar{1}]$, $\{\bar{2}\}$ takes $[0\bar{1}0]$ go back to $[1\bar{0}]$.

$[\bar{0}12] = N\bar{0}12N = \{N\bar{0}12, N\bar{2}10, N\bar{0}21, N\bar{1}02, N\bar{0}12, N\bar{1}20\}$, and coset stabilizer of $N\bar{0}12$ given by $N^{\bar{0}12} = N^{(\bar{0}12)} = \{e\}$, thus; $N^{\bar{0}12}$ has orbits $\{0\}$, $\{1\}$, $\{2\}$, $\{\bar{0}\}$, $\{\bar{1}\}$, $\{\bar{2}\}$. Hence, the number of the single cosets in the double coset $[\bar{0}12]$ is at most 6.

Taking representative from each orbit:

$[\bar{0}120] = [0\bar{1}\bar{2}]$

$[\bar{0}121] = [\overline{012}]$

$[\bar{0}122] = (0,2)\bar{1}0\bar{1} \in [\bar{0}1\bar{0}]$

$[\bar{0}12\bar{0}] = [\bar{0}12\bar{0}]$

$[\bar{0}12\bar{1}] = [0\bar{1}\bar{2}]$

$[\bar{0}12\bar{2}] = [\bar{0}1]$

Therefore, $\{0\}$ takes $[\bar{0}12]$ go to $[0\bar{1}\bar{2}]$, $\{1\}$ takes $[\bar{0}12]$ go to $[\bar{0}\bar{1}\bar{2}]$, $\{2\}$ takes $[\bar{0}12]$ go to $[\bar{0}1\bar{0}]$, $\{\bar{0}\}$ takes $[\bar{0}12]$ go to $[\bar{0}12\bar{0}]$, $\{\bar{1}\}$ takes $[\bar{0}12]$ go to $[0\bar{1}\bar{2}]$, $\{\bar{2}\}$ takes $[\bar{0}12]$ go back to $[\bar{0}1]$.

$[0\bar{1}\bar{2}] = N0\bar{1}\bar{2}N = \{N0\bar{1}\bar{2}, N0\bar{2}\bar{1}, N1\bar{2}\bar{0}, N1\bar{2}\bar{1}, N2\bar{0}\bar{1}, N2\bar{1}\bar{0}\}$, and coset stabilizer of $N0\bar{1}\bar{2}$ given by $N^{0\bar{1}\bar{2}} = N^{(0\bar{1}\bar{2})} = \{e\}$, thus; $N^{0\bar{1}\bar{2}}$ has orbits $\{0\}$, $\{1\}$, $\{2\}$, $\{\bar{0}\}$, $\{\bar{1}\}, \{\bar{2}\}$. Hence, the number of the single cosets in the double coset $[0\bar{1}\bar{2}]$ is at most 6.

Taking representative from each orbit:

$[0\bar{1}\bar{2}0] = [0\bar{1}\bar{2}0]$

$[0\bar{1}\bar{2}1] = (1,2)\bar{2}10 \in [\bar{0}12]$

$[0\bar{1}\bar{2}2] = [0\bar{1}]$

$[0\bar{1}\bar{2}\bar{0}] = [\bar{0}12]$

$[0\bar{1}\bar{2}\bar{1}] = [012]$

$[0\bar{1}\bar{2}\bar{2}] = [0\bar{1}2]$

Therefore, $\{0\}$ takes $[0\bar{1}\bar{2}]$ go to $[0\bar{1}\bar{2}0]$, $\{1\}$ takes $[0\bar{1}\bar{2}]$ go to $[\bar{0}12]$, $\{2\}$ takes $[0\bar{1}\bar{2}]$ go back to $[0\bar{1}]$, $\{\bar{0}\}$ takes $[0\bar{1}\bar{2}]$ go to $[\bar{0}12]$, $\{\bar{1}\}$ takes $[0\bar{1}\bar{2}]$ go to $[012]$, $\{\bar{2}\}$ takes $[0\bar{1}\bar{2}]$ go to $[0\bar{1}2]$.

Next, we considering the double coset with a words length 4.

$[0\bar{1}\bar{2}0] = N0\bar{1}\bar{2}0N = \{N0\bar{1}\bar{2}0, N1\bar{0}\bar{2}1, N2\bar{1}\bar{0}2, N0\bar{2}\bar{1}0, N1\bar{2}\bar{0}1, N2\bar{0}\bar{1}2\}$, and coset stabilizer of $N0\bar{1}\bar{2}0$ given by $N^{0\bar{1}\bar{2}0} = N^{(0\bar{1}\bar{2}0)} = \{e\}$, thus; $N^{0\bar{1}\bar{2}0}$ has orbits $\{0\}$, $\{1\}$, $\{2\}$, $\{\bar{0}\}$, $\{\bar{1}\}, \{\bar{2}\}$. Hence, the number of the single cosets in the double coset $[0\bar{1}\bar{2}0]$ is at most 6.

Taking representative from each orbit:

$[0\bar{1}\bar{2}00] = [0\bar{1}\bar{2}\bar{0}] = [0021] = [\bar{0}21] \in [\bar{0}12]$

$[0\bar{1}\bar{2}01] = (0,2)10\bar{2} \in [10\bar{2}]$

$[0\bar{1}\bar{2}02] = [01\bar{0}\bar{1}]$

$[0\bar{1}\bar{2}0\bar{0}] = [0\bar{1}\bar{2}]$

$[0\bar{1}\bar{2}0\bar{1}] = [\bar{0}1\bar{0}\bar{1}]$

$[0\bar{1}\bar{2}0\bar{2}] = [\bar{2}\bar{0}1] \in [\bar{0}\bar{1}2]$

Therefore, $\{0\}$ takes $[0\bar{1}\bar{2}0]$ go back to $[\bar{0}12]$, $\{1\}$ takes $[0\bar{1}\bar{2}0]$ go back to $[01\bar{2}]$, $\{2\}$ takes $[0\bar{1}\bar{2}0]$ go to $[01\bar{0}\bar{1}]$, $\{\bar{0}\}$ takes $[0\bar{1}\bar{2}0]$ go back to $[0\bar{1}\bar{2}]$, $\{\bar{1}\}$ takes $[0\bar{1}\bar{2}0]$ go to $[\bar{0}1\bar{0}\bar{1}]$, $\{\bar{2}\}$

takes $[0\bar{1}\bar{2}0]$ go back to $[\bar{0}12]$.

$[01\bar{0}\bar{1}] = N01\bar{0}\bar{1}N = \{N01\bar{0}\bar{1}, N10\bar{1}\bar{0}, N21\bar{2}\bar{1}, N02\bar{0}\bar{2}, N12\bar{1}\bar{2}, N21\bar{2}\bar{1}\}$, and coset stabilizer of $N01\bar{0}\bar{1}$ given by $N^{01\bar{0}\bar{1}} = N^{(01\bar{0}\bar{1})} = \{e\}$, thus; $N^{01\bar{0}\bar{1}}$ has orbits $\{0\}$, $\{1\}$, $\{2\}$, $\{\bar{0}\}$, $\{\bar{1}\}, \{\bar{2}\}$. Hence, the number of the single cosets in the double coset $[01\bar{0}\bar{1}]$ is at most 6.
Taking representative from each orbit:

$[01\bar{0}\bar{1}0] = [1\bar{0}1\bar{0}] \in [0\bar{1}0\bar{1}$

$[01\bar{0}\bar{1}1] = [1\bar{0}1] \in [0\bar{1}0]$

$[01\bar{0}\bar{1}2] = [\bar{2}01] \in [\bar{0}12]$

$[01\bar{0}\bar{1}\bar{0}] = [1\bar{0}1] \in [0\bar{1}0]$

$[01\bar{0}\bar{1}\bar{1}] = [\bar{0}\bar{1}0\bar{1}]$

$[01\bar{0}\bar{1}\bar{2}] = [0\bar{1}\bar{2}0]$

Therefore, $\{0\}$ takes $[01\bar{0}\bar{1}]$ go to $[0\bar{1}0\bar{1}]$, $\{1\}$ takes $[01\bar{0}\bar{1}]$ go back to $[0\bar{1}0]$, $\{2\}$ takes $[01\bar{0}\bar{1}]$ go back to $[\bar{0}12]$, $\{\bar{0}\}$ takes $[01\bar{0}\bar{1}]$ go back to $[0\bar{1}0]$, $\{\bar{1}\}$ takes $[01\bar{0}\bar{1}]$ go to $[\bar{0}101]$, $\{\bar{2}\}$ takes $[01\bar{0}\bar{1}0]$ go to $[0\bar{1}\bar{2}0]$.

$[\bar{0}1\bar{0}\bar{1}] = N\bar{0}1\bar{0}\bar{1}N = \{N\bar{0}1\bar{0}\bar{1}, N\bar{1}0\bar{1}\bar{0}, N\bar{2}1\bar{2}\bar{1}, N\bar{0}2\bar{0}\bar{2}, N\bar{1}2\bar{1}\bar{2}, N\bar{2}1\bar{2}\bar{1}\}$, and coset stabilizer of $N\bar{0}1\bar{0}\bar{1}$ given by $N^{\bar{0}1\bar{0}\bar{1}} = N^{(\bar{0}1\bar{0}\bar{1})} = \{e\}$, thus; $N^{\bar{0}1\bar{0}\bar{1}}$ has orbits $\{0\}$, $\{1\}$, $\{2\}$, $\{\bar{0}\}$, $\{\bar{1}\}, \{\bar{2}\}$. Hence, the number of the single cosets in the double coset $[\bar{0}1\bar{0}\bar{1}]$ is at most 6.
Taking representative from each orbit:

$[\bar{0}1\bar{0}\bar{1}0] = [\bar{1}0\bar{1}0] \in [\bar{0}10\bar{1}]$

$[\bar{0}1\bar{0}\bar{1}1] = [\bar{0}1\bar{0}]$

$[\bar{0}1\bar{0}\bar{1}2] = [0\bar{2}\bar{1}0] \in [0\bar{1}\bar{2}0]$

$[\bar{0}1\bar{0}\bar{1}\bar{0}] = [\bar{1}0\bar{1}] \in [\bar{0}\bar{1}0]$

$[\bar{0}1\bar{0}\bar{1}\bar{1}] = [\bar{0}2\bar{0}2] \in [\bar{0}1\bar{0}1]$

$[\bar{0}1\bar{0}\bar{1}\bar{2}] = [12\bar{0}] \in [01\bar{2}$

Therefore, $\{0\}$ takes $[\bar{0}1\bar{0}\bar{1}]$ go to $[\bar{0}\bar{1}0\bar{1}]$, $\{1\}$ takes $[\bar{0}1\bar{0}\bar{1}]$ go back to $[\bar{0}1\bar{0}]$, $\{2\}$ takes $[\bar{0}1\bar{0}\bar{1}]$ go to $[0\bar{1}\bar{2}0]$, $\{\bar{0}\}$ takes $[\bar{0}1\bar{0}\bar{1}]$ go back to $[\bar{0}\bar{1}0]$, $\{\bar{1}\}$ takes $[\bar{0}1\bar{0}\bar{1}]$ go to $[\bar{0}1\bar{0}1]$, $\{\bar{2}\}$ takes $[\bar{0}1\bar{0}\bar{1}0]$ go back to $[01\bar{2}]$.

$\bar{0}1\bar{0}1 = \bar{0}2\bar{0}2 \qquad 0\bar{1}0\bar{1} = 0\bar{2}0\bar{2}$

$\bar{1}0\bar{1}0 = \bar{1}2\bar{1}2 \qquad 1\bar{0}1\bar{0} = 1\bar{2}1\bar{2}$

$\bar{2}0\bar{2}0 = \bar{2}1\bar{2}1 \qquad 2\bar{0}2\bar{0} = 2\bar{1}2\bar{1}$

$[\bar{0}1\bar{0}1] = N\bar{0}1\bar{0}1N = \{N\bar{0}1\bar{0}1, N\bar{1}0\bar{1}0, N\bar{2}0\bar{2}0\}$, and coset stabilizer of $N\bar{0}1\bar{0}1$ given by $N^{\bar{0}1\bar{0}1} = N^{(\bar{0}1\bar{0}1)} = \{e, (1,2)\}$, thus; $N^{\bar{0}1\bar{0}1}$ has orbits $\{0\}$, $\{1,\ 2\}$ on T, $\{\bar{0}\}$, $\{\bar{1},\bar{2}\}$ on $\overline{T}$.
Hence, the number of the single cosets in the double coset $[\bar{0}1\bar{0}1]$ is at most 3.
Taking representative from each orbit:

$[\bar{0}1\bar{0}10] = [\bar{1}01\bar{0}] \in [\bar{0}10\bar{1}]$

$[\bar{0}1\bar{0}11] = [\bar{0}1\bar{0}\bar{1}]$

$[\bar{0}1\bar{0}1\bar{0}] = [\bar{0}1\bar{0}1\bar{0}]$

$[\bar{0}1\bar{0}1\bar{1}] = [\bar{0}1\bar{0}]$

Therefore, $\{0\}$ takes $[\bar{0}1\bar{0}1]$ go to $[\bar{0}1\bar{0}\bar{1}]$, $\{1,\ 2\}$ takes $[\bar{0}1\bar{0}1]$ go to $[0\bar{1}\bar{0}\bar{1}]$, $\{\bar{0}\}$ takes $[\bar{0}1\bar{0}1]$ go to $[\bar{0}1\bar{0}1\bar{0}]$, $\{\bar{1},\bar{2}\}$ takes $[\bar{0}1\bar{0}1]$ go to $[\bar{0}1\bar{0}]$.

$[0\bar{1}0\bar{1}] = N0\bar{1}0\bar{1}N = \{N0\bar{1}0\bar{0}, N1\bar{0}1\bar{0}, N2\bar{0}2\bar{0}\}$, and coset stabilizer of $N0\bar{1}0\bar{1}$ given by $N^{0\bar{1}0\bar{1}} = N^{(0\bar{1}0\bar{1})} = \{e, (1,2)\}$, thus; $N^{0\bar{1}0\bar{1}}$ has orbits $\{0\}$, $\{1,\ 2\}$ on T, $\{\bar{0}\}$, $\{\bar{1},\bar{2}\}$ on $\overline{T}$.
Hence, the number of the single cosets in the double coset $[0\bar{1}0\bar{1}]$ is at most 3.
Taking representative from each orbit:

$[0\bar{1}0\bar{1}0] = [0\bar{1}0\bar{1}0]$

$[0\bar{1}0\bar{1}1] = [0\bar{1}0]$

$[0\bar{1}0\bar{1}\bar{0}] = [01\bar{0}\bar{1}]$

$[0\bar{1}0\bar{1}\bar{1}] = [0\bar{1}01] \in [0\bar{1}01]$

Therefore, $\{0\}$ takes $[0\bar{1}0\bar{1}]$ go to $[0\bar{1}0\bar{1}0]$, $\{1,\ 2\}$ take $[0\bar{1}0\bar{1}]$ go back to $[0\bar{1}0]$, $\{\bar{0}\}$ takes $[0\bar{1}0\bar{1}]$ go to $[01\bar{0}\bar{1}]$, $\{\bar{1},\bar{2}\}$ take $[0\bar{1}0\bar{1}]$ go to $[0\bar{1}01]$.
Applying the relation 6:

$\bar{0}1\bar{0}1 = \bar{0}(0,2)\bar{1}2 = \bar{2}0\bar{1}2 = \bar{0}2\bar{0}2 \Rightarrow \overline{\bar{0}1\bar{0}1} = \overline{\bar{0}(0,2)\bar{1}2} = \overline{\bar{2}0\bar{1}2} = \overline{\bar{0}2\bar{0}2} \Rightarrow 0\bar{1}0\bar{1} = 0\bar{2}0\bar{2}$

Conjugating with all element in S_3, then we will get:

$\bar{0}1\bar{0}1 = \bar{0}2\bar{0}2 \qquad 0\bar{1}0\bar{1} = 0\bar{2}0\bar{2}$

$\bar{1}0\bar{1}0 = \bar{1}2\bar{1}2 \qquad 1\bar{0}1\bar{0} = 1\bar{2}1\bar{2}$

$\bar{2}0\bar{2}0 = \bar{2}1\bar{2}1 \qquad 2\bar{0}2\bar{0} = 2\bar{1}2\bar{1}$

$[\bar{0}1\bar{0}1] = N\bar{0}1\bar{0}1N = \{N\bar{0}1\bar{0}1, N\bar{1}0\bar{1}0, N\bar{2}0\bar{2}0\}$, and coset stabilizer of $N\bar{0}1\bar{0}1$ given by $N^{\bar{0}1\bar{0}1} = N^{(\bar{0}1\bar{0}1)} = \{e, (1,2)\}$, thus; $N^{\bar{0}1\bar{0}1}$ has orbits $\{0\}$, $\{1,\ 2\}$ on T, $\{\bar{0}\}$, $\{\bar{1},\bar{2}\}$ on $\overline{T}$.
Hence, the number of the single cosets in the double coset $[\bar{0}1\bar{0}1]$ is at most 3.

Taking representative from each orbit:

$[\bar{0}1\bar{0}10] = [\bar{1}01\bar{0}] \in [\bar{0}\bar{1}0\bar{1}]$

$[\bar{0}1\bar{0}11] = [\bar{0}1\bar{0}\bar{1}]$

$[\bar{0}1\bar{0}1\bar{0}] = [\bar{0}1\bar{0}1\bar{0}]$

$[\bar{0}1\bar{0}1\bar{1}] = [\bar{0}1\bar{0}]$

Therefore, $\{0\}$ take $[\bar{0}1\bar{0}1]$ go to $[\bar{0}1\bar{0}\bar{1}]$, $\{1, 2\}$ take $[\bar{0}1\bar{0}1]$ go to $[0\bar{1}0\bar{1}]$, $\{\bar{0}\}$ take $[\bar{0}1\bar{0}1]$ go to $[\bar{0}1\bar{0}1\bar{0}]$, $\{\bar{1},\bar{2}\}$ take $[\bar{0}1\bar{0}1]$ go to $[\bar{0}1\bar{0}]$.

$[0\bar{1}0\bar{1}] = N0\bar{1}0\bar{1}N = \{N0\bar{1}0\bar{0}, N1\bar{0}1\bar{0}, N2\bar{0}2\bar{0}\}$, and coset stabilizer of $N0\bar{1}0\bar{1}$ given by $N^{0\bar{1}0\bar{1}} = N^{(0\bar{1}0\bar{1})} = \{e,(1,2)\}$, thus; $N^{0\bar{1}0\bar{1}}$ has orbits $\{0\}$, $\{1, 2\}$ on T, $\{\bar{0}\}$, $\{\bar{1},\bar{2}\}$ on $\overline{T}$. Hence, the number of the single cosets in the double coset $[0\bar{1}0\bar{1}]$ is at most 3.

Taking representative from each orbit:

$[0\bar{1}0\bar{1}0] = [0\bar{1}0\bar{1}0]$

$[0\bar{1}0\bar{1}1] = [0\bar{1}0]$

$[0\bar{1}0\bar{1}\bar{0}] = [01\bar{0}\bar{1}]$

$[0\bar{1}0\bar{1}\bar{1}] = [0\bar{1}01] \in [0\bar{1}01]$

Therefore, $\{0\}$ take $[0\bar{1}0\bar{1}]$ go to $[0\bar{1}0\bar{1}0]$, $\{1, 2\}$ take $[0\bar{1}0\bar{1}]$ go back to $[0\bar{1}0]$, $\{\bar{0}\}$ take $[0\bar{1}0\bar{1}]$ go to $[01\bar{0}\bar{1}]$, $\{\bar{1},\bar{2}\}$ take $[0\bar{1}0\bar{1}]$ go to $[0\bar{1}01]$.

Next, we considering the double coset with a words length 5:

By relation 2 and 6 then:

$[\bar{0}1\bar{0}1\bar{0}] = \bar{0}1\bar{0}1\bar{0} = 0\bar{1}0\bar{1}0 = \bar{2}0\bar{2}0\bar{2} = 2\bar{0}2\bar{0}2 = \bar{1}0\bar{1}0\bar{1} = 1\bar{0}1\bar{0}1$

Therefore, $\{0, 1, 2\}$ take $[0\bar{1}0\bar{1}0]$ go back to $[\bar{0}1\bar{0}1]$, and $\{\bar{0},\bar{1},\bar{2}\}$ take $[0\bar{1}0\bar{1}0]$ go back to $[0\bar{1}0\bar{1}]$.

Our double coset enumeration must be complete since the set of right coset is close under right multiplication by the symmetric generators. The Double coset enumeration shows that the index of $N \cong L_2(11)$ in G is at most: $\frac{|N|}{|N|}+\frac{|N|}{|N^{(0)}|}+\frac{|N|}{|N^{(\bar{0})}|}+\frac{|N|}{|N^{(01)}|}+\frac{|N|}{|N^{(\bar{0}\bar{1})}|}+\frac{|N|}{|N^{(\bar{0}1)}|}+\frac{|N|}{|N^{(0\bar{1})}|}$ $+\frac{|N|}{|N^{(012)}|}+\frac{|N|}{|N^{(01\bar{2})}|}+\frac{|N|}{|N^{(01\bar{0})}|}+\frac{|N|}{|N^{(\bar{0}12)}|}+\frac{|N|}{|N^{(\bar{0}\bar{1}2)}|}+\frac{|N|}{|N^{(010)}|}+\frac{|N|}{|N^{(0\bar{1}\bar{2})}|}+\frac{|N|}{|N^{(0\bar{0}\bar{1})}|}+\frac{|N|}{|N^{(\bar{0}1\bar{0})}|}+\frac{|N|}{|N^{(\bar{0}1\bar{0}1)}|}$ $+\frac{|N|}{|N^{(0\bar{1}\bar{2}0)}|}+\frac{|N|}{|N^{(01\bar{0}\bar{1})}|}+\frac{|N|}{|N^{(\bar{0}1\bar{0}1)}|}+\frac{|N|}{|N^{(0\bar{1}0\bar{1})}|}+\frac{|N|}{|N^{(\bar{0}1\bar{0}1\bar{0})}|} = 1+6+6+6+6+6+6+6+6+$ $6+6+6+6+6+6+6+6+6+3+3+1 = 110$

$\Longrightarrow |G| \leq 110|N| = 110 \times 6 = 660$

In order to show $|G| = 660$, we consider G as a subgroup of S_{110} acting on 110 coset the we have found, and labeled as follows:

Permutation Representation

Coset	$x \sim (0,1,2)$	$y \sim (1,2)$	t_0
1 N	1 N	1 N	2 $N0$
2 $N0$	3 $N1$	2 $N0$	5 $N\bar{0}$
3 $N1$	4 $N2$	4 $N2$	10 $N10$
4 $N2$	2 $N0$	3 $N1$	12 $N20$
5 $N\bar{0}$	6 $N\bar{1}$	5 $N\bar{0}$	1 N
6 $N\bar{1}$	7 $N\bar{2}$	7 $N\bar{2}$	28 $N\bar{1}0$
7 $N\bar{2}$	5 $N\bar{0}$	6 $N\bar{1}$	30 $N\bar{2}0$
8 $N01$	11 $N12$	9 $N02$	14 $N\overline{01}$
9 $N02$	10 $N10$	8 $N01$	15 $N\overline{02}$
10 $N10$	13 $N21$	12 $N20$	22 $N1\bar{0}$
11 $N12$	12 $N20$	13 $N21$	41 $N120$
12 $N20$	8 $N01$	10 $N10$	24 $N2\bar{0}$
13 $N21$	9 $N02$	11 $N12$	39 $N210$
14 $N\overline{01}$	17 $N\overline{12}$	15 $N\overline{02}$	44 $N01\bar{0}$
15 $N\overline{02}$	16 $N\overline{10}$	14 $N\overline{01}$	46 $N02\bar{0}$
16 $N\overline{10}$	19 $N\overline{21}$	18 $N\overline{20}$	6 $N\bar{1}$
17 $N\overline{12}$	18 $N\overline{20}$	19 $N\overline{21}$	54 $N\overline{12}0$
18 $N\overline{20}$	14 $N\overline{01}$	16 $N\overline{10}$	7 $N\bar{2}$
19 $N\overline{21}$	15 $N\overline{02}$	17 $N\overline{12}$	52 $N\overline{21}0$
20 $N0\bar{1}$	23 $N1\bar{2}$	21 $N0\bar{2}$	68 $N0\bar{1}0$
21 $N0\bar{2}$	22 $N1\bar{0}$	20 $N0\bar{1}$	69 $N0\bar{2}0$
22 $N1\bar{0}$	25 $N2\bar{1}$	24 $N2\bar{0}$	3 $N1$
23 $N1\bar{2}$	24 $N2\bar{0}$	25 $N2\bar{1}$	73 $N2\bar{1}$
24 $N2\bar{0}$	20 $N0\bar{1}$	22 $N1\bar{0}$	4 $N2$
25 $N2\bar{1}$	21 $N0\bar{2}$	23 $N1\bar{2}$	71 $N1\bar{2}1$
26 $N\bar{0}1$	29 $N\bar{1}2$	27 $N\bar{0}2$	56 $N\bar{0}10$
27 $N\bar{0}2$	28 $N\bar{1}0$	26 $N\bar{0}1$	59 $N\bar{0}20$
28 $N\bar{1}0$	31 $N\bar{2}1$	30 $N\bar{2}0$	16 $N\overline{10}$
29 $N\bar{1}2$	30 $N\bar{2}0$	31 $N\bar{2}1$	84 $N\bar{1}20$
30 $N\bar{2}0$	26 $N\bar{0}1$	28 $N\bar{1}0$	18 $N\overline{20}$
31 $N\bar{2}1$	27 $N\bar{0}2$	29 $N\bar{1}2$	82 $N\bar{2}10$
32 $N01\bar{2}$	37 $N12\bar{0}$	36 $N02\bar{1}$	95 $N\bar{2}1\overline{21}$
33 $N10\bar{2}$	35 $N21\bar{0}$	34 $N20\bar{1}$	55 $N\overline{20}1$
34 $N20\bar{1}$	32 $N01\bar{2}$	33 $N10\bar{2}$	51 $N\overline{10}2$
35 $N21\bar{0}$	36 $N02\bar{1}$	37 $N12\bar{0}$	13 $N21$
36 $N02\bar{1}$	33 $N10\bar{2}$	32 $N01\bar{2}$	96 $N\bar{1}2\overline{12}$
37 $N12\bar{0}$	34 $N20\bar{1}$	35 $N21\bar{0}$	11 $N12$

38 $N012$	41 $N120$	42 $N021$	19 $N\bar{2}\bar{1}$
39 $N210$	42 $N021$	41 $N120$	35 $N21\bar{0}$
40 $N201$	38 $N012$	43 $N102$	64 $N2\bar{0}\bar{1}$
41 $N120$	40 $N201$	39 $N210$	37 $N12\bar{0}$
42 $N021$	43 $N102$	38 $N012$	17 $N\bar{1}\bar{2}$
43 $N102$	39 $N210$	40 $N201$	67 $N1\bar{0}\bar{2}$
44 $N01\bar{0}$	48 $N12\bar{1}$	46 $N02\bar{0}$	8 $N01$
45 $N10\bar{1}$	49 $N21\bar{2}$	47 $N20\bar{2}$	92 $N\bar{0}1\bar{0}\bar{1}$
46 $N02\bar{0}$	45 $N10\bar{1}$	44 $N01\bar{0}$	9 $N02$
47 $N20\bar{2}$	44 $N01\bar{0}$	45 $N10\bar{1}$	93 $N\bar{0}2\bar{0}\bar{2}$
48 $N12\bar{1}$	47 $N20\bar{2}$	49 $N21\bar{2}$	48 $N12\bar{1}$
49 $N21\bar{2}$	46 $N02\bar{0}$	48 $N12\bar{1}$	49 $N21\bar{2}$
50 $N\bar{0}\bar{1}2$	54 $N\bar{1}\bar{2}0$	53 $N\bar{0}\bar{2}1$	90 $N1\bar{2}\bar{0}1$
51 $N\bar{1}\bar{0}2$	52 $N\bar{2}\bar{1}0$	55 $N\bar{2}\bar{0}1$	60 $N\bar{2}12$
52 $N\bar{2}\bar{1}0$	52 $N\bar{2}\bar{1}0$	54 $N\bar{1}\bar{2}0$	38 $N012$
53 $N\bar{0}\bar{2}1$	55 $N\bar{2}\bar{0}1$	52 $N\bar{2}\bar{1}0$	42 $N021$
54 $N\bar{1}\bar{2}0$	51 $N\bar{1}\bar{0}2$	50 $N\bar{0}\bar{1}2$	89 $N2\bar{1}\bar{0}2$
55 $N\bar{2}\bar{0}1$	50 $N\bar{0}\bar{1}2$	51 $N\bar{1}\bar{0}2$	58 $N\bar{1}21$
56 $N\bar{0}10$	58 $N\bar{1}21$	59 $N\bar{0}20$	74 $N\bar{0}1\bar{0}$
57 $N\bar{1}01$	60 $N\bar{2}12$	61 $N\bar{2}02$	20 $N0\bar{1}$
58 $N\bar{1}21$	61 $N\bar{2}02$	60 $N\bar{2}12$	33 $N10\bar{2}$
59 $N\bar{0}20$	57 $N\bar{1}01$	56 $N\bar{0}10$	75 $N\bar{0}2\bar{0}$
60 $N\bar{2}12$	59 $N\bar{0}20$	58 $N\bar{1}21$	34 $N20\bar{1}$
61 $N\bar{2}02$	56 $N\bar{0}10$	57 $N\bar{1}01$	21 $N0\bar{2}$
62 $N0\bar{1}\bar{2}$	65 $N1\bar{2}\bar{0}$	66 $N0\bar{2}\bar{1}$	86 $N0\bar{1}\bar{2}0$
63 $N2\bar{1}\bar{0}$	66 $N0\bar{2}\bar{1}$	65 $N1\bar{2}\bar{0}$	25 $N2\bar{1}$
64 $N2\bar{0}\bar{1}$	62 $N0\bar{1}\bar{2}$	67 $N1\bar{0}\bar{2}$	81 $N\bar{1}02$
65 $N1\bar{2}\bar{0}$	64 $N2\bar{0}\bar{1}$	63 $N2\bar{1}\bar{0}$	23 $N1\bar{2}$
66 $N0\bar{2}\bar{1}$	67 $N1\bar{0}\bar{2}$	62 $N0\bar{1}\bar{2}$	87 $N0\bar{2}\bar{1}0$
67 $N1\bar{0}\bar{2}$	63 $N2\bar{1}\bar{0}$	64 $N2\bar{0}\bar{1}$	85 $N\bar{2}01$
68 $N0\bar{1}0$	71 $N1\bar{2}1$	69 $N0\bar{2}0$	57 $N\bar{1}01$
69 $N0\bar{2}0$	70 $N1\bar{0}1$	68 $N0\bar{1}0$	61 $N\bar{2}02$
70 $N1\bar{0}1$	73 $N2\bar{1}$	72 $N2\bar{0}2$	98 $N01\bar{0}\bar{1}$
71 $N1\bar{2}1$	72 $N2\bar{0}2$	73 $N2\bar{1}$	63 $N2\bar{1}\bar{0}$
72 $N2\bar{0}2$	68 $N0\bar{1}0$	70 $N1\bar{0}1$	99 $N02\bar{0}\bar{2}$
73 $N2\bar{1}$	69 $N0\bar{2}0$	71 $N1\bar{2}1$	65 $N1\bar{2}\bar{0}$
74 $N\bar{0}1\bar{0}$	77 $N\bar{1}2\bar{1}$	75 $N\bar{0}2\bar{0}$	26 $N\bar{0}1$
75 $N\bar{0}2\bar{0}$	76 $N\bar{1}0\bar{1}$	74 $N\bar{0}1\bar{0}$	27 $N\bar{0}2$
76 $N\bar{1}0\bar{1}$	79 $N\bar{2}1\bar{2}$	78 $N\bar{2}0\bar{2}$	105 $N\bar{1}2\bar{1}2$
77 $N\bar{1}2\bar{1}$	78 $N\bar{2}0\bar{2}$	79 $N\bar{2}0\bar{2}$	31 $N\bar{2}1$
78 $N\bar{2}0\bar{2}$	74 $N\bar{0}1\bar{0}$	76 $N\bar{1}0\bar{1}$	106 $N\bar{2}1\bar{2}1$

79 $N\bar{2}1\bar{2}$	75 $N\bar{0}2\bar{0}$	77 $N\bar{1}2\bar{1}$	29 $N\bar{1}2$
80 $N\bar{0}12$	84 $N\bar{1}20$	83 $N\bar{0}21$	66 $N0\overline{21}$
81 $N\bar{1}02$	82 $N\bar{2}10$	85 $N\bar{2}01$	40 $N201$
82 $N\bar{2}10$	83 $N\bar{0}21$	84 $N\bar{1}20$	77 $N\bar{1}2\bar{1}$
83 $N\bar{0}21$	81 $N\bar{1}02$	80 $N\bar{0}12$	62 $N0\overline{12}$
84 $N\bar{1}20$	85 $N\bar{2}01$	82$N\bar{2}10$	79 $N\bar{2}1\bar{2}$
85 $N\bar{2}01$	80 $N\bar{0}12$	81$N\bar{1}02$	43 $N102$
86 $N0\overline{12}0$	90 $N1\overline{20}1$	87$N0\overline{21}0$	83 $N\bar{0}21$
87 $N0\overline{21}0$	88 $N1\overline{02}1$	86$N0\overline{12}0$	80 $N\bar{0}12$
88 $N1\overline{02}1$	89 $N2\overline{10}2$	91$N2\overline{01}2$	32 $N01\bar{2}$
89 $N2\overline{10}2$	87 $N0\overline{21}0$	90$N1\overline{20}1$	101 $N21\overline{21}$
90 $N1\overline{20}1$	91 $N2\overline{01}2$	89$N2\overline{10}2$	102 $N12\overline{12}$
91 $N2\overline{01}2$	86 $N0\overline{12}0$	88$N1\overline{02}1$	36 $N02\bar{1}$
92 $N\bar{0}1\overline{01}$	96 $N\bar{1}2\overline{12}$	93 $N\bar{0}2\overline{02}$	100 $N10\overline{10}$
93 $N\bar{0}2\overline{02}$	94 $N\bar{1}0\overline{10}$	92 $N\bar{0}1\overline{01}$	103 $N20\overline{20}$
94 $N\bar{1}0\overline{10}$	95 $N\bar{2}1\overline{21}$	97 $N\bar{2}0\overline{20}$	76 $N\bar{1}0\bar{1}$
95 $N\bar{2}1\overline{21}$	93 $N\bar{0}2\overline{02}$	96 $N\bar{1}2\overline{12}$	88 $N1\overline{02}1$
96 $N\bar{1}2\overline{12}$	97 $N\bar{2}0\overline{20}$	95 $N\bar{2}1\overline{21}$	91 $N2\overline{01}2$
97 $N\bar{2}0\overline{20}$	92 $N\bar{0}1\overline{01}$	94$N\bar{1}0\overline{10}$	78 $N\bar{2}0\bar{2}$
98 $N01\overline{01}$	102 $N12\overline{12}$	99 $N02\overline{02}$	108 $N1\bar{2}1\bar{2}$
99 $N02\overline{02}$	100 $N10\overline{10}$	98 $N01\overline{01}$	109 $N2\bar{1}2\bar{1}$
100 $N10\overline{10}$	101 $N21\overline{21}$	103 $N20\overline{20}$	45 $N10\bar{1}$
101 $N21\overline{21}$	99 $N02\overline{02}$	102 $N12\overline{12}$	53 $N\overline{02}1$
102 $N12\overline{12}$	103 $N20\overline{20}$	101 $N21\overline{21}$	50 $N\overline{01}2$
103 $N20\overline{20}$	98 $N01\overline{01}$	100 $N10\overline{10}$	47 $N20\bar{2}$
104 $N\bar{0}1\bar{0}1$	105 $N\bar{1}2\bar{1}2$	104 $N\bar{0}1\bar{0}1$	107 $N0\bar{1}0\bar{1}$
105 $N\bar{1}2\bar{1}2$	106 $N\bar{2}1\bar{2}1$	106 $N\bar{2}1\bar{2}1$	94 $N\bar{1}0\overline{10}$
106 $N\bar{2}1\bar{2}1$	104 $N\bar{0}1\bar{0}1$	105 $N\bar{1}2\bar{1}2$	97 $N\bar{2}0\overline{20}$
107 $N0\bar{1}0\bar{1}$	108 $N1\bar{2}1\bar{2}$	107 $N0\bar{1}0\bar{1}$	110 $N\bar{0}1\bar{0}1\bar{0}$
108 $N1\bar{2}1\bar{2}$	109 $N2\bar{1}2\bar{1}$	109 $N2\bar{1}2\bar{1}$	70 $N1\bar{0}1$
109 $N2\bar{1}2\bar{1}$	107 $N0\bar{1}0\bar{1}$	108 $N1\bar{2}1\bar{2}$	72 $N2\bar{0}2$
110 $N\bar{0}1\bar{0}1\bar{0}$	110 $N\bar{0}1\bar{0}1\bar{0}$	110 $N\bar{0}1\bar{0}1\bar{0}$	104 $N\bar{0}1\bar{0}1$

$f(x) = (2,3,4)(5,6,7)(8,11,12)(9,10,13)(14,17,18)(15,16,19)$
$(20,23,24)(21,22,25)(26,29,30)(27,28,31)(32,37,34)(33,35,36)(38,41,40)$
$(39,42,43)(44,48,47)(45,49,46)(50,54,55)(51,52,53)(56,58,61)(57,60,59)$
$(62,65,64)(66,67,63)(68,71,72)(69,70,73)(74,77,78)(75,76,79)(80,84,85)$
$(81,82,83)(86,90,91)(87,88,89)(92,96,97)(93,94,95)(98,102,103)(99,100,101)$
$(104,105,106)(107,108,109)$.

$f(y) = (3,4)(6,7)(8,9)(10,12)(11,13)(14,15)(16,18)(17,19)(20,21)$
$(22,24)(23,25)(26,27)(28,30)(29,31)(32,36)(33,34)(35,37)(38,42)(39,41)$
$(40,43)(44,46)(45,47)(48,49)(50,53)(51,55)(52,54)(56,59)(57,61)(58,60)$

(62, 66)(63, 65)(64, 67)(68, 69)(70, 72)(71, 73)(74, 75)(76, 78)(77, 79)(80, 83)
(81, 85)(82, 84)(86, 87)(88, 91)(89, 90)(92, 93)(94, 97)(95, 96)(98, 99)(100, 103)
(101, 102)(105, 106)(108, 109).

Addition with $f(t_0)$.

$f(t_0) = (1,2,5)(3,10,22)(4,12,24)(6,28,16)(7,30,18)(8,14,44)$
$(9,15,46)(11,41,37)(13,39,35)(17,54,42)(19,52,38)(20,68,57)(21,69,61)$
$(23,73,65)(25,71,63)(26,56,74)(27,59,75)(29,84,79)(31,82,77)(32,95,88)$
$(33,55,58)(34,51,60)(36,96,91)(40,64,81)(43,67,85)(45,92,100)(47,93,103)$
$(50,90,102)(70,98,108)(53,89,101)(62,86,83)(66,87,80)(76,105,94)(78,106,97)$
$(99,109,72)(104,107,110)$.

So it is imply:

$f(t_1) = f({t_0}^x) = f({t_0}^{f(x)}) = (1,3,6)(2,8,20)$
$(4,13,25)(5,26,14)(7,31,19)(9,42,36)(10,16,45)(11,17,48)$
$(12,40,34)(15,53,41)(18,55,43)(21,72,66)(22,70,56)(23,71,60)$
$(24,69,64)(27,83,78)(28,57,76)(29,58,77)(30,85,75)(32,52,59)$
$(33,97,86)(35,50,61)(37,93,89)(38,62,82)(39,63,80)(44,94,98)$
$(49,96,101)(51,87,99)(54,91,103)(65,90,81)(67,88,84)(68,100,107)$
$(73,102,109)(74,104,92)(79,106,95)(105,108,110)$.

$f(t_2) = f({t_1}^x) = f({t_1}^{f(x)}) = (1,4,7)(2,9,21)(3,11,23)$
$(5,27,15)(6,29,17)(8,38,32)(10,43,33)(12,18,47)(13,19,49)(14,50,39)$
$(16,51,40)(20,70,62)(22,68,67)(24,72,59)(25,73,58)(26,80,76)(28,81,74)$
$(30,61,78)(31,60,79)(34,94,87)(35,92,90)(36,54,56)(37,53,57)(41,65,83)$
$(42,66,84)(46,97,99)(48,95,102)(52,88,100)(55,86,98)(63,89,85)(64,91,82)$
$(69,103,107)(71,101,108)(75,104,93)(77,105,96)(106,109,110)$.

It readily checks that the order of $\langle x, y, t\rangle$, a subgroup of a symmetric group S_{110} action on the 110 right cosets of N in G, is 660. Visibly $|x| = 3$ and $|y| = 2$, additionally $|xy| = 2$ and $[x,y]^2 = 1$, hence $\langle x, y\rangle \cong S_3$. If we conjugate t by S_3 we see that t has exactly 3 conjugates. We conclude that $\langle x, y, t\rangle$ is a homomorphic image of the progenitor $3^{*3} : S_3$. Thus if the original six relation hold in $\langle x, y, t\rangle$, then $\langle x, y, t\rangle$ is a homomorphic image of G and this will give $|G| \geq |\langle x, y, t\rangle| = 660$.

Verify the relation (1) $t_2t_1t_0t_2t_1t_0 = \{e\} = 1$

Verify the relation (2) $t_0t_1t_0t_1t_0 = (0,1)$. By multiply the permutations listed above,

$t_0t_1t_0t_1t_0 = (2,3)(5,6)(8,10)(9,11)(12,13)(14,16)(15,17)(18,19)(20,22)(21,23)$
$(24,25)(26,28)(27,29)(30,31)(32,33)(34,35)(36,37)(38,43)(39,40)(41,42)(44,45)(46,48)$
$(47,49)(50,51)(52,55)(53,54)(56,57)(58,59)(60,61)(62,67)(63,64)(65,66)(68,70)(69,71)$
$(72,73)(74,76)(75,77)(78,79)(80,81)(82,85)(83,84)(86,88)(87,90)(89,91)(92,94)(93,96)$
$(95,97)(98,100)(99,102)(101,103)(104,105)(107,108)$
$t_0^{t_0t_1t_0t_1t_0} = t_1 \qquad t_1^{t_0t_1t_0t_1t_0} = t_0 \qquad t_2^{t_0t_1t_0t_1t_0} = t_2$

So $t_0t_1t_0t_1t_0$ acts as the permutation (0,1).

Verify the relation (3) $t_0t_1t_1t_0t_1t_1t_0t_1t_1t_0t_1t_1t_0t_1t_1 = \{e\} = 1$

Verify the relation (4) $t_1t_2t_2t_0t_1t_1t_2t_1t_1t_1t_2t_2t_0t_1t_1 = (0,1,2)$. By multiply the permutations listed above,

$t_1t_2t_2t_0t_1t_1t_2t_1t_1t_1t_2t_2t_0t_1t_1 = (2,3,4)(5,6,7)$
$(8,11,12)(9,10,13)(14,17,18)(15,16,19)(20,23,24)(21,22,25)(26,29,30)(27,28,31)$
$(32,37,34)(33,35,36)(38,41,40)(39,42,43)(44,48,47)(45,49,46)(50,54,55)(51,52,53)$
$(56,58,61)(57,60,59)(62,65,64)(63,66,67)(68,71,72)(69,70,73)(74,77,78)(75,76,79)$
$(80,84,85)(81,82,83)(86,90,91)(87,88,89)(92,96,97)(93,94,95)(98,102,103)(99,100,101)$
$(104,105,106)(107,108,109)$
$t_0^{t_1t_2t_2t_0t_1t_1t_2t_1t_1t_1t_2t_2t_0t_1t_1} = t_1 \qquad t_1^{t_1t_2t_2t_0t_1t_1t_2t_1t_1t_1t_2t_2t_0t_1t_1} = t_2$
$t_2^{t_1t_2t_2t_0t_1t_1t_2t_1t_1t_1t_2t_2t_0t_1t_1} = t_0$

So, $t_1t_2t_2t_0t_1t_1t_2t_1t_1t_1t_2t_2t_0t_1t_1$ acts as the permutation (0, 1, 2).

Verify the relation (5) $t_0t_2t_1t_2t_0t_2t_1t_2t_0t_2 = (0,1)$. By multiply the permutations listed above,

$t_0t_2t_1t_2t_0t_2t_1t_2t_0t_2 = (2,3)(5,6)(8,10)(9,11)(12,13)(14,16)(15,17)$
$(18,19)(20,22)(21,23)(24,25)(26,28)(27,29)(30,31)(32,33)(34,35)(36,37)(38,43)$
$(39,40)(41,42)(44,45)(46,48)(47,49)(50,51)(52,55)(53,54)(56,57)(58,59)(60,61)$
$(62,67)(63,64)(65,66)(68,70)(69,71)(72,73)(74,76)(75,77)(78,79)(80,81)(82,85)(83,84)$
$(86,88)(87,90)(89,91)(92,94)(93,96)(95,97)(98,100)(99,102)(101,103)(104,105)(107,108)$
$t_0^{t_0t_2t_1t_2t_0t_2t_1t_2t_0t_2} = t_1 \qquad t_1^{t_0t_2t_1t_2t_0t_2t_1t_2t_0t_2} = t_0 \qquad t_2^{t_0t_2t_1t_2t_0t_2t_1t_2t_0t_2} = t_2$

So, $t_0t_2t_1t_2t_0t_2t_1t_2t_0t_2$ acts as the permutation (0,1).

Verify the relation (6) $t_0t_2t_2t_1t_2t_2t_0t_2t_2 = (0,1)$. By multiply the permutations listed above,

$t_0t_2t_2t_1t_2t_2t_0t_2t_2 = (2,3)(5,6)(8,10)(9,11)(12,13)(14,16)(15,17)$
$(18,19)(20,22)(21,23)(24,25)(26,28)(27,29)(30,31)(32,33)(34,35)(36,37)(38,43)$

(39, 40)(41, 42)(44, 45)(46, 48)(47, 49)(50, 51)(52, 55)(53, 54)(56, 57)(58, 59)(60, 61)
(62, 67)(63, 64)(65, 66)(68, 70)(69, 71)(72, 73)(74, 76)(75, 77)(78, 79)(80, 81)(82, 85)(83, 84)
(86, 88)(87, 90)(89, 91)(92, 94)(93, 96)(95, 97)(98, 100)(99, 102)(101, 103)(104, 105)(107, 108)
$t_0^{t_0t_2t_2t_1t_2t_2t_0t_2t_2} = t_1$ $\quad t_1^{t_0t_2t_2t_1t_2t_2t_0t_2t_2} = t_0$ $\quad t_2^{t_0t_2t_2t_1t_2t_2t_0t_2t_2} = t_2$
So, $t_0t_2t_2t_1t_2t_2t_0t_2t_2$ acts as the permutation (0,1).

Now, we want to show that $| G |= L_2(11)$.

The Homomorphism Image Of G

$\bar{G} = f(3^{*3} : S_3) = \langle f(x), f(y), f(t_0), f(t_1), f(t_2)\rangle$.
However;
$\{f(t_1), f(t_2)\} \in \{f(x), f(y), f(t_0)\}$.
So it is imply that: $\bar{G} = f(3^{*3} : S_3) = \langle f(x), f(y), f(t_0)\rangle$.
If the addition relation hold in $\langle f(x), f(y), f(t_0)\rangle$ then:

$$f\left(\frac{3^{*3} : S_3}{[(0,1)t_0]^5 = 1}\right) = \langle f(x), f(y), f(t_0)\rangle.$$

The addition relation, namely $(0,1)f(t_1)f(t_2^{-1})f(t_0)f(t_2^{-1})$ must equal to an identity
Let $\mu = f(t_0)f(t_1)f(t_0)f(t_1)f(t_0) = (2,3,4)(5,6,7)(8,11,12)(9,10,13)$
$(14,17,18)(15,16,19)(20,23,24)(21,22,25)(26,29,30)(27,28,31)(32,37,34)(33,35,36)$
$(38,41,40)(39,42,43)(44,48,47)(45,49,46)(50,54,55)(51,52,53)(56,58,61)(57,60,59)$
$(62,65,64)(63,66,67)(68,71,72)(69,70,73)(74,77,78)(75,76,79)(80,84,85)(81,82,83)$
$(86,90,91)(87,88,89)(92,96,97)(93,94,95)(98,102,103)(99,100,101)(104,105,106)$
$(107,108,109)$.
Then $(0,1)\mu = \{e\} = 1$ Thus the relation holds in $\bar{G}$.
$\Longrightarrow \bar{G}$ is a homomorphism image of G.
Now, by the First Isomorphic Theorem $|G/Kerf| \cong \bar{G}$.
$\Longrightarrow |G| \geq |\bar{G}|$. It is easily verified that $|\bar{G}| = |\langle f(x), f(y), f(t_0)\rangle| = 660$.
$\Longrightarrow |G| \geq 660$
But we have seen that $|G| \leq 660$. Hence; $|G| = 660$.

Next, we need to show that $G = \frac{3^{*3}:S_3}{[(0,1)t_0]^5}$ is isomorphic to $L_2(11)$.

$$G = \frac{3^{*3}:S_3}{[(0,1)t_0]^5} \cong L_2(11)$$

$S_3 = \langle x, y\rangle$, where $x \sim (0,1,2)$ and $y \sim (1,2)$.

We construct a homomorphism ϕ from the progenitor $3^{*3} : S_3$ to $L_2(11)$ be defining

$\phi(x) = (\frac{\eta+3}{-\eta+1}) = (1,\infty,10)(2,6,7)(3,8,0)(4,5,9)$

$\phi(y) = (\frac{-\eta+9}{3\eta+1}) = (1,2)(3,5)(4,8)(6,10)(7,\infty)(9,0)$

Since the order of $\phi(x), \phi(y)$, and $\phi(x)\phi(y)$ are 3, 2 and 2, respectively,

$N = \langle\phi(x), \phi(x)\rangle \cong S_3$

We now let

$\phi(t_0) = (\frac{8\eta+4}{5\eta+4}) = (6,7,\infty)(1,5,0)(2,3,9)(4,7,10)$

It is readily verified that $L_2(11) \cong \langle\phi(x), \phi(y), \phi(t_0)\rangle$

We now show that ϕ preserves the operation of $3^{*3} : S_3$.

We find that $|\phi(t_0)^N| = 3$ and

$\phi(t) = \phi(t_0) = (6,8,\infty)(1,5,0)(2,3,9)(4,7,10)$

$\phi(t_1) = \phi(t_0^x) = (1,5,2)(3,\infty,9)(4,6,8)(7,0,10)$

$\phi(t_2) = \phi(t_1^x) = (1,2,3)(4,8,10)(5,7,0)(6,\infty,9)$

and the N permutes the three images of t_0, by conjugation, as the group S_3 given by

$\phi(x) : (\phi(t_0), \phi(t_1), \phi(t_2)$ and

$\phi(y) : (\phi(t_1), \phi(t_2)$. Thus $\phi(3^{*3} : S_3) = L_2(11)$.

Now the addition relation given by $[(0,1)t_0]^5 = 1 \Leftrightarrow t_0t_1t_0t_1t_0 = (0,1)$ is satisfied in $L_2(11)$, because $\phi(t_0)\phi(t_1)\phi(t_0)\phi(t_1)\phi(t_0) = (1,6)(2,12)(3,9)(4,11)(5,8)(7,10)$ acts as $(\phi(t_0), \phi(t_1))$, by conjugation,on the images of the three symmetric generators. This shows that $L_2(11)$ is an image of G Thus $|G| \geq |L_2(11)|$; but $|G| \leq 660 = L_2(11)$; and so the equality holds and $G \cong L_2(11)$.

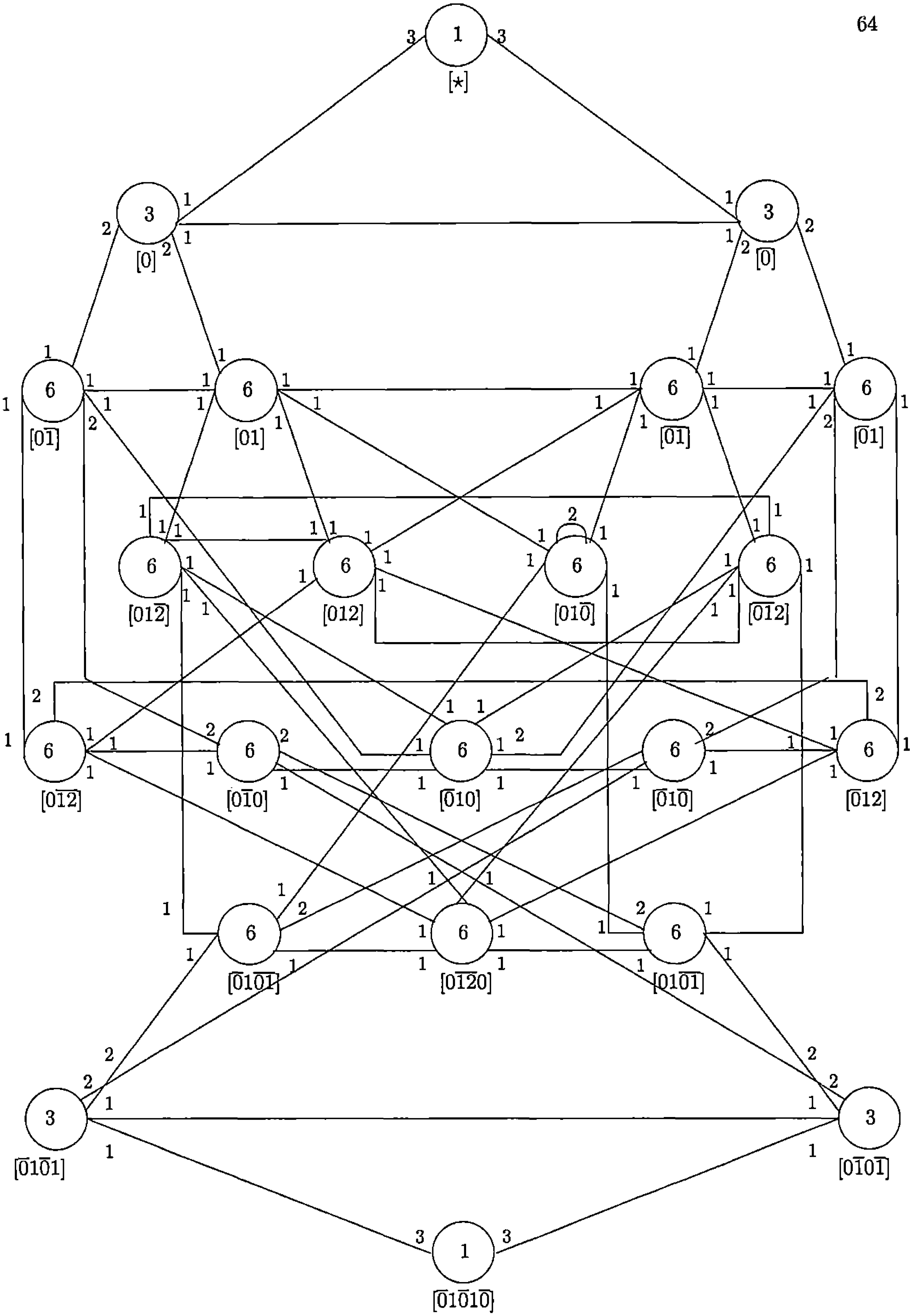

Figure 8.1: The Cayley Diagram of $L_2(11)$ over $3^{*3} : S_3$

Chapter 9

Construction of $M_{12}:2$

We consider the progenitor:

$$2^{*4}:S_4 \cong \langle\, x,y,t|x^4=y^2=(yx)^3=t^2=[t,y]=[t^x,y]\rangle$$

where $x\sim(0,1,2,3)$, $y\sim(2,3)$, $t\sim t_0$. The following relation may used for the purpose of manual double coset enumeration:

1. $[(0,1,2,3)t_0]^{10}=1$
2. $[(0,1,2)t_0]^{11}=1$
3. $[(0,1)(2,3)t_0t_1^{-1}]^{10}=1$
4. $[(0,1)t_0]^{10}=1$
5. $[(0,1)t_0t_2]^{12}=1$

We now perform the manual double coset enumeration of,

$$G\cong\frac{2^{*4}:S_4}{[(0,1)t_0]^{10}=1}$$

The relations 1, 2, and 3 is very helpful to find the manual double coset enumeration. Expanding relation 1: $[(0,1,2,3)t_0]^{10}=1$.

Let $x = (0,1,2,3)$ then $[(0,1,2,3)t_0]^{10} = 1 = [xt_0]^{10}$

$$\begin{aligned}
[xt_0]^{10} &= 1 \\
xt_0xt_0xt_0xt_0xt_0xt_0xt_0xt_0xt_0xt_0 &= 1 \\
x^{10}t_0^{x^9}t_0^{x^8}t_0^{x^7}t_0^{x^6}t_0^{x^5}t_0^{x^4}t_0^{x^3}t_0^{x^2}t_0^{x}t_0 &= 1 \\
(0,2)(1,3)t_1t_0t_3t_2t_1t_0t_3t_2t_1t_0 &= 1 \\
(0,2)(1,3)t_1t_0t_3t_2t_1 &= t_0t_1t_2t_3t_0 \\
Nt_1t_0t_3t_2t_1 &= Nt_0t_1t_2t_3t_0
\end{aligned}$$

Expanding relation 2: $[(0,1,2)t_0]^{11} = 1$.
Let $y = (0,1,2)$ then $[(0,1,2)t_0]^{11} = 1 = [yt_0]^{11}$

$$\begin{aligned}
[yt_0]^{11} &= 1 \\
yt_0yt_0yt_0yt_0yt_0yt_0yt_0yt_0yt_0yt_0yt_0 &= 1 \\
y^{11}t_0^{y^{10}}t_0^{y^9}t_0^{y^8}t_0^{y^7}t_0^{y^6}t_0^{y^5}t_0^{y^4}t_0^{y^3}t_0^{y^2}t_0^{y}t_0 &= 1 \\
(0,2,1)t_1t_0t_2t_1t_0t_2t_1t_0t_2t_1t_0 &= 1 \\
(0,2,1)t_1t_0t_2t_1t_0t_2 &= t_0t_1t_2t_0t_1 \\
Nt_1t_1t_0t_2t_1t_0t_2 &= Nt_0t_1t_2t_0t_1
\end{aligned}$$

Expanding relation 3: $[(0,1)t_0]^{10} = 1$.
Let $z = (0,1)$ then $[(0,1)t_0]^{10} = 1 = [zt_0]^{10}$

$$\begin{aligned}
[zt_0]^{10} &= 1 \\
zt_0zt_0zt_0zt_0zt_0zt_0zt_0zt_0zt_0zt_0 &= 1 \\
z^{10}t_0^{z^9}t_0^{z^8}t_0^{z^7}t_0^{z^6}t_0^{z^5}t_0^{z^4}t_0^{z^3}t_0^{z^2}t_0^{z}t_0 &= 1 \\
t_1t_0t_1t_0t_1t_0t_1t_0t_1t_0 &= 1 \\
t_1t_0t_1t_0t_1 &= t_0t_1t_0t_1t_0 \\
Nt_1t_0t_1t_0t_1 &= Nt_0t_1t_0t_1t_0
\end{aligned}$$

Manual double coset enumeration

We note that; $NeN = \{Nen|n \in N\} = \{Nn|n \in N\} = \{N\}$. Thus; NeN, denoted by $[\star]$, contains one single coset. N is transitive on $\{0,1,2,3\}$, so it has a single orbit $\{0, 1, 2, 3\}$. We takes a representative, say $\{0\}$ from the orbit, and find to which the double cosets Nt_0 belong? Clearly; $Nt_0 \in Nt_0N = \{Nt_0^n | n \in N\} = \{Nt_0, Nt_1, Nt_2, Nt_3\}$.

Now consider the cosets stabilizer of $N^{(0)}$. The cosets stabilizer of Nt_0 is equal to the point stabilizer N^0, given by:

$$\begin{aligned} N^{(0)} &= N^0. \\ &= \langle e, (2,3), (1,2,3) \rangle. \\ &= \{e, (2,3), (1,2), (1,3), (1,2,3), (1,3,2)\}. \end{aligned}$$

Then the number of the singles coset in the double coset Nt_0N is at most: $\frac{|N|}{|N^{(0)}|} = \frac{24}{6} = 4$.

The orbit of $N^{(0)}$ on $\{0, 1, 2, 3\}$ are $\{0\}$ and $\{1, 2, 3\}$. We takes a representative from each orbit, and find to which the double cosets Nt_0t_0 and Nt_0t_1 belong? However, $Nt_0t_0 = N \in [\star]$, then one symmetric generator goes back to the double coset NeN, and three symmetric generators go to the next double coset $Nt_0t_1 \in [01]$.

Length 2.

1. Now, we consider the cosets stabilizer $N^{(01)}$.

$N^{(01)} = \{e, (2,3)\}$, then the number of the single cosets in the double coset Nt_0t_1N is at most: $\frac{|N|}{|N^{(01)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(01)}$ on $\{0, 1, 2, 3\}$ are $\{2, 3\}$, $\{0\}$ and $\{1\}$. We take a representatives from each orbit, and find to which the double cosets $Nt_0t_1t_0$, $Nt_0t_1t_1$ and $Nt_0t_1t_2$ belong? However, $Nt_0t_1t_1 = Nt_0 \in [0]$, then one symmetric generator goes back to the double coset Nt_0N, one symmetric generator goes to $Nt_0t_1t_0$ and two symmetric generator go to $Nt_0t_1t_2$.

Length 3.

1. Now, we consider the cosets stabilizer $N^{(010)}$.

$N^{(010)} = \{e, (2,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0N$ is at most: $\frac{|N|}{|N^{(010)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(010)}$ on $\{0, 1, 2, 3\}$ are $\{2, 3\}$, $\{0\}$ and $\{1\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_0$, $Nt_0t_1t_0t_1$ and $Nt_0t_1t_0t_2$ belong? However, $Nt_0t_1t_0t_0 = Nt_0t_1 \in [01]$, then one symmetric generator goes back to the double coset Nt_0t_1N, one symmetric generator goes to $Nt_0t_1t_0t_1$ and two symmetric generator go to $Nt_0t_1t_0t_2$.

2. Now, we consider the cosets stabilizer $N^{(012)}$.

$N^{(012)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2N$ is at most: $\frac{|N|}{|N^{(012)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_2$, $Nt_0t_1t_2t_0$, $Nt_0t_1t_2t_1$ and $Nt_0t_1t_2t_3$ belong? However, $Nt_0t_1t_2t_2 = Nt_0t_1 \in [01]$, then one symmetric generator goes back to the double coset Nt_0t_1N, one symmetric generator goes to $Nt_0t_1t_2t_0$, one symmetric generator goes to $Nt_0t_1t_2t_1$ and one symmetric generator goes to $Nt_0t_1t_2t_3$.

Length 4.

1. Now, we consider the cosets stabilizer $N^{(0101)}$.

$N^{(0101)} = \{e, (2,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1N$ is at most: $\frac{|N|}{|N^{(0101)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(0101)}$ on $\{0,1,2,3\}$ are $\{2,3\}$, $\{0\}$ and $\{1\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_1$, $Nt_0t_1t_0t_1t_0$ and $Nt_0t_1t_0t_1t_2$ belong? However, $Nt_0t_1t_0t_1t_1 = Nt_0t_1t_0 \in [010]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_0$ and two symmetric generators go to $Nt_0t_1t_0t_1t_2$.

2. Now, we consider the cosets stabilizer $N^{(0102)}$.

$N^{(0102)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2N$ is at most: $\frac{|N|}{|N^{(0102)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0102)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_2$, $Nt_0t_1t_0t_2t_0$ $Nt_0t_1t_0t_2t_1$ and $Nt_0t_1t_0t_2t_3$ belong? However, $Nt_0t_1t_0t_2t_2 = Nt_0t_1t_0 \in [010]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3$.

3. Now, we consider the cosets stabilizer $N^{(0120)}$.
$N^{(0120)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0N$ is at most: $\frac{|N|}{|N^{(0120)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0120)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_0$, $Nt_0t_1t_2t_0t_1$, $Nt_0t_1t_2t_0t_2$ and $Nt_0t_1t_2t_0t_3$ belong? However, $Nt_0t_1t_2t_0t_0 = Nt_0t_1t_2 \in [012]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2$ and one symmetric generator goes to $Nt_0t_1t_2t_0t_3$.

4. Now, we consider the cosets stabilizer $N^{(0121)}$.
$N^{(0121)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0N$ is at most: $\frac{|N|}{|N^{(0121)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0121)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_0$, $Nt_0t_1t_2t_1t_1$, $Nt_0t_1t_2t_1t_2$ and $Nt_0t_1t_2t_1t_3$ belong? However, $Nt_0t_1t_2t_1t_1 = Nt_0t_1t_2 \in [012]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2$ and one symmetric generator goes to $Nt_0t_1t_2t_1t_3$.

5. Now, we consider the cosets stabilizer $N^{(0123)}$.
$N^{(0123)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0N$ is at most: $\frac{|N|}{|N^{(0123)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0123)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_0$, $Nt_0t_1t_2t_3t_1$, $Nt_0t_1t_2t_3t_2$ and $Nt_0t_1t_2t_3t_3$ belong? However, $Nt_0t_1t_2t_3t_3 = Nt_0t_1t_2 \in [012]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_0$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1$ and one symmetric generator goes to $Nt_0t_1t_2t_3t_2$.

Length 5.

1. Now, we consider the cosets stabilizer $N^{(01010)}$.
We know, $t_1t_0t_1t_0t_1 = t_0t_1t_0t_1t_0$ then $Nt_1t_0t_1t_0t_1 = Nt_0t_1t_0t_1t_0$.
$N^{(01010)} = \{e, (0,1), (2,3), (0,1)(2,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_0N$ is at most: $\frac{|N|}{|N^{(01010)}|} = \frac{24}{4} = 6$.

The orbit of $N^{(0101)}$ on $\{0,1,2,3\}$ are $\{2,3\}$, and $\{0,1\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_0t_0$ and $Nt_0t_1t_0t_1t_0t_2$? However, $Nt_0t_1t_0t_1t_0t_0 = Nt_0t_1t_0t_1 \in [0101]$, then two symmetric generators go back to the double coset $Nt_0t_1t_0t_1N$, two symmetric generator go to $Nt_0t_1t_0t_1t_0t_2$.

2. Now, we consider the cosets stabilizer $N^{(01012)}$.
$N^{(01012)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2N$ is at most: $\frac{|N|}{|N^{(01012)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01012)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_0$, $Nt_0t_1t_0t_1t_2t_1$, $Nt_0t_1t_0t_1t_2t_2$ and $Nt_0t_1t_0t_1t_2t_3$ belong? However, $Nt_0t_1t_0t_1t_2t_2 = Nt_0t_1t_0t_1 \in [0101]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3$.

3. Now, we consider the cosets stabilizer $N^{(01020)}$.
$N^{(01024)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0N$ is at most: $\frac{|N|}{|N^{(01020)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01024)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_0$, $Nt_0t_1t_0t_2t_0t_1$, $Nt_0t_1t_0t_2t_0t_2$ and $Nt_0t_1t_0t_2t_0t_3$ belong? However, $Nt_0t_1t_0t_2t_0t_0 = Nt_0t_1t_0t_2 \in [0102]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3$.

4. Now, we consider the cosets stabilizer $N^{(01021)}$.
$N^{(01021)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_1N$ is at most: $\frac{|N|}{|N^{(01021)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01021)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_0$, $Nt_0t_1t_0t_2t_1t_1$, $Nt_0t_1t_0t_2t_1t_2$ and $Nt_0t_1t_0t_2t_1t_3$ belong? However, $Nt_0t_1t_0t_2t_1t_1 = Nt_0t_1t_0t_2 \in [0102]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3$.

5. Now, we consider the cosets stabilizer $N^{(01023)}$.

$N^{(01023)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3N$ is at most: $\frac{|N|}{|N^{(01023)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01023)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_0$, $Nt_0t_1t_0t_2t_3t_1$, $Nt_0t_1t_0t_2t_3t_2$ and $Nt_0t_1t_0t_2t_3t_3$ belong? However, $Nt_0t_1t_0t_2t_3t_3 = Nt_0t_1t_0t_2 \in [0102]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2$.

6. Now, we consider the cosets stabilizer $N^{(01201)}$.

$N^{(01201)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_1N$ is at most: $\frac{|N|}{|N^{(01201)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01241)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_1t_0$, $Nt_0t_1t_2t_0t_1t_1$, $Nt_0t_1t_2t_0t_1t_2$ and $Nt_0t_1t_2t_0t_1t_3$ belong? However, $Nt_0t_1t_2t_0t_1t_1 = Nt_0t_1t_0t_2 \in [0120]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0N$,
and we know, $t_0t_1t_2t_0t_1t_2 = (1,0,2)t_1t_0t_2t_1t_0$ then $Nt_0t_1t_2t_0t_1t_2 = Nt_1t_0t_2t_1t_0 \in [01201]$ one symmetric generator goes back to itself $Nt_0t_1t_2t_0t_1$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3$.

7. Now, we consider the cosets stabilizer $N^{(01202)}$.

$N^{(01202)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_1N$ is at most: $\frac{|N|}{|N^{(01202)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01202)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_2t_0$, $Nt_0t_1t_2t_0t_2t_1$, $Nt_0t_1t_2t_0t_2t_2$ and $Nt_0t_1t_2t_0t_2t_3$ belong? However, $Nt_0t_1t_2t_0t_2t_2 = Nt_0t_1t_0t_2 \in [0120]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_0$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3$.

8. Now, we consider the cosets stabilizer $N^{(01203)}$.
$N^{(01203)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_3N$ is at most: $\frac{|N|}{|N^{(01203)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01203)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_3t_0$, $Nt_0t_1t_2t_0t_3t_1$, $Nt_0t_1t_2t_0t_3t_2$ and $Nt_0t_1t_2t_0t_3t_3$ belong? However, $Nt_0t_1t_2t_0t_3t_3 = Nt_0t_1t_2t_0 \in [0120]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_0$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_2$.

9. Now, we consider the cosets stabilizer $N^{(01210)}$.
$N^{(01210)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_0N$ is at most: $\frac{|N|}{|N^{(01210)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01210)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_0t_0$, $Nt_0t_1t_2t_1t_0t_1$, $Nt_0t_1t_2t_1t_0t_2$ and $Nt_0t_1t_2t_1t_0t_3$ belong? However, $Nt_0t_1t_2t_1t_0t_0 = Nt_0t_1t_0t_2 \in [0121]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_1$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_3$.

10. Now, we consider the cosets stabilizer $N^{(01212)}$.
$N^{(01212)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_2N$ is at most: $\frac{|N|}{|N^{(01212)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01212)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_2t_0$, $Nt_0t_1t_2t_1t_2t_1$, $Nt_0t_1t_2t_1t_2t_2$ and $Nt_0t_1t_2t_1t_2t_3$ belong? However, $Nt_0t_1t_2t_1t_2t_2 = Nt_0t_1t_0t_2 \in [0121]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_0$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3$.

11. Now, we consider the cosets stabilizer $N^{(01213)}$.
$N^{(01213)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_3N$ is at most: $\frac{|N|}{|N^{(01213)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01213)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_3t_0$, $Nt_0t_1t_2t_1t_3t_1$, $Nt_0t_1t_2t_1t_3t_2$ and $Nt_0t_1t_2t_1t_3t_3$ belong? However, $Nt_0t_1t_2t_1t_3t_3 = Nt_0t_1t_0t_2 \in [0121]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_0$.

12. Now, we consider the cosets stabilizer $N^{(01230)}$.
We know, $t_0t_1t_2t_3t_0 = (1,3)(2,0)t_1t_0t_3t_2t_1$ then,
$Nt_1t_0t_1t_0t_1 = Nt_0t_1t_0t_1t_0$ $N^{(01213)} = \{e, (0,1)(2,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_0N$ is at most: $\frac{|N|}{|N^{(01213)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(01230)}$ on $\{0,1,2,3\}$ are $\{0,1\}$, and $\{2,3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_0t_0$, $Nt_0t_1t_2t_3t_0t_2$, belong? However, $Nt_0t_1t_2t_3t_0t_0 = Nt_0t_1t_0t_2 \in [0123]$, then two symmetric generators go back to the double coset $Nt_0t_1t_2t_3N$, two symmetric generator go to $Nt_0t_1t_2t_3t_0t_2$.

13. Now, we consider the cosets stabilizer $N^{(01231)}$.
$N^{(01231)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_1N$ is at most: $\frac{|N|}{|N^{(01231)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01231)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_1t_0$, $Nt_0t_1t_2t_3t_1t_1$, $Nt_0t_1t_2t_3t_1t_2$ and $Nt_0t_1t_2t_3t_1t_3$ belong? However, $Nt_0t_1t_2t_3t_1t_1 = Nt_0t_1t_2t_3 \in [0123]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_0$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_3$.

14. Now, we consider the cosets stabilizer $N^{(01232)}$.
$N^{(01232)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_2N$ is at most: $\frac{|N|}{|N^{(01232)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01232)}$ on $\{0, 1, 2, 3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_2t_0$, $Nt_0t_1t_2t_3t_2t_1$, $Nt_0t_1t_2t_3t_2t_2$ and $Nt_0t_1t_2t_3t_2t_3$ belong? However, $Nt_0t_1t_2t_3t_2t_2 = Nt_0t_1t_2t_3 \in [0123]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_0$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_3$.

Length 6.

1. Now, we consider the cosets stabilizer $N^{(010102)}$.
We know, $t_1t_0t_1t_0t_1t_2 = t_0t_1t_0t_1t_0t_2$ then $Nt_1t_0t_1t_0t_1t_2 = Nt_0t_1t_0t_1t_0t_2$. $N^{(010102)} = \{e, (0, 1)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_0t_2N$ is at most: $\frac{|N|}{|N^{(010102)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(0101)}$ on $\{0, 1, 2, 3\}$ are $\{2\}$, $\{3\}$and $\{0, 1\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_0t_2t_0$, $Nt_0t_1t_0t_1t_0t_2t_2$ and $Nt_0t_1t_0t_1t_0t_2t_3$?
However, $Nt_0t_1t_0t_1t_0t_2t_2 = Nt_0t_1t_0t_1t_0 \in [01010]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_0N$, two symmetric generator go to $Nt_0t_1t_0t_1t_0t_2t_0$, one symmetric generator goes to $Nt_0t_1t_0t_1t_0t_2t_3$.

2. Now, we consider the cosets stabilizer $N^{(010120)}$.
$N^{(010120)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_0N$ is at most: $\frac{|N|}{|N^{(010120)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010120)}$ on $\{0, 1, 2, 3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_0t_0$, $Nt_0t_1t_0t_1t_2t_0t_1$, $Nt_0t_1t_0t_1t_2t_0t_2$ and $Nt_0t_1t_0t_1t_2t_0t_3$ belong? However, $Nt_0t_1t_0t_1t_2t_0t_0 = Nt_0t_1t_0t_1t_2 \in [01012]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_1$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_3$.

3. Now, we consider the cosets stabilizer $N^{(010121)}$.

$N^{(010121)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_1N$ is at most: $\frac{|N|}{|N^{(010121)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010121)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_1t_0$, $Nt_0t_1t_0t_1t_2t_1t_1$, $Nt_0t_1t_0t_1t_2t_1t_2$ and $Nt_0t_1t_0t_1t_2t_1t_3$ belong? However, $Nt_0t_1t_0t_1t_2t_1t_1 = Nt_0t_1t_0t_1t_2 \in [01012]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_0$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_2$, and one symmetric generator goes toto $Nt_0t_1t_0t_1t_2t_1t_3$.

4. Now, we consider the cosets stabilizer $N^{(010123)}$.

$N^{(010123)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_3N$ is at most: $\frac{|N|}{|N^{(010123)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010123)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_0$, $Nt_0t_1t_0t_1t_2t_3t_1$, $Nt_0t_1t_0t_1t_2t_3t_2$ and $Nt_0t_1t_0t_1t_2t_3t_3$ belong? However, $Nt_0t_1t_0t_1t_2t_3t_3 = Nt_0t_1t_0t_1t_2 \in [01012]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_0$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_2$.

5. Now, we consider the cosets stabilizer $N^{(010201)}$.

$N^{(010201)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_1N$ is at most: $\frac{|N|}{|N^{(010201)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010201)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_1t_0$, $Nt_0t_1t_0t_2t_0t_1t_1$, $Nt_0t_1t_0t_2t_0t_1t_2$ and $Nt_0t_1t_0t_2t_0t_1t_3$ belong? However, $Nt_0t_1t_0t_2t_0t_1t_1 = Nt_0t_1t_0t_1t_2 \in [01020]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_3$.

6. Now, we consider the cosets stabilizer $N^{(010202)}$.

$N^{(010202)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_2N$ is at most: $\frac{|N|}{|N^{(010202)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010202)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_2t_0$, $Nt_0t_1t_0t_2t_0t_2t_1$, $Nt_0t_1t_0t_2t_0t_2t_2$ and $Nt_0t_1t_0t_2t_0t_2t_3$ belong? However, $Nt_0t_1t_0t_2t_0t_2t_2 = Nt_0t_1t_0t_1t_2 \in [01020]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_3$.

7. Now, we consider the cosets stabilizer $N^{(010203)}$.

$N^{(010203)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_3N$ is at most: $\frac{|N|}{|N^{(010203)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010203)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_3t_0$, $Nt_0t_1t_0t_2t_0t_3t_1$, $Nt_0t_1t_0t_2t_0t_3t_2$ and $Nt_0t_1t_0t_2t_0t_3t_3$ belong? However, $Nt_0t_1t_0t_2t_0t_3t_3 = Nt_0t_1t_0t_1t_2 \in [01020]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_2$.

8. Now, we consider the cosets stabilizer $N^{(010210)}$.

$N^{(010210)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_1t_0N$ is at most: $\frac{|N|}{|N^{(010210)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010210)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_0t_0$, $Nt_0t_1t_0t_2t_1t_0t_1$, $Nt_0t_1t_0t_2t_1t_0t_2$ and $Nt_0t_1t_0t_2t_1t_0t_3$ belong? However, $Nt_0t_1t_0t_2t_1t_0t_0 = Nt_0t_1t_0t_2t_0 \in [01021]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_0t_3$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_0t_2$.

9. Now, we consider the cosets stabilizer $N^{(010212)}$.

$N^{(010212)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_1t_2N$

is at most: $\frac{|N|}{|N^{(010212)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010212)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_2t_0$, $Nt_0t_1t_0t_2t_1t_2t_1$, $Nt_0t_1t_0t_2t_1t_2t_2$ and $Nt_0t_1t_0t_2t_1t_2t_3$ belong? However, $Nt_0t_1t_0t_2t_1t_2t_2 = Nt_0t_1t_0t_2t_0 \in [01021]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_2t_3$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_2t_0$.

10. Now, we consider the cosets stabilizer $N^{(010213)}$.

$N^{(010213)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_1t_3N$ is at most: $\frac{|N|}{|N^{(010213)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010213)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_3t_0$, $Nt_0t_1t_0t_2t_1t_3t_1$, $Nt_0t_1t_0t_2t_1t_3t_2$ and $Nt_0t_1t_0t_2t_1t_3t_3$ belong? However, $Nt_0t_1t_0t_2t_1t_3t_3 = Nt_0t_1t_0t_2t_0 \in [01021]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_2$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_0$.

11. Now, we consider the cosets stabilizer $N^{(010230)}$.

$N^{(010230)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_0N$ is at most: $\frac{|N|}{|N^{(010230)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010230)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_0t_0$, $Nt_0t_1t_0t_2t_3t_0t_1$, $Nt_0t_1t_0t_2t_3t_0t_2$ and $Nt_0t_1t_0t_2t_3t_0t_3$ belong? However, $Nt_0t_1t_0t_2t_3t_0t_0 = Nt_0t_1t_0t_2t_3 \in [01023]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_2$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_3$.

12. Now, we consider the cosets stabilizer $N^{(010231)}$.

We know, $t_0t_1t_0t_2t_3t_1 = (0,3)(1,2)(t_0t_1t_2t_0t_3t_1)^{(1,0,3,2)}$ then

$Nt_0t_1t_0t_2t_3t_1 = (0,1)(2,3)(t_3t_0t_1t_3t_2t_0) \in Nt_0t_1t_2t_0t_3t_1$.

$N^{(010231)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_1N$

is at most: $\frac{|N|}{|N^{(010231)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010231)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_1t_0$, $Nt_0t_1t_0t_2t_3t_1t_1$, $Nt_0t_1t_0t_2t_3t_1t_2$ and $Nt_0t_1t_0t_2t_3t_1t_3$ belong? However, $Nt_0t_1t_0t_2t_3t_1t_1 = Nt_0t_1t_0t_2t_3 \in [01023]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3N$, and $Nt_0t_1t_0t_2t_3t_1t_0 = Nt_3t_0t_1t_3t_2 \in [01203]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_3$.

13. Now, we consider the cosets stabilizer $N^{(010232)}$.

$N^{(010232)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_2N$ is at most: $\frac{|N|}{|N^{(010232)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010232)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_2t_0$, $Nt_0t_1t_0t_2t_3t_2t_1$, $Nt_0t_1t_0t_2t_3t_2t_2$ and $Nt_0t_1t_0t_2t_3t_2t_3$ belong? However, $Nt_0t_1t_0t_2t_3t_2t_2 = Nt_0t_1t_0t_2t_3 \in [01023]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_3$.

14. Now, we consider the cosets stabilizer $N^{(012013)}$.

$N^{(012013)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_1t_0N$ is at most: $\frac{|N|}{|N^{(012013)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012013)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_1t_0t_0$, $Nt_0t_1t_2t_0t_1t_0t_1$, $Nt_0t_1t_2t_0t_1t_0t_2$ and $Nt_0t_1t_2t_0t_1t_0t_3$ belong? However, $Nt_0t_1t_2t_0t_1t_0t_0 = Nt_0t_1t_2t_0t_1 \in [01201]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_0t_2$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_0t_3$.

15. Now, we consider the cosets stabilizer $N^{(012013)}$.

$N^{(012013)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_1t_3N$ is at most: $\frac{|N|}{|N^{(012013)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012013)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_1t_3t_0$, $Nt_0t_1t_2t_0t_1t_3t_1$, $Nt_0t_1t_2t_0t_1t_3t_2$ and $Nt_0t_1t_2t_0t_1t_3t_3$ belong? However, $Nt_0t_1t_2t_0t_1t_3t_3 = Nt_0t_1t_2t_0t_1 \in [01201]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_2$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_0$.

16. Now, we consider the cosets stabilizer $N^{(012020)}$.
$N^{(012020)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_2t_0N$ is at most: $\frac{|N|}{|N^{(012020)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012020)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_2t_0t_0$, $Nt_0t_1t_2t_0t_2t_0t_1$, $Nt_0t_1t_2t_0t_2t_0t_2$ and $Nt_0t_1t_2t_0t_2t_0t_3$ belong? However, $Nt_0t_1t_2t_0t_2t_0t_0 = Nt_0t_1t_2t_0t_2 \in [01202]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_0t_2$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_0t_2$.

17. Now, we consider the cosets stabilizer $N^{(012021)}$.
We know, $t_0t_1t_2t_0t_2t_1 = (t_0t_1t_2t_1t_0t_2)^{(1,2,0)}$ then
$Nt_0t_1t_2t_0t_2t_1 = Nt_1t_2t_0t_2t_1t_0 \in Nt_0t_1t_2t_1t_0t_2$.
$N^{(012021)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_2t_1N$ is at most: $\frac{|N|}{|N^{(012021)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012021)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_2t_1t_0$, $Nt_0t_1t_2t_0t_2t_1t_1$, $Nt_0t_1t_2t_0t_2t_1t_2$ and $Nt_0t_1t_2t_0t_2t_1t_3$ belong? However, $Nt_0t_1t_2t_0t_2t_1t_1 = Nt_0t_1t_2t_0t_2 \in [01202]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_2N$, and $Nt_0t_1t_2t_0t_2t_1t_0 = Nt_1t_2t_0t_2t_1 \in [01201]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_1t_3$.

18. Now, we consider the cosets stabilizer $N^{(012023)}$.
$N^{(012023)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_2t_3N$

is at most: $\frac{|N|}{|N^{(012023)}|} = \frac{24}{1} = 24.$

The orbit of $N^{(012023)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_2t_3t_0$, $Nt_0t_1t_2t_0t_2t_3t_1$, $Nt_0t_1t_2t_0t_2t_3t_2$ and $Nt_0t_1t_2t_0t_2t_3t_3$ belong? However, $Nt_0t_1t_2t_0t_2t_3t_3 = Nt_0t_1t_2t_0t_2 \in [01202]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_0$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_2$.

19. Now, we consider the cosets stabilizer $N^{(012030)}$.

$N^{(012030)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_3t_0N$ is at most: $\frac{|N|}{|N^{(012030)}|} = \frac{24}{1} = 24.$

The orbit of $N^{(012030)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_3t_0t_0$, $Nt_0t_1t_2t_0t_3t_0t_1$, $Nt_0t_1t_2t_0t_3t_0t_2$ and $Nt_0t_1t_2t_0t_3t_0t_3$ belong? However, $Nt_0t_1t_2t_0t_3t_0t_0 = Nt_0t_1t_2t_0t_3 \in [01203]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_0t_3$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_0t_2$.

20. Now, we consider the cosets stabilizer $N^{(012032)}$.

$N^{(012032)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_3t_2N$ is at most: $\frac{|N|}{|N^{(012032)}|} = \frac{24}{1} = 24.$

The orbit of $N^{(012032)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_3t_2t_0$, $Nt_0t_1t_2t_0t_3t_2t_1$, $Nt_0t_1t_2t_0t_3t_2t_2$ and $Nt_0t_1t_2t_0t_3t_2t_3$ belong?

However, $Nt_0t_1t_2t_0t_3t_2t_2 = Nt_0t_1t_2t_0t_3 \in [01203]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_2t_3$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_2t_0$.

21. Now, we consider the cosets stabilizer $N^{(012101)}$.

$N^{(012101)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_0t_1N$ is at most: $\frac{|N|}{|N^{(012101)}|} = \frac{24}{1} = 24.$

The orbit of $N^{(012101)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_0t_1t_0$, $Nt_0t_1t_2t_1t_0t_1t_1$, $Nt_0t_1t_2t_1t_0t_1t_2$ and $Nt_0t_1t_2t_1t_0t_1t_3$ belong?
However, $Nt_0t_1t_2t_1t_0t_1t_1 = Nt_0t_1t_2t_1t_0 \in [01210]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_1t_3$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_1t_0$.

22. Now, we consider the cosets stabilizer $N^{(012103)}$.
$N^{(012103)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_0t_3N$ is at most: $\frac{|N|}{|N^{(012103)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012103)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_0t_3t_0$, $Nt_0t_1t_2t_1t_0t_3t_1$, $Nt_0t_1t_2t_1t_0t_3t_2$ and $Nt_0t_1t_2t_1t_0t_3t_3$ belong?
However, $Nt_0t_1t_2t_1t_0t_3t_3 = Nt_0t_1t_2t_1t_0 \in [01210]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_3t_1$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_3t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_3t_0$.

23. Now, we consider the cosets stabilizer $N^{(012120)}$.
$N^{(012120)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_2t_0N$ is at most: $\frac{|N|}{|N^{(012120)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012120)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_2t_0t_0$, $Nt_0t_1t_2t_1t_2t_0t_1$, $Nt_0t_1t_2t_1t_2t_0t_2$ and $Nt_0t_1t_2t_1t_2t_0t_3$ belong?
However, $Nt_0t_1t_2t_1t_2t_0t_0 = Nt_0t_1t_2t_1t_2 \in [01212]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_0t_3$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_0t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_0t_1$.
24. Now, we consider the cosets stabilizer $N^{(012121)}$.
We know, $t_0t_1t_2t_1t_2t_1 = t_0t_2t_1t_2t_1t_2$ then $N^{(012121)} = \{e,(1,2)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_2t_1N$ is at most: $\frac{|N|}{|N^{(012121)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(012121)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1,2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_2t_1t_0$, $Nt_0t_1t_2t_0t_2t_1t_1$, and $Nt_0t_1t_2t_0t_2t_1t_3$ belong? However, $Nt_0t_1t_2t_1t_2t_1t_1 = Nt_0t_1t_2t_1t_2 \in [01212]$, then two symmetric generators go back to the double coset $Nt_0t_1t_2t_1t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_1t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_1t_3$.

25. Now, we consider the cosets stabilizer $N^{(012123)}$.
$N^{(012123)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_2t_3N$ is at most: $\frac{|N|}{|N^{(012123)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012123)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_2t_3t_0$, $Nt_0t_1t_2t_1t_2t_3t_1$, $Nt_0t_1t_2t_1t_2t_3t_2$ and $Nt_0t_1t_2t_1t_2t_3t_3$ belong? However, $Nt_0t_1t_2t_1t_2t_3t_3 = Nt_0t_1t_2t_1t_2 \in [01212]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_2$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_1$.

26. Now, we consider the cosets stabilizer $N^{(012130)}$.
$N^{(012130)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_3t_0N$ is at most: $\frac{|N|}{|N^{(012130)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012130)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_3t_0t_0$, $Nt_0t_1t_2t_1t_3t_0t_1$, $Nt_0t_1t_2t_1t_3t_0t_2$ and $Nt_0t_1t_2t_1t_3t_0t_3$ belong? However, $Nt_0t_1t_2t_1t_3t_0t_0 = Nt_0t_1t_2t_1t_3 \in [01213]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_0t_2$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_0t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_0t_1$.

27. Now, we consider the cosets stabilizer $N^{(012131)}$.
$N^{(012131)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_3t_1N$ is at most: $\frac{|N|}{|N^{(012131)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012131)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_3t_1t_0$, $Nt_0t_1t_2t_1t_3t_1t_1$, $Nt_0t_1t_2t_1t_3t_1t_2$ and $Nt_0t_1t_2t_1t_3t_1t_3$ belong? However, $Nt_0t_1t_2t_1t_3t_1t_1 =$

$Nt_0t_1t_2t_1t_3 \in [01213]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1t_2$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1t_0$.

28. Now, we consider the cosets stabilizer $N^{(012132)}$.
$N^{(012132)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_3t_2N$ is at most: $\frac{|N|}{|N^{(012132)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012132)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_3t_2t_0$, $Nt_0t_1t_2t_1t_3t_2t_1$, $Nt_0t_1t_2t_1t_3t_2t_2$ and $Nt_0t_1t_2t_1t_3t_2t_3$ belong? However, $Nt_0t_1t_2t_1t_3t_2t_2 = Nt_0t_1t_2t_1t_3 \in [01213]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_2t_1$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_2t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_2t_0$.

29. Now, we consider the cosets stabilizer $N^{(012302)}$.
$N^{(012302)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_0t_2N$ is at most: $\frac{|N|}{|N^{(012302)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012302)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_0t_2t_0$, $Nt_0t_1t_2t_3t_0t_2t_1$, $Nt_0t_1t_2t_3t_0t_2t_2$ and $Nt_0t_1t_2t_3t_0t_2t_3$ belong? However, $Nt_0t_1t_2t_3t_0t_2t_2 = Nt_0t_1t_2t_3t_0 \in [01230]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_0t_2t_1$, one symmetric generator goes to $Nt_0t_1t_2t_3t_0t_2t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_0t_2t_0$.

30. Now, we consider the cosets stabilizer $N^{(012310)}$.
$N^{(012310)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_1t_0N$ is at most: $\frac{|N|}{|N^{(012310)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012310)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_1t_0t_0$, $Nt_0t_1t_2t_3t_1t_0t_1$, $Nt_0t_1t_2t_3t_1t_0t_2$ and $Nt_0t_1t_2t_3t_1t_0t_3$ belong? However, $Nt_0t_1t_2t_3t_1t_0t_0 = Nt_0t_1t_2t_3t_1 \in [01231]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_0t_1$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_0t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_0t_2$.

31. Now, we consider the cosets stabilizer $N^{(012312)}$.
$N^{(012312)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_1t_2N$ is at most: $\frac{|N|}{|N^{(012312)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012312)}$ on $\{0, 1, 2, 3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_1t_2t_0$, $Nt_0t_1t_2t_3t_1t_2t_1$, $Nt_0t_1t_2t_3t_1t_2t_2$ and $Nt_0t_1t_2t_3t_1t_2t_3$ belong? However, $Nt_0t_1t_2t_3t_1t_2t_2 = Nt_0t_1t_2t_3t_1 \in [01231]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_2t_1$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_2t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_2t_0$.

32. Now, we consider the cosets stabilizer $N^{(012313)}$.
$N^{(012313)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_1t_3N$ is at most: $\frac{|N|}{|N^{(012313)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012313)}$ on $\{0, 1, 2, 3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_1t_3t_0$, $Nt_0t_1t_2t_3t_1t_3t_1$, $Nt_0t_1t_2t_3t_1t_3t_2$ and $Nt_0t_1t_2t_3t_1t_3t_3$ belong? However, $Nt_0t_1t_2t_3t_1t_3t_3 = Nt_0t_1t_2t_3t_1 \in [01231]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_3t_1$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_3t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_3t_0$.

33. Now, we consider the cosets stabilizer $N^{(012320)}$.
$N^{(012320)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_2t_0N$ is at most: $\frac{|N|}{|N^{(012320)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012320)}$ on $\{0, 1, 2, 3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_2t_0t_0$, $Nt_0t_1t_2t_3t_2t_0t_1$, $Nt_0t_1t_2t_3t_2t_0t_2$ and $Nt_0t_1t_2t_3t_2t_0t_3$ belong? However, $Nt_0t_1t_2t_3t_2t_0t_0 = Nt_0t_1t_2t_3t_2 \in [01232]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_0t_1$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_0t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_0t_3$.

34. Now, we consider the cosets stabilizer $N^{(012321)}$.

$N^{(012321)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_2t_1N$ is at most: $\frac{|N|}{|N^{(012321)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012321)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_2t_1t_0$, $Nt_0t_1t_2t_3t_2t_1t_1$, $Nt_0t_1t_2t_3t_2t_1t_2$ and $Nt_0t_1t_2t_3t_2t_1t_3$ belong? However, $Nt_0t_1t_2t_3t_2t_1t_1 = Nt_0t_1t_2t_3t_2 \in [01232]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_1t_0$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_1t_3$.

35. Now, we consider the cosets stabilizer $N^{(012323)}$.

$N^{(012323)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_2t_3N$ is at most: $\frac{|N|}{|N^{(012323)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012323)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_2t_3t_0$, $Nt_0t_1t_2t_3t_2t_3t_1$, $Nt_0t_1t_2t_3t_2t_3t_2$ and $Nt_0t_1t_2t_3t_2t_3t_3$ belong? However, $Nt_0t_1t_2t_3t_2t_3t_3 = Nt_0t_1t_2t_3t_2 \in [01232]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_3t_0$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_3t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_3t_1$.

Length 7.

1. Now, we consider coset $N^{(0101020)}$.

We know, $t_0t_1t_0t_1t_0t_2t_0 = t_1t_0t_1t_0t_1t_2t_0$ then $Nt_0t_1t_0t_1t_0t_2t_0 = Nt_1t_0t_1t_0t_1t_2t_0 \in Nt_0t_1t_0t_1t_2t_0$. However, $t_0t_1t_0t_1t_0t_2t_0 = t_2t_1t_2t_1t_0t_2 = (t_0t_1t_0t_1t_2t_0)^{(0,2)} \in t_0t_1t_0t_1t_2t_0$. Therefore, t_0 takes $Nt_0t_1t_0t_1t_0t_2$ go to $Nt_0t_1t_0t_1t_2t_0$.

2. Now, we consider the cosets stabilizer $N^{(0101023)}$.

We know, $t_0t_1t_0t_1t_0t_2t_3 = t_1t_0t_1t_0t_1t_2t_3$ then $N^{(0101023)} = \{e, (0,1)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_0t_2t_3N$ is at most: $\frac{|N|}{|N^{(0101023)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(0101023)}$ on $\{0,1,2,3\}$ are $\{0,1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_0t_2t_3t_0$,

$Nt_0t_1t_0t_1t_0t_2t_3t_2$, and $Nt_0t_1t_0t_1t_0t_2t_3t_3$ belong? However, $Nt_0t_1t_0t_1t_0t_2t_3t_3 = Nt_0t_1t_0t_1t_0t_2$ $\in [010102]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_0t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_0t_2t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_0t_2t_3t_2$.

3. Now, we consider the cosets stabilizer $N^{(0101201)}$.
$N^{(0101201)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_0t_1N$ is at most: $\frac{|N|}{|N^{(0101201)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0101201)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_0t_1t_0$, $Nt_0t_1t_0t_1t_2t_0t_1t_1$, $Nt_0t_1t_0t_1t_2t_0t_1t_2$ and $Nt_0t_1t_0t_1t_2t_0t_1t_3$ belong? However, $Nt_0t_1t_0t_1t_2t_0t_1t_1$ $= Nt_0t_1t_0t_1t_2t_0 \in [010120]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_1t_0$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_1t_3$.

4. Now, we consider coset $N^{(0101202)}$.
We know, $t_0t_1t_0t_1t_2t_0t_2 = t_0t_2t_0t_2t_1t_0t_1$ then $Nt_0t_1t_0t_1t_0t_2t_0 = Nt_0t_2t_0t_2t_1t_0t_1$ $\in Nt_0t_1t_0t_1t_2t_0t_2$. However, $t_0t_1t_0t_1t_2t_0t_2 = t_1t_2t_1t_2t_1t_0 = (t_0t_1t_0t_1t_0t_2)^{(0,1,2)} \in$ $t_0t_1t_0t_1t_0t_2$. Therefore, t_2 takes $Nt_0t_1t_0t_1t_2t_0$ go to $Nt_0t_1t_0t_1t_0t_2$.

5. Now, we consider the cosets stabilizer $N^{(0101203)}$.
$N^{(0101203)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_0t_3N$ is at most: $\frac{|N|}{|N^{(0101203)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0101203)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_0t_3t_0$, $Nt_0t_1t_0t_1t_2t_0t_3t_1$, $Nt_0t_1t_0t_1t_2t_0t_3t_2$ and $Nt_0t_1t_0t_1t_2t_0t_3t_3$ belong? However, $Nt_0t_1t_0t_1t_2t_0t_3t_3$ $= Nt_0t_1t_0t_1t_2t_0 \in [010120]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_3t_0$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_3t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_3t_1$.

6. Now, we consider the cosets stabilizer $N^{(0101210)}$.
We know, $t_0t_1t_0t_1t_2t_1t_0 = (0,2,1)(t_0t_1t_2t_1t_0t_1t_0)^{(1,2,0)}$ then

$Nt_0t_1t_0t_1t_2t_1t_0 = Nt_1t_2t_0t_2t_1t_2t_1 \in Nt_0t_1t_2t_1t_0t_1t_0$.
$N^{(0101210)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_1t_0N$ is at most: $\frac{|N|}{|N^{(0101210)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0101210)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_1t_0t_0$, $Nt_0t_1t_0t_1t_2t_1t_0t_1$, $Nt_0t_1t_0t_1t_2t_1t_0t_2$ and $Nt_0t_1t_0t_1t_2t_1t_0t_3$ belong? However, $Nt_0t_1t_0t_1t_2t_1t_0t_0$ $= Nt_0t_1t_0t_1t_2t_1 \in [010121]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_1N$, and $Nt_0t_1t_0t_1t_2t_1t_0t_1 = Nt_1t_2t_0t_2t_1t_2 \in [012101]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_0t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_0t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_0t_3$.

7. Now, we consider coset $N^{(0101212)}$.
We know, $t_0t_1t_0t_1t_2t_1t_2 = t_1t_2t_0t_2t_0t_1 = (t_0t_1t_2t_1t_2t_0)^{(0,1,2)} \in t_0t_1t_2t_1t_2t_0$. Therefore, t_2 takes $Nt_0t_1t_0t_1t_2t_1$ go to$Nt_0t_1t_2t_1t_2t_0$.

8. Now, we consider the cosets stabilizer $N^{(0101213)}$.
$N^{(0101213)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_1t_3N$ is at most: $\frac{|N|}{|N^{(0101213)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0101213)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_1t_3t_0$, $Nt_0t_1t_0t_1t_2t_1t_3t_1$, $Nt_0t_1t_0t_1t_2t_1t_3t_2$ and $Nt_0t_1t_0t_1t_2t_1t_3t_3$ belong? However, $Nt_0t_1t_0t_1t_2t_1t_3t_3$ $= Nt_0t_1t_0t_1t_2t_1 \in [010121]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_3t_0$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_3t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_3t_1$.

9. Now, we consider the cosets stabilizer $N^{(0101230)}$.
We know, $t_0t_1t_0t_1t_2t_3t_0 = (0,2)(1,3)(t_0t_1t_2t_3t_1t_0t_2)^{(1,3,0,2)}$ then
$Nt_0t_1t_0t_1t_2t_3t_0 = Nt_2t_3t_1t_0t_3t_2t_1 \in Nt_0t_1t_2t_3t_1t_0t_2$.
$N^{(0101230)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_3t_0N$ is at most: $\frac{|N|}{|N^{(0101230)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0101230)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_0t_0$,

$Nt_0t_1t_0t_1t_2t_3t_0t_1$, $Nt_0t_1t_0t_1t_2t_3t_0t_2$ and $Nt_0t_1t_0t_1t_2t_3t_0t_3$ belong? However, $Nt_0t_1t_0t_1t_2t_3t_0t_0$ $= Nt_0t_1t_0t_1t_2t_1 \in [010123]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_3N$, and $Nt_0t_1t_0t_1t_2t_3t_0t_1 = Nt_2t_3t_1t_0t_3t_2 \in [012310]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_1t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_0t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_0t_3$.

10. Now, we consider the cosets stabilizer $N^{(0101231)}$.
$N^{(0101231)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_3t_1N$ is at most: $\frac{|N|}{|N^{(0101231)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0101231)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_1t_0$, $Nt_0t_1t_0t_1t_2t_3t_1t_1$, $Nt_0t_1t_0t_1t_2t_3t_1t_2$ and $Nt_0t_1t_0t_1t_2t_3t_1t_3$ belong? However, $Nt_0t_1t_0t_1t_2t_3t_1t_1$ $= Nt_0t_1t_0t_1t_2t_3 \in [010123]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_1t_0$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_1t_3$.

11. Now, we consider the cosets stabilizer $N^{(0101232)}$.
$N^{(0101232)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_3t_2N$ is at most: $\frac{|N|}{|N^{(0101232)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0101232)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_2t_0$, $Nt_0t_1t_0t_1t_2t_3t_2t_1$, $Nt_0t_1t_0t_1t_2t_3t_2t_2$ and $Nt_0t_1t_0t_1t_2t_3t_2t_3$ belong? However, $Nt_0t_1t_0t_1t_2t_3t_2t_2$ $= Nt_0t_1t_0t_1t_2t_3 \in [010123]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_2t_0$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_2t_3$.

12. Now, we consider the cosets stabilizer $N^{(0102010)}$.
We know, $t_0t_1t_0t_2t_0t_1t_0 = t_1t_2t_1t_0t_1t_2t_1 = t_2t_0t_2t_1t_2t_0t_2$ then
$N^{(0102010)} = \{e, (0,1,2), (0,2,1)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_1t_0N$ is at most: $\frac{|N|}{|N^{(0102010)}|} = \frac{24}{3} = 8$

The orbit of $N^{(0102010)}$ on $\{0,1,2,3\}$ are $\{0,1,2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_1t_0t_0$, and

$Nt_0t_1t_0t_2t_0t_1t_0t_3$ belong? However, $Nt_0t_1t_0t_2t_0t_1t_0t_0$
$= Nt_0t_1t_0t_2t_0t_1 \in [010201]$, then three symmetric generators go back to the double coset $Nt_0t_1t_0t_2t_0t_1N$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_0t_3$.

13. Now, we consider the cosets stabilizer $N^{(0102012)}$.
We know, $t_0t_1t_0t_2t_0t_1t_2 = (t_0t_1t_2t_0t_1t_0t_2)^{(1,2)}$ then
$Nt_0t_1t_0t_2t_0t_1t_2 = Nt_0t_2t_1t_0t_2t_0t_1 \in Nt_0t_1t_2t_0t_1t_0t_2$.
$N^{(0102012)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_1t_2N$ is at most: $\frac{|N|}{|N^{(0102012)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0102012)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_1t_2t_0$, $Nt_0t_1t_0t_2t_0t_1t_2t_1$, $Nt_0t_1t_0t_2t_0t_1t_2t_2$ and $Nt_0t_1t_0t_2t_0t_1t_2t_3$ belong? However, $Nt_0t_1t_0t_2t_0t_1t_2t_2$ $= Nt_0t_1t_0t_2t_0t_1 \in [010201]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_1N$, and $Nt_0t_1t_0t_2t_0t_1t_2t_1 = Nt_0t_2t_1t_0t_2t_0 \in [021020]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_1t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_2t_3$.

14. Now, we consider the cosets stabilizer $N^{(0102013)}$.
$N^{(0102013)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_1t_3N$ is at most: $\frac{|N|}{|N^{(0102013)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0102013)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_1t_3t_0$, $Nt_0t_1t_0t_2t_0t_1t_3t_1$, $Nt_0t_1t_0t_2t_0t_1t_3t_2$ and $Nt_0t_1t_0t_2t_0t_1t_3t_3$ belong? However, $Nt_0t_1t_0t_2t_0t_1t_3t_3$ $= Nt_0t_1t_0t_2t_0t_1 \in [010201]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_3t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_3t_2$.

15. Now, we consider coset $N^{(0102020)}$.
We know, $t_0t_1t_0t_2t_0t_2t_0 = t_0t_1t_2t_0t_2t_0t_2$ then $Nt_0t_1t_0t_2t_0t_2t_0 = Nt_0t_1t_2t_0t_2t_0t_2$ $\in Nt_0t_1t_0t_2t_0t_2t_0$. However, $t_0t_1t_0t_2t_0t_2t_0 = t_1t_0t_2t_1t_2t_1 = (t_0t_1t_2t_0t_2t_0)^{(0,1)} \in t_0t_1t_2t_0t_2t_0$. Therefore, t_0 takes $Nt_0t_1t_0t_2t_0t_2$ go to $Nt_0t_1t_2t_0t_2t_0$.

16. Now, we consider the cosets stabilizer $N^{(0102021)}$.

We know, $t_0t_1t_0t_2t_0t_2t_1 = (0,1,2)(t_0t_1t_2t_1t_2t_0t_2)$ then
$Nt_0t_1t_0t_2t_0t_2t_1 = Nt_0t_1t_2t_1t_2t_0t_2 \in Nt_0t_1t_2t_1t_2t_0t_2$.
$N^{(0102021)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_2t_1N$ is at most: $\frac{|N|}{|N^{(0102021)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0102021)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_2t_1t_0$, $Nt_0t_1t_0t_2t_0t_2t_1t_1$, $Nt_0t_1t_0t_2t_0t_2t_1t_2$ and $Nt_0t_1t_0t_2t_0t_2t_1t_3$ belong? However, $Nt_0t_1t_0t_2t_0t_2t_1t_1 = Nt_0t_1t_0t_2t_0t_2 \in [010202]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_2N$, and $Nt_0t_1t_0t_2t_0t_2t_1t_2 = Nt_0t_1t_2t_1t_2t_0 \in [012120]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_1t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_1t_3$.

17. Now, we consider the cosets stabilizer $N^{(0102023)}$.

$N^{(0102023)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_2t_3N$ is at most: $\frac{|N|}{|N^{(0102023)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0102023)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_2t_3t_0$, $Nt_0t_1t_0t_2t_0t_2t_3t_1$, $Nt_0t_1t_0t_2t_0t_2t_3t_2$ and $Nt_0t_1t_0t_2t_0t_2t_3t_3$ belong? However, $Nt_0t_1t_0t_2t_0t_2t_3t_3 = Nt_0t_1t_0t_2t_0t_2 \in [010202]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_3t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_3t_2$.

18. Now, we consider the cosets stabilizer $N^{(0102030)}$.

$N^{(0102030)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_3t_0N$ is at most: $\frac{|N|}{|N^{(0102030)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0102030)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_3t_0t_0$, $Nt_0t_1t_0t_2t_0t_3t_0t_1$, $Nt_0t_1t_0t_2t_0t_3t_0t_2$ and $Nt_0t_1t_0t_2t_0t_3t_0t_3$ belong? However, $Nt_0t_1t_0t_2t_0t_3t_0t_0 = Nt_0t_1t_0t_2t_0t_3 \in [010203]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_0t_3$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_0t_2$.

19. Now, we consider the cosets stabilizer $N^{(0102031)}$.
$N^{(0102031)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_3t_1N$ is at most: $\frac{|N|}{|N^{(0102031)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0102031)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_3t_1t_0$, $Nt_0t_1t_0t_2t_0t_3t_1t_1$, $Nt_0t_1t_0t_2t_0t_3t_1t_2$ and $Nt_0t_1t_0t_2t_0t_3t_1t_3$ belong? However, $Nt_0t_1t_0t_2t_0t_3t_1t_1$ $= Nt_0t_1t_0t_2t_0t_3 \in$ [010203], then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_1t_3$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_1t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_1t_2$.

20. Now, we consider the cosets stabilizer $N^{(0102032)}$.
$N^{(0102032)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_3t_2N$ is at most: $\frac{|N|}{|N^{(0102032)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0102032)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_3t_2t_0$, $Nt_0t_1t_0t_2t_0t_3t_2t_1$, $Nt_0t_1t_0t_2t_0t_3t_2t_2$ and $Nt_0t_1t_0t_2t_0t_3t_2t_3$ belong? However, $Nt_0t_1t_0t_2t_0t_3t_2t_2$ $= Nt_0t_1t_0t_2t_0t_3 \in$ [010203], then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_2t_3$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_2t_1$.

21. Now, we consider the cosets stabilizer $N^{(0102101)}$.
We know, $t_0t_1t_0t_2t_1t_0t_1 = (0,1,2)(t_0t_1t_2t_0t_2t_3t_0)^{(1,0,3,2)}$ then
$Nt_0t_1t_0t_2t_1t_0t_1 = Nt_3t_0t_1t_3t_1t_2t_3 \in Nt_0t_1t_0t_2t_2t_3t_0$.
$N^{(0102101)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_1t_0t_1N$ is at most: $\frac{|N|}{|N^{(0102101)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0102101)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_0t_1t_0$, $Nt_0t_1t_0t_2t_1t_0t_1t_1$, $Nt_0t_1t_0t_2t_1t_0t_1t_2$ and $Nt_0t_1t_0t_2t_1t_0t_1t_3$ belong? However, $Nt_0t_1t_0t_2t_1t_0t_1t_1$ $= Nt_0t_1t_0t_2t_1t_0 \in$ [010210], then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_1t_0N$, and $Nt_0t_1t_0t_2t_1t_0t_1t_3 = Nt_0t_1t_2t_0t_2t_3 \in$ [012023], then one symmetric generator goes back to $Nt_0t_1t_2t_0t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_0t_1t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_0t_1t_2$.

22. Now, we consider coset $N^{(0102102)}$.
However, $t_0t_1t_0t_2t_1t_0t_2 = (1,2,0)(t_1t_0t_1t_2t_0t_1 = (1,2,0)(t_0t_1t_2t_0t_2t_0)^{(0,1)} \in Nt_0t_1t_0t_2t_1t_0$. Therefore, t_2 takes $Nt_0t_1t_0t_2t_1t_0t_2$ go to $Nt_0t_1t_0t_2t_1t_0$.

23. Now, we consider the cosets stabilizer $N^{(0102103)}$.
We know, $t_0t_1t_0t_2t_1t_0t_3 = (2,0,3)(t_0t_1t_2t_3t_2t_0t_3)^{(0,2,3)}$ then
$Nt_0t_1t_0t_2t_1t_0t_3 = Nt_2t_1t_3t_0t_3t_2t_0 \in Nt_0t_1t_2t_3t_2t_0t_3$.
$N^{(0102103)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_1t_0t_3N$ is at most: $\frac{|N|}{|N^{(0102103)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0102103)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_0t_3t_0$, $Nt_0t_1t_0t_2t_1t_0t_3t_1$, $Nt_0t_1t_0t_2t_1t_0t_3t_2$ and $Nt_0t_1t_0t_2t_1t_0t_3t_3$ belong? However, $Nt_0t_1t_0t_2t_1t_0t_3t_3 = Nt_0t_1t_0t_2t_1t_0 \in$ [010210], then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_1t_0N$, and $Nt_0t_1t_0t_2t_1t_0t_3t_0 = Nt_2t_1t_3t_0t_3t_2 \in$ [012320], then one symmetric generator goes back to $Nt_0t_1t_2t_3t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_0t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_0t_3t_2$.

24. Now, we consider coset $N^{(0102121)}$.
We know, $t_0t_1t_0t_2t_1t_2t_1 = t_2t_1t_2t_0t_1t_0t_1$ then
$Nt_0t_1t_0t_1t_0t_2t_0 = Nt_0t_2t_0t_2t_1t_0t_1 \in Nt_0t_1t_0t_2t_1t_2t_1$.
However, $t_0t_1t_0t_2t_1t_2t_1 = t_1t_2t_0t_2t_0t_1 = (t_0t_1t_2t_1t_2t_1)^{(0,1,2)} \in Nt_0t_1t_0t_2t_1t_0$. Therefore, t_1 takes $Nt_0t_1t_0t_2t_1t_2t_1$ go to $Nt_0t_1t_2t_1t_2t_1$.

25. Now, we consider the cosets stabilizer $N^{(0102123)}$.
We know, $t_0t_1t_0t_2t_1t_2t_3 = t_2t_1t_2t_0t_1t_0t_3$ then
$N^{(0102123)} = \{e,(0,2)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_1t_2t_3N$ is at most: $\frac{|N|}{|N^{(0102123)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(0102123)}$ on $\{0,1,2,3\}$ are $\{0,2\}$,$\{1\}$ and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_2t_3t_0$, $Nt_0t_1t_0t_2t_1t_2t_3t_1$ and $Nt_0t_1t_0t_2t_1t_2t_3t_3$ belong? However, $Nt_0t_1t_0t_2t_1t_2t_3t_3 = Nt_0t_1t_0t_2t_1t_2 \in$ [010212], then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_1t_2N$, and two symmetric generator go to $Nt_0t_1t_0t_2t_1t_2t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_2t_3t_1$.

26. Now, we consider the cosets stabilizer $N^{(0102130)}$.
We know, $t_0t_1t_0t_2t_1t_3t_0 = (0,1)(2,3)(t_0t_1t_2t_1t_3t_0t_2)^{(0,1)}$ then
$Nt_0t_1t_0t_2t_1t_3t_0 = Nt_1t_0t_2t_0t_3t_1t_2 \in Nt_0t_1t_2t_1t_3t_0t_2$.
$N^{(0102130)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_1t_3t_0N$ is at most: $\frac{|N|}{|N^{(0102130)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0102130)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_3t_0t_0$, $Nt_0t_1t_0t_2t_1t_3t_0t_1$, $Nt_0t_1t_0t_2t_1t_3t_0t_2$ and $Nt_0t_1t_0t_2t_1t_3t_0t_3$ belong?
However, $Nt_0t_1t_0t_2t_1t_3t_0t_0 = Nt_0t_1t_0t_2t_1t_3 \in [010213]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_1t_3N$, and $Nt_0t_1t_0t_2t_1t_3t_0t_2 = Nt_1t_0t_2t_0t_3t_1 \in [012130]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_0t_3$.

27. Now, we consider the cosets stabilizer $N^{(0102131)}$.
$N^{(0102131)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_1t_3t_1N$ is at most: $\frac{|N|}{|N^{(0102131)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0102131)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_3t_1t_0$, $Nt_0t_1t_0t_2t_1t_3t_1t_1$, $Nt_0t_1t_0t_2t_1t_3t_1t_2$ and $Nt_0t_1t_0t_2t_1t_3t_1t_3$ belong?
However, $Nt_0t_1t_0t_2t_1t_3t_1t_1 = Nt_0t_1t_0t_2t_1t_0 \in [010213]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_1t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_1t_3$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_1t_0$.

28. Now, we consider the cosets stabilizer $N^{(0102132)}$.
$N^{(0102132)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_1t_3t_2N$ is at most: $\frac{|N|}{|N^{(0102132)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0102132)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_3t_2t_0$,

$Nt_0t_1t_0t_2t_1t_3t_2t_1$, $Nt_0t_1t_0t_2t_1t_3t_2t_2$ and $Nt_0t_1t_0t_2t_1t_3t_2t_3$ belong?
However, $Nt_0t_1t_0t_2t_1t_3t_2t_2 = Nt_0t_1t_0t_2t_1t_0 \in [010213]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_1t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_2t_3$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_2t_0$.

29. Now, we consider the cosets stabilizer $N^{(0102301)}$.
$N^{(0102301)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_0t_1N$ is at most: $\frac{|N|}{|N^{(0102301)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0102301)}$ on $\{0, 1, 2, 3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_0t_1t_0$, $Nt_0t_1t_0t_2t_3t_0t_1t_1$, $Nt_0t_1t_0t_2t_3t_0t_1t_2$ and $Nt_0t_1t_0t_2t_3t_0t_1t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_0t_1t_1 = Nt_0t_1t_0t_2t_3t_0 \in [010230]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_1t_3$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_1t_0$.

30. Now, we consider the cosets stabilizer $N^{(0102302)}$.
We know, $t_0t_1t_0t_2t_3t_0t_2 = (0, 3, 1)(t_0t_1t_2t_1t_3t_0t_1)^{(0,1)(2,3)}$ then
$Nt_0t_1t_0t_2t_3t_0t_2 = Nt_1t_0t_3t_0t_2t_1t_0 \in Nt_0t_1t_2t_1t_3t_0t_1$.
$N^{(0102302)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_0t_2N$ is at most: $\frac{|N|}{|N^{(0102302)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0102302)}$ on $\{0, 1, 2, 3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_0t_2t_0$, $Nt_0t_1t_0t_2t_3t_0t_2t_1$, $Nt_0t_1t_0t_2t_3t_0t_2t_2$ and $Nt_0t_1t_0t_2t_3t_0t_2t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_0t_2t_2 = Nt_0t_1t_0t_2t_3t_0 \in [010230]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_0N$, and $Nt_0t_1t_0t_2t_3t_0t_2t_0 = Nt_1t_0t_3t_0t_2t_1 \in [012130]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_2t_3$.

31. Now, we consider the cosets stabilizer $N^{(0102303)}$.
$N^{(0102303)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_0t_3N$

is at most: $\frac{|N|}{|N^{(0102303)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0102303)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_0t_3t_0$, $Nt_0t_1t_0t_2t_3t_0t_3t_1$, $Nt_0t_1t_0t_2t_3t_0t_3t_2$ and $Nt_0t_1t_0t_2t_3t_0t_3t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_0t_3t_3 = Nt_0t_1t_0t_2t_3t_0 \in [010230]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_3t_1$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_3t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_3t_0$.

32. Now, we consider the cosets stabilizer $N^{(0102312)}$.
$N^{(0102312)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_1t_2N$ is at most: $\frac{|N|}{|N^{(0102312)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0102312)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_1t_2t_0$, $Nt_0t_1t_0t_2t_3t_1t_2t_1$, $Nt_0t_1t_0t_2t_3t_1t_2t_2$ and $Nt_0t_1t_0t_2t_3t_1t_2t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_1t_2t_2 = Nt_0t_1t_0t_2t_3t_1 \in [010231]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_2t_1$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_2t_3$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_2t_0$.

33. Now, we consider the cosets stabilizer $N^{(0102313)}$.
$N^{(0102313)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_1t_3N$ is at most: $\frac{|N|}{|N^{(0102313)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0102313)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_1t_3t_0$, $Nt_0t_1t_0t_2t_3t_1t_3t_1$, $Nt_0t_1t_0t_2t_3t_1t_3t_2$ and $Nt_0t_1t_0t_2t_3t_1t_3t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_1t_3t_3 = Nt_0t_1t_0t_2t_3t_1 \in [010231]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_3t_1$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_3t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_3t_0$.

34. Now, we consider the cosets stabilizer $N^{(0102320)}$.
$N^{(0102320)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_2t_0N$ is at most: $\frac{|N|}{|N^{(0102320)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0102320)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_2t_0t_0$, $Nt_0t_1t_0t_2t_3t_2t_0t_1$, $Nt_0t_1t_0t_2t_3t_2t_0t_2$ and $Nt_0t_1t_0t_2t_3t_2t_0t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_2t_0t_0 = Nt_0t_1t_0t_2t_3t_2 \in [010232]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_0t_1$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_0t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_0t_3$.

35. Now, we consider the cosets stabilizer $N^{(0102321)}$.
$N^{(0102321)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_2t_1N$ is at most: $\frac{|N|}{|N^{(0102321)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0102321)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_2t_1t_0$, $Nt_0t_1t_0t_2t_3t_2t_1t_1$, $Nt_0t_1t_0t_2t_3t_2t_1t_2$ and $Nt_0t_1t_0t_2t_3t_2t_1t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_2t_1t_1 = Nt_0t_1t_0t_2t_3t_2 \in [010232]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_1t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_1t_3$.

36. Now, we consider the cosets stabilizer $N^{(0102323)}$.
$N^{(0102323)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_2t_3N$ is at most: $\frac{|N|}{|N^{(0102323)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0102323)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_2t_3t_0$, $Nt_0t_1t_0t_2t_3t_2t_3t_1$, $Nt_0t_1t_0t_2t_3t_2t_3t_2$ and $Nt_0t_1t_0t_2t_3t_2t_3t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_2t_3t_3 = Nt_0t_1t_0t_2t_3t_2 \in [010232]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_3t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_3t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_3t_1$.

37. Now, we consider the cosets stabilizer $N^{(0120101)}$.
$N^{(0120101)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_1t_0t_1N$ is at most: $\frac{|N|}{|N^{(0120101)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0120101)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_1t_0t_1t_0$, $Nt_0t_1t_2t_0t_1t_0t_1t_1$, $Nt_0t_1t_2t_0t_1t_0t_1t_2$ and $Nt_0t_1t_2t_0t_1t_0t_1t_3$ belong?
However, $Nt_0t_1t_2t_0t_1t_0t_1t_1 = Nt_0t_1t_2t_0t_1t_0 \in [012010]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_1t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_0t_1t_0$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_0t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_0t_1t_3$.

38. [0120102] Same as 13

39. Now, we consider the cosets stabilizer $N^{(0120103)}$.
We know, $t_0t_1t_2t_0t_1t_0t_3 = t_0t_3t_1t_0t_3t_0t_2 = t_0t_2t_3t_0t_2t_0t_1$ then
$N^{(0120103)} = \{e, (1,3,2), (1,2,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_1t_0t_3N$ is at most: $\frac{|N|}{|N^{(0120103)}|} = \frac{24}{3} = 8$

The orbit of $N^{(0120103)}$ on $\{0,1,2,3\}$ are $\{1,2,3\}$, and $\{0\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_1t_0t_3t_0$, and $Nt_0t_1t_2t_0t_1t_0t_3t_3$ belong? However, $Nt_0t_1t_2t_0t_1t_0t_3t_3 = Nt_0t_1t_0t_2t_1t_0 \in [012010]$, then three symmetric generators go back to the double coset $Nt_0t_1t_2t_0t_1t_0N$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_0t_3t_0$.

40. Now, we consider the cosets stabilizer $N^{(0120130)}$.
$N^{(0120130)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_1t_3t_0N$ is at most: $\frac{|N|}{|N^{(0120130)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0120130)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_1t_3t_0t_0$, $Nt_0t_1t_2t_0t_1t_3t_0t_1$, $Nt_0t_1t_2t_0t_1t_3t_0t_2$ and $Nt_0t_1t_2t_0t_1t_3t_0t_3$ belong?
However, $Nt_0t_1t_2t_0t_1t_3t_0t_0 = Nt_0t_1t_2t_0t_1t_3 \in [012013]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_1t_3N$, one symmetric generator goes to

$Nt_0t_1t_2t_0t_1t_3t_0t_1$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_0t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_0t_3$.

41. Now, we consider the cosets stabilizer $N^{(0120131)}$.
We know, $t_0t_1t_2t_0t_1t_3t_1 = (1,3,2)(t_0t_1t_2t_0t_3t_0t_1)^{(1,2,0,3)}$ then
$Nt_0t_1t_2t_0t_1t_3t_1 = Nt_3t_2t_0t_3t_1t_3t_2 \in Nt_0t_1t_2t_0t_3t_0t_1$.
$N^{(0120131)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_1t_3t_1N$ is at most: $\frac{|N|}{|N^{(0120131)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0120131)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_1t_3t_1t_0$, $Nt_0t_1t_2t_0t_1t_3t_1t_1$, $Nt_0t_1t_2t_0t_1t_3t_1t_2$ and $Nt_0t_1t_2t_0t_1t_3t_1t_3$ belong?
However, $Nt_0t_1t_2t_0t_1t_3t_1t_1 = Nt_0t_1t_2t_0t_1t_3 \in [012013]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_1t_3N$, and $Nt_0t_1t_2t_0t_1t_3t_1t_2 = Nt_3t_2t_0t_3t_1t_3 \in [012030]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_1t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_1t_3$.

42. Now, we consider the cosets stabilizer $N^{(0120132)}$.
We know, $t_0t_1t_2t_0t_1t_3t_2 = (1,2)(0,3)(t_0t_1t_2t_1t_0t_3t_2)^{(1,2,0,3)}$ then
$Nt_0t_1t_2t_0t_1t_3t_2 = Nt_3t_2t_0t_2t_3t_1t_0 \in Nt_0t_1t_2t_1t_0t_3t_2$.
$N^{(0120132)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_1t_3t_2N$ is at most: $\frac{|N|}{|N^{(0120132)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0120132)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_1t_3t_2t_0$, $Nt_0t_1t_2t_0t_1t_3t_2t_1$, $Nt_0t_1t_2t_0t_1t_3t_2t_2$ and $Nt_0t_1t_2t_0t_1t_3t_2t_3$ belong?
However, $Nt_0t_1t_2t_0t_1t_3t_2t_2 = Nt_0t_1t_2t_0t_1t_3 \in [012013]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_1t_3N$, and $Nt_0t_1t_2t_0t_1t_3t_2t_0 = Nt_3t_2t_0t_2t_3t_1 \in [012103]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_2t_3$.

43. Now, we consider coset $N^{(0120201)}$.
However, $t_0t_1t_2t_0t_2t_0t_1 = t_1t_0t_1t_2t_1t_2 = (t_0t_1t_0t_2t_0t_2)^{(0,1)} \in Nt_0t_1t_0t_2t_0t_2$. Therefore, t_1 takes $Nt_0t_1t_2t_0t_2t_0t_1$ go to $Nt_0t_1t_0t_2t_0t_2$.

44. Now, we consider coset $N^{(0120202)}$.

However, $t_0t_1t_2t_0t_2t_0t_2 = t_1t_0t_2t_1t_2t_1 = (t_0t_1t_2t_0t_2t_0)^{(0,1)} \in Nt_0t_1t_2t_0t_2t_0$. Therefore, t_2 takes $Nt_0t_1t_2t_0t_2t_0t_2$ go to $Nt_0t_1t_2t_0t_2t_0$.

45. Now, we consider the cosets stabilizer $N^{(0120203)}$.

$N^{(0120203)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_2t_0t_3N$ is at most: $\frac{|N|}{|N^{(0120203)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0120203)}$ on $\{0, 1, 2, 3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_2t_0t_3t_0$, $Nt_0t_1t_2t_0t_2t_0t_3t_1$, $Nt_0t_1t_2t_0t_2t_0t_3t_2$ and $Nt_0t_1t_2t_0t_2t_0t_3t_3$ belong?

However, $Nt_0t_1t_2t_0t_2t_0t_3t_3 = Nt_0t_1t_2t_0t_2t_0 \in [012020]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_0t_3t_1$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_0t_3t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_0t_3t_0$.

46. Now, we consider the cosets stabilizer $N^{(0120212)}$.

We know, $t_0t_1t_2t_0t_2t_1t_2 = (1,3)(2,4)(t_0t_1t_2t_3t_1t_2t_1)$ then

$Nt_0t_1t_2t_0t_2t_1t_2 = Nt_0t_1t_2t_3t_1t_2t_1 \in Nt_0t_1t_2t_3t_1t_2t_1$.

$N^{(0120212)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_2t_1t_2N$ is at most: $\frac{|N|}{|N^{(0120212)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0120212)}$ on $\{0, 1, 2, 3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_2t_1t_2t_0$, $Nt_0t_1t_2t_0t_2t_1t_2t_1$, $Nt_0t_1t_2t_0t_2t_1t_2t_2$ and $Nt_0t_1t_2t_0t_2t_1t_2t_3$ belong?

However, $Nt_0t_1t_2t_0t_2t_1t_2t_2 = Nt_0t_1t_2t_0t_2t_1 \in [012021]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_2t_1N$, and $Nt_0t_1t_2t_0t_2t_1t_2t_1 = Nt_0t_1t_2t_3t_1t_2 \in [012312]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_1t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_1t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_1t_2t_3$.

47. Now, we consider the cosets stabilizer $N^{(0120213)}$.

$N^{(0120213)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_2t_1t_3N$ is at most: $\frac{|N|}{|N^{(0120213)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0120213)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_2t_1t_3t_0$, $Nt_0t_1t_2t_0t_2t_1t_3t_1$, $Nt_0t_1t_2t_0t_2t_1t_3t_2$ and $Nt_0t_1t_2t_0t_2t_1t_3t_3$ belong?
However, $Nt_0t_1t_2t_0t_2t_1t_3t_3 = Nt_0t_1t_2t_0t_2t_1 \in [012021]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_2t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_1t_3t_1$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_1t_3t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_1t_3t_0$.

48. [0120230] Same as 21

49. Now, we consider the cosets stabilizer $N^{(0120231)}$.
$N^{(0120231)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_2t_3t_1N$ is at most: $\frac{|N|}{|N^{(0120231)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0120231)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_2t_3t_1t_0$, $Nt_0t_1t_2t_0t_2t_3t_1t_1$, $Nt_0t_1t_2t_0t_2t_3t_1t_2$ and $Nt_0t_1t_2t_0t_2t_3t_1t_3$ belong?
However, $Nt_0t_1t_2t_0t_2t_3t_1t_1 = Nt_0t_1t_2t_0t_2t_3 \in [012023]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_1t_3$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_1t_0$.

50. Now, we consider the cosets stabilizer $N^{(0120232)}$.
$N^{(0120232)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_2t_3t_2N$ is at most: $\frac{|N|}{|N^{(0120232)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0120232)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_2t_3t_2t_0$, $Nt_0t_1t_2t_0t_2t_3t_2t_1$, $Nt_0t_1t_2t_0t_2t_3t_2t_2$ and $Nt_0t_1t_2t_0t_2t_3t_2t_3$ belong?
However, $Nt_0t_1t_2t_0t_2t_3t_2t_2 = Nt_0t_1t_2t_0t_2t_3 \in [012023]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_2t_3$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_2t_0$.

51. [0120301] Same as 41

52. Now, we consider the cosets stabilizer $N^{(0120302)}$.
$N^{(0120302)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_3t_0t_2N$ is at most: $\frac{|N|}{|N^{(0120302)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0120302)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_3t_0t_2t_0$, $Nt_0t_1t_2t_0t_3t_0t_2t_1$, $Nt_0t_1t_2t_0t_3t_0t_2t_2$ and $Nt_0t_1t_2t_0t_3t_0t_2t_3$ belong?
However, $Nt_0t_1t_2t_0t_3t_0t_2t_2 = Nt_0t_1t_2t_0t_3t_0 \in$ [012030], then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_0t_2t_3$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_0t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_0t_2t_0$.

53. Now, we consider the cosets stabilizer $N^{(0120303)}$.
$N^{(0120303)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_3t_0t_3N$ is at most: $\frac{|N|}{|N^{(0120303)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0120303)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_3t_0t_3t_0$, $Nt_0t_1t_2t_0t_3t_0t_3t_1$, $Nt_0t_1t_2t_0t_3t_0t_3t_2$ and $Nt_0t_1t_2t_0t_3t_0t_3t_3$ belong?
However, $Nt_0t_1t_2t_0t_3t_0t_3t_3 = Nt_0t_1t_2t_0t_3t_0 \in$ [012030], then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_0t_3t_2$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_0t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_0t_3t_0$.

54. Now, we consider the cosets stabilizer $N^{(0120320)}$.
$N^{(0120320)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_3t_2t_0N$ is at most: $\frac{|N|}{|N^{(0120320)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0120320)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_3t_2t_0t_0$, $Nt_0t_1t_2t_0t_3t_2t_0t_1$, $Nt_0t_1t_2t_0t_3t_2t_0t_2$ and $Nt_0t_1t_2t_0t_3t_2t_0t_3$ belong?
However, $Nt_0t_1t_2t_0t_3t_2t_0t_0 = Nt_0t_1t_2t_0t_3t_2 \in$ [012032], then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_3t_2N$, one symmetric generator goes to

$Nt_0t_1t_2t_0t_3t_2t_0t_2$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_2t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_2t_0t_3$.

55. Now, we consider the cosets stabilizer $N^{(0120321)}$.

$N^{(0120321)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_3t_2t_1N$ is at most: $\frac{|N|}{|N^{(0120321)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0120321)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_3t_2t_1t_0$, $Nt_0t_1t_2t_0t_3t_2t_1t_1$, $Nt_0t_1t_2t_0t_3t_2t_1t_2$ and $Nt_0t_1t_2t_0t_3t_2t_1t_3$ belong?

However, $Nt_0t_1t_2t_0t_3t_2t_1t_1 = Nt_0t_1t_2t_0t_3t_2 \in [012032]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_2t_1t_2$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_2t_1t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_2t_1t_3$.

56. Now, we consider the cosets stabilizer $N^{(0120323)}$.

$N^{(0120323)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_3t_2t_3N$ is at most: $\frac{|N|}{|N^{(0120323)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0120323)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_3t_2t_3t_0$, $Nt_0t_1t_2t_0t_3t_2t_3t_1$, $Nt_0t_1t_2t_0t_3t_2t_3t_2$ and $Nt_0t_1t_2t_0t_3t_2t_3t_3$ belong?

However, $Nt_0t_1t_2t_0t_3t_2t_3t_3 = Nt_0t_1t_2t_0t_3t_2 \in [012032]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_2t_3t_2$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_2t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_2t_3t_1$.

57. [0121010] Same as 6

58. Now, we consider the cosets stabilizer $N^{(0121012)}$.

We know, $t_0t_1t_2t_1t_0t_1t_2 = t_0t_2t_1t_2t_0t_2t_1$ then

$N^{(0121012)} = \{e,(1,2)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_0t_1t_2N$ is at most: $\frac{|N|}{|N^{(0121012)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(0121012)}$ on $\{0,1,2,3\}$ are $\{1,2\}$,$\{0\}$ and $\{3\}$. We take a repre-

sentative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_0t_1t_2t_0$, $Nt_0t_1t_2t_1t_0t_1t_2t_2$ and $Nt_0t_1t_2t_1t_0t_1t_2t_3$ belong?
However, $Nt_0t_1t_2t_1t_0t_1t_2t_2 = Nt_0t_1t_2t_1t_0t_1 \in [012101]$, then two symmetric generators go back to the double coset $Nt_0t_1t_2t_1t_0t_1N$, one symmetric generator go to $Nt_0t_1t_2t_1t_0t_1t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_1t_2t_3$.

59. Now, we consider the cosets stabilizer $N^{(0121013)}$.
$N^{(0121013)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_0t_1t_3N$ is at most: $\frac{|N|}{|N^{(0121013)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0121013)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_0t_1t_3t_0$, $Nt_0t_1t_2t_1t_0t_1t_3t_1$, $Nt_0t_1t_2t_1t_0t_1t_3t_2$ and $Nt_0t_1t_2t_1t_0t_1t_3t_3$ belong?
However, $Nt_0t_1t_2t_1t_0t_1t_3t_3 = Nt_0t_1t_2t_1t_0t_1 \in [012101]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_0t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_1t_3t_2$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_1t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_1t_3t_1$.

60. Now, we consider the cosets stabilizer $N^{(0121030)}$.
$N^{(0121030)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_0t_3t_0N$ is at most: $\frac{|N|}{|N^{(0121030)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0121030)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_0t_3t_0t_0$, $Nt_0t_1t_2t_1t_0t_3t_0t_1$, $Nt_0t_1t_2t_1t_0t_3t_0t_2$ and $Nt_0t_1t_2t_1t_0t_3t_0t_3$ belong?
However, $Nt_0t_1t_2t_1t_0t_3t_0t_0 = Nt_0t_1t_2t_1t_0t_3 \in [012103]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_3t_0t_2$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_3t_0t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_3t_0t_1$.

61. Now, we consider the cosets stabilizer $N^{(0121031)}$.
$N^{(0121031)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_0t_3t_1N$ is at most: $\frac{|N|}{|N^{(0121031)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0121031)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a

representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_0t_3t_1t_0$, $Nt_0t_1t_2t_1t_0t_3t_1t_1$, $Nt_0t_1t_2t_1t_0t_3t_1t_2$ and $Nt_0t_1t_2t_1t_0t_3t_1t_3$ belong?
However, $Nt_0t_1t_2t_1t_0t_3t_1t_1 = Nt_0t_1t_2t_1t_0t_3 \in [012103]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_3t_1t_2$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_3t_1t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_3t_1t_0$.

62. [0121032] Same as 42

63. Now, we consider coset $N^{(0121201)}$.
However, $t_0t_1t_2t_1t_2t_0t_1 = t_2t_0t_2t_0t_1t_0 = (t_0t_1t_0t_1t_2t_1)^{(0,2,1)} \in Nt_0t_1t_0t_1t_2t_1$. Therefore, t_1 takes $Nt_0t_1t_2t_1t_2t_0t_1$ go to $Nt_0t_1t_0t_1t_2t_1$.

64. [0121202] Same as 16

65. Now, we consider the cosets stabilizer $N^{(0121203)}$.
$N^{(0121203)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_2t_0t_3N$ is at most: $\frac{|N|}{|N^{(0121203)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0121203)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_2t_0t_3t_0$, $Nt_0t_1t_2t_1t_2t_0t_3t_1$, $Nt_0t_1t_2t_1t_2t_0t_3t_2$ and $Nt_0t_1t_2t_1t_2t_0t_3t_3$ belong?
However, $Nt_0t_1t_2t_1t_2t_0t_3t_3 = Nt_0t_1t_2t_1t_2t_0 \in [012120]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_0t_3t_2$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_0t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_0t_3t_0$.

66. Now, we consider coset $N^{(0121210)}$.
However, $t_0t_1t_2t_1t_2t_1t_0 = t_1t_0t_1t_2t_0t_2 = (t_0t_1t_0t_2t_1t_2)^{(0,1)} \in Nt_0t_1t_0t_2t_1t_2$. Therefore, t_0 takes $Nt_0t_1t_2t_1t_2t_1t_0$ go to $Nt_0t_1t_0t_2t_1t_2$.

67. Now, we consider the cosets stabilizer $N^{(0121213)}$.
We know, $t_0t_1t_2t_1t_2t_1t_3 = t_0t_2t_1t_2t_1t_2t_3$ then $N^{(0121213)} = \{e, (1,2)\}$, then the number of

the single cosets in the double coset $Nt_0t_1t_2t_1t_2t_1t_3N$ is at most: $\frac{|N|}{|N^{(0121213)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(0121213)}$ on $\{0,1,2,3\}$ are $\{1,2\}$, $\{0\}$ and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_2t_1t_3t_0$, $Nt_0t_1t_2t_1t_2t_1t_3t_2$ and $Nt_0t_1t_2t_1t_2t_1t_3t_3$ belong?
However, $Nt_0t_1t_2t_1t_2t_1t_3t_3 = Nt_0t_1t_2t_1t_2t_1 \in [012121]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_2t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_1t_3t_0$, and two symmetric generators go to $Nt_0t_1t_2t_1t_2t_1t_3t_1$.

68. Now, we consider the cosets stabilizer $N^{(0121230)}$.
$N^{(0121230)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_2t_3t_0N$ is at most: $\frac{|N|}{|N^{(0121230)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0121230)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_2t_3t_0t_0$, $Nt_0t_1t_2t_1t_2t_3t_0t_1$, $Nt_0t_1t_2t_1t_2t_3t_0t_2$ and $Nt_0t_1t_2t_1t_2t_3t_0t_3$ belong?
However, $Nt_0t_1t_2t_1t_2t_3t_0t_0 = Nt_0t_1t_2t_1t_2t_3 \in [012123]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_0t_2$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_0t_3$.

69. Now, we consider the cosets stabilizer $N^{(0121231)}$.
$N^{(0121231)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_2t_3t_1N$ is at most: $\frac{|N|}{|N^{(0121231)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0121231)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_2t_3t_1t_0$, $Nt_0t_1t_2t_1t_2t_3t_1t_1$, $Nt_0t_1t_2t_1t_2t_3t_1t_2$ and $Nt_0t_1t_2t_1t_2t_3t_1t_3$ belong?
However, $Nt_0t_1t_2t_1t_2t_3t_1t_1 = Nt_0t_1t_2t_1t_2t_3 \in [012123]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_1t_2$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_1t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_1t_3$.

70. Now, we consider the cosets stabilizer $N^{(0121232)}$.
$N^{(0121232)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_2t_3t_2N$

is at most: $\frac{|N|}{|N^{(0121232)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0121232)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_2t_3t_2t_0$, $Nt_0t_1t_2t_1t_2t_3t_2t_1$, $Nt_0t_1t_2t_1t_2t_3t_2t_2$ and $Nt_0t_1t_2t_1t_2t_3t_2t_3$ belong?
However, $Nt_0t_1t_2t_1t_2t_3t_2t_2 = Nt_0t_1t_2t_1t_2t_3 \in [012123]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_2t_1$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_2t_3$.

71. [0121301] Same as 30

72. [0121302] Same as 26

73. Now, we consider the cosets stabilizer $N^{(0121303)}$.
$N^{(0121303)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_3t_0t_3N$ is at most: $\frac{|N|}{|N^{(0121303)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0121303)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_3t_0t_3t_0$, $Nt_0t_1t_2t_1t_3t_0t_3t_1$, $Nt_0t_1t_2t_1t_3t_0t_3t_2$ and $Nt_0t_1t_2t_1t_3t_0t_3t_3$ belong?
However, $Nt_0t_1t_2t_1t_3t_0t_3t_3 = Nt_0t_1t_2t_1t_3t_0 \in [012130]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_0t_3t_1$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_0t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_0t_3t_2$.

74. Now, we consider the cosets stabilizer $N^{(0121310)}$.
$N^{(0121310)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_3t_1t_0N$ is at most: $\frac{|N|}{|N^{(0121310)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0121310)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_3t_1t_0t_0$, $Nt_0t_1t_2t_1t_3t_1t_0t_1$, $Nt_0t_1t_2t_1t_3t_1t_0t_2$ and $Nt_0t_1t_2t_1t_3t_1t_0t_3$ belong?
However, $Nt_0t_1t_2t_1t_3t_1t_0t_0 = Nt_0t_1t_2t_1t_3t_1 \in [012131]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_3t_1N$, one symmetric generator goes to

$Nt_0t_1t_2t_1t_3t_1t_0t_3$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1t_0t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1t_0t_1$.

75. Now, we consider the cosets stabilizer $N^{(0121312)}$.
$N^{(0121312)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_3t_1t_2N$ is at most: $\frac{|N|}{|N^{(0121312)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0121312)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_3t_1t_2t_0$, $Nt_0t_1t_2t_1t_3t_1t_2t_1$, $Nt_0t_1t_2t_1t_3t_1t_2t_2$ and $Nt_0t_1t_2t_1t_3t_1t_2t_3$ belong?
However, $Nt_0t_1t_2t_1t_3t_1t_2t_2 = Nt_0t_1t_2t_1t_3t_1 \in [012131]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_3t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1t_2t_3$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1t_2t_1$.

76. Now, we consider the cosets stabilizer $N^{(0121313)}$.
$N^{(0121313)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_3t_1t_3N$ is at most: $\frac{|N|}{|N^{(0121313)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0121313)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_3t_1t_3t_0$, $Nt_0t_1t_2t_1t_3t_1t_3t_1$, $Nt_0t_1t_2t_1t_3t_1t_3t_2$ and $Nt_0t_1t_2t_1t_3t_1t_3t_3$ belong?
However, $Nt_0t_1t_2t_1t_3t_1t_3t_3 = Nt_0t_1t_2t_1t_3t_1 \in [012131]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_3t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1t_3t_2$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1t_3t_1$.

77. Now, we consider the cosets stabilizer $N^{(0121320)}$.
$N^{(0121320)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_3t_2t_0N$ is at most: $\frac{|N|}{|N^{(0121320)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0121320)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_3t_2t_0t_0$, $Nt_0t_1t_2t_1t_3t_2t_0t_1$, $Nt_0t_1t_2t_1t_3t_2t_0t_2$ and $Nt_0t_1t_2t_1t_3t_2t_0t_3$ belong?

However, $Nt_0t_1t_2t_1t_3t_2t_0t_0 = Nt_0t_1t_2t_1t_3t_2 \in [012132]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_2t_0t_1$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_2t_0t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_2t_0t_2$.

78. Now, we consider the cosets stabilizer $N^{(0121321)}$.
We know, $t_0t_1t_2t_1t_3t_2t_1 = t_0t_3t_1t_0t_3t_0t_2 = t_0t_2t_3t_0t_2t_0t_1$ then
$N^{(0121321)} = \{e, (1,0,2), (1,2,0)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_3t_2t_1N$ is at most: $\frac{|N|}{|N^{(0121321)}|} = \frac{24}{3} = 8$

The orbit of $N^{(0121321)}$ on $\{0,1,2,3\}$ are $\{1,2,0\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_3t_2t_1t_0$, and $Nt_0t_1t_2t_1t_3t_2t_1t_3$ belong?
However, $Nt_0t_1t_2t_1t_3t_2t_1t_1 = Nt_0t_1t_2t_1t_3t_2 \in [012132]$, then three symmetric generators go back to the double coset $Nt_0t_1t_2t_1t_3t_2N$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_2t_1t_3$.
79. Now, we consider the cosets stabilizer $N^{(0121323)}$.
We know, $t_0t_1t_2t_1t_3t_2t_3 = t_0t_2t_1t_2t_1t_2t_3$ then $N^{(0121323)} = \{e, (1,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_3t_2t_3N$ is at most: $\frac{|N|}{|N^{(0121323)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(0121323)}$ on $\{0,1,2,3\}$ are $\{1,3\}$,$\{0\}$ and $\{2\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_3t_2t_3t_0$, $Nt_0t_1t_2t_1t_3t_2t_3t_2$ and $Nt_0t_1t_2t_1t_3t_2t_3t_3$ belong?
However, $Nt_0t_1t_2t_1t_3t_2t_3t_3 = Nt_0t_1t_2t_1t_3t_2 \in [012132]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_3t_2N$, two symmetric generator go to $Nt_0t_1t_2t_1t_3t_2t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_2t_3t_1$.

80. Now, we consider the cosets stabilizer $N^{(0123020)}$.
$N^{(0123020)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_0t_2t_0N$ is at most: $\frac{|N|}{|N^{(0123020)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0123020)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_0t_2t_0t_0$, $Nt_0t_1t_2t_3t_0t_2t_0t_1$, $Nt_0t_1t_2t_3t_0t_2t_0t_2$ and $Nt_0t_1t_2t_3t_0t_2t_0t_3$ belong?

However, $Nt_0t_1t_2t_3t_0t_2t_0t_0 = Nt_0t_1t_2t_3t_0t_2 \in [012302]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_0t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_0t_2t_0t_1$, one symmetric generator goes to $Nt_0t_1t_2t_3t_0t_2t_0t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_0t_2t_0t_2$.

81. Now, we consider the cosets stabilizer $N^{(0123021)}$.

$N^{(0123021)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_0t_2t_1N$ is at most: $\frac{|N|}{|N^{(0123021)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0123021)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_0t_2t_1t_0$, $Nt_0t_1t_2t_3t_0t_2t_1t_1$, $Nt_0t_1t_2t_3t_0t_2t_1t_2$ and $Nt_0t_1t_2t_3t_0t_2t_1t_3$ belong?

However, $Nt_0t_1t_2t_3t_0t_2t_1t_1 = Nt_0t_1t_2t_3t_0t_2 \in [012302]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_0t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_0t_2t_1t_0$, one symmetric generator goes to $Nt_0t_1t_2t_3t_0t_2t_1t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_0t_2t_1t_2$.

82. Now, we consider the cosets stabilizer $N^{(0123023)}$.

$N^{(0123023)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_0t_2t_3N$ is at most: $\frac{|N|}{|N^{(0123023)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0123023)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_0t_2t_3t_0$, $Nt_0t_1t_2t_3t_0t_2t_3t_1$, $Nt_0t_1t_2t_3t_0t_2t_3t_2$ and $Nt_0t_1t_2t_3t_0t_2t_3t_3$ belong?

However, $Nt_0t_1t_2t_3t_0t_2t_3t_3 = Nt_0t_1t_2t_3t_0t_2 \in [012302]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_0t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_0t_2t_3t_0$, one symmetric generator goes to $Nt_0t_1t_2t_3t_0t_2t_3t_1$, and one symmetric generator goes toto $Nt_0t_1t_2t_3t_0t_2t_3t_2$.

83. Now, we consider the cosets stabilizer $N^{(0123101)}$.

$N^{(0123101)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_1t_0t_1N$ is at most: $\frac{|N|}{|N^{(0123101)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0123101)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_1t_0t_1t_0$,

$Nt_0t_1t_2t_3t_1t_0t_1t_1$, $Nt_0t_1t_2t_3t_1t_0t_1t_2$ and $Nt_0t_1t_2t_3t_1t_0t_1t_3$ belong?
However, $Nt_0t_1t_2t_3t_1t_0t_1t_1 = Nt_0t_1t_2t_3t_1t_0 \in$ [012310], then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_1t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_0t_1t_0$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_0t_1t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_0t_1t_2$.

84. [0123102] Same as 9

85. Now, we consider the cosets stabilizer $N^{(0123130)}$.
$N^{(0123130)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_1t_0t_3N$ is at most: $\frac{|N|}{|N^{(0123130)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0123130)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_1t_0t_3t_0$, $Nt_0t_1t_2t_3t_1t_0t_3t_1$, $Nt_0t_1t_2t_3t_1t_0t_3t_2$ and $Nt_0t_1t_2t_3t_1t_0t_3t_3$ belong?
However, $Nt_0t_1t_2t_3t_1t_0t_3t_3 = Nt_0t_1t_2t_3t_1t_0 \in$ [012310], then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_1t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_0t_3t_0$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_0t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_0t_3t_2$.

86. Now, we consider the cosets stabilizer $N^{(0123121)}$.
$N^{(0123121)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_1t_2t_1N$ is at most: $\frac{|N|}{|N^{(0123121)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0123121)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_1t_2t_1t_0$, $Nt_0t_1t_2t_3t_1t_2t_1t_1$, $Nt_0t_1t_2t_3t_1t_2t_1t_2$ and $Nt_0t_1t_2t_3t_1t_2t_1t_3$ belong?
However, $Nt_0t_1t_2t_3t_1t_2t_1t_1 = Nt_0t_1t_2t_3t_1t_2 \in$ [012312], then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_1t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_2t_1t_2$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_2t_1t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_2t_1t_3$.

87. [0123121] Same as 46

88. Now, we consider coset $N^{(0123123)}$.
However, $t_0t_1t_2t_3t_1t_2t_3 = (1,2)(t_0t_3t_1t_2t_3t_1) = (1,2)(t_0t_1t_2t_3t_1t_2)^{(0,1)} \in Nt_0t_1t_0t_2t_1t_2$. Therefore, t_3 takes $Nt_0t_1t_2t_3t_1t_2t_3$ go to itself $Nt_0t_1t_2t_3t_1t_2$.

89. Now, we consider the cosets stabilizer $N^{(0123130)}$.
$N^{(0123130)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_1t_3t_0N$ is at most: $\frac{|N|}{|N^{(0123130)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0123130)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_1t_3t_0t_0$, $Nt_0t_1t_2t_3t_1t_3t_0t_1$, $Nt_0t_1t_2t_3t_1t_3t_0t_2$ and $Nt_0t_1t_2t_3t_1t_3t_0t_3$ belong?
However, $Nt_0t_1t_2t_3t_1t_3t_0t_0 = Nt_0t_1t_2t_3t_1t_3 \in [012313]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_1t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_3t_0t_3$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_3t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_3t_0t_2$.

90. Now, we consider the cosets stabilizer $N^{(0123131)}$.
$N^{(0123131)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_1t_3t_1N$ is at most: $\frac{|N|}{|N^{(0123131)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0123131)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_1t_3t_1t_0$, $Nt_0t_1t_2t_3t_1t_3t_1t_1$, $Nt_0t_1t_2t_3t_1t_3t_1t_2$ and $Nt_0t_1t_2t_3t_1t_3t_1t_3$ belong?
However, $Nt_0t_1t_2t_3t_1t_3t_1t_1 = Nt_0t_1t_2t_3t_1t_3 \in [012313]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_1t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_3t_1t_3$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_3t_1t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_3t_1t_2$.

91. Now, we consider the cosets stabilizer $N^{(0123132)}$.
We know, $t_0t_1t_2t_3t_1t_3t_2 = (t_0t_1t_2t_3t_2t_1t_3)^{(1,2,3)}$ then
$Nt_0t_1t_2t_3t_1t_3t_2 = Nt_0t_3t_2t_1t_3t_2t_1 \in Nt_0t_1t_2t_3t_2t_1t_3$.
$N^{(0123132)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_1t_3t_2N$ is at most: $\frac{|N|}{|N^{(0123132)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0123132)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a

representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_1t_3t_2t_0$, $Nt_0t_1t_2t_3t_1t_3t_2t_1$, $Nt_0t_1t_2t_3t_1t_3t_2t_2$ and $Nt_0t_1t_2t_3t_1t_3t_2t_3$ belong?
However, $Nt_0t_1t_2t_3t_1t_3t_2t_2 = Nt_0t_1t_2t_3t_1t_3 \in [012313]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_1t_3N$, and $Nt_0t_1t_2t_3t_1t_3t_2t_1 = Nt_0t_1t_2t_3t_2t_1 \in [012321]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_2t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_3t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_3t_2t_3$.

92. Now, we consider the cosets stabilizer $N^{(0123201)}$.
$N^{(0123201)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_2t_0t_1N$ is at most: $\frac{|N|}{|N^{(0123201)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0123201)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_2t_0t_1t_0$, $Nt_0t_1t_2t_3t_2t_0t_1t_1$, $Nt_0t_1t_2t_3t_2t_0t_1t_2$ and $Nt_0t_1t_2t_3t_2t_0t_1t_3$ belong?
However, $Nt_0t_1t_2t_3t_2t_0t_1t_1 = Nt_0t_1t_2t_3t_2t_0 \in [012320]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_0t_1t_3$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_0t_1t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_0t_1t_2$.

93. Now, we consider the cosets stabilizer $N^{(0123202)}$.
$N^{(0123202)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_2t_0t_2N$ is at most: $\frac{|N|}{|N^{(0123202)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0123202)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_2t_0t_2t_0$, $Nt_0t_1t_2t_3t_2t_0t_2t_1$, $Nt_0t_1t_2t_3t_2t_0t_2t_2$ and $Nt_0t_1t_2t_3t_2t_0t_2t_3$ belong?
However, $Nt_0t_1t_2t_3t_2t_0t_2t_2 = Nt_0t_1t_2t_3t_2t_0 \in [012320]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_0t_2t_3$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_0t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_0t_2t_1$.

94. [0123203] Same as 23

95. Now, we consider the cosets stabilizer $N^{(0123210)}$.
$N^{(0123210)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_2t_1t_0N$ is at most: $\frac{|N|}{|N^{(0123210)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0123210)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_2t_1t_0t_0$, $Nt_0t_1t_2t_3t_2t_1t_0t_1$, $Nt_0t_1t_2t_3t_2t_1t_0t_2$ and $Nt_0t_1t_2t_3t_2t_1t_0t_3$ belong?
However, $Nt_0t_1t_2t_3t_2t_1t_0t_0 = Nt_0t_1t_2t_3t_2t_1 \in [012321]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_2t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_1t_0t_3$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_1t_0t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_1t_0t_1$.

96. Now, we consider the cosets stabilizer $N^{(0123212)}$.
$N^{(0123212)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_2t_1t_2N$ is at most: $\frac{|N|}{|N^{(0123212)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0123212)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_2t_1t_2t_0$, $Nt_0t_1t_2t_3t_2t_1t_2t_1$, $Nt_0t_1t_2t_3t_2t_1t_2t_2$ and $Nt_0t_1t_2t_3t_2t_1t_2t_3$ belong?
However, $Nt_0t_1t_2t_3t_2t_1t_2t_2 = Nt_0t_1t_2t_3t_2t_1 \in [012321]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_2t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_1t_2t_3$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_1t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_1t_2t_1$.

97. [0123213] Same as 88

98. Now, we consider the cosets stabilizer $N^{(0123230)}$.
$N^{(0123230)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_2t_3t_0N$ is at most: $\frac{|N|}{|N^{(0123230)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0123230)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_2t_3t_0t_0$, $Nt_0t_1t_2t_3t_2t_3t_0t_1$, $Nt_0t_1t_2t_3t_2t_3t_0t_2$ and $Nt_0t_1t_2t_3t_2t_3t_0t_3$ belong?
However, $Nt_0t_1t_2t_3t_2t_3t_0t_0 = Nt_0t_1t_2t_3t_2t_3 \in [012323]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_2t_3N$, one symmetric generator goes to

$Nt_0t_1t_2t_3t_2t_3t_0t_3$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_3t_0t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_3t_0t_1$.

99. Now, we consider the cosets stabilizer $N^{(0123231)}$.
$N^{(0123231)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_2t_3t_1N$ is at most: $\frac{|N|}{|N^{(0123231)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0123231)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_2t_3t_1t_0$, $Nt_0t_1t_2t_3t_2t_3t_1t_1$, $Nt_0t_1t_2t_3t_2t_3t_1t_2$ and $Nt_0t_1t_2t_3t_2t_3t_1t_3$ belong?
However, $Nt_0t_1t_2t_3t_2t_3t_1t_1 = Nt_0t_1t_2t_3t_2t_3 \in [012323]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_3t_1t_3$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_3t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_3t_1t_0$.

100. Now, we consider the cosets stabilizer $N^{(0123232)}$.
We know, $t_0t_1t_2t_1t_3t_2t_3 = t_0t_1t_3t_2t_3t_2t_3$ then
$N^{(0123232)} = \{e, (2,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_3t_2t_3N$ is at most: $\frac{|N|}{|N^{(0123232)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(0123232)}$ on $\{0,1,2,3\}$ are $\{2,3\}$, $\{0\}$ and $\{1\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_3t_2t_3t_0$, $Nt_0t_1t_2t_1t_3t_2t_3t_1$ and $Nt_0t_1t_2t_1t_3t_2t_3t_3$ belong?
However, $Nt_0t_1t_2t_1t_3t_2t_3t_3 = Nt_0t_1t_2t_1t_3t_2 \in [012323]$, then two symmetric generators go back to the double coset $Nt_0t_1t_2t_3t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_2t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_2t_3t_1$.

Length 8.

1. Now, we consider the cosets stabilizer $N^{(01010230)}$.
We know, $t_0t_1t_0t_1t_0t_2t_3t_0 = (1,3)(2,0)(t_0t_1t_0t_2t_3t_0t_1t_2)^{(1,2)(0,3)}$ then
$Nt_0t_1t_0t_1t_0t_2t_3t_0 = Nt_3t_2t_3t_1t_0t_3t_2t_1 \in Nt_0t_1t_0t_2t_3t_0t_1t_2$.
$N^{(01010230)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_0t_2t_3t_0N$ is at most: $\frac{|N|}{|N^{(01010230)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01010230)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_0t_2t_3t_0t_0$, $Nt_0t_1t_0t_1t_0t_2t_3t_0t_1$, $Nt_0t_1t_0t_1t_0t_2t_3t_0t_2$ and $Nt_0t_1t_0t_1t_0t_2t_3t_0t_3$ belong?
However, $Nt_0t_1t_0t_1t_0t_2t_3t_0t_0 = Nt_0t_1t_0t_1t_0t_2t_3 \in [0101023]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_0t_2t_3N$, and $Nt_0t_1t_0t_1t_0t_2t_3t_0t_1 = Nt_0t_1t_0t_2t_3t_0t_1$ $\in [0101301]$, then one symmetric generator goes back to $Nt_0t_1t_0t_2t_3t_0t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_0t_2t_3t_0t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_0t_2t_3t_0t_3$.

2. Now, we consider the cosets stabilizer $N^{(01010232)}$.
We know, $t_0t_1t_0t_1t_0t_2t_3t_2 = t_1t_0t_1t_0t_1t_2t_3t_2$ then
$N^{(01010232)} = \{e,(0,1)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_0t_2t_3t_2N$ is at most: $\frac{|N|}{|N^{(01010232)}|} = \frac{24}{2} = 12$.
The orbit of $N^{(01010232)}$ on $\{0,1,2,3\}$ are $\{0,1\}$,$\{2\}$ and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_0t_2t_3t_2t_0$, $Nt_0t_1t_0t_1t_0t_2t_3t_2t_1$ and $Nt_0t_1t_0t_1t_0t_2t_3t_2t_3$ belong?
However, $Nt_0t_1t_0t_1t_0t_2t_3t_2t_2 = Nt_1t_0t_1t_0t_1t_2t_3 \in [0101023]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_0t_2t_3N$, two symmetric generator go to $Nt_0t_1t_0t_1t_0t_2t_3t_2t_0$, andone symmetric generator goes to $Nt_0t_1t_0t_1t_0t_2t_3t_2t_2$.

3. Now, we consider the cosets stabilizer $N^{(01012010)}$.
We know, $t_0t_1t_0t_1t_2t_0t_1t_0 = (1,2,0)(t_0t_1t_0t_2t_0t_1t_2t_0)^{(1,0,2)}$
$= (1,0,2)(t_0t_1t_0t_2t_1t_0t_1t_0)^{(1,2,0)} = (1,0,2)(t_0t_1t_2t_3t_1t_3t_0t_1)^{(1,3,0,2)}$, and $N^{(01012010)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_0t_1t_0N$ is at most: $\frac{|N|}{|N^{(01012010)}|} = \frac{24}{1} = 24$.
The orbit of $N^{(01012010)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_0t_1t_0t_0$, $Nt_0t_1t_0t_1t_2t_0t_1t_0t_1$, $Nt_0t_1t_0t_1t_2t_0t_1t_0t_2$ and $Nt_0t_1t_0t_1t_2t_0t_1t_0t_3$ belong?
However, $Nt_0t_1t_0t_1t_2t_0t_1t_0t_0 = Nt_0t_1t_0t_1t_0t_2t_3 \in [0101023]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_0t_2t_3N$, and $Nt_0t_1t_0t_1t_2t_0t_1t_0t_1 = Nt_1t_2t_1t_0t_1t_2t_1$ $\in [0102101]$, then one symmetric generator goes back to $Nt_0t_1t_0t_2t_1t_0t_1N$,
$Nt_0t_1t_0t_1t_2t_0t_1t_0t_2 = Nt_2t_0t_2t_1t_2t_0t_1 \in [0102012]$, then one symmetric generator goes back to $Nt_0t_1t_0t_2t_0t_1t_2N$.

$Nt_0t_1t_0t_1t_2t_0t_1t_0t_3 = Nt_2t_3t_1t_0t_3t_0t_2 \in [0123130]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_1t_3t_0N$.

4. Now, we consider coset $N^{(01012012)}$.
However, $t_0t_1t_0t_1t_2t_0t_1t_2 = (1,3,2)(t_1t_2t_0t_1t_0t_2t_0) = (1,2)(t_0t_1t_2t_0t_2t_1t_2)^{(0,1,2)}$ $\in Nt_0t_1t_2t_0t_2t_1t_2$. Therefore, t_2 takes $Nt_0t_1t_0t_1t_2t_0t_1t_2$ go to $Nt_0t_1t_2t_0t_2t_1t_2$.

5. Now, we consider the cosets stabilizer $N^{(01012013)}$.
We know, $t_0t_1t_0t_1t_2t_0t_1t_3 = (1,3,2)(t_0t_1t_0t_2t_3t_2t_1t_3)^{(1,2,3)}$ then $Nt_0t_1t_0t_1t_2t_0t_1t_3 = Nt_0t_2t_0t_3t_1t_3t_2t_1 \in Nt_0t_1t_0t_2t_3t_2t_1t_3$.
$N^{(01012013)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_0t_1t_3N$ is at most: $\frac{|N|}{|N^{(01012013)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01012013)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_0t_1t_3t_0$, $Nt_0t_1t_0t_1t_2t_0t_1t_3t_1$, $Nt_0t_1t_0t_1t_2t_0t_1t_3t_2$ and $Nt_0t_1t_0t_1t_2t_0t_1t_3t_3$ belong?
However, $Nt_0t_1t_0t_1t_2t_0t_1t_3t_3 = Nt_0t_1t_0t_1t_2t_0t_1 \in [0101201]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_0t_1N$, and $Nt_0t_1t_0t_1t_2t_0t_1t_3t_1 = Nt_0t_2t_0t_3t_1t_3t_2$ $\in [0102321]$, then one symmetric generator goes back to $Nt_0t_1t_0t_2t_3t_2t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_1t_3t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_1t_3t_0$.

6. Now, we consider the cosets stabilizer $N^{(01012030)}$.
$N^{(01012030)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_0t_3t_0N$ is at most: $\frac{|N|}{|N^{(01012030)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01012030)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_0t_3t_0t_0$, $Nt_0t_1t_0t_1t_2t_0t_3t_0t_1$, $Nt_0t_1t_0t_1t_2t_0t_3t_0t_2$ and $Nt_0t_1t_0t_1t_2t_0t_3t_0t_3$ belong?
However, $Nt_0t_1t_0t_1t_2t_0t_3t_0t_0 = Nt_0t_1t_0t_1t_2t_0t_3 \in [0101203]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_3t_0t_2$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_3t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_3t_0t_3$.

7. Now, we consider the cosets stabilizer $N^{(01012031)}$.

We know, $t_0t_1t_0t_1t_2t_0t_3t_1 = (0,1)(2,3)(t_0t_1t_0t_2t_0t_3t_1t_2)^{(1,0)}$ then $Nt_0t_1t_0t_1t_2t_0t_3t_1 = Nt_1t_0t_1t_2t_1t_3t_0t_2 \in Nt_0t_1t_0t_2t_0t_3t_1t_2$.

$N^{(01012031)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_0t_3t_1N$ is at most: $\frac{|N|}{|N^{(01012031)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01012031)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_0t_3t_1t_0$, $Nt_0t_1t_0t_1t_2t_0t_3t_1t_1$, $Nt_0t_1t_0t_1t_2t_0t_3t_1t_2$ and $Nt_0t_1t_0t_1t_2t_0t_3t_1t_3$ belong?

However, $Nt_0t_1t_0t_1t_2t_0t_3t_1t_1 = Nt_0t_1t_0t_1t_2t_0t_3 \in [0101203]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_0t_3N$, and $Nt_0t_1t_0t_1t_2t_0t_3t_1t_2 = Nt_0t_2t_0t_3t_1t_3t_2$ $\in [0102031]$, then one symmetric generator goes back to $Nt_0t_1t_0t_2t_0t_3t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_3t_1t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_3t_1t_3$.

8. Now, we consider the cosets stabilizer $N^{(01012032)}$.

We know, $t_0t_1t_0t_1t_2t_0t_3t_2 = (t_0t_1t_2t_1t_2t_3t_1t_0)^{(2,0,3)}$ then $Nt_0t_1t_0t_1t_2t_0t_3t_2 = Nt_3t_1t_0t_1t_0t_2t_1t_3 \in Nt_0t_1t_2t_1t_2t_3t_1t_0$.

$N^{(01012032)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_0t_3t_2N$ is at most: $\frac{|N|}{|N^{(01012032)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01012032)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_0t_3t_2t_0$, $Nt_0t_1t_0t_1t_2t_0t_3t_2t_1$, $Nt_0t_1t_0t_1t_2t_0t_3t_2t_2$ and $Nt_0t_1t_0t_1t_2t_0t_3t_2t_3$ belong?

However, $Nt_0t_1t_0t_1t_2t_0t_3t_2t_2 = Nt_0t_1t_0t_1t_2t_0t_3 \in [0101203]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_0t_3N$, and $Nt_0t_1t_0t_1t_2t_0t_3t_2t_3 = Nt_3t_1t_0t_1t_0t_2t_1$ $\in [0102031]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_2t_3t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_3t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_3t_2t_3$.

9. Now, we consider coset $N^{(01012102)}$.

However, $t_0t_1t_0t_1t_2t_1t_0t_2 = (1,0,2)(t_1t_0t_2t_1t_0t_1t_0) = (1,0,2)(t_0t_1t_2t_0t_1t_0t_1)^{(0,1)} \in Nt_0t_1t_2t_0t_2t_1t_2$. Therefore, t_2 takes $Nt_0t_1t_0t_1t_2t_1t_0t_2$ go to $Nt_0t_1t_2t_0t_1t_0t_1$.

10. Now, we consider the cosets stabilizer $N^{(01012103)}$.

$N^{(01012103)} = \{e\}$, then the number of the single cosets in the double coset

$Nt_0t_1t_0t_1t_2t_1t_0t_3N$ is at most: $\frac{|N|}{|N^{(01012103)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01012103)}$ on $\{0, 1, 2, 3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_1t_0t_3t_0$, $Nt_0t_1t_0t_1t_2t_1t_0t_3t_1$, $Nt_0t_1t_0t_1t_2t_1t_0t_3t_2$ and $Nt_0t_1t_0t_1t_2t_1t_0t_3t_3$ belong?
However, $Nt_0t_1t_0t_1t_2t_1t_0t_3t_3 = Nt_0t_1t_0t_1t_2t_0t_3 \in [0101210]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_0t_3t_2$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_0t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_0t_3t_0$.

11. Now, we consider the cosets stabilizer $N^{(01012130)}$.
$N^{(01012130)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_1t_3t_0N$ is at most: $\frac{|N|}{|N^{(01012130)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01012130)}$ on $\{0, 1, 2, 3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_1t_3t_0t_0$, $Nt_0t_1t_0t_1t_2t_1t_3t_0t_1$, $Nt_0t_1t_0t_1t_2t_1t_3t_0t_2$ and $Nt_0t_1t_0t_1t_2t_1t_3t_0t_3$ belong?
However, $Nt_0t_1t_0t_1t_2t_1t_3t_0t_0 = Nt_0t_1t_0t_1t_2t_1t_3 \in [0101213]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_3t_0t_2$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_3t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_3t_0t_3$.

12. Now, we consider the cosets stabilizer $N^{(01012131)}$.
$N^{(01012131)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_1t_3t_1N$ is at most: $\frac{|N|}{|N^{(01012131)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01012131)}$ on $\{0, 1, 2, 3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_1t_3t_1t_0$, $Nt_0t_1t_0t_1t_2t_1t_3t_1t_1$, $Nt_0t_1t_0t_1t_2t_1t_3t_1t_2$ and $Nt_0t_1t_0t_1t_2t_1t_3t_1t_3$ belong?
However, $Nt_0t_1t_0t_1t_2t_1t_3t_1t_1 = Nt_0t_1t_0t_1t_2t_1t_3 \in [0101213]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_3t_1t_2$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_3t_1t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_3t_1t_3$.

13. Now, we consider the cosets stabilizer $N^{(01012132)}$.
$N^{(01012132)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_1t_3t_2N$ is at most: $\frac{|N|}{|N^{(01012132)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01012132)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_1t_3t_2t_0$, $Nt_0t_1t_0t_1t_2t_1t_3t_2t_1$, $Nt_0t_1t_0t_1t_2t_1t_3t_2t_2$ and $Nt_0t_1t_0t_1t_2t_1t_3t_2t_3$ belong?
However, $Nt_0t_1t_0t_1t_2t_1t_3t_2t_2 = Nt_0t_1t_0t_1t_2t_1t_3 \in [0101213]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_3t_2t_1$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_3t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_3t_2t_3$.

14. Now, we consider the cosets stabilizer $N^{(01012302)}$.
$N^{(01012302)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_3t_0t_2N$ is at most: $\frac{|N|}{|N^{(01012302)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01012302)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_0t_2t_0$, $Nt_0t_1t_0t_1t_2t_3t_0t_2t_1$, $Nt_0t_1t_0t_1t_2t_3t_0t_2t_2$ and $Nt_0t_1t_0t_1t_2t_3t_0t_2t_3$ belong?
However, $Nt_0t_1t_0t_1t_2t_3t_0t_2t_2 = Nt_0t_1t_0t_1t_2t_3t_0 \in [0101230]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_0t_2t_1$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_0t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_0t_2t_3$.

15. Now, we consider the cosets stabilizer $N^{(01012303)}$.
$N^{(01012303)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_3t_0t_3N$ is at most: $\frac{|N|}{|N^{(01012303)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01012303)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_0t_3t_0$, $Nt_0t_1t_0t_1t_2t_3t_0t_3t_1$, $Nt_0t_1t_0t_1t_2t_3t_0t_3t_2$ and $Nt_0t_1t_0t_1t_2t_3t_0t_3t_3$ belong?
However, $Nt_0t_1t_0t_1t_2t_3t_0t_3t_3 = Nt_0t_1t_0t_1t_2t_3t_0 \in [0101230]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_0t_3t_1$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_0t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_0t_3t_2$.

16. Now, we consider the cosets stabilizer $N^{(01012310)}$.

We know, $t_0t_1t_0t_1t_2t_3t_1t_0 = t_2t_3t_2t_3t_0t_1t_3t_2$ then

$N^{(01012310)} = \{e, (0,2)(1,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_3t_1t_0N$ is at most: $\frac{|N|}{|N^{(01012310)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(01012310)}$ on $\{0,1,2,3\}$ are $\{0,1\}$ and $\{2,3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_1t_0t_0$,and $Nt_0t_1t_0t_1t_2t_3t_1t_0t_2$ belong?

However, $Nt_0t_1t_0t_1t_2t_3t_1t_0t_0 = Nt_2t_3t_2t_3t_0t_1t_3 \in [0101231]$, then two symmetric generators go back to the double coset $Nt_0t_1t_0t_1t_2t_3t_1N$, two symmetric generator go to $Nt_0t_1t_0t_1t_2t_3t_1t_0t_1$.

17. Now, we consider the cosets stabilizer $N^{(01012312)}$.

We know, $t_0t_1t_0t_1t_2t_3t_1t_2 = (1,3,0)(t_0t_1t_0t_2t_0t_3t_1t_0)^{(0,1)(2,3)}$ then

$Nt_0t_1t_0t_1t_2t_3t_1t_2 = Nt_1t_0t_1t_3t_1t_2t_0t_1 \in Nt_0t_1t_0t_2t_0t_3t_1t_0$.

$N^{(01012312)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_3t_1t_2N$ is at most: $\frac{|N|}{|N^{(01012312)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01012312)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_1t_2t_0$, $Nt_0t_1t_0t_1t_2t_3t_1t_2t_1$, $Nt_0t_1t_0t_1t_2t_3t_1t_2t_2$ and $Nt_0t_1t_0t_1t_2t_3t_1t_2t_3$ belong?

However, $Nt_0t_1t_0t_1t_2t_3t_1t_2t_2 = Nt_0t_1t_0t_1t_2t_3t_1 \in [0101231]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_3t_1N$, and $Nt_0t_1t_0t_1t_2t_3t_1t_2t_1 = Nt_1t_0t_1t_3t_1t_2t_0 \in [0102031]$, then one symmetric generator goes back to $Nt_0t_1t_0t_2t_0t_3t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_1t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_1t_2t_3$.

18. Now, we consider the cosets stabilizer $N^{(01012313)}$.

$N^{(01012313)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_3t_1t_3N$ is at most: $\frac{|N|}{|N^{(01012313)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01012313)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_1t_3t_0$, $Nt_0t_1t_0t_1t_2t_3t_1t_3t_1$, $Nt_0t_1t_0t_1t_2t_3t_1t_3t_2$ and $Nt_0t_1t_0t_1t_2t_3t_1t_3t_3$ belong?

However, $Nt_0t_1t_0t_1t_2t_3t_1t_3t_3 = Nt_0t_1t_0t_1t_2t_3t_1 \in [0101231]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_3t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_1t_3t_1$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_1t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_1t_3t_2$.

19. Now, we consider the cosets stabilizer $N^{(01012320)}$.
$N^{(01012320)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_3t_2t_0N$ is at most: $\frac{|N|}{|N^{(01012320)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01012320)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_2t_0t_0$, $Nt_0t_1t_0t_1t_2t_3t_2t_0t_1$, $Nt_0t_1t_0t_1t_2t_3t_2t_0t_2$ and $Nt_0t_1t_0t_1t_2t_3t_2t_0t_3$ belong?
However, $Nt_0t_1t_0t_1t_2t_3t_2t_0t_0 = Nt_0t_1t_0t_1t_2t_3t_2 \in [0101232]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_2t_0t_1$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_2t_0t_3$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_2t_0t_2$.

20. Now, we consider the cosets stabilizer $N^{(01012321)}$.
We know, $t_0t_1t_0t_1t_2t_3t_2t_1 = (t_0t_1t_0t_2t_1t_3t_2t_0)^{(1,2,0,3)}$ then
$Nt_0t_1t_0t_1t_2t_3t_2t_1 = Nt_3t_2t_3t_0t_2t_1t_0t_3 \in Nt_0t_1t_0t_2t_1t_3t_2t_0$.
$N^{(01012321)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_3t_2t_1N$ is at most: $\frac{|N|}{|N^{(01012321)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01012321)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_2t_1t_0$, $Nt_0t_1t_0t_1t_2t_3t_2t_1t_1$, $Nt_0t_1t_0t_1t_2t_3t_2t_1t_2$ and $Nt_0t_1t_0t_1t_2t_3t_2t_1t_3$ belong?
However, $Nt_0t_1t_0t_1t_2t_3t_2t_1t_1 = Nt_0t_1t_0t_1t_2t_3t_2 \in [0101232]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_3t_2N$, and $Nt_0t_1t_0t_1t_2t_3t_2t_1t_3 = Nt_3t_2t_3t_0t_2t_1t_0$ $\in [0102132]$, then one symmetric generator goes back to $Nt_0t_1t_0t_2t_1t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_2t_1t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_2t_1t_2$.

21. Now, we consider the cosets stabilizer $N^{(01012323)}$.
We know, $t_0t_1t_0t_1t_2t_3t_2t_3 = t_3t_1t_3t_1t_2t_0t_2t_0$ then
$N^{(01012323)} = \{e, (0,3)\}$, then the number of the single cosets in the double coset

$Nt_0t_1t_0t_1t_2t_3t_2t_3N$ is at most: $\frac{|N|}{|N^{(01012323)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(01012323)}$ on $\{0,1,2,3\}$ are $\{0,3\}$, $\{1\}$ and $\{2\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_2t_3t_0$, $Nt_0t_1t_0t_1t_2t_3t_2t_3t_1$, and $Nt_0t_1t_0t_1t_2t_3t_2t_3t_2$ belong?
However, $Nt_0t_1t_0t_1t_2t_3t_2t_3t_0 = Nt_3t_1t_3t_1t_2t_0t_2 \in [0101232]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_2t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_2t_3t_2$.

22. Now, we consider the cosets stabilizer $N^{(01020103)}$.
We know, $t_0t_1t_0t_2t_0t_1t_0t_3 = t_2t_0t_2t_1t_2t_0t_2t_3 = t_1t_2t_1t_0t_1t_2t_1t_3$ then $N^{(01020103)} = \{e, (0,2,1), (0,1,2)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_1t_0t_3N$ is at most: $\frac{|N|}{|N^{(01020103)}|} = \frac{24}{3} = 8$

The orbit of $N^{(01020103)}$ on $\{0,1,2,3\}$ are $\{0,1,2\}$ and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_1t_0t_3t_0$, and $Nt_0t_1t_0t_2t_0t_1t_0t_3t_3$ belong?
However, $Nt_0t_1t_0t_2t_0t_1t_0t_3t_3 = Nt_1t_2t_1t_0t_1t_2t_1 = Nt_2t_0t_2t_1t_2t_0t_2 \in [0102010]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_1t_0N$, three symmetric generators go to $Nt_0t_1t_0t_2t_0t_1t_0t_3t_0$.

23. [01020120] Same as 3.

24. Now, we consider the cosets stabilizer $N^{(01020123)}$.
$N^{(01020123)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_1t_2t_3N$ is at most: $\frac{|N|}{|N^{(01020123)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01020123)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_1t_2t_3t_0$, $Nt_0t_1t_0t_2t_0t_1t_2t_3t_1$, $Nt_0t_1t_0t_2t_0t_1t_2t_3t_2$ and $Nt_0t_1t_0t_2t_0t_1t_2t_3t_3$ belong?
However, $Nt_0t_1t_0t_2t_0t_1t_2t_3t_3 = Nt_0t_1t_0t_2t_0t_1t_2 \in [0102012]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_1t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_2t_3t_1$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_2t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_2t_3t_2$.

25. Now, we consider the cosets stabilizer $N^{(01020130)}$.
We know, $t_0t_1t_0t_2t_0t_1t_3t_0 = (1,0,2)(t_0t_1t_2t_0t_3t_0t_2t_0)^{(1,3,0)}$ then
$Nt_0t_1t_0t_2t_0t_1t_3t_0 = Nt_1t_3t_2t_1t_0t_1t_2t_1 \in Nt_0t_1t_2t_0t_3t_0t_2t_0$.
$N^{(01020130)} = \{e\}$, then the number of the single cosets in the double coset
$Nt_0t_1t_0t_2t_0t_1t_3t_0N$ is at most: $\frac{|N|}{|N^{(01020130)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01020130)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_1t_3t_0t_0$, $Nt_0t_1t_0t_2t_0t_1t_3t_0t_1$, $Nt_0t_1t_0t_2t_0t_1t_3t_0t_2$ and $Nt_0t_1t_0t_2t_0t_1t_3t_0t_3$ belong?
However, $Nt_0t_1t_0t_2t_0t_1t_3t_0t_0 = Nt_0t_1t_0t_1t_2t_3t_2 \in$ [0102013], then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_1t_3N$, and $Nt_0t_1t_0t_2t_0t_1t_3t_0t_1 = Nt_1t_3t_2t_1t_0t_1t_2$ $\in$ [0120302], then one symmetric generator goes back to $Nt_0t_1t_2t_0t_3t_0t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_3t_0t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_3t_0t_3$.

26. Now, we consider the cosets stabilizer $N^{(01020131)}$.
We know, $t_0t_1t_0t_2t_0t_1t_3t_1 = (t_0t_1t_0t_2t_3t_2t_0t_2)^{(1,3)(2,0)}$ then
$Nt_0t_1t_0t_2t_0t_1t_3t_1 = Nt_2t_3t_2t_0t_1t_0t_2t_0 \in Nt_0t_1t_0t_2t_3t_2t_0t_2$.
$N^{(01020131)} = \{e\}$, then the number of the single cosets in the double coset
$Nt_0t_1t_0t_2t_0t_1t_3t_1N$ is at most: $\frac{|N|}{|N^{(01020131)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01020131)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_1t_3t_1t_0$, $Nt_0t_1t_0t_2t_0t_1t_3t_1t_1$, $Nt_0t_1t_0t_2t_0t_1t_3t_1t_2$ and $Nt_0t_1t_0t_2t_0t_1t_3t_1t_3$ belong?
However, $Nt_0t_1t_0t_2t_0t_1t_3t_1t_1 = Nt_0t_1t_0t_2t_0t_1t_3 \in$ [0102013], then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_1t_3N$, and $Nt_0t_1t_0t_2t_0t_1t_3t_1t_0 = Nt_2t_3t_2t_0t_1t_0t_2$ $\in$ [0102320], then one symmetric generator goes back to $Nt_0t_1t_0t_2t_3t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_3t_1t_3$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_3t_1t_2$.

27. Now, we consider the cosets stabilizer $N^{(01020132)}$.
$t_0t_1t_0t_2t_0t_1t_3t_2 = (1,2,0)(t_0t_1t_0t_2t_0t_3t_2t_3)^{(1,2,3)} = (1,2)(0,3)(t_0t_1t_0t_2t_1t_0t_3t_2)^{(1,2,0,3)}$
$N^{(01020132)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_1t_3t_2N$ is at most: $\frac{|N|}{|N^{(01020132)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01020132)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_1t_3t_2t_0$,

$Nt_0t_1t_0t_2t_0t_1t_3t_2t_1$, $Nt_0t_1t_0t_2t_0t_1t_3t_2t_2$ and $Nt_0t_1t_0t_2t_0t_1t_3t_2t_3$ belong?
However, $Nt_0t_1t_0t_2t_0t_1t_3t_2t_2 = Nt_0t_1t_0t_2t_0t_1t_3$
$\in$ [0102013], then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_1t_3N$, and $Nt_0t_1t_0t_2t_0t_1t_3t_2t_1 = Nt_0t_2t_0t_3t_0t_1t_3 \in$ [0102032], then one symmetric generator goes back to $Nt_0t_1t_0t_2t_0t_3t_2N$, $Nt_0t_1t_0t_2t_0t_1t_3t_2t_0$
$= Nt_3t_2t_3t_0t_2t_3t_1 \in$ [0102103], then one symmetric generator goes back to $Nt_0t_1t_0t_2t_1t_0t_3N$ and one symmetric generator goes back to $Nt_0t_1t_0t_2t_0t_1t_3t_2t_3$.

28. Now, we consider coset $N^{(01020210)}$.
$t_0t_1t_0t_2t_0t_2t_1t_0 = (1,2,0)(t_0t_2t_0t_1t_0t_1t_2) = (1,2,0)(t_0t_1t_0t_2t_0t_2t_1)^{(1,2)} \in Nt_0t_1t_0t_2t_0t_2t_1$.
Therefore, t_0 takes $Nt_0t_1t_0t_2t_0t_2t_1t_0$ go to $Nt_0t_1t_0t_2t_0t_2t_1$.

29. Now, we consider the cosets stabilizer $N^{(01020213)}$.
$N^{(01020213)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_2t_1t_3N$ is at most: $\frac{|N|}{|N^{(01020213)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01020213)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_2t_1t_3t_0$, $Nt_0t_1t_0t_2t_0t_2t_1t_3t_1$, $Nt_0t_1t_0t_2t_0t_2t_1t_3t_2$ and $Nt_0t_1t_0t_2t_0t_2t_1t_3t_3$ belong?
However, $Nt_0t_1t_0t_2t_0t_2t_1t_3t_3 = Nt_0t_1t_0t_2t_0t_2t_1 \in$ [0102021], then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_2t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_1t_3t_1$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_1t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_1t_3t_2$.

30. Now, we consider the cosets stabilizer $N^{(01020230)}$.
$N^{(01020230)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_2t_3t_0N$ is at most: $\frac{|N|}{|N^{(01020230)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01020230)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_2t_3t_0t_0$, $Nt_0t_1t_0t_2t_0t_2t_3t_0t_1$, $Nt_0t_1t_0t_2t_0t_2t_3t_0t_2$ and $Nt_0t_1t_0t_2t_0t_2t_3t_0t_3$ belong?
However, $Nt_0t_1t_0t_2t_0t_2t_3t_0t_0 = Nt_0t_1t_0t_2t_0t_2t_3 \in$ [0102023], then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_3t_0t_1$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_3t_0t_3$, and one sym-

metric generator goes to $Nt_0t_1t_0t_2t_0t_2t_3t_0t_2$.

31. Now, we consider the cosets stabilizer $N^{(01020231)}$.
$N^{(01020231)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_2t_3t_1N$ is at most: $\frac{|N|}{|N^{(01020231)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01020231)}$ on $\{0, 1, 2, 3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_2t_3t_1t_0$, $Nt_0t_1t_0t_2t_0t_2t_3t_1t_1$, $Nt_0t_1t_0t_2t_0t_2t_3t_1t_2$ and $Nt_0t_1t_0t_2t_0t_2t_3t_1t_3$ belong?
However, $Nt_0t_1t_0t_2t_0t_2t_3t_1t_1 = Nt_0t_1t_0t_2t_0t_2t_3 \in [0102023]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_3t_1t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_3t_1t_3$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_3t_1t_2$.

32. Now, we consider the cosets stabilizer $N^{(01020232)}$.
$N^{(01020232)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_2t_3t_2N$ is at most: $\frac{|N|}{|N^{(01020232)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01020232)}$ on $\{0, 1, 2, 3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_2t_3t_2t_0$, $Nt_0t_1t_0t_2t_0t_2t_3t_2t_1$, $Nt_0t_1t_0t_2t_0t_2t_3t_2t_2$ and $Nt_0t_1t_0t_2t_0t_2t_3t_2t_3$ belong?
However, $Nt_0t_1t_0t_2t_0t_2t_3t_2t_2 = Nt_0t_1t_0t_2t_0t_2t_3 \in [0102023]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_3t_2t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_3t_2t_3$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_3t_2t_1$.

33. Now, we consider the cosets stabilizer $N^{(01020301)}$.
$N^{(01020301)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_3t_0t_1N$ is at most: $\frac{|N|}{|N^{(01020301)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01020301)}$ on $\{0, 1, 2, 3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0$, $Nt_0t_1t_0t_2t_0t_3t_0t_1t_1$, $Nt_0t_1t_0t_2t_0t_3t_0t_1t_2$ and $Nt_0t_1t_0t_2t_0t_3t_0t_1t_3$ belong?
However, $Nt_0t_1t_0t_2t_0t_3t_0t_1t_1 = Nt_0t_1t_0t_2t_0t_2t_3$
$\in [0102030]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_3t_0N$,

one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_0t_1t_3$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_0t_1t_2$.

34. Now, we consider the cosets stabilizer $N^{(01020302)}$.
We know, $t_0t_1t_0t_2t_0t_3t_0t_2 = (2,0,3)(t_0t_1t_0t_2t_1t_3t_0t_3)$ then
$Nt_0t_1t_0t_2t_0t_3t_0t_2 = Nt_0t_1t_0t_2t_1t_3t_0t_3 \in Nt_0t_1t_0t_2t_1t_3t_0t_3$.
$N^{(01020302)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_3t_0t_2N$ is at most: $\frac{|N|}{|N^{(01020302)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01020302)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_3t_0t_2t_0$, $Nt_0t_1t_0t_2t_0t_3t_0t_2t_1$, $Nt_0t_1t_0t_2t_0t_3t_0t_2t_2$ and $Nt_0t_1t_0t_2t_0t_3t_0t_2t_3$ belong?
However, $Nt_0t_1t_0t_2t_0t_3t_0t_2t_2 = Nt_0t_1t_0t_2t_0t_3t_0 \in$ [0102030], then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_3t_0N$, and $Nt_0t_1t_0t_2t_0t_3t_0t_2t_3 = Nt_0t_1t_0t_2t_1t_3t_0$ $\in$ [0102130], then one symmetric generator goes back to $Nt_0t_1t_0t_2t_1t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_0t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_0t_2t_0$.

35. Now, we consider the cosets stabilizer $N^{(01020303)}$.
$N^{(01020303)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_3t_0t_3N$ is at most: $\frac{|N|}{|N^{(01020303)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01020303)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_3t_0t_3t_0$, $Nt_0t_1t_0t_2t_0t_3t_0t_3t_1$, $Nt_0t_1t_0t_2t_0t_3t_0t_3t_2$ and $Nt_0t_1t_0t_2t_0t_3t_0t_3t_3$ belong?
However, $Nt_0t_1t_0t_2t_0t_3t_0t_3t_3 = Nt_0t_1t_0t_2t_0t_2t_3 \in$ [0102030], then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_0t_3t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_0t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_0t_3t_2$.

36. [01020310] Same as 17.

37. [01020312] Same as 7.

38. Now, we consider the cosets stabilizer $N^{(01020313)}$.
$N^{(01020313)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_3t_1t_3N$ is at most: $\frac{|N|}{|N^{(01020313)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01020313)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_3t_1t_3t_0$, $Nt_0t_1t_0t_2t_0t_3t_1t_3t_1$, $Nt_0t_1t_0t_2t_0t_3t_1t_3t_2$ and $Nt_0t_1t_0t_2t_0t_3t_1t_3t_3$ belong?
However, $Nt_0t_1t_0t_2t_0t_3t_1t_3t_3 = Nt_0t_1t_0t_2t_0t_3t_1 \in [0102031]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_3t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_1t_3t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_1t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_1t_3t_2$.

39. Now, we consider the cosets stabilizer $N^{(01020320)}$.
We know, $t_0t_1t_0t_2t_0t_3t_2t_0 = t_2t_0t_2t_1t_2t_3t_1t_2 = t_1t_2t_1t_0t_1t_3t_0t_1$ then
$N^{(01020320)} = \{e, (0,2,1), (0,1,2)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_3t_2t_0N$ is at most: $\frac{|N|}{|N^{(01020320)}|} = \frac{24}{3} = 8$

The orbit of $N^{(01020320)}$ on $\{0,1,2,3\}$ are $\{0,1,2\}$ and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_3t_2t_0t_0$, and $Nt_0t_1t_0t_2t_0t_3t_2t_0t_3$ belong?
However, $Nt_0t_1t_0t_2t_0t_3t_2t_0t_0 = Nt_2t_0t_2t_1t_2t_3t_1 = Nt_1t_2t_1t_0t_1t_3t_0 \in [0102032]$, then three symmetric generators go back to the double coset $Nt_0t_1t_0t_2t_0t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_2t_0t_3$.

40. Now, we consider the cosets stabilizer $N^{(01020321)}$.
$N^{(01020321)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_3t_2t_1N$ is at most: $\frac{|N|}{|N^{(01020321)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01020321)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_3t_2t_1t_0$, $Nt_0t_1t_0t_2t_0t_3t_2t_1t_1$, $Nt_0t_1t_0t_2t_0t_3t_2t_1t_2$ and $Nt_0t_1t_0t_2t_0t_3t_2t_1t_3$ belong?
However, $Nt_0t_1t_0t_2t_0t_3t_2t_1t_1 = Nt_0t_1t_0t_2t_0t_3t_2 \in [0102032]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_2t_1t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_2t_1t_3$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_2t_1t_2$.

41. [01020323] Same as 27.

42. [01021010] Same as 3.

43. Now, we consider the cosets stabilizer $N^{(01021012)}$.
We know, $t_0t_1t_0t_2t_1t_0t_1t_2 = t_0t_2t_0t_1t_2t_0t_2t_1$ then
$N^{(01021012)} = \{e, (1,2)\}$, thus the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_1t_0t_1t_2N$ is at most: $\frac{|N|}{|N^{(01021012)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(01021012)}$ on $\{0,1,2,3\}$ are $\{0\},\{1,2\}$ and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_0t_1t_2t_0$, $Nt_0t_1t_0t_2t_1t_0t_1t_2t_2$ and $Nt_0t_1t_0t_2t_1t_0t_1t_2t_3$ belong?
Moreover, $t_0t_1t_0t_2t_1t_0t_1t_2 = (1,0,2)(t_0t_1t_2t_1t_0t_1t_2t_0)^{(1,2)} = (1,0,2)(t_0t_2t_1t_2t_0t_2t_1t_0) \in$ [01210120].
However, $Nt_0t_1t_0t_2t_1t_0t_1t_2t_1 = Nt_0t_2t_0t_1t_2t_0t_2 \in$ [0102032], then two symmetric generators go back to the double coset $Nt_0t_1t_0t_2t_0t_3t_2N$, $Nt_0t_1t_0t_2t_1t_0t_1t_2t_0 = Nt_0t_1t_2t_1t_0t_1t_2 \in$ [0121012] one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_0t_1t_2t_3$.

44. Now, we consider the cosets stabilizer $N^{(01021031)}$.
$N^{(01021031)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_1t_0t_3t_1N$ is at most: $\frac{|N|}{|N^{(01021031)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01021031)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_0t_3t_1t_0$, $Nt_0t_1t_0t_2t_1t_0t_3t_1t_1$, $Nt_0t_1t_0t_2t_1t_0t_3t_1t_2$ and $Nt_0t_1t_0t_2t_1t_0t_3t_1t_3$ belong?
However, $Nt_0t_1t_0t_2t_1t_0t_3t_1t_1 = Nt_0t_1t_0t_2t_1t_0t_3 \in$ [0102103], then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_1t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_0t_3t_1t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_0t_3t_1t_3$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_0t_3t_1t_2$.

45. [01021032] Same as 27.

46. Now, we consider the cosets stabilizer $N^{(01021230)}$.
We know, $t_0t_1t_0t_2t_1t_2t_3t_0 = (2,0,1)(t_0t_1t_2t_3t_0t_2t_3t_2)^{(0,1)}$ then
$Nt_0t_1t_0t_2t_1t_2t_3t_0 = Nt_1t_0t_2t_3t_1t_2t_3t_2 \in Nt_0t_1t_2t_3t_0t_2t_3t_2$.
$N^{(01021230)} = \{e\}$, then the number of the single cosets in the double coset
$Nt_0t_1t_0t_2t_1t_2t_3t_0N$ is at most: $\frac{|N|}{|N^{(01021230)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01021230)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_2t_3t_0t_0$, $Nt_0t_1t_0t_2t_1t_2t_3t_0t_1$, $Nt_0t_1t_0t_2t_1t_2t_3t_0t_2$ and $Nt_0t_1t_0t_2t_1t_2t_3t_0t_3$ belong?
However, $Nt_0t_1t_0t_2t_1t_2t_3t_0t_0 = Nt_0t_1t_0t_2t_1t_2t_3 \in [0102123]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_1t_2t_3N$, and $Nt_0t_1t_0t_2t_1t_2t_3t_0t_2 = Nt_1t_0t_2t_3t_0t_2t_3$ $\in [0123023]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_0t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_2t_3t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_2t_3t_0t_3$.

47. Now, we consider the cosets stabilizer $N^{(01021231)}$.
We know, $t_0t_1t_0t_2t_1t_2t_3t_1 = t_2t_1t_2t_0t_1t_0t_3t_1$ then
$N^{(01021231)} = \{e,(0,2)\}$, then the number of the single cosets in the double coset
$Nt_0t_1t_0t_2t_1t_2t_3t_1N$ is at most: $\frac{|N|}{|N^{(01021231)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(01021231)}$ on $\{0,1,2,3\}$ are $\{0,2\},\{1\}$ and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_2t_3t_1t_0$, $Nt_0t_1t_0t_2t_1t_2t_3t_1t_1$ and $Nt_0t_1t_0t_2t_1t_2t_3t_1t_3$ belong?
Moreover, $t_0t_1t_0t_2t_1t_2t_3t_1 = (2,3,0)(t_0t_1t_2t_0t_2t_1t_2t_0)^{(1,3)} = (2,3,0)(t_0t_3t_2t_0t_2t_3t_2t_0) \in [01202120]$.
However, $Nt_0t_1t_0t_2t_1t_2t_3t_1t_1 = Nt_0t_1t_0t_2t_1t_2t_3 \in [0102123]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_1t_2t_3N$, $Nt_0t_1t_0t_2t_1t_2t_3t_1t_0 = Nt_0t_1t_2t_0t_2t_1t_2 \in [0120212]$ two symmetric generator go to $Nt_0t_1t_2t_1t_0t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_2t_3t_1t_3$.

48. Now, we consider the cosets stabilizer $N^{(01021301)}$.
$N^{(01021301)} = \{e\}$, then the number of the single cosets in the double coset
$Nt_0t_1t_0t_2t_1t_3t_0t_1N$ is at most: $\frac{|N|}{|N^{(01021301)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01021301)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a

representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_3t_0t_1t_0$, $Nt_0t_1t_0t_2t_1t_3t_0t_1t_1$, $Nt_0t_1t_0t_2t_1t_3t_0t_1t_2$ and $Nt_0t_1t_0t_2t_1t_3t_0t_1t_3$ belong?
However, $Nt_0t_1t_0t_2t_1t_3t_0t_1t_1 = Nt_0t_1t_0t_2t_1t_3t_0 \in$ [0102130], then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_1t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_0t_1t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_0t_1t_3$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_0t_1t_2$.

49. [01021303] Same as 34.

50. Now, we consider the cosets stabilizer $N^{(01021310)}$.
$t_0t_1t_0t_2t_1t_3t_1t_0 = (1,3,0)(t_0t_1t_2t_3t_1t_2t_0t_2)^{(1,2,3,0)} = (1,2)(0,3)(t_0t_1t_0t_2t_3t_0t_2t_1)^{(1,2,3,0)}$
$N^{(01021310)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_1t_3t_1t_0N$ is at most: $\frac{|N|}{|N^{(01021310)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01021310)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_3t_1t_0t_0$, $Nt_0t_1t_0t_2t_1t_3t_1t_0t_1$, $Nt_0t_1t_0t_2t_1t_3t_1t_0t_2$ and $Nt_0t_1t_0t_2t_1t_3t_1t_0t_3$ belong?
However, $Nt_0t_1t_0t_2t_1t_3t_1t_0t_0 = Nt_0t_1t_0t_2t_1t_3t_1 \in$ [0102131], then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_1t_3t_1N$, and $Nt_0t_1t_0t_2t_1t_3t_1t_0t_3 = Nt_1t_2t_3t_0t_2t_3t_1$ $\in$ [0123120], then one symmetric generator goes back to $Nt_0t_1t_2t_3t_1t_2t_0N$, $Nt_0t_1t_0t_2t_1t_3t_1t_0t_2 = Nt_1t_2t_1t_3t_1t_0t_3 \in$ [0102302], then one symmetric generator goes back to $Nt_0t_1t_0t_2t_3t_0t_2N$ and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_1t_0t_1$.

51. Now, we consider the cosets stabilizer $N^{(01021312)}$.
$N^{(01021312)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_1t_3t_1t_2N$ is at most: $\frac{|N|}{|N^{(01021312)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01021312)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_3t_1t_2t_0$, $Nt_0t_1t_0t_2t_1t_3t_1t_2t_1$, $Nt_0t_1t_0t_2t_1t_3t_1t_2t_2$ and $Nt_0t_1t_0t_2t_1t_3t_1t_2t_3$ belong?
However, $Nt_0t_1t_0t_2t_1t_3t_1t_2t_2 = Nt_0t_1t_0t_2t_1t_3t_1 \in$ [0102131], then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_1t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_1t_2t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_1t_2t_3$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_1t_2t_1$.

52. Now, we consider the cosets stabilizer $N^{(01021313)}$.
$N^{(01021313)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_1t_3t_1t_3N$ is at most: $\frac{|N|}{|N^{(01021313)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01021313)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_3t_1t_3t_0$, $Nt_0t_1t_0t_2t_1t_3t_1t_3t_1$, $Nt_0t_1t_0t_2t_1t_3t_1t_3t_2$ and $Nt_0t_1t_0t_2t_1t_3t_1t_3t_3$ belong?
However, $Nt_0t_1t_0t_2t_1t_3t_1t_3t_3 = Nt_0t_1t_0t_2t_1t_3t_1 \in [0102131]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_1t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_1t_3t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_1t_3t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_1t_3t_1$.

53. [01021320] Same as 20.

54. Now, we consider the cosets stabilizer $N^{(01021321)}$.
$N^{(01021321)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_1t_3t_2t_1N$ is at most: $\frac{|N|}{|N^{(01021321)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01021321)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_3t_2t_1t_0$, $Nt_0t_1t_0t_2t_1t_3t_2t_1t_1$, $Nt_0t_1t_0t_2t_1t_3t_2t_1t_2$ and $Nt_0t_1t_0t_2t_1t_3t_2t_1t_3$ belong?
However, $Nt_0t_1t_0t_2t_1t_3t_2t_1t_1 = Nt_0t_1t_0t_2t_1t_3t_1 \in [0102132]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_1t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_2t_1t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_2t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_2t_1t_3$.

55. Now, we consider the cosets stabilizer $N^{(01021323)}$.
We know, $t_0t_1t_0t_2t_1t_3t_2t_3 = t_2t_3t_2t_0t_3t_1t_0t_1$ then
$N^{(01021323)} = \{e,(0,2)(1,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_1t_3t_2t_3N$ is at most: $\frac{|N|}{|N^{(01021323)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(01021323)}$ on $\{0,1,2,3\}$ are $\{0,2\}$ and $\{1,3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_3t_2t_3t_0$, $Nt_0t_1t_0t_2t_1t_3t_2t_3t_3$and belong?

Moreover, $t_0t_1t_0t_2t_1t_3t_2t_3 = (t_0t_1t_2t_0t_2t_0t_3t_2)^{(1,3,2,0)} = t_1t_3t_0t_1t_0t_1t_2t_0 \in [01202032]$. However, $Nt_0t_1t_0t_2t_1t_3t_2t_3t_3 = Nt_0t_1t_0t_2t_1t_2t_3 \in [0102132]$, then two symmetric generators go back to the double coset $Nt_0t_1t_0t_2t_1t_3t_2N$, $Nt_0t_1t_0t_2t_1t_3t_2t_3t_0 = Nt_1t_3t_0t_1t_0t_1t_2 \in [0120203]$ two symmetric generator go to $Nt_0t_1t_2t_0t_2t_0t_3$.

56. Now, we consider coset $N^{(01023010)}$.
However, $t_0t_1t_0t_2t_3t_0t_1t_0 = (1,2,3)(t_1t_3t_1t_0t_2t_3t_2) = (1,2,3)(t_0t_1t_0t_2t_3t_1t_3)^{(1,3,2,0)}$ $\in Nt_0t_1t_0t_2t_0t_2t_1$. Therefore, t_0 takes $Nt_0t_1t_0t_2t_3t_0t_1t_0$ go to $Nt_0t_1t_0t_2t_3t_1t_3$.

57. [01023012] Same as 1.

58. Now, we consider the cosets stabilizer $N^{(01023013)}$.
$N^{(01023013)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_0t_1t_3N$ is at most: $\frac{|N|}{|N^{(01023013)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01023013)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_0t_1t_3t_0$, $Nt_0t_1t_0t_2t_3t_0t_1t_3t_1$, $Nt_0t_1t_0t_2t_3t_0t_1t_3t_2$ and $Nt_0t_1t_0t_2t_3t_0t_1t_3t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_0t_1t_3t_3 = Nt_0t_1t_0t_2t_3t_0t_1 \in [0102301]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_0t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_1t_3t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_1t_3t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_1t_3t_1$.

59. [01023021] Same as 50.

60. Now, we consider coset $N^{(01023023)}$.
However, $t_0t_1t_0t_2t_3t_0t_2t_3 = (0,2,3)(t_3t_1t_2t_0t_2t_2t_0) = (0,2,3)(t_0t_1t_2t_3t_0t_2t_3)^{(3,0)}$ $\in Nt_0t_1t_0t_2t_0t_2t_1$. Therefore, t_3 takes $Nt_0t_1t_0t_2t_3t_0t_2t_3$ go to $Nt_0t_1t_2t_3t_0t_2t_3$.

61. Now, we consider the cosets stabilizer $N^{(01023030)}$.
$N^{(01023030)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_0t_3t_0N$ is at most: $\frac{|N|}{|N^{(01023030)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01023030)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a

representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_0t_3t_0t_0$, $Nt_0t_1t_0t_2t_3t_0t_3t_0t_1$, $Nt_0t_1t_0t_2t_3t_0t_3t_0t_2$ and $Nt_0t_1t_0t_2t_3t_0t_3t_0t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_0t_3t_0t_0 = Nt_0t_1t_0t_2t_3t_0t_3 \in [0102303]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_3t_0t_1$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_3t_0t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_3t_0t_3$.

62. Now, we consider the cosets stabilizer $N^{(01023031)}$.
We know, $t_0t_1t_0t_2t_3t_0t_3t_1 = (1,3,2)(t_0t_1t_2t_1t_3t_2t_0t_2)^{(0,2)}$ then
$Nt_0t_1t_0t_2t_3t_0t_3t_1 = Nt_2t_1t_0t_1t_3t_0t_2t_0 \in Nt_0t_1t_2t_1t_3t_2t_0t_2$.
$N^{(01023031)} = \{e\}$, then the number of the single cosets in the double coset
$Nt_0t_1t_0t_2t_3t_0t_3t_1N$ is at most: $\frac{|N|}{|N^{(01023031)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01023031)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_0t_3t_1t_0$, $Nt_0t_1t_0t_2t_3t_0t_3t_1t_1$, $Nt_0t_1t_0t_2t_3t_0t_3t_1t_2$ and $Nt_0t_1t_0t_2t_3t_0t_3t_1t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_0t_3t_1t_1 = Nt_0t_1t_0t_2t_3t_0t_3 \in [0102303]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_0t_3N$, and $Nt_0t_1t_0t_2t_3t_0t_3t_1t_0 = Nt_2t_1t_0t_1t_3t_0t_2$ $\in [0121320]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_3t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_3t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_3t_1t_3$.

63. Now, we consider the cosets stabilizer $N^{(01023032)}$.
We know, $t_0t_1t_0t_2t_3t_0t_3t_2 = (1,3)(0,2)(t_0t_1t_2t_3t_0t_2t_3t_1)^{(0,1)(2,3)}$ then
$Nt_0t_1t_0t_2t_3t_0t_3t_2 = Nt_1t_0t_3t_2t_1t_3t_2t_0 \in Nt_0t_1t_2t_3t_0t_2t_3t_1$.
$N^{(01023032)} = \{e\}$, then the number of the single cosets in the double coset
$Nt_0t_1t_0t_2t_3t_0t_3t_2N$ is at most: $\frac{|N|}{|N^{(01023032)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01023032)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_0t_3t_2t_0$, $Nt_0t_1t_0t_2t_3t_0t_3t_2t_1$, $Nt_0t_1t_0t_2t_3t_0t_3t_2t_2$ and $Nt_0t_1t_0t_2t_3t_0t_3t_2t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_0t_3t_2t_2 = Nt_0t_1t_0t_2t_3t_0t_3 \in [0102303]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_0t_3N$, and $Nt_0t_1t_0t_2t_3t_0t_3t_2t_0 = Nt_1t_0t_3t_2t_1t_3t_2$ $\in [0123023]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_0t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_3t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_3t_2t_3$.

64. Now, we consider the cosets stabilizer $N^{(01023120)}$.
$N^{(01023120)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_1t_2t_0N$ is at most: $\frac{|N|}{|N^{(01023120)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01023120)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_1t_2t_0t_0$, $Nt_0t_1t_0t_2t_3t_1t_2t_0t_1$, $Nt_0t_1t_0t_2t_3t_1t_2t_0t_2$ and $Nt_0t_1t_0t_2t_3t_1t_2t_0t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_1t_2t_0t_0 = Nt_0t_1t_0t_2t_3t_1t_2 \in [0102312]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_1t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_2t_0t_1$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_2t_0t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_2t_0t_3$.

65. Now, we consider the cosets stabilizer $N^{(01023121)}$.
We know, $t_0t_1t_0t_2t_3t_1t_2t_1 = (1,0,3)(t_0t_1t_2t_1t_0t_1t_3t_1)^{(0,1)(2,3)}$ then $Nt_0t_1t_0t_2t_3t_1t_2t_1 = Nt_1t_0t_3t_0t_1t_0t_2t_0 \in Nt_0t_1t_2t_1t_0t_1t_3t_1$.
$N^{(01023121)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_1t_2t_1N$ is at most: $\frac{|N|}{|N^{(01023121)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01023121)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_1t_2t_1t_0$, $Nt_0t_1t_0t_2t_3t_1t_2t_1t_1$, $Nt_0t_1t_0t_2t_3t_1t_2t_1t_2$ and $Nt_0t_1t_0t_2t_3t_1t_2t_1t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_1t_2t_1t_1 = Nt_0t_1t_0t_2t_3t_0t_3 \in [0102312]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_0t_3N$, and $Nt_0t_1t_0t_2t_3t_1t_2t_1t_0 = Nt_1t_0t_3t_0t_1t_0t_2$ $\in [0121013]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_0t_1t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_2t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_2t_1t_3$.

66. Now, we consider the cosets stabilizer $N^{(01023123)}$.
We know, $t_0t_1t_0t_2t_3t_1t_2t_3 = (1,0,2)(t_0t_1t_2t_0t_2t_3t_2t_1)^{(1,2,0)}$ then $Nt_0t_1t_0t_2t_3t_1t_2t_3 = Nt_1t_2t_0t_1t_0t_3t_0t_2 \in Nt_0t_1t_2t_0t_2t_3t_2t_1$.
$N^{(01023123)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_1t_2t_3N$ is at most: $\frac{|N|}{|N^{(01023123)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01023123)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_1t_2t_3t_0$,

$Nt_0t_1t_0t_2t_3t_1t_2t_3t_1$, $Nt_0t_1t_0t_2t_3t_1t_2t_3t_2$ and $Nt_0t_1t_0t_2t_3t_1t_2t_3t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_1t_2t_3t_3 = Nt_0t_1t_0t_2t_3t_0t_3 \in$ [0102312], then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_0t_3N$, and $Nt_0t_1t_0t_2t_3t_1t_2t_3t_2 = Nt_1t_2t_0t_1t_0t_3t_0$ $\in$ [0120232], then one symmetric generator goes back to $Nt_0t_1t_2t_0t_2t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_2t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_2t_3t_1$.

67. Now, we consider the cosets stabilizer $N^{(01023130)}$.
$N^{(01023130)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_1t_3t_0N$ is at most: $\frac{|N|}{|N^{(01023130)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01023130)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_1t_3t_0t_0$, $Nt_0t_1t_0t_2t_3t_1t_3t_0t_1$, $Nt_0t_1t_0t_2t_3t_1t_3t_0t_2$ and $Nt_0t_1t_0t_2t_3t_1t_3t_0t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_1t_3t_0t_0 = Nt_0t_1t_0t_2t_3t_1t_3 \in$ [0102313], then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_1t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_3t_0t_1$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_3t_0t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_3t_0t_3$.

68. Now, we consider the cosets stabilizer $N^{(01023131)}$.
$N^{(01023131)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_1t_3t_1N$ is at most: $\frac{|N|}{|N^{(01023131)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01023131)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_1t_3t_1t_0$, $Nt_0t_1t_0t_2t_3t_1t_3t_1t_1$, $Nt_0t_1t_0t_2t_3t_1t_3t_1t_2$ and $Nt_0t_1t_0t_2t_3t_1t_3t_1t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_1t_3t_1t_1 = Nt_0t_1t_0t_2t_3t_1t_3 \in$ [0102313], then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_1t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_3t_1t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_3t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_3t_1t_3$.

69. Now, we consider coset $N^{(01023132)}$.
However, $t_0t_1t_0t_2t_3t_1t_3t_2 = (1,3,0)(t_2t_0t_2t_3t_2t_1t_0) = (1,3,0)(t_0t_1t_0t_2t_3t_0t_1)^{(1,0,2,3)}$ $\in Nt_0t_1t_0t_2t_3t_0t_1$. Therefore, t_2 takes $Nt_0t_1t_0t_2t_3t_1t_3t_2$ go to $Nt_0t_1t_0t_2t_3t_0t_1$.

70. Now, we consider coset $N^{(01023201)}$.
However, $t_0t_1t_0t_2t_3t_2t_0t_1 = (1,2,3)(t_1t_3t_0t_2t_0t_1t_3) = (1,2,3)(t_0t_1t_2t_3t_2t_0t_1)^{(1,3,0,2)}$ $\in Nt_0t_1t_2t_3t_2t_0t_1$. Therefore, t_1 takes $Nt_0t_1t_0t_2t_3t_2t_0t_1$ go to $Nt_0t_1t_2t_3t_2t_0t_1$.

71. [01023202] Same as 26.

72. Now, we consider the cosets stabilizer $N^{(01023203)}$.
We know, $t_0t_1t_0t_2t_3t_2t_0t_3 = (t_0t_1t_2t_0t_2t_1t_3t_1)^{(2,3,0)} = (t_0t_1t_2t_0t_3t_2t_3t_0)^{(2,3)}$
$N^{(01023203)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_2t_0t_3N$ is at most: $\frac{|N|}{|N^{(01023203)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01023203)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_2t_0t_3t_0$, $Nt_0t_1t_0t_2t_3t_2t_0t_3t_1$, $Nt_0t_1t_0t_2t_3t_2t_0t_3t_2$ and $Nt_0t_1t_0t_2t_3t_2t_0t_3t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_2t_0t_3t_3 = Nt_0t_1t_0t_2t_3t_2t_0 \in [0102320]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_2t_0N$, and $Nt_0t_1t_0t_2t_3t_2t_0t_3t_1 = Nt_2t_1t_3t_2t_3t_1t_0$ $\in [0120213]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_2t_1t_3N$, $Nt_0t_1t_0t_2t_3t_2t_0t_3t_0 = Nt_0t_1t_3t_0t_2t_3t_2 \in [0120323]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_3t_2t_3N$ and one symmetric generator goes back to $Nt_0t_1t_0t_2t_3t_2t_0t_3t_2$.

73. Now, we consider coset $N^{(01023210)}$.
However, $t_0t_1t_0t_2t_3t_2t_1t_0 = (1,0,3)(t_2t_1t_0t_1t_2t_3t_2) = (1,2,3)(t_0t_1t_2t_1t_0t_3t_0)^{(0,2)}$ $\in Nt_0t_1t_2t_1t_0t_3t_0$. Therefore, t_0 takes $Nt_0t_1t_0t_2t_3t_2t_1t_0$ go to $Nt_0t_1t_2t_1t_0t_3t_0$.

74. Now, we consider the cosets stabilizer $N^{(01023212)}$.
$N^{(01023212)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_2t_1t_2N$ is at most: $\frac{|N|}{|N^{(01023212)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01023212)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_2t_1t_2t_0$, $Nt_0t_1t_0t_2t_3t_2t_1t_2t_1$, $Nt_0t_1t_0t_2t_3t_2t_1t_2t_2$ and $Nt_0t_1t_0t_2t_3t_2t_1t_2t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_2t_1t_2t_2 = Nt_0t_1t_0t_2t_3t_2t_1 \in [0102321]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_2t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_1t_2t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_1t_2t_1$, and one sym-

metric generator goes to $Nt_0t_1t_0t_2t_3t_2t_1t_2t_3$.

75. [01023213] Same as 25.

76. Now, we consider the cosets stabilizer $N^{(01023230)}$.
$N^{(01023230)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_2t_3t_0N$ is at most: $\frac{|N|}{|N^{(01023230)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01023230)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_2t_3t_0t_0$, $Nt_0t_1t_0t_2t_3t_2t_3t_0t_1$, $Nt_0t_1t_0t_2t_3t_2t_3t_0t_2$ and $Nt_0t_1t_0t_2t_3t_2t_3t_0t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_2t_3t_0t_0 = Nt_0t_1t_0t_2t_3t_2t_3 \in [0102323]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_3t_0t_2$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_3t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_3t_0t_3$.

77. Now, we consider the cosets stabilizer $N^{(01023231)}$.
We know, $t_0t_1t_0t_2t_3t_2t_3t_1 = (t_0t_1t_2t_0t_3t_2t_1t_3)^{(2,3,0)} = (t_0t_1t_2t_1t_2t_3t_0t_3)^{(1,3,2,0)}$
$N^{(01023231)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_2t_3t_1N$ is at most: $\frac{|N|}{|N^{(01023231)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01023231)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_2t_3t_1t_0$, $Nt_0t_1t_0t_2t_3t_2t_3t_1t_1$, $Nt_0t_1t_0t_2t_3t_2t_3t_1t_2$ and $Nt_0t_1t_0t_2t_3t_2t_3t_1t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_2t_3t_1t_1 = Nt_0t_1t_0t_2t_3t_2t_0 \in [0102323]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_2t_0N$, and $Nt_0t_1t_0t_2t_3t_2t_3t_1t_0 = Nt_2t_1t_3t_2t_0t_3t_1 \in [0120321]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_3t_2t_1N$, $Nt_0t_1t_0t_2t_3t_2t_3t_1t_2 = Nt_1t_3t_0t_3t_0t_2t_1 \in [0121230]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_2t_3t_0N$ and one symmetric generator goes back to $Nt_0t_1t_0t_2t_3t_2t_3t_1t_3$.

78. Now, we consider the cosets stabilizer $N^{(01023232)}$.
We know, $t_0t_1t_0t_2t_3t_2t_3t_2 = t_0t_1t_0t_3t_2t_3t_2t_3$ then
$N^{(01023232)} = \{e, (0,1)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_2t_3t_2N$ is at most: $\frac{|N|}{|N^{(01023232)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(01023232)}$ on $\{0,1,2,3\}$ are $\{2,3\}$,$\{0\}$ and $\{1\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_2t_3t_2t_0$, $Nt_0t_1t_0t_2t_3t_2t_3t_2t_1$ and $Nt_0t_1t_0t_2t_3t_2t_3t_2t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_2t_3t_2t_2 = Nt_0t_1t_0t_3t_2t_3t_2 \in$ [0102323], then two symmetric generators go back to the double coset $Nt_0t_1t_0t_2t_3t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_3t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_3t_2t_1$.

79. Now, we consider coset $N^{(01201010)}$.
However, $t_0t_1t_2t_0t_1t_0t_1t_0 = (t_0t_2t_1t_0t_2t_0t_2) = (t_0t_1t_2t_0t_1t_0t_1)^{(1,2)} \in Nt_0t_1t_2t_0t_1t_0t_1$. Therefore, t_0 takes $Nt_0t_1t_2t_0t_1t_0t_1t_0$ go to $Nt_0t_1t_2t_0t_1t_0t_1$.

80. Now, we consider coset $N^{(01201012)}$.
However, $t_0t_1t_2t_0t_1t_0t_1t_2 = (1,0,2)(t_0t_2t_1t_0t_2t_0t_2) = (1,0,2)(t_0t_1t_0t_1t_2t_1t_0)^{(1,0)}$
$\in Nt_0t_1t_0t_1t_2t_1t_0$. Therefore, t_2 takes $Nt_0t_1t_2t_0t_1t_0t_1t_2$ go to $Nt_0t_1t_0t_1t_2t_1t_0$.

81. Now, we consider the cosets stabilizer $N^{(01201013)}$.
We know, $t_0t_1t_2t_0t_1t_0t_1t_3 = (t_0t_1t_2t_0t_3t_2t_3t_1)^{(1,0)(2,3)}$ then
$Nt_0t_1t_2t_0t_1t_0t_1t_3 = Nt_1t_0t_3t_1t_2t_3t_2t_0 \in Nt_0t_1t_2t_0t_3t_2t_3t_1$.
$N^{(01201013)} = \{e\}$, then the number of the single cosets in the double coset
$Nt_0t_1t_2t_0t_1t_0t_1t_3N$ is at most: $\frac{|N|}{|N^{(01201013)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01201013)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_1t_0t_1t_3t_0$, $Nt_0t_1t_2t_0t_1t_0t_1t_3t_1$, $Nt_0t_1t_2t_0t_1t_0t_1t_3t_2$ and $Nt_0t_1t_2t_0t_1t_0t_1t_3t_3$ belong?
However, $Nt_0t_1t_2t_0t_1t_0t_1t_3t_3 = Nt_0t_1t_0t_2t_3t_0t_3 \in$ [0120101], then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_1t_0t_1N$, and $Nt_0t_1t_2t_0t_1t_0t_1t_3t_0 = Nt_1t_0t_3t_1t_2t_3t_2$ $\in$ [0120323], then one symmetric generator goes back to $Nt_0t_1t_2t_0t_3t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_0t_1t_3t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_0t_1t_3t_1$.

82. Now, we consider the cosets stabilizer $N^{(01201030)}$.
We know, $t_0t_1t_2t_0t_1t_0t_3t_0 = t_0t_2t_3t_0t_2t_0t_1t_0 = t_0t_3t_1t_0t_3t_0t_2t_0$ then
$N^{(01201030)} = \{e,(3,2,1),(3,1,2)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_1t_0t_3t_0N$ is at most: $\frac{|N|}{|N^{(01201030)}|} = \frac{24}{3} = 8$

The orbit of $N^{(01201030)}$ on $\{0,1,2,3\}$ are $\{3,1,2\}$ and $\{0\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_1t_0t_3t_0t_0$ and $Nt_0t_1t_2t_0t_1t_0t_3t_0t_3$ belong?
However, $Nt_0t_1t_2t_0t_1t_0t_3t_0t_0 = Nt_0t_2t_3t_0t_2t_0t_1 = Nt_0t_3t_1t_0t_3t_0t_2 \in [0120103]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_1t_0t_3N$, three symmetric generators go to $Nt_0t_1t_2t_0t_1t_0t_3t_0t_1$.

83. Now, we consider the cosets stabilizer $N^{(01201301)}$.
$N^{(01201301)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_1t_3t_0t_1N$ is at most: $\frac{|N|}{|N^{(01201301)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01201301)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_1t_3t_0t_1t_0$, $Nt_0t_1t_2t_0t_1t_3t_0t_1t_1$, $Nt_0t_1t_2t_0t_1t_3t_0t_1t_2$ and $Nt_0t_1t_2t_0t_1t_3t_0t_1t_3$ belong?
However, $Nt_0t_1t_2t_0t_1t_3t_0t_1t_1 = Nt_0t_1t_2t_0t_1t_3t_0 \in [0120130]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_1t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_0t_1t_2$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_0t_1t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_0t_1t_3$.

84. Now, we consider the cosets stabilizer $N^{(01201302)}$.
$N^{(01201302)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_1t_3t_0t_2N$ is at most: $\frac{|N|}{|N^{(01201302)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01201302)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_1t_3t_0t_2t_0$, $Nt_0t_1t_2t_0t_1t_3t_0t_2t_1$, $Nt_0t_1t_2t_0t_1t_3t_0t_2t_2$ and $Nt_0t_1t_2t_0t_1t_3t_0t_2t_3$ belong?
However, $Nt_0t_1t_2t_0t_1t_3t_0t_2t_2 = Nt_0t_1t_2t_0t_1t_3t_0 \in [0120130]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_1t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_0t_2t_1$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_0t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_0t_2t_3$.

85. Now, we consider the cosets stabilizer $N^{(01201303)}$.
$N^{(01201303)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_1t_3t_0t_3N$ is at most: $\frac{|N|}{|N^{(01201303)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01201303)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_1t_3t_0t_3t_0$, $Nt_0t_1t_2t_0t_1t_3t_0t_3t_1$, $Nt_0t_1t_2t_0t_1t_3t_0t_3t_2$ and $Nt_0t_1t_2t_0t_1t_3t_0t_3t_3$ belong?
However, $Nt_0t_1t_2t_0t_1t_3t_0t_3t_3 = Nt_0t_1t_2t_0t_1t_3t_0 \in [0120130]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_1t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_0t_3t_1$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_0t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_0t_3t_2$.

86. Now, we consider coset $N^{(01201310)}$.
However, $t_0t_1t_2t_0t_1t_3t_1t_0 = (1,3,0)(t_2t_3t_1t_0t_1t_3t_2) = (1,3,0)(t_0t_1t_2t_3t_2t_1t_0)^{(1,3,0,2)}$ $\in Nt_0t_1t_2t_3t_2t_1t_0$. Therefore, t_0 takes $Nt_0t_1t_2t_0t_1t_3t_1t_0$ go to $Nt_0t_1t_2t_3t_2t_1t_0$.

87. Now, we consider the cosets stabilizer $N^{(01201313)}$.
$N^{(01201313)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_1t_3t_1t_3N$ is at most: $\frac{|N|}{|N^{(01201313)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01201313)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_1t_3t_1t_3t_0$, $Nt_0t_1t_2t_0t_1t_3t_1t_3t_1$, $Nt_0t_1t_2t_0t_1t_3t_1t_3t_2$ and $Nt_0t_1t_2t_0t_1t_3t_1t_3t_3$ belong?
However, $Nt_0t_1t_2t_0t_1t_3t_1t_3t_3 = Nt_0t_1t_2t_0t_1t_3t_1 \in [0120131]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_1t_3t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_1t_3t_1$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_1t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_1t_3t_2$.

88. Now, we consider coset $N^{(01201321)}$.
However, $t_0t_1t_2t_0t_1t_3t_2t_1 = (1,0,3)(t_2t_3t_1t_0t_3t_2t_3) = (1,0,3)(t_0t_1t_2t_3t_1t_0t_1)^{(1,2,0,3)}$ $\in Nt_0t_1t_2t_3t_1t_0t_1$. Therefore, t_1 takes $Nt_0t_1t_2t_0t_1t_3t_2t_1$ go to $Nt_0t_1t_2t_3t_1t_0t_1$.

89. Now, we consider the cosets stabilizer $N^{(01201323)}$.
We know, $t_0t_1t_2t_0t_1t_3t_2t_3 = (1,3,2)(t_0t_1t_2t_1t_3t_1t_0t_3)^{(1,2,0,3)}$ then
$Nt_0t_1t_2t_0t_1t_3t_2t_3 = Nt_3t_2t_0t_2t_1t_2t_3t_1 \in Nt_0t_1t_2t_1t_3t_1t_0t_3$.
$N^{(01201323)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_1t_3t_2t_3N$ is at most: $\frac{|N|}{|N^{(01201323)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01201323)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_1t_3t_2t_3t_0$, $Nt_0t_1t_2t_0t_1t_3t_2t_3t_1$, $Nt_0t_1t_2t_0t_1t_3t_2t_3t_2$ and $Nt_0t_1t_2t_0t_1t_3t_2t_3t_3$ belong?
However, $Nt_0t_1t_2t_0t_1t_3t_2t_3t_3 = Nt_0t_1t_0t_2t_3t_0t_3 \in [0120132]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_1t_0t_1N$, and $Nt_0t_1t_2t_0t_1t_3t_2t_3t_1 = Nt_3t_2t_0t_2t_1t_2t_3$ $\in [0121310]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_3t_1t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_2t_3t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_2t_3t_0$.

90. Now, we consider the cosets stabilizer $N^{(01202030)}$.
$N^{(01202030)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_2t_0t_3t_0N$ is at most: $\frac{|N|}{|N^{(01202030)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01202030)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_2t_0t_3t_0t_0$, $Nt_0t_1t_2t_0t_2t_0t_3t_0t_1$, $Nt_0t_1t_2t_0t_2t_0t_3t_0t_2$ and $Nt_0t_1t_2t_0t_2t_0t_3t_0t_3$ belong?
However, $Nt_0t_1t_2t_0t_2t_0t_3t_0t_0 = Nt_0t_1t_2t_0t_2t_0t_3 \in [0120203]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_2t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_0t_3t_0t_1$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_0t_3t_0t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_0t_3t_0t_2$.

91. Now, we consider the cosets stabilizer $N^{(01202031)}$.
We know, $t_0t_1t_2t_0t_2t_0t_3t_1 = (t_0t_1t_2t_3t_2t_3t_0t_2)^{(1,0)(2,3)}$ then
$Nt_0t_1t_2t_0t_2t_0t_3t_1 = Nt_1t_0t_3t_2t_3t_2t_1t_3 \in Nt_0t_1t_2t_3t_2t_3t_0t_2$.
$N^{(01202031)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_2t_0t_3t_1N$ is at most: $\frac{|N|}{|N^{(01202031)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01202031)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_2t_0t_3t_1t_0$, $Nt_0t_1t_2t_0t_2t_0t_3t_1t_1$, $Nt_0t_1t_2t_0t_2t_0t_3t_1t_2$ and $Nt_0t_1t_2t_0t_2t_0t_3t_1t_3$ belong?
However, $Nt_0t_1t_2t_0t_2t_0t_3t_1t_1 = Nt_0t_1t_2t_0t_2t_0t_3 \in [0120203]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_2t_0t_3N$, and $Nt_0t_1t_2t_0t_2t_0t_3t_1t_3 = Nt_1t_0t_3t_2t_3t_2t_1$ $\in [0123230]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_2t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_0t_3t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_0t_3t_1t_0$.

92. [01202032] Same as 55.

93. [01202120] Same as 47.

94. Now, we consider the cosets stabilizer $N^{(01202123)}$.
We know, $t_0t_1t_2t_0t_2t_1t_2t_3 = (1,0,2)(t_0t_1t_2t_3t_0t_2t_0t_1)^{(0,1,2,3)}$ then
$Nt_0t_1t_2t_0t_2t_1t_2t_3 = Nt_1t_2t_3t_0t_1t_3t_1t_2 \in Nt_0t_1t_2t_3t_0t_2t_0t_1$.
$N^{(01202123)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_2t_1t_2t_3N$ is at most: $\frac{|N|}{|N^{(01202123)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01202123)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_2t_1t_2t_3t_0$, $Nt_0t_1t_2t_0t_2t_1t_2t_3t_1$, $Nt_0t_1t_2t_0t_2t_1t_2t_3t_2$ and $Nt_0t_1t_2t_0t_2t_1t_2t_3t_3$ belong?
However, $Nt_0t_1t_2t_0t_2t_1t_2t_3t_3 = Nt_0t_1t_2t_0t_2t_1t_2 \in$ [0120212], then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_2t_1t_2N$, and $Nt_0t_1t_2t_0t_2t_1t_2t_3t_2 = Nt_1t_2t_3t_0t_1t_3t_1$ $\in$ [0123020], then one symmetric generator goes back to $Nt_0t_1t_2t_3t_0t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_1t_2t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_1t_2t_3t_0$.

95. Now, we consider the cosets stabilizer $N^{(01202130)}$.
We know, $t_0t_1t_2t_0t_2t_1t_3t_0 = (1,0)(3,2)(t_0t_1t_2t_3t_1t_2t_0t_3)^{(0,1)(2,3)}$ then
$Nt_0t_1t_2t_0t_2t_1t_3t_0 = Nt_1t_0t_3t_2t_0t_3t_1t_2 \in Nt_0t_1t_2t_3t_1t_2t_0t_3$.
$N^{(01202130)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_2t_1t_3t_0N$ is at most: $\frac{|N|}{|N^{(01202130)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01202130)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_2t_1t_3t_0t_0$, $Nt_0t_1t_2t_0t_2t_1t_3t_0t_1$, $Nt_0t_1t_2t_0t_2t_1t_3t_0t_2$ and $Nt_0t_1t_2t_0t_2t_1t_3t_0t_3$ belong?
However, $Nt_0t_1t_2t_0t_2t_1t_3t_0t_0 = Nt_0t_1t_2t_0t_2t_1t_3 \in$ [0120213], then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_2t_1t_3N$, and $Nt_0t_1t_2t_0t_2t_1t_3t_0t_2 = Nt_1t_0t_3t_2t_0t_3t_1$ $\in$ [0123120], then one symmetric generator goes back to $Nt_0t_1t_2t_3t_1t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_1t_3t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_1t_3t_0t_2$.

96. [01202131] Same as 72.

97. Now, we consider the cosets stabilizer $N^{(01202132)}$.
We know, $t_0t_1t_2t_0t_2t_1t_3t_2 = (1,3)(0,2)(t_0t_1t_2t_3t_2t_0t_1t_3)^{(1,0,3)}$ then
$Nt_0t_1t_2t_0t_2t_1t_3t_2 = Nt_3t_0t_2t_1t_2t_3t_0t_1 \in Nt_0t_1t_2t_3t_2t_0t_1t_3$.
$N^{(01202132)} = \{e\}$, then the number of the single cosets in the double coset
$Nt_0t_1t_2t_0t_2t_1t_3t_2N$ is at most: $\frac{|N|}{|N^{(01202132)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01202132)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_2t_1t_3t_2t_0$, $Nt_0t_1t_2t_0t_2t_1t_3t_2t_1$, $Nt_0t_1t_2t_0t_2t_1t_3t_2t_2$ and $Nt_0t_1t_2t_0t_2t_1t_3t_2t_3$ belong?
However, $Nt_0t_1t_2t_0t_2t_1t_3t_2t_2 = Nt_0t_1t_2t_0t_2t_1t_3 \in [0120213]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_2t_1t_3N$, and $Nt_0t_1t_2t_0t_2t_1t_3t_2t_1 = Nt_3t_0t_2t_1t_2t_3t_0$ $\in [0123201]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_2t_0t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_1t_3t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_1t_3t_2t_2$.

98. Now, we consider the cosets stabilizer $N^{(01202310)}$.
$t_0t_1t_2t_0t_2t_3t_1t_0 = (1,3)(2,0)(t_0t_1t_2t_1t_3t_2t_0t_1)^{(1,2)(3,0)} = (1,0,2)(t_0t_1t_2t_1t_3t_1t_2t_3)^{(1,2,0)}$
$N^{(01202310)} = \{e\}$, then the number of the single cosets in the double coset
$Nt_0t_1t_2t_0t_2t_3t_1t_0N$ is at most: $\frac{|N|}{|N^{(01202310)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01202310)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_2t_3t_1t_0t_0$, $Nt_0t_1t_2t_0t_2t_3t_1t_0t_1$, $Nt_0t_1t_2t_0t_2t_3t_1t_0t_2$ and $Nt_0t_1t_2t_0t_2t_3t_1t_0t_3$ belong?
However, $Nt_0t_1t_2t_0t_2t_3t_1t_0t_0 = Nt_0t_1t_2t_0t_2t_3t_1 \in [0120231]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_2t_3t_1N$, and $Nt_0t_1t_2t_0t_2t_3t_1t_0t_2 = Nt_3t_2t_1t_2t_0t_1t_3$ $\in [0121320]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_3t_2t_0N$, $Nt_0t_1t_2t_0t_2t_3t_1t_0t_3$ $= Nt_1t_2t_0t_2t_3t_2t_0 \in [0121312]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_3t_1t_2N$ and one symmetric generator goes back to $Nt_0t_1t_2t_0t_2t_3t_1t_0t_1$.

99. Now, we consider coset $N^{(01202312)}$.
However, $t_0t_1t_2t_0t_2t_3t_1t_2 = (1,3,2)(t_1t_3t_2t_0t_3t_2t_0) = (1,3,2)(t_0t_1t_2t_3t_1t_2t_3)^{(1,3,0)}$
$\in Nt_0t_1t_2t_3t_1t_2t_3$. Therefore, t_2 takes $Nt_0t_1t_2t_0t_2t_3t_1t_2$ go to $Nt_0t_1t_2t_3t_1t_2t_3$.

100. Now, we consider the cosets stabilizer $N^{(01202313)}$.
$N^{(01202313)} = \{e\}$, then the number of the single cosets in the double coset

$Nt_0t_1t_2t_0t_2t_3t_1t_3N$ is at most: $\frac{|N|}{|N^{(01202313)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01202313)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_2t_3t_1t_3t_0$, $Nt_0t_1t_2t_0t_2t_3t_1t_3t_1$, $Nt_0t_1t_2t_0t_2t_3t_1t_3t_2$ and $Nt_0t_1t_2t_0t_2t_3t_1t_3t_3$ belong?

However, $Nt_0t_1t_2t_0t_2t_3t_1t_3t_3 = Nt_0t_1t_2t_0t_2t_3t_1 \in [0120231]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_2t_3t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_1t_3t_1$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_1t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_1t_3t_2$.

101. Now, we consider the cosets stabilizer $N^{(01202320)}$.

$N^{(01202320)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_2t_3t_2t_0N$ is at most: $\frac{|N|}{|N^{(01202320)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01202320)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_2t_3t_2t_0t_0$, $Nt_0t_1t_2t_0t_2t_3t_2t_0t_1$, $Nt_0t_1t_2t_0t_2t_3t_2t_0t_2$ and $Nt_0t_1t_2t_0t_2t_3t_2t_0t_3$ belong?

However, $Nt_0t_1t_2t_0t_2t_3t_2t_0t_0 = Nt_0t_1t_2t_0t_2t_3t_2 \in [0120232]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_2t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_2t_0t_1$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_2t_0t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_2t_0t_2$.

102. [01202321] Same as 66.

103. Now, we consider the cosets stabilizer $N^{(01202323)}$.

$N^{(01202323)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_2t_3t_2t_3N$ is at most: $\frac{|N|}{|N^{(01202323)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01202323)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_2t_3t_2t_3t_0$, $Nt_0t_1t_2t_0t_2t_3t_2t_3t_1$, $Nt_0t_1t_2t_0t_2t_3t_2t_3t_2$ and $Nt_0t_1t_2t_0t_2t_3t_2t_3t_3$ belong?

However, $Nt_0t_1t_2t_0t_2t_3t_2t_3t_3 = Nt_0t_1t_2t_0t_2t_3t_2 \in [0120232]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_2t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_2t_3t_1$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_2t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_2t_3t_2$.

104. [01203020] Same as 25.

105. Now, we consider coset $N^{(01203021)}$.
However, $t_0t_1t_2t_0t_3t_0t_2t_1 = (2,0,3)(t_0t_1t_3t_0t_2t_0t_3) = (2,0,3)(t_0t_1t_2t_0t_3t_0t_2)^{(2,3)}$ $\in Nt_0t_1t_2t_0t_3t_0t_2$. Therefore, t_1 takes $Nt_0t_1t_2t_0t_3t_0t_2t_1$ go to itself $Nt_0t_1t_2t_0t_3t_0t_2$.

106. Now, we consider the cosets stabilizer $N^{(01203023)}$.
We know, $t_0t_1t_2t_0t_3t_0t_2t_3 = (t_0t_1t_2t_3t_0t_2t_0t_3)^{(2,3)}$ then
$Nt_0t_1t_2t_0t_3t_0t_2t_3 = Nt_0t_1t_3t_2t_0t_3t_0t_2 \in Nt_0t_1t_2t_3t_0t_2t_0t_3$.
$N^{(01203023)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_3t_0t_2t_3N$ is at most: $\frac{|N|}{|N^{(01203023)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01203023)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_3t_0t_2t_3t_0$, $Nt_0t_1t_2t_0t_3t_0t_2t_3t_1$, $Nt_0t_1t_2t_0t_3t_0t_2t_3t_2$ and $Nt_0t_1t_2t_0t_3t_0t_2t_3t_3$ belong?
However, $Nt_0t_1t_2t_0t_3t_0t_2t_3t_2 = Nt_0t_1t_2t_0t_3t_0t_2 \in$ [0120302], then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_3t_0t_2N$, and $Nt_0t_1t_2t_0t_3t_0t_2t_3t_2 = Nt_0t_1t_3t_2t_0t_3t_0$ $\in$ [0123020], then one symmetric generator goes back to $Nt_0t_1t_2t_3t_0t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_0t_2t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_0t_2t_3t_1$.

107. Now, we consider the cosets stabilizer $N^{(01203030)}$.
We know, $t_0t_1t_2t_0t_3t_0t_3t_0 = (1,3)(2,0)(t_0t_1t_2t_3t_0t_2t_1t_2)^{(0,1)(2,3)}$ then
$Nt_0t_1t_2t_0t_3t_0t_3t_0 = Nt_1t_0t_3t_2t_1t_3t_0t_3 \in Nt_0t_1t_2t_3t_0t_2t_1t_2$.
$N^{(01203030)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_3t_0t_3t_0N$ is at most: $\frac{|N|}{|N^{(01203030)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01203030)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_3t_0t_3t_0t_0$, $Nt_0t_1t_2t_0t_3t_0t_3t_0t_1$, $Nt_0t_1t_2t_0t_3t_0t_3t_0t_2$ and $Nt_0t_1t_2t_0t_3t_0t_3t_0t_3$ belong?
However, $Nt_0t_1t_2t_0t_3t_0t_3t_0t_0 = Nt_0t_1t_2t_0t_3t_0t_2 \in$ [0120303], then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_3t_0t_2N$, and $Nt_0t_1t_2t_0t_3t_0t_3t_0t_3 = Nt_1t_0t_3t_2t_1t_3t_0$ $\in$ [0123021], then one symmetric generator goes back to $Nt_0t_1t_2t_3t_0t_2t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_0t_3t_0t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_0t_3t_0t_1$.

108. Now, we consider the cosets stabilizer $N^{(01203031)}$.
$N^{(01203031)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_3t_0t_3t_1N$ is at most: $\frac{|N|}{|N^{(01203031)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01203031)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_3t_0t_3t_1t_0$, $Nt_0t_1t_2t_0t_3t_0t_3t_1t_1$, $Nt_0t_1t_2t_0t_3t_0t_3t_1t_2$ and $Nt_0t_1t_2t_0t_3t_0t_3t_1t_3$ belong?
However, $Nt_0t_1t_2t_0t_3t_0t_3t_1t_1 = Nt_0t_1t_2t_0t_3t_0t_3 \in [0120303]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_3t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_0t_3t_1t_3$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_0t_3t_1t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_0t_3t_1t_2$.

109. Now, we consider the cosets stabilizer $N^{(01203032)}$.
$N^{(01203032)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_3t_0t_3t_2N$ is at most: $\frac{|N|}{|N^{(01203032)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01203032)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_3t_0t_3t_2t_0$, $Nt_0t_1t_2t_0t_3t_0t_3t_2t_1$, $Nt_0t_1t_2t_0t_3t_0t_3t_2t_2$ and $Nt_0t_1t_2t_0t_3t_0t_3t_2t_3$ belong?
However, $Nt_0t_1t_2t_0t_3t_0t_3t_2t_2 = Nt_0t_1t_2t_0t_3t_0t_3 \in [0120303]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_3t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_0t_3t_2t_3$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_0t_3t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_0t_3t_2t_1$.

110. Now, we consider the cosets stabilizer $N^{(01203210)}$.
$N^{(01203210)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_3t_2t_0t_1N$ is at most: $\frac{|N|}{|N^{(01203210)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01203210)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_3t_2t_0t_1t_0$, $Nt_0t_1t_2t_0t_3t_2t_0t_1t_1$, $Nt_0t_1t_2t_0t_3t_2t_0t_1t_2$ and $Nt_0t_1t_2t_0t_3t_2t_0t_1t_3$ belong?
However, $Nt_0t_1t_2t_0t_3t_2t_0t_1t_1 = Nt_0t_1t_2t_0t_3t_2t_0 \in [0120321]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_3t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_2t_0t_1t_3$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_2t_0t_1t_0$, and one sym-

metric generator goes to $Nt_0t_1t_2t_0t_3t_2t_0t_1t_2$.

111. Now, we consider the cosets stabilizer $N^{(01203202)}$.
We know, $t_0t_1t_2t_0t_3t_2t_0t_2 = (1,2,3)(t_0t_1t_2t_3t_1t_3t_0t_2)^{(1,3,2)}$ then
$Nt_0t_1t_2t_0t_3t_2t_0t_2 = Nt_0t_3t_1t_2t_3t_2t_0t_1 \in Nt_0t_1t_2t_3t_1t_3t_0t_2$.
$N^{(01203202)} = \{e\}$, then the number of the single cosets in the double coset
$Nt_0t_1t_2t_0t_3t_2t_0t_2N$ is at most: $\frac{|N|}{|N^{(01203202)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01203202)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_3t_2t_0t_2t_0$, $Nt_0t_1t_2t_0t_3t_2t_0t_2t_1$, $Nt_0t_1t_2t_0t_3t_2t_0t_2t_2$ and $Nt_0t_1t_2t_0t_3t_2t_0t_2t_3$ belong?
However, $Nt_0t_1t_2t_0t_3t_2t_0t_2t_2 = Nt_0t_1t_2t_0t_3t_0t_2 \in [0120320]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_3t_0t_2N$, and $Nt_0t_1t_2t_0t_3t_2t_0t_2t_1 = Nt_0t_3t_1t_2t_3t_2t_0$ $\in [0123130]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_1t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_2t_0t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_2t_0t_2t_3$.

112. Now, we consider coset $N^{(01203203)}$.
However, $t_0t_1t_2t_0t_3t_2t_0t_3 = (2,3,0)(t_2t_1t_0t_2t_3t_0t_2) = (2,3,0)(t_0t_1t_2t_0t_3t_2t_0)^{(2,0)}$ $\in Nt_0t_1t_2t_0t_3t_2t_0$. Therefore, t_3 takes $Nt_0t_1t_2t_0t_3t_2t_0t_3$ go to itself $Nt_0t_1t_2t_0t_3t_2t_0$.

113. Now, we consider the cosets stabilizer $N^{(01203210)}$.
We know, $t_0t_1t_2t_0t_3t_2t_1t_0 = (1,3)(0,2)(t_0t_1t_2t_1t_2t_3t_0t_1)^{(1,3)(0,2)}$ then
$Nt_0t_1t_2t_0t_3t_2t_1t_0 = Nt_2t_3t_0t_3t_0t_1t_2t_3 \in Nt_0t_1t_2t_1t_2t_3t_0t_1$.
$N^{(01203210)} = \{e\}$, then the number of the single cosets in the double coset
$Nt_0t_1t_2t_0t_3t_2t_1t_0N$ is at most: $\frac{|N|}{|N^{(01203210)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01203210)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_3t_2t_1t_0t_0$, $Nt_0t_1t_2t_0t_3t_2t_1t_0t_1$, $Nt_0t_1t_2t_0t_3t_2t_1t_0t_2$ and $Nt_0t_1t_2t_0t_3t_2t_1t_0t_3$ belong?
However, $Nt_0t_1t_2t_0t_3t_2t_1t_0t_0 = Nt_0t_1t_2t_0t_3t_2t_1 \in [0120321]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_3t_0t_2N$, and $Nt_0t_1t_2t_0t_3t_2t_1t_0t_3 = Nt_2t_3t_0t_3t_0t_1t_2$ $\in [0121230]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_2t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_2t_1t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_2t_1t_0t_2$.

114. Now, we consider the cosets stabilizer $N^{(01203212)}$.
$N^{(01203212)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_3t_2t_1t_2N$ is at most: $\frac{|N|}{|N^{(01203212)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01203212)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_3t_2t_1t_2t_0$, $Nt_0t_1t_2t_0t_3t_2t_1t_2t_1$, $Nt_0t_1t_2t_0t_3t_2t_1t_2t_2$ and $Nt_0t_1t_2t_0t_3t_2t_1t_2t_3$ belong?
However, $Nt_0t_1t_2t_0t_3t_2t_1t_2t_2 = Nt_0t_1t_2t_0t_3t_2t_1 \in [0120321]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_3t_2t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_2t_1t_2t_3$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_2t_1t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_2t_1t_2t_1$.

115. [01203213] Same as 45.

116. [01203230] Same as 72.

117. [01203231] Same as 81.

118. Now, we consider the cosets stabilizer $N^{(01203232)}$.
We know, $t_0t_1t_2t_0t_3t_2t_3t_2 = (t_0t_1t_2t_3t_1t_3t_1t_0)^{(1,3,2,0)}$ then
$Nt_0t_1t_2t_0t_3t_2t_3t_2 = Nt_1t_3t_0t_2t_3t_2t_3t_1 \in Nt_0t_1t_2t_3t_1t_3t_1t_0$.
$N^{(01203232)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_3t_2t_3t_2N$ is at most: $\frac{|N|}{|N^{(01203232)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01203232)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_3t_2t_3t_2t_0$, $Nt_0t_1t_2t_0t_3t_2t_3t_2t_1$, $Nt_0t_1t_2t_0t_3t_2t_3t_2t_2$ and $Nt_0t_1t_2t_0t_3t_2t_3t_2t_3$ belong?
However, $Nt_0t_1t_2t_0t_3t_2t_3t_2t_2 = Nt_0t_1t_2t_0t_3t_2t_3 \in [0120323]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_3t_0t_2N$, and $Nt_0t_1t_2t_0t_3t_2t_3t_2t_1 = Nt_1t_3t_0t_2t_3t_2t_3$ $\in [0123131]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_1t_3t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_2t_3t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_2t_3t_2t_2$.

119. [01210120] Same as 43.

120. Now, we consider the cosets stabilizer $N^{(01210123)}$.
We know, $t_0t_1t_2t_1t_0t_1t_2t_3 = t_0t_2t_1t_2t_0t_2t_1t_3$ then
$N^{(01210123)} = \{e, (2,1)\}$, then the number of the single cosets in the double coset
$Nt_0t_1t_2t_1t_0t_1t_2t_3N$ is at most: $\frac{|N|}{|N^{(01210123)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(01210123)}$ on $\{0,1,2,3\}$ are $\{2,1\}$, $\{0\}$ and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_0t_1t_2t_3t_0$, $Nt_0t_1t_2t_1t_0t_1t_2t_3t_1$, $Nt_0t_1t_2t_1t_0t_1t_2t_3t_3$and belong?
Moreover, $t_0t_1t_2t_1t_0t_1t_2t_3 = (1,2,0)(t_0t_1t_2t_1t_3t_2t_3t_0)^{(2,3)} = t_0t_1t_3t_1t_2t_3t_2t_0 \in [01213230]$ and $t_0t_1t_2t_1t_0t_1t_2t_3 = (1,0,2)(t_0t_1t_2t_3t_2t_1t_2t_3)^{(1,0,3)} = t_3t_0t_2t_1t_2t_0t_2t_1 \in [01232123]$.
However, $Nt_0t_1t_2t_1t_0t_1t_2t_3t_3 = Nt_0t_1t_2t_1t_0t_1t_2 \in [0121012]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_0t_1t_2N$, and $Nt_0t_1t_2t_1t_0t_1t_2t_3t_0 = Nt_0t_1t_3t_1t_2t_3t_2$ $\in [0121323]$ one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_2t_3$. $Nt_0t_1t_2t_1t_0t_1t_2t_3t_1 = Nt_0t_1t_2t_1t_0t_1t_2 \in [0123212]$ two symmetric generator go to $Nt_0t_1t_2t_3t_2t_1t_2$.

121. Now, we consider the cosets stabilizer $N^{(01210130)}$.
$N^{(01210130)} = \{e\}$, then the number of the single cosets in the double coset
$Nt_0t_1t_2t_1t_0t_1t_3t_0N$ is at most: $\frac{|N|}{|N^{(01210130)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01210130)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_0t_1t_3t_0t_0$, $Nt_0t_1t_2t_1t_0t_1t_3t_0t_1$, $Nt_0t_1t_2t_1t_0t_1t_3t_0t_2$ and $Nt_0t_1t_2t_1t_0t_1t_3t_0t_3$ belong?
However, $Nt_0t_1t_2t_1t_0t_1t_3t_0t_0 = Nt_0t_1t_2t_1t_0t_1t_3 \in [0121013]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_0t_1t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_1t_3t_0t_3$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_1t_3t_0t_2$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_1t_3t_0t_1$.

122. [01210131] Same as 65.

123. Now, we consider the cosets stabilizer $N^{(01210132)}$.
$N^{(01210132)} = \{e\}$, then the number of the single cosets in the double coset
$Nt_0t_1t_2t_1t_0t_1t_3t_2N$ is at most: $\frac{|N|}{|N^{(01210132)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01210132)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_0t_1t_3t_2t_0$,

$Nt_0t_1t_2t_1t_0t_1t_3t_2t_1$, $Nt_0t_1t_2t_1t_0t_1t_3t_2t_2$ and $Nt_0t_1t_2t_1t_0t_1t_3t_2t_3$ belong?
However, $Nt_0t_1t_2t_1t_0t_1t_3t_2t_2 = Nt_0t_1t_2t_1t_0t_1t_3 \in [0121013]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_0t_1t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_1t_3t_2t_3$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_1t_3t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_1t_3t_2t_1$.

124. Now, we consider the cosets stabilizer $N^{(01210301)}$.
We know, $t_0t_1t_2t_1t_0t_3t_0t_1 = t_2t_3t_0t_3t_2t_1t_2t_3 = t_3t_0t_1t_0t_3t_2t_3t_0 = t_1t_2t_3t_2t_1t_0t_1t_2$ then $N^{(01210301)} = \{e, (0,2)(1,3), (0,3,2,1), (0,1,2,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_0t_3t_0t_1N$ is at most: $\frac{|N|}{|N^{(01210301)}|} = \frac{24}{4} = 6$

The orbit of $N^{(01210301)}$ on $\{0,1,2,3\}$ is $\{0,1,2,3\}$ then $Nt_0t_1t_2t_1t_0t_3t_0t_1t_1 = Nt_0t_1t_2t_1t_0t_3t_0 \in [0121030]$, then four symmetric generators go back to the double coset $Nt_0t_1t_2t_1t_0t_3t_0N$

125. Now, we consider coset $N^{(01210302)}$.
$t_0t_1t_2t_1t_0t_3t_0t_2 = (1,3,2)(t_2t_1t_2t_0t_3t_0t_1) = (1,3,2)(t_0t_1t_0t_2t_3t_2t_1)^{(0,2)} \in Nt_0t_1t_0t_2t_3t_2t_1$.
Therefore, t_2 takes $Nt_0t_1t_2t_1t_0t_3t_0t_2$ go to $Nt_0t_1t_0t_2t_3t_2t_1$.

126. Now, we consider the cosets stabilizer $N^{(01210303)}$.
We know, $t_0t_1t_2t_1t_0t_3t_0t_3 = (t_0t_1t_2t_3t_1t_0t_3t_0)^{(1,3)(2,0)}$ then
$Nt_0t_1t_2t_1t_0t_3t_0t_3 = Nt_2t_3t_0t_1t_3t_2t_1t_2 \in Nt_0t_1t_2t_3t_1t_0t_3t_0$.
$N^{(01210303)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_0t_3t_0t_3N$ is at most: $\frac{|N|}{|N^{(01210303)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01210303)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_0t_3t_0t_3t_0$, $Nt_0t_1t_2t_1t_0t_3t_0t_3t_1$, $Nt_0t_1t_2t_1t_0t_3t_0t_3t_2$ and $Nt_0t_1t_2t_1t_0t_3t_0t_3t_3$ belong?
However, $Nt_0t_1t_2t_1t_0t_3t_0t_3t_3 = Nt_0t_1t_2t_0t_3t_2t_3 \in [0121030]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_3t_0t_2N$, and $Nt_0t_1t_2t_1t_0t_3t_0t_3t_2 = Nt_2t_3t_0t_1t_3t_2t_1$ $\in [0123103]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_1t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_3t_0t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_3t_0t_3t_1$.

127. Now, we consider coset $N^{(01210310)}$.
However, $t_0t_1t_2t_1t_0t_3t_1t_0 = (1,0,2)(t_3t_1t_0t_2t_0t_1t_3) = (1,0,2)(t_0t_1t_2t_3t_2t_1t_0)^{(2,0,3)}$ $\in Nt_0t_1t_2t_3t_2t_1t_0$. Therefore, t_0 takes $Nt_0t_1t_2t_1t_0t_3t_1t_0$ go to $Nt_0t_1t_2t_3t_2t_1t_0$.

128. Now, we consider the cosets stabilizer $N^{(01210312)}$.
$N^{(01210312)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_0t_3t_1t_2N$ is at most: $\frac{|N|}{|N^{(01210312)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01210312)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_0t_3t_1t_2t_0$, $Nt_0t_1t_2t_1t_0t_3t_1t_2t_1$, $Nt_0t_1t_2t_1t_0t_3t_1t_2t_2$ and $Nt_0t_1t_2t_1t_0t_3t_1t_2t_3$ belong?
However, $Nt_0t_1t_2t_1t_0t_3t_1t_2t_2 = Nt_0t_1t_2t_1t_0t_3t_1 \in [0121031]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_0t_3t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_3t_1t_2t_3$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_3t_1t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_3t_1t_2t_1$.

129. Now, we consider the cosets stabilizer $N^{(01210313)}$.
$N^{(01210313)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_0t_3t_1t_3N$ is at most: $\frac{|N|}{|N^{(01210313)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01210313)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_0t_3t_1t_3t_0$, $Nt_0t_1t_2t_1t_0t_3t_1t_3t_1$, $Nt_0t_1t_2t_1t_0t_3t_1t_3t_2$ and $Nt_0t_1t_2t_1t_0t_3t_1t_3t_3$ belong?
However, $Nt_0t_1t_2t_1t_0t_3t_1t_3t_3 = Nt_0t_1t_2t_1t_0t_3t_1 \in [0121031]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_0t_3t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_3t_1t_3t_2$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_3t_1t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_3t_1t_3t_1$.

130. Now, we consider the cosets stabilizer $N^{(01212030)}$.
$N^{(01212030)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_2t_0t_3t_0N$ is at most: $\frac{|N|}{|N^{(01212030)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01212030)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_2t_0t_3t_0t_0$, $Nt_0t_1t_2t_1t_2t_0t_3t_0t_1$, $Nt_0t_1t_2t_1t_2t_0t_3t_0t_2$ and $Nt_0t_1t_2t_1t_2t_0t_3t_0t_3$ belong?

However, $Nt_0t_1t_2t_1t_2t_0t_3t_0t_0 = Nt_0t_1t_2t_1t_2t_0t_3 \in [0121203]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_2t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_0t_3t_0t_2$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_0t_3t_0t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_0t_3t_0t_1$.

131. Now, we consider the cosets stabilizer $N^{(01212031)}$.
We know, $t_0t_1t_2t_1t_2t_0t_3t_1 = (0,1)(2,3)(t_0t_1t_2t_3t_0t_2t_1t_3)^{(1,0)(2,3)}$ then
$Nt_0t_1t_2t_1t_2t_0t_3t_1 = Nt_1t_0t_3t_2t_1t_3t_0t_2 \in Nt_0t_1t_2t_3t_0t_2t_1t_3$.
$N^{(01212031)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_2t_0t_3t_1N$ is at most: $\frac{|N|}{|N^{(01212031)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01212031)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_2t_0t_3t_1t_0$, $Nt_0t_1t_2t_1t_2t_0t_3t_1t_1$, $Nt_0t_1t_2t_1t_2t_0t_3t_1t_2$ and $Nt_0t_1t_2t_1t_2t_0t_3t_1t_3$ belong?
However, $Nt_0t_1t_2t_1t_2t_0t_3t_1t_1 = Nt_0t_1t_2t_0t_3t_2t_3 \in [0121203]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_3t_0t_2N$, and $Nt_0t_1t_2t_1t_2t_0t_3t_1t_2 = Nt_1t_0t_3t_2t_1t_3t_0$ $\in [0123021]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_0t_2t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_0t_3t_1t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_0t_3t_1t_3$.

132. Now, we consider the cosets stabilizer $N^{(01212032)}$.
$N^{(01212032)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_2t_0t_3t_2N$ is at most: $\frac{|N|}{|N^{(01212032)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01212032)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_2t_0t_3t_2t_0$, $Nt_0t_1t_2t_1t_2t_0t_3t_2t_1$, $Nt_0t_1t_2t_1t_2t_0t_3t_2t_2$ and $Nt_0t_1t_2t_1t_2t_0t_3t_2t_3$ belong?
However, $Nt_0t_1t_2t_1t_2t_0t_3t_2t_2 = Nt_0t_1t_2t_1t_2t_0t_3 \in [0121203]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_2t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_0t_3t_2t_0$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_0t_3t_2t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_0t_3t_2t_1$.

133. Now, we consider the cosets stabilizer $N^{(01212130)}$.
We know, $t_0t_1t_2t_1t_2t_1t_3t_0 = t_0t_2t_1t_2t_1t_2t_3t_0$ then
$N^{(01212130)} = \{e,(2,1)\}$, then the number of the single cosets in the double coset

$Nt_0t_1t_2t_1t_2t_1t_3t_0N$ is at most: $\frac{|N|}{|N^{(01212130)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(01212130)}$ on $\{0,1,2,3\}$ are $\{2,1\}$,$\{0\}$ and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_2t_1t_3t_0t_0$, $Nt_0t_1t_2t_1t_2t_1t_3t_0t_1$ and $Nt_0t_1t_2t_1t_2t_1t_3t_0t_3$ belong?
However, $Nt_0t_1t_2t_1t_2t_1t_3t_0t_0 = Nt_0t_2t_1t_2t_1t_2t_3 \in [0102323]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_2t_1t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_1t_3t_0t_3$, and two symmetric generators go to $Nt_0t_1t_2t_1t_2t_1t_3t_0t_1$.

134. Now, we consider coset $N^{(01212131)}$.
However, $t_0t_1t_2t_1t_2t_1t_3t_1 = (t_0t_3t_2t_3t_2t_1t_3) \in Nt_0t_1t_2t_1t_2t_3t_1$.
Therefore, t_1 takes $Nt_0t_1t_2t_1t_2t_1t_3t_1$ go to itself $Nt_0t_1t_2t_1t_2t_3t_1$.

135. [01212301] Same as 113.

136. Now, we consider the cosets stabilizer $N^{(01212302)}$.
$N^{(01212302)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_2t_3t_0t_2N$ is at most: $\frac{|N|}{|N^{(01212302)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01212302)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_2t_3t_0t_2t_0$, $Nt_0t_1t_2t_1t_2t_3t_0t_2t_1$, $Nt_0t_1t_2t_1t_2t_3t_0t_2t_2$ and $Nt_0t_1t_2t_1t_2t_3t_0t_2t_3$ belong?
However, $Nt_0t_1t_2t_1t_2t_3t_0t_2t_2 = Nt_0t_1t_2t_1t_2t_3t_0$
$\in [0121230]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_2t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_0t_2t_0$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_0t_2t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_0t_2t_1$.

137. [01212303] Same as 45.

138. [01212310] Same as 8.

139. Now, we consider the cosets stabilizer $N^{(01212312)}$.
We know, $t_0t_1t_2t_1t_2t_3t_1t_2 = (t_0t_1t_2t_1t_3t_2t_0t_3)^{(1,3)(2,0)}$ then
$Nt_0t_1t_2t_1t_2t_3t_1t_2 = Nt_2t_3t_0t_3t_1t_0t_2t_1 \in Nt_0t_1t_2t_1t_3t_2t_0t_3$.

$N^{(01212312)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_2t_3t_1t_2N$ is at most: $\frac{|N|}{|N^{(01212312)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01212312)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_2t_3t_1t_2t_0$, $Nt_0t_1t_2t_1t_2t_3t_1t_2t_1$, $Nt_0t_1t_2t_1t_2t_3t_1t_2t_2$ and $Nt_0t_1t_2t_1t_2t_3t_1t_2t_3$ belong?
However, $Nt_0t_1t_2t_1t_2t_3t_1t_2t_2 = Nt_0t_1t_2t_0t_3t_2t_3 \in [0121231]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_3t_0t_2N$, and $Nt_0t_1t_2t_1t_2t_3t_1t_2t_1 = Nt_2t_3t_0t_3t_1t_0t_2$ $\in [0121320]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_3t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_1t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_1t_2t_3$.

140. Now, we consider coset $N^{(01212313)}$.
$t_0t_1t_2t_1t_2t_3t_1t_3 = (t_0t_3t_2t_3t_2t_3t_1) = (t_0t_1t_2t_1t_2t_1t_3)^{(1,3)} \in Nt_0t_1t_2t_1t_2t_1t_3$. Therefore, t_3 takes $Nt_0t_1t_2t_1t_2t_3t_1t_3$ go to itself $Nt_0t_1t_2t_1t_2t_1t_3$.

141. Now, we consider the cosets stabilizer $N^{(01212320)}$.
$N^{(01212320)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_2t_3t_2t_0N$ is at most: $\frac{|N|}{|N^{(01212320)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01212320)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_2t_3t_2t_0t_0$, $Nt_0t_1t_2t_1t_2t_3t_2t_0t_1$, $Nt_0t_1t_2t_1t_2t_3t_2t_0t_2$ and $Nt_0t_1t_2t_1t_2t_3t_2t_0t_3$ belong?
However, $Nt_0t_1t_2t_1t_2t_3t_2t_0t_0 = Nt_0t_1t_2t_1t_2t_3t_2 \in [0121232]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_2t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_2t_0t_2$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_2t_0t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_2t_0t_1$.

142. Now, we consider the cosets stabilizer $N^{(01212321)}$.
We know, $t_0t_1t_2t_1t_2t_3t_2t_1 = (1,3,2)(t_0t_1t_2t_3t_2t_1t_2t_1)^{(1,2,3)}$ then
$Nt_0t_1t_2t_1t_2t_3t_2t_1 = Nt_0t_2t_3t_1t_3t_2t_3t_2 \in Nt_0t_1t_2t_3t_2t_1t_2t_1$.
$N^{(01212321)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_2t_3t_2t_1N$ is at most: $\frac{|N|}{|N^{(01212321)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01212321)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_2t_3t_2t_1t_0$,

$Nt_0t_1t_2t_1t_2t_3t_2t_1t_1$, $Nt_0t_1t_2t_1t_2t_3t_2t_1t_2$ and $Nt_0t_1t_2t_1t_2t_3t_2t_1t_3$ belong?
However, $Nt_0t_1t_2t_1t_2t_3t_2t_1t_1 = Nt_0t_1t_2t_1t_2t_3t_2 \in [0121232]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_3t_0t_2N$, and $Nt_0t_1t_2t_1t_2t_3t_2t_1t_2 = Nt_0t_2t_3t_1t_3t_2t_3$ $\in [0123212]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_2t_1t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_2t_1t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_2t_1t_3$.

143. Now, we consider coset $N^{(01212323)}$.
$t_0t_1t_2t_1t_2t_3t_2t_3 = (t_0t_2t_3t_1t_3t_1t_2) = (t_0t_1t_2t_3t_2t_3t_1)^{(1,2,3)} \in Nt_0t_1t_2t_3t_2t_3t_1$. Therefore, t_3 takes $Nt_0t_1t_2t_1t_2t_3t_2t_3$ go to $Nt_0t_1t_2t_3t_2t_3t_1$.

144. Now, we consider the cosets stabilizer $N^{(01213030)}$.
$N^{(01213030)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_3t_0t_3t_0N$ is at most: $\frac{|N|}{|N^{(01213030)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01213030)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_3t_0t_3t_0t_0$, $Nt_0t_1t_2t_1t_3t_0t_3t_0t_1$, $Nt_0t_1t_2t_1t_3t_0t_3t_0t_2$ and $Nt_0t_1t_2t_1t_3t_0t_3t_0t_3$ belong?
However, $Nt_0t_1t_2t_1t_3t_0t_3t_0t_0 = Nt_0t_1t_2t_1t_3t_0t_3 \in [0121303]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_3t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_0t_3t_0t_2$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_0t_3t_0t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_0t_3t_0t_1$.

145. Now, we consider coset $N^{(01213031)}$.
However, $t_0t_1t_2t_1t_3t_0t_3t_1 = (2,3,0)(t_0t_2t_3t_1t_3t_1t_2) = (2,3,0)(t_0t_1t_2t_1t_3t_0t_3)^{(2,3)}$ $\in Nt_0t_1t_2t_1t_3t_0t_3$. Therefore, t_1 takes $Nt_0t_1t_2t_1t_3t_0t_3t_1$ go to $Nt_0t_1t_2t_1t_3t_0t_3$.

146. Now, we consider the cosets stabilizer $N^{(01213032)}$.
$N^{(01213032)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_3t_0t_3t_2N$ is at most: $\frac{|N|}{|N^{(01213032)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01213032)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_3t_0t_3t_2t_0$, $Nt_0t_1t_2t_1t_3t_0t_3t_2t_1$, $Nt_0t_1t_2t_1t_3t_0t_3t_2t_2$ and $Nt_0t_1t_2t_1t_3t_0t_3t_2t_3$ belong?

However, $Nt_0t_1t_2t_1t_3t_0t_3t_2t_2 = Nt_0t_1t_2t_1t_3t_0t_3 \in [0121303]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_3t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_0t_3t_2t_0$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_0t_3t_2t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_0t_3t_2t_1$.

147. [01213201] Same as 98.

148. [01213202] Same as 62.

149. [01213203] Same as 139.

150. Now, we consider coset $N^{(01213213)}$.
However, $t_0t_1t_2t_1t_3t_2t_1t_3 = (1,2,3)(t_0t_2t_1t_2t_3t_1t_2) = (1,2,3)(t_0t_1t_2t_1t_3t_2t_1)^{(2,1)}$ $\in Nt_0t_1t_2t_1t_3t_2t_1$. Therefore, t_3 takes $Nt_0t_1t_2t_1t_3t_2t_1t_3$ go to itself $Nt_0t_1t_2t_1t_3t_2t_1$.

151. [01213230] Same as 120.

152. Now, we consider coset $N^{(01213232)}$.
However, $t_0t_1t_2t_1t_3t_2t_3t_2 = (t_0t_2t_1t_3t_1t_3t_1) = (t_0t_1t_2t_3t_2t_3t_2)^{(2,1)} \in Nt_0t_1t_2t_3t_2t_3t_2$. Therefore, t_3 takes $Nt_0t_1t_2t_1t_3t_2t_3t_2$ go to $Nt_0t_1t_2t_3t_2t_3t_2$.

153. [01230201] Same as 94.

154. Now, we consider the cosets stabilizer $N^{(01230202)}$.
$N^{(01230202)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_0t_2t_0t_2N$ is at most: $\frac{|N|}{|N^{(01230202)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01230202)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_0t_2t_0t_2t_0$, $Nt_0t_1t_2t_3t_0t_2t_0t_2t_1$, $Nt_0t_1t_2t_3t_0t_2t_0t_2t_2$ and $Nt_0t_1t_2t_3t_0t_2t_0t_2t_3$ belong?
However, $Nt_0t_1t_2t_3t_0t_2t_0t_2t_2 = Nt_0t_1t_2t_3t_0t_2t_0 \in [0123020]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_0t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_0t_2t_0t_2t_0$, one symmetric generator goes to $Nt_0t_1t_2t_3t_0t_2t_0t_2t_3$, and one sym-

metric generator goes to $Nt_0t_1t_2t_3t_0t_2t_0t_2t_1$.

155. [01230203] Same as 106.

156. Now, we consider the cosets stabilizer $N^{(01230210)}$.
$N^{(01230210)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_0t_2t_1t_0N$ is at most: $\frac{|N|}{|N^{(01230210)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01230210)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_0t_2t_1t_0t_0$, $Nt_0t_1t_2t_3t_0t_2t_1t_0t_1$, $Nt_0t_1t_2t_3t_0t_2t_1t_0t_2$ and $Nt_0t_1t_2t_3t_0t_2t_1t_0t_3$ belong?
However, $Nt_0t_1t_2t_3t_0t_2t_1t_0t_0 = Nt_0t_1t_2t_3t_0t_2t_1 \in [0123021]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_0t_2t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_0t_2t_1t_0t_2$, one symmetric generator goes to $Nt_0t_1t_2t_3t_0t_2t_1t_0t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_0t_2t_1t_0t_1$.

157. [01230212] Same as 107.

158. [01230213] Same as 131.

159. Now, we consider coset $N^{(01230230)}$.
However, $t_0t_1t_2t_3t_0t_2t_3t_0 = (1,3,2)(t_1t_3t_0t_3t_2t_1t_3) = (1,3,2)(t_0t_1t_2t_1t_3t_0t_1)^{(1,3,2,0)}$ $\in Nt_0t_1t_2t_1t_3t_0t_1$. Therefore, t_0 takes $Nt_0t_1t_2t_3t_0t_2t_3t_0$ go to $Nt_0t_1t_2t_1t_3t_0t_1$.

160. [01230231] Same as 63.

161. [01230232] Same as 46.

162. Now, we consider the cosets stabilizer $N^{(01231010)}$.
We know, $t_0t_1t_2t_3t_1t_0t_1t_0 = t_1t_0t_3t_2t_0t_1t_0t_1$ then
$N^{(01231010)} = \{e, (0,1)(2,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_1t_0t_1t_0N$ is at most: $\frac{|N|}{|N^{(01231010)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(01231010)}$ on $\{0,1,2,3\}$ are $\{0,1\}$ and $\{2,3\}$. We take a repre-

sentative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_1t_0t_1t_0t_0$, and $Nt_0t_1t_2t_3t_1t_0t_1t_0t_2$ belong?
However, $Nt_0t_1t_2t_3t_1t_0t_1t_0t_0 = Nt_1t_0t_3t_2t_0t_1t_0 \in [0102323]$, then two symmetric generators go back to the double coset $Nt_0t_1t_2t_3t_1t_0t_1N$, two symmetric generator go to $Nt_0t_1t_2t_3t_1t_0t_1t_0t_2$.

163. Now, we consider coset $N^{(01231012)}$.
However, $t_0t_1t_2t_3t_1t_0t_1t_2 = (2,3,0)(t_1t_0t_3t_0t_1t_2t_3) = (2,3,0)(t_0t_1t_2t_1t_0t_3t_2)^{(1,0)(2,3)}$ $\in Nt_0t_1t_2t_1t_0t_3t_2$. Therefore, t_2 takes $Nt_0t_1t_2t_3t_1t_0t_1t_2$ go to $Nt_0t_1t_2t_1t_0t_3t_2$.

164. Now, we consider the cosets stabilizer $N^{(01231013)}$.
$N^{(01231013)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_1t_0t_1t_3N$ is at most: $\frac{|N|}{|N^{(01231013)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01231013)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_1t_0t_1t_3t_0$, $Nt_0t_1t_2t_3t_1t_0t_1t_3t_1$, $Nt_0t_1t_2t_3t_1t_0t_1t_3t_2$ and $Nt_0t_1t_2t_3t_1t_0t_1t_3t_3$ belong?
However, $Nt_0t_1t_2t_3t_1t_0t_1t_3t_3 = Nt_0t_1t_2t_3t_1t_0t_1 \in [0123101]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_1t_0t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_0t_1t_3t_2$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_0t_1t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_0t_1t_3t_1$.

165. [01231039] Same as 126.

166. Now, we consider the cosets stabilizer $N^{(01231031)}$.
$N^{(01231031)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_1t_0t_3t_1N$ is at most: $\frac{|N|}{|N^{(01231031)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01231031)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_1t_0t_3t_1t_0$, $Nt_0t_1t_2t_3t_1t_0t_3t_1t_1$, $Nt_0t_1t_2t_3t_1t_0t_3t_1t_2$ and $Nt_0t_1t_2t_3t_1t_0t_3t_1t_3$ belong?
However, $Nt_0t_1t_2t_3t_1t_0t_3t_1t_1 = Nt_0t_1t_2t_3t_1t_0t_3 \in [0123103]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_1t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_0t_3t_1t_2$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_0t_3t_1t_0$, and one sym-

metric generator goes to $Nt_0t_1t_2t_3t_1t_0t_3t_1t_3$.

167. Now, we consider the cosets stabilizer $N^{(01231032)}$.
We know, $t_0t_1t_2t_3t_1t_0t_3t_2 = (t_0t_1t_2t_3t_2t_3t_0t_1)^{(1,0,2)}$ then
$Nt_0t_1t_2t_3t_1t_0t_3t_2 = Nt_2t_0t_1t_3t_1t_3t_2t_0 \in Nt_0t_1t_2t_3t_2t_3t_0t_1$.
$N^{(01231032)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_1t_0t_3t_2N$ is at most: $\frac{|N|}{|N^{(01231032)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01231032)}$ on $\{0, 1, 2, 3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_1t_0t_3t_2t_0$, $Nt_0t_1t_2t_3t_1t_0t_3t_2t_1$, $Nt_0t_1t_2t_3t_1t_0t_3t_2t_2$ and $Nt_0t_1t_2t_3t_1t_0t_3t_2t_3$ belong?
However, $Nt_0t_1t_2t_3t_1t_0t_3t_2t_2 = Nt_0t_1t_2t_3t_1t_0t_3 \in$ [0123103], then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_3t_0t_2N$, and $Nt_0t_1t_2t_3t_1t_0t_3t_2t_0 = Nt_2t_0t_1t_3t_1t_3t_2$ $\in$ [0123230], then one symmetric generator goes back to $Nt_0t_1t_2t_3t_2t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_0t_3t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_0t_3t_2t_3$.

168. Now, we consider the cosets stabilizer $N^{(01231201)}$.
$N^{(01231201)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_1t_2t_0t_1N$ is at most: $\frac{|N|}{|N^{(01231201)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01231201)}$ on $\{0, 1, 2, 3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_1t_2t_0t_1t_0$, $Nt_0t_1t_2t_3t_1t_2t_0t_1t_1$, $Nt_0t_1t_2t_3t_1t_2t_0t_1t_2$ and $Nt_0t_1t_2t_3t_1t_2t_0t_1t_3$ belong?
However, $Nt_0t_1t_2t_3t_1t_2t_0t_1t_1 = Nt_0t_1t_2t_3t_1t_2t_0 \in$ [0123120], then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_1t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_2t_0t_1t_2$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_2t_0t_1t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_2t_0t_1t_3$.

169. [01231202] Same as 50.

170. [01231203] Same as 95.

171. [01231301] Same as 3.

172. [01231302] Same as 111.

173. Now, we consider the cosets stabilizer $N^{(01231303)}$.
$N^{(01231303)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_1t_3t_0t_3N$ is at most: $\frac{|N|}{|N^{(01231303)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01231303)}$ on $\{0, 1, 2, 3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_1t_3t_0t_3t_0$, $Nt_0t_1t_2t_3t_1t_3t_0t_3t_1$, $Nt_0t_1t_2t_3t_1t_3t_0t_3t_2$ and $Nt_0t_1t_2t_3t_1t_3t_0t_3t_3$ belong?
However, $Nt_0t_1t_2t_3t_1t_3t_0t_3t_3 = Nt_0t_1t_2t_3t_1t_3t_0 \in [0123130]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_1t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_3t_0t_3t_2$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_3t_0t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_3t_0t_3t_1$.

174. [01231310] Same as 118.

175. Now, we consider coset $N^{(01231312)}$.
However, $t_0t_1t_2t_3t_1t_3t_1t_2 = (t_0t_2t_1t_2t_3t_2t_3) = (t_0t_1t_2t_1t_3t_1t_3)^{(1,2)} \in Nt_0t_1t_2t_1t_3t_1t_3$. Therefore, t_2 takes $Nt_0t_1t_2t_3t_1t_3t_1t_2$ go to $Nt_0t_1t_2t_1t_3t_1t_3$.

176. Now, we consider coset $N^{(01231313)}$.
However, $t_0t_1t_2t_3t_1t_3t_1t_3 = (t_0t_2t_1t_2t_3t_2t_3) = (t_0t_1t_2t_1t_3t_1t_3)^{(1,2)} \in Nt_0t_1t_2t_1t_3t_1t_3$. Therefore, t_3 takes $Nt_0t_1t_2t_3t_1t_3t_1t_3$ go to $Nt_0t_1t_2t_1t_3t_1t_3$.

177. Now, we consider the cosets stabilizer $N^{(01231320)}$.
We know, $t_0t_1t_2t_3t_1t_3t_2t_0 = t_2t_3t_0t_1t_3t_1t_0t_2$ then
$N^{(01231320)} = \{e, (0,2)(1,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_1t_3t_2t_0N$ is at most: $\frac{|N|}{|N^{(01231320)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(01231320)}$ on $\{0, 1, 2, 3\}$ are $\{0, 2\}$, and $\{1, 3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_1t_3t_2t_0t_0$, $Nt_0t_1t_2t_3t_1t_3t_2t_0t_1$, and belong?
However, $Nt_0t_1t_2t_3t_1t_3t_2t_0t_0 = Nt_0t_1t_3t_2t_3t_2t_3 \in [0123132]$, then two symmetric generators go back to the double coset $Nt_0t_1t_2t_3t_1t_3t_2N$, and two symmetric generator go to

$Nt_0t_1t_2t_3t_1t_3t_2t_0t_1$.

178. Now, we consider the cosets stabilizer $N^{(01231323)}$.
$N^{(01231323)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_1t_3t_2t_3N$ is at most: $\frac{|N|}{|N^{(01231323)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01231323)}$ on $\{0, 1, 2, 3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_1t_3t_2t_3t_0$, $Nt_0t_1t_2t_3t_1t_3t_2t_3t_1$, $Nt_0t_1t_2t_3t_1t_3t_2t_3t_2$ and $Nt_0t_1t_2t_3t_1t_3t_2t_3t_3$ belong?
However, $Nt_0t_1t_2t_3t_1t_3t_2t_3t_3 = Nt_0t_1t_2t_3t_1t_3t_2 \in [0123132]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_1t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_3t_2t_3t_2$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_3t_2t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_3t_2t_3t_1$.

179. Now, we consider coset $N^{(01232010)}$.
However, $t_0t_1t_2t_3t_2t_0t_1t_0 = (1, 3, 0)(t_2t_0t_2t_3t_1t_3t_2) = (1, 3, 0)(t_0t_1t_0t_2t_3t_2t_0)^{(1,0,2,3)}$ $\in Nt_0t_1t_0t_2t_3t_2t_0$. Therefore, t_0 takes $Nt_0t_1t_2t_3t_2t_0t_1t_0$ go to $Nt_0t_1t_0t_2t_3t_2t_0$.

180. Now, we consider coset $N^{(01232012)}$.
However, $t_0t_1t_2t_3t_2t_0t_1t_2 = (1, 3, 0)(t_3t_2t_0t_1t_0t_2t_3) = (1, 3, 0)(t_0t_1t_2t_3t_2t_1t_0)^{(1,2,0,3)}$ $\in Nt_0t_1t_2t_3t_2t_1t_0$. Therefore, t_2 takes $Nt_0t_1t_2t_3t_2t_0t_1t_2$ go to $Nt_0t_1t_2t_3t_2t_1t_0$.

181. [01232013] Same as 97.

182. Now, we consider the cosets stabilizer $N^{(01232020)}$.
$N^{(01232020)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_2t_0t_2t_0N$ is at most: $\frac{|N|}{|N^{(01232020)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01232020)}$ on $\{0, 1, 2, 3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_2t_0t_2t_0t_0$, $Nt_0t_1t_2t_3t_2t_0t_2t_0t_1$, $Nt_0t_1t_2t_3t_2t_0t_2t_0t_2$ and $Nt_0t_1t_2t_3t_2t_0t_2t_0t_3$ belong?
However, $Nt_0t_1t_2t_3t_2t_0t_2t_0t_0 = Nt_0t_1t_2t_3t_2t_0t_2 \in [0123202]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_2t_0t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_0t_2t_0t_2$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_0t_2t_0t_3$, and one sym-

metric generator goes to $Nt_0t_1t_2t_3t_2t_0t_2t_0t_1$.

183. Now, we consider the cosets stabilizer $N^{(01232021)}$.
$N^{(01232021)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_2t_0t_2t_1N$ is at most: $\frac{|N|}{|N^{(01232021)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01232021)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_2t_0t_2t_1t_0$, $Nt_0t_1t_2t_3t_2t_0t_2t_1t_1$, $Nt_0t_1t_2t_3t_2t_0t_2t_1t_2$ and $Nt_0t_1t_2t_3t_2t_0t_2t_1t_3$ belong?
However, $Nt_0t_1t_2t_3t_2t_0t_2t_1t_1 = Nt_0t_1t_2t_3t_2t_0t_2 \in [0123202]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_2t_0t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_0t_2t_1t_2$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_0t_2t_1t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_0t_2t_1t_0$.

184. Now, we consider the cosets stabilizer $N^{(01232023)}$.
$N^{(01232023)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_2t_0t_2t_3N$ is at most: $\frac{|N|}{|N^{(01232023)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01232023)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_2t_0t_2t_3t_0$, $Nt_0t_1t_2t_3t_2t_0t_2t_3t_1$, $Nt_0t_1t_2t_3t_2t_0t_2t_3t_2$ and $Nt_0t_1t_2t_3t_2t_0t_2t_3t_3$ belong?
However, $Nt_0t_1t_2t_3t_2t_0t_2t_3t_3 = Nt_0t_1t_2t_3t_2t_0t_2 \in [0123202]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_2t_0t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_0t_2t_3t_2$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_0t_2t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_0t_2t_3t_0$.

185. Now, we consider coset $N^{(01232101)}$.
However, $t_0t_1t_2t_3t_2t_1t_0t_1 = (2,0,3)(t_2t_3t_1t_0t_1t_2t_3) = (2,0,3)(t_0t_1t_2t_3t_2t_0t_1)^{(1,2,0,3)}$ $\in Nt_0t_1t_2t_3t_2t_0t_1$. Therefore, t_1 takes $Nt_0t_1t_2t_3t_2t_1t_0t_1$ go to $Nt_0t_1t_2t_3t_2t_0t_1$.

186. Now, we consider coset $N^{(01232102)}$.
However, $t_0t_1t_2t_3t_2t_1t_0t_2 = (2,1,3)(t_2t_1t_3t_1t_2t_0t_1) = (2,1,3)(t_0t_1t_2t_1t_0t_3t_1)^{(2,0,3)}$ $\in Nt_0t_1t_2t_1t_0t_3t_1$. Therefore, t_2 takes $Nt_0t_1t_2t_3t_2t_1t_0t_2$ go to $Nt_0t_1t_2t_1t_0t_3t_1$.

187. Now, we consider coset $N^{(01232103)}$.
However, $t_0t_1t_2t_3t_2t_1t_0t_3 = (2,3,0)(t_1t_0t_3t_1t_2t_1t_0) = (2,3,0)(t_0t_1t_2t_0t_3t_0t_1)^{(1,0)(2,3)}$ $\in Nt_0t_1t_2t_0t_3t_0t_1$. Therefore, t_3 takes $Nt_0t_1t_2t_3t_2t_1t_0t_3$ go to $Nt_0t_1t_2t_0t_3t_0t_1$.

188. Now, we consider the cosets stabilizer $N^{(01232120)}$.
$N^{(01232120)} = \{e\}$, then the number of the single cosets in the double coset

$Nt_0t_1t_2t_3t_2t_1t_2t_0N$ is at most: $\frac{|N|}{|N^{(01232120)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01232120)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_2t_1t_2t_0t_0$, $Nt_0t_1t_2t_3t_2t_1t_2t_0t_1$, $Nt_0t_1t_2t_3t_2t_1t_2t_0t_2$ and $Nt_0t_1t_2t_3t_2t_1t_2t_0t_3$ belong?
However, $Nt_0t_1t_2t_3t_2t_1t_2t_0t_0 = Nt_0t_1t_2t_3t_2t_1t_2 \in [0123212]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_2t_1t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_1t_2t_0t_2$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_1t_2t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_1t_2t_0t_3$.

189. [01232121] Same as 142.

190. [01232123] Same as 120.

191. [01232301] Same as 167.

192. [01232302] Same as 91.

193. Now, we consider the cosets stabilizer $N^{(01232303)}$.
$N^{(01232303)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_2t_3t_0t_3N$ is at most: $\frac{|N|}{|N^{(01232303)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01232303)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_2t_3t_0t_3t_0$, $Nt_0t_1t_2t_3t_2t_3t_0t_3t_1$, $Nt_0t_1t_2t_3t_2t_3t_0t_3t_2$ and $Nt_0t_1t_2t_3t_2t_3t_0t_3t_3$ belong?
However, $Nt_0t_1t_2t_3t_2t_3t_0t_3t_3 = Nt_0t_1t_2t_3t_2t_3t_0 \in [0123230]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_2t_3t_0N$, one symmetric generator goes to

$Nt_0t_1t_2t_3t_2t_3t_0t_3t_2$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_3t_0t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_3t_0t_3t_0$.

194. Now, we consider the cosets stabilizer $N^{(01232310)}$.
We know, $t_0t_1t_2t_3t_2t_3t_1t_0 = t_2t_3t_0t_1t_0t_1t_3t_2$ then
$N^{(01232310)} = \{e, (0,2)(1,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_2t_3t_1t_0N$ is at most: $\frac{|N|}{|N^{(01232310)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(01232310)}$ on $\{0,1,2,3\}$ are $\{0,2\}$ and $\{1,3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_2t_3t_1t_0t_0$, and $Nt_0t_1t_2t_3t_2t_3t_1t_0t_1$ belong?
However, $Nt_0t_1t_2t_3t_2t_3t_1t_0t_0 = Nt_0t_1t_2t_3t_2t_3t_1 \in [0123231]$, then two symmetric generators go back to the double coset $Nt_0t_1t_2t_3t_2t_3t_1N$, two symmetric generator go to $Nt_0t_1t_2t_3t_2t_3t_1t_0t_1$.

195. Now, we consider coset $N^{(01232312)}$.
However, $t_0t_1t_2t_3t_2t_3t_1t_2 = (t_0t_3t_1t_3t_1t_2t_1) = (t_0t_1t_2t_1t_2t_3t_2)^{(1,3,2)} \in Nt_0t_1t_2t_1t_2t_3t_2$. Therefore, t_2 takes $Nt_0t_1t_2t_3t_2t_3t_1t_2$ go to $Nt_0t_1t_2t_1t_2t_3t_2$.

196. Now, we consider the cosets stabilizer $N^{(01232313)}$.
We know, $t_0t_1t_2t_3t_2t_3t_1t_3 = (1,3,2)(t_0t_1t_2t_1t_3t_1t_3t_2)^{(1,0,2)}$ then
$Nt_0t_1t_2t_3t_2t_3t_1t_3 = Nt_0t_1t_2t_1t_3t_1t_3t_2 \in Nt_0t_1t_2t_1t_3t_1t_3t_2$.
$N^{(01232313)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_2t_3t_1t_3N$ is at most: $\frac{|N|}{|N^{(01232313)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01232313)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_2t_3t_1t_3t_0$, $Nt_0t_1t_2t_3t_2t_3t_1t_3t_1$, $Nt_0t_1t_2t_3t_2t_3t_1t_3t_2$ and $Nt_0t_1t_2t_3t_2t_3t_1t_3t_3$ belong?
However, $Nt_0t_1t_2t_3t_2t_3t_1t_3t_3 = Nt_0t_1t_2t_3t_2t_3t_1 \in [0123231]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_2t_3t_1N$, and $Nt_0t_1t_2t_3t_2t_3t_1t_3t_2 = Nt_2t_0t_1t_3t_1t_3t_2$ $\in [0121313]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_3t_1t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_3t_1t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_3t_1t_3t_0$.

197. Now, we consider the cosets stabilizer $N^{(01232320)}$.
We know, $t_0t_1t_2t_3t_2t_3t_2t_0 = t_0t_1t_3t_2t_3t_2t_3t_0$ then
$N^{(01232320)} = \{e, (0,2)(1,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_2t_3t_2t_0N$ is at most: $\frac{|N|}{|N^{(01232320)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(01232320)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$ and $\{2,3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_2t_3t_2t_0t_0$, $Nt_0t_1t_2t_3t_2t_3t_2t_0t_1$, and $Nt_0t_1t_2t_3t_2t_3t_2t_0t_2$ belong?
However, $Nt_0t_1t_2t_3t_2t_3t_2t_0t_0 = Nt_0t_1t_3t_2t_3t_2t_3 \in [0123232]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_2t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_3t_2t_0t_1$, and two symmetric generator go to $Nt_0t_1t_2t_3t_2t_3t_2t_0t_2$.

198. Now, we consider coset $N^{(01232321)}$.
However, $t_0t_1t_2t_3t_2t_3t_2t_1 = (t_0t_2t_1t_2t_3t_1t_3) = (t_0t_1t_2t_1t_3t_2t_3)^{(1,2)} \in Nt_0t_1t_2t_1t_3t_2t_3$. Therefore, t_1 takes $Nt_0t_1t_2t_3t_2t_3t_2t_1$ go to $Nt_0t_1t_2t_1t_3t_2t_3$.

199. Now, we consider the cosets stabilizer $N^{(01213101)}$.
$N^{(01213101)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_3t_1t_0t_1N$ is at most: $\frac{|N|}{|N^{(01213101)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01213101)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_3t_1t_0t_1t_0$, $Nt_0t_1t_2t_1t_3t_1t_0t_1t_1$, $Nt_0t_1t_2t_1t_3t_1t_0t_1t_2$ and $Nt_0t_1t_2t_1t_3t_1t_0t_1t_3$ belong?
However, $Nt_0t_1t_2t_1t_3t_1t_0t_1t_1 = Nt_0t_1t_2t_1t_3t_1t_0 \in [0121310]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_3t_1t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1t_0t_1t_2$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1t_0t_1t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1t_0t_1t_0$.

200. Now, we consider the cosets stabilizer $N^{(01213102)}$.
$N^{(01213102)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_3t_1t_0t_2N$ is at most: $\frac{|N|}{|N^{(01213102)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01213102)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_3t_1t_0t_2t_0$, $Nt_0t_1t_2t_1t_3t_1t_0t_2t_1$, $Nt_0t_1t_2t_1t_3t_1t_0t_2t_2$ and $Nt_0t_1t_2t_1t_3t_1t_0t_2t_3$ belong?

However, $Nt_0t_1t_2t_1t_3t_1t_0t_2t_2 = Nt_0t_1t_2t_1t_3t_1t_0 \in [0121310]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_3t_1t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1t_0t_2t_1$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1t_0t_2t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1t_0t_2t_0$.

201. [01213103] Same as 89.

202. Now, we consider the cosets stabilizer $N^{(01213120)}$.
$N^{(01213120)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_3t_1t_2t_0N$ is at most: $\frac{|N|}{|N^{(01213120)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01213120)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_3t_1t_2t_0t_0$, $Nt_0t_1t_2t_1t_3t_1t_2t_0t_1$, $Nt_0t_1t_2t_1t_3t_1t_2t_0t_2$ and $Nt_0t_1t_2t_1t_3t_1t_2t_0t_3$ belong?
However, $Nt_0t_1t_2t_1t_3t_1t_2t_0t_0 = Nt_0t_1t_2t_1t_3t_1t_2 \in [0121312]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_3t_1t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1t_2t_0t_1$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1t_2t_0t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1t_2t_0t_2$.

203. Now, we consider the cosets stabilizer $N^{(01213121)}$.
We know, $t_0t_1t_2t_1t_3t_1t_2t_1 = t_0t_3t_1t_3t_2t_3t_1t_3 = t_0t_2t_3t_2t_1t_2t_3t_2$ then
$N^{(01213121)} = \{e, (3,2,1), (3,1,2)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_3t_1t_2t_1N$ is at most: $\frac{|N|}{|N^{(01213121)}|} = \frac{24}{3} = 8$

The orbit of $N^{(01213121)}$ on $\{0,1,2,3\}$ are $\{3,1,2\}$ and $\{0\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_3t_1t_2t_1t_0$ and $Nt_0t_1t_2t_1t_3t_1t_2t_1t_3$ belong?
However, $Nt_0t_1t_2t_1t_3t_1t_2t_1t_1 = Nt_0t_3t_1t_3t_2t_3t_1 = Nt_0t_2t_3t_2t_1t_2t_3 \in [0121312]$, then three symmetric generators go back to the double coset $Nt_0t_1t_2t_1t_3t_1t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1t_2t_1t_0$.

204. [01213123] Same as 98.

205. Now, we consider the cosets stabilizer $N^{(01213130)}$.
$N^{(01213130)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_3t_1t_3t_0N$ is at most: $\frac{|N|}{|N^{(01213130)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(01213130)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_3t_1t_3t_0t_0$, $Nt_0t_1t_2t_1t_3t_1t_3t_0t_1$, $Nt_0t_1t_2t_1t_3t_1t_3t_0t_2$ and $Nt_0t_1t_2t_1t_3t_1t_3t_0t_3$ belong?
However, $Nt_0t_1t_2t_1t_3t_1t_3t_0t_0 = Nt_0t_1t_2t_1t_3t_1t_3 \in [0121313]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_3t_1t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1t_3t_0t_1$, one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1t_3t_0t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_3t_1t_3t_0t_2$.

206. Now, we consider coset $N^{(01213131)}$.
However, $t_0t_1t_2t_1t_3t_1t_3t_1 = (t_0t_2t_1t_3t_2t_3t_2) = (t_0t_1t_2t_3t_1t_3t_1)^{(1,2)} \in Nt_0t_1t_2t_3t_1t_3t_1$. Therefore, t_1 takes $Nt_0t_1t_2t_1t_3t_1t_3t_1$ go to $Nt_0t_1t_2t_3t_1t_3t_1$.

207. [01213132] Same as 196.

Length 9.

1. Now, we consider the cosets stabilizer $N^{(010102302)}$.
$N^{(010102302)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_0t_2t_3t_0t_2N$ is at most: $\frac{|N|}{|N^{(010102302)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010102302)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_0t_2t_3t_0t_2t_0$, $Nt_0t_1t_0t_1t_0t_2t_3t_0t_2t_1$, $Nt_0t_1t_0t_1t_0t_2t_3t_0t_2t_2$ and $Nt_0t_1t_0t_1t_0t_2t_3t_0t_2t_3$ belong?
We know, $t_0t_1t_0t_1t_0t_2t_3t_0t_2 = (1,0,3)(t_0t_1t_2t_1t_0t_1t_3t_2t_1)^{(1,0,3,2)}$.
$Nt_0t_1t_0t_1t_0t_2t_3t_0t_2 = Nt_3t_0t_1t_0t_3t_0t_2t_1t_0$.
$Nt_0t_1t_0t_1t_0t_2t_3t_0t_2t_0 = Nt_3t_0t_1t_0t_3t_0t_2t_1 \in [01210132]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_0t_1t_3t_2N$.

Moreover, $Nt_0t_1t_0t_1t_0t_2t_3t_0t_2t_2 = Nt_0t_1t_0t_1t_0t_2t_3t_0 \in [01010230]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_0t_2t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_0t_2t_3t_0t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_0t_2t_3t_0t_2t_3$.

2. Now, we consider the cosets stabilizer $N^{(010102303)}$.
$N^{(010102303)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_0t_2t_3t_0t_3N$ is at most: $\frac{|N|}{|N^{(010102303)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010102303)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_0t_2t_3t_0t_3t_0$, $Nt_0t_1t_0t_1t_0t_2t_3t_0t_3t_1$, $Nt_0t_1t_0t_1t_0t_2t_3t_0t_3t_2$ and $Nt_0t_1t_0t_1t_0t_2t_3t_0t_3t_3$ belong?
We know, $t_0t_1t_0t_1t_0t_2t_3t_0t_3 = (1,0)(2,3)(t_0t_1t_2t_1t_0t_3t_1t_2t_0)^{(1,3,0,2)}$.
$Nt_0t_1t_0t_1t_0t_2t_3t_0t_3 = Nt_2t_3t_1t_3t_2t_0t_3t_1t_2$.
$Nt_0t_1t_0t_1t_0t_2t_3t_0t_3t_2 = Nt_2t_3t_1t_3t_2t_0t_3t_1 \in [01210312]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_0t_3t_1t_2N$.

We know, $t_0t_1t_0t_1t_0t_2t_3t_0t_3 = (t_0t_1t_2t_1t_2t_1t_3t_0t_1)^{(1,0,3)}$.
$Nt_0t_1t_0t_1t_0t_2t_3t_0t_3 = Nt_3t_0t_2t_0t_2t_0t_1t_3t_0$.
$Nt_0t_1t_0t_1t_0t_2t_3t_0t_3t_0 = Nt_3t_0t_2t_0t_2t_0t_1t_3 \in [01212130]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_2t_1t_3t_0N$.

Moreover, $Nt_0t_1t_0t_1t_0t_2t_3t_0t_3t_3 = Nt_0t_1t_0t_1t_0t_2t_3t_0 \in [01010230]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_0t_2t_3t_0N$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_0t_2t_3t_0t_3t_3$.

3. Now, we consider coset $N^{(010102320)}$.
However, $t_0t_1t_0t_1t_0t_2t_3t_2t_0 = (1,3,0)(t_2t_0t_3t_2t_0t_2t_3t_2) = (1,3,0)(t_0t_1t_2t_3t_1t_3t_2t_3)^{(1,0,2,3)}$ $\in Nt_0t_1t_2t_3t_1t_3t_2t_3$. Therefore, t_0 takes $Nt_0t_1t_0t_1t_0t_2t_3t_2t_0$ go to $Nt_0t_1t_2t_3t_1t_3t_2t_3$.

4. Now, we consider the cosets stabilizer $N^{(010102323)}$.
We know, $t_0t_1t_0t_1t_0t_2t_3t_2t_3 = t_1t_0t_1t_0t_1t_2t_3t_2t_3 = t_3t_0t_3t_0t_3t_2t_1t_2t_1$
$= t_3t_1t_3t_1t_3t_2t_0t_2t_0 = t_0t_3t_0t_3t_0t_2t_1t_2t_1 = t_1t_3t_1t_3t_1t_2t_0t_2t_0$ then
$N^{(010102323)} = \{e,(0,1),(0,3),(1,3),(0,3,1),(0,1,3)\}$, then the number of the single cosets in the double coset
$Nt_0t_1t_0t_1t_0t_2t_3t_2t_3N$ is at most: $\frac{|N|}{|N^{(010102323)}|} = \frac{24}{6} = 4$

The orbit of $N^{(010102323)}$ on $\{0,1,2,3\}$ are $\{2\}$, and $\{0,1,3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_0t_2t_3t_2t_3t_0$, $Nt_0t_1t_0t_1t_0t_2t_3t_2t_3t_2$, and belong?

However, $Nt_0t_1t_0t_1t_0t_2t_3t_2t_3t_0 = Nt_0t_1t_0t_1t_0t_2t_3t_2 \in [01010232]$, then three symmetric generators go back to the double coset $Nt_0t_1t_0t_1t_0t_2t_3t_2N$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_0t_2t_3t_2t_3t_2$.

5. Now, we consider the cosets stabilizer $N^{(010120130)}$.

We know, $t_0t_1t_0t_1t_2t_0t_1t_3t_0 = t_2t_3t_2t_3t_0t_2t_3t_1t_2$ then

$N^{(010120130)} = \{e, (0,2)(1,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_0t_1t_3t_0N$ is at most: $\frac{|N|}{|N^{(010120130)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(010120130)}$ on $\{0,1,2,3\}$ are $\{0,2\}$, and $\{1,3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_0t_1t_3t_0t_0$, and $Nt_0t_1t_0t_1t_2t_0t_1t_3t_0t_1$ belong?

However, $Nt_0t_1t_0t_1t_2t_0t_1t_3t_0t_0 = Nt_0t_1t_0t_1t_2t_0t_1t_3 \in [01012013]$, then two symmetric generators go back to the double coset $Nt_0t_1t_0t_1t_2t_0t_1t_3N$, and two symmetric generator go to $Nt_0t_1t_0t_1t_2t_0t_1t_3t_0t_1$.

6. Now, we consider the cosets stabilizer $N^{(010120132)}$.

$N^{(010120132)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_0t_1t_3t_2N$ is at most: $\frac{|N|}{|N^{(010120132)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010120132)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_0t_1t_3t_2t_0$, $Nt_0t_1t_0t_1t_2t_0t_1t_3t_2t_1$, $Nt_0t_1t_0t_1t_2t_0t_1t_3t_2t_2$ and $Nt_0t_1t_0t_1t_2t_0t_1t_3t_2t_3$ belong?

We know, $t_0t_1t_0t_1t_2t_0t_1t_3t_2 = (1,2)(0,3)(t_0t_1t_0t_1t_2t_1t_0t_3t_2)^{(1,2,0,3)}$.

$Nt_0t_1t_0t_1t_2t_0t_1t_3t_2 = Nt_3t_2t_3t_2t_0t_2t_3t_1t_0$.

$Nt_0t_1t_0t_1t_0t_2t_3t_0t_3t_0 = Nt_3t_2t_3t_2t_0t_2t_3t_1 \in [01012103]$, then one symmetric generator goes back to $Nt_0t_1t_0t_1t_2t_1t_0t_3N$.

We know, $t_0t_1t_0t_1t_2t_0t_1t_3t_2 = (1,3,2)(t_0t_1t_0t_1t_2t_1t_3t_2t_3)^{(1,0,3)}$.

$Nt_0t_1t_0t_1t_2t_0t_1t_3t_2 = Nt_0t_2t_0t_2t_1t_2t_3t_1t_3$.

$Nt_0t_1t_0t_1t_0t_2t_3t_0t_3t_3 = Nt_0t_2t_0t_2t_1t_2t_3t_1 \in [01012132]$, then one symmetric generator goes back to $Nt_0t_1t_0t_1t_2t_1t_3t_2N$.

Moreover, $Nt_0t_1t_0t_1t_2t_0t_1t_3t_2t_2 = Nt_0t_1t_0t_1t_2t_0t_1t_3 \in [01012013]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_0t_1t_3N$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_1t_3t_2t_1$.

7. Now, we consider the cosets stabilizer $N^{(010120301)}$.
$N^{(010120301)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_0t_3t_0t_1N$ is at most: $\frac{|N|}{|N^{(010120301)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010120301)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_0t_3t_0t_1t_0$, $Nt_0t_1t_0t_1t_2t_0t_3t_0t_1t_1$, $Nt_0t_1t_0t_1t_2t_0t_3t_0t_1t_2$ and $Nt_0t_1t_0t_1t_2t_0t_3t_0t_1t_3$ belong?
We know, $t_0t_1t_0t_1t_2t_0t_3t_0t_1 = (1,3)(2,0)(t_0t_1t_0t_1t_2t_3t_1t_2t_0)^{(1,0,2,3)}$.
$Nt_0t_1t_0t_1t_2t_0t_3t_0t_1 = Nt_2t_0t_2t_0t_3t_1t_0t_3t_2$.
$Nt_0t_1t_0t_1t_0t_2t_3t_0t_1t_2 = Nt_2t_0t_2t_0t_3t_1t_0t_3 \in [01012312]$, then one symmetric generator goes back to $Nt_0t_1t_0t_1t_2t_3t_1t_2N$.

We know, $t_0t_1t_0t_1t_2t_0t_3t_0t_1 = (1,0,3)(t_0t_1t_2t_0t_3t_2t_0t_1t_0)^{(1,0,3)}$.
$Nt_0t_1t_0t_1t_2t_0t_3t_0t_1 = Nt_3t_0t_2t_3t_1t_2t_3t_0t_3$.
$Nt_0t_1t_0t_1t_0t_2t_3t_0t_1t_3 = Nt_3t_0t_2t_3t_1t_2t_3t_0 \in [01203201]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_3t_2t_0t_1N$.

Moreover, $Nt_0t_1t_0t_1t_2t_0t_3t_0t_1t_1 = Nt_0t_1t_0t_1t_2t_0t_3t_0 \in [01012030]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_0t_3t_0N$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_3t_0t_1t_0$.

8. Now, we consider the cosets stabilizer $N^{(010120302)}$.
$N^{(010120302)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_0t_3t_0t_2N$ is at most: $\frac{|N|}{|N^{(010120302)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010120302)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_0t_3t_0t_2t_0$, $Nt_0t_1t_0t_1t_2t_0t_3t_0t_2t_1$, $Nt_0t_1t_0t_1t_2t_0t_3t_0t_2t_2$ and $Nt_0t_1t_0t_1t_2t_0t_3t_0t_2t_3$ belong?

However, $Nt_0t_1t_0t_1t_2t_0t_3t_0t_2t_2 = Nt_0t_1t_0t_1t_0t_2t_3t_0 \in [01012030]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_0t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_3t_0t_2t_1$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_3t_0t_2t_3$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_3t_0t_2t_0$.

9. Now, we consider coset $N^{(010120303)}$.
However, $t_0t_1t_0t_1t_2t_0t_3t_0t_3 = (t_1t_3t_1t_2t_1t_2t_0t_1) = (t_0t_1t_0t_2t_0t_2t_3t_0)^{(1,3,0)} \in Nt_0t_1t_0t_2t_0t_2t_3t_0$.
Therefore, t_3 takes $Nt_0t_1t_0t_1t_2t_0t_3t_0t_3$ go to $Nt_0t_1t_0t_2t_0t_2t_3t_0$.

10. Now, we consider coset $N^{(010120310)}$.
However, $t_0t_1t_0t_1t_2t_0t_3t_1t_0 = (1,2)(3,0)(t_1t_3t_1t_2t_1t_2t_0t_1) = (1,2)(3,0)(t_0t_1t_0t_1t_2t_1t_3t_0)^{(2,0)}$ $\in Nt_0t_1t_0t_1t_2t_1t_3t_0$. Therefore, t_0 takes $Nt_0t_1t_0t_1t_2t_0t_3t_1t_0$ go to $Nt_0t_1t_0t_1t_2t_1t_3t_0$.

11. Now, we consider the cosets stabilizer $N^{(010120313)}$.
$N^{(010120313)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_0t_3t_1t_3N$ is at most: $\frac{|N|}{|N^{(010120313)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010120313)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_0t_3t_1t_3t_0$, $Nt_0t_1t_0t_1t_2t_0t_3t_1t_3t_1$, $Nt_0t_1t_0t_1t_2t_0t_3t_1t_3t_2$ and $Nt_0t_1t_0t_1t_2t_0t_3t_1t_3t_3$ belong?
We know, $t_0t_1t_0t_1t_2t_0t_3t_1t_3 = (1,2,3)(t_0t_1t_0t_1t_2t_1t_3t_1t_2)$.
$Nt_0t_1t_0t_1t_2t_0t_3t_1t_3 = Nt_0t_1t_0t_1t_2t_1t_3t_1t_2$.
$Nt_0t_1t_0t_1t_2t_0t_3t_1t_3t_2 = Nt_0t_1t_0t_1t_2t_1t_3t_1 \in [01012131]$, then one symmetric generator goes back to $Nt_0t_1t_0t_1t_2t_1t_3t_1N$.

Moreover, $Nt_0t_1t_0t_1t_2t_0t_3t_1t_3t_3 = Nt_0t_1t_0t_1t_2t_0t_3t_1 \in [01012031]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_0t_3t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_3t_1t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_3t_1t_3t_0$.

12. Now, we consider the cosets stabilizer $N^{(010120320)}$.
$N^{(010120320)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_0t_3t_2t_0N$ is at most: $\frac{|N|}{|N^{(010120320)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010120320)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a

representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_0t_3t_2t_0t_0$, $Nt_0t_1t_0t_1t_2t_0t_3t_2t_0t_1$, $Nt_0t_1t_0t_1t_2t_0t_3t_2t_0t_2$ and $Nt_0t_1t_0t_1t_2t_0t_3t_2t_0t_3$ belong?
We know, $t_0t_1t_0t_1t_2t_0t_3t_2t_0 = (1,0)(2,3)(t_0t_1t_2t_1t_0t_1t_3t_2t_0)^{(1,0,2)}$.
$Nt_0t_1t_0t_1t_2t_0t_3t_2t_0 = Nt_2t_0t_1t_0t_2t_0t_3t_1t_2$.
$Nt_0t_1t_0t_1t_2t_0t_3t_2t_0t_2 = Nt_2t_0t_1t_0t_2t_0t_3t_1 \in [01012131]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_0t_1t_3t_2N$.

Moreover, $Nt_0t_1t_0t_1t_2t_0t_3t_2t_0t_0 = Nt_0t_1t_0t_1t_2t_0t_3t_2 \in [01012032]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_0t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_3t_2t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_3t_2t_0t_3$.

13. Now, we consider the cosets stabilizer $N^{(010120321)}$.
$N^{(010120321)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_0t_3t_2t_1N$ is at most: $\frac{|N|}{|N^{(010120321)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010120321)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_0t_3t_2t_1t_0$, $Nt_0t_1t_0t_1t_2t_0t_3t_2t_1t_1$, $Nt_0t_1t_0t_1t_2t_0t_3t_2t_1t_2$ and $Nt_0t_1t_0t_1t_2t_0t_3t_2t_1t_3$ belong?
We know, $t_0t_1t_0t_1t_2t_0t_3t_2t_1 = (t_0t_1t_2t_1t_2t_0t_3t_0t_2)^{(1,2,3)}$.
$Nt_0t_1t_0t_1t_2t_0t_3t_2t_1 = Nt_0t_2t_3t_2t_3t_0t_1t_0t_3$.
$Nt_0t_1t_0t_1t_2t_0t_3t_2t_1t_2 = Nt_0t_2t_3t_2t_3t_0t_1t_0 \in [01212030]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_2t_0t_3t_0N$.

Moreover, $Nt_0t_1t_0t_1t_2t_0t_3t_2t_1t_1 = Nt_0t_1t_0t_1t_2t_0t_3t_2 \in [01012032]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_0t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_3t_2t_1t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_0t_3t_2t_1t_2$.

14. Now, we consider coset $N^{(010121030)}$.
However, $t_0t_1t_0t_1t_2t_1t_0t_3t_0 = (1,0,3)(t_0t_2t_1t_2t_3t_0t_3t_1) = (1,0,3)(t_0t_1t_2t_1t_3t_0t_3t_2)^{(1,2)}$ $\in Nt_0t_1t_2t_1t_3t_0t_3t_2$. Therefore, t_0 takes $Nt_0t_1t_0t_1t_2t_1t_0t_3t_0$ go to $Nt_0t_1t_2t_1t_3t_0t_3t_2$.

15. Now, we consider the cosets stabilizer $N^{(010121031)}$.
$N^{(010121031)} = \{e\}$, then the number of the single cosets in the double coset

$Nt_0t_1t_0t_1t_2t_1t_0t_3t_1N$ is at most: $\frac{|N|}{|N^{(010121031)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010121031)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_1t_0t_3t_1t_0$, $Nt_0t_1t_0t_1t_2t_1t_0t_3t_1t_1$, $Nt_0t_1t_0t_1t_2t_1t_0t_3t_1t_2$ and $Nt_0t_1t_0t_1t_2t_1t_0t_3t_1t_3$ belong?
We know, $t_0t_1t_0t_1t_2t_1t_0t_3t_1 = (1,2,0)(t_0t_1t_2t_3t_1t_0t_1t_3t_1)^{(1,0)(2,3)}$.
$Nt_0t_1t_0t_1t_2t_1t_0t_3t_1 = Nt_1t_0t_3t_2t_0t_1t_0t_2t_0$.
$Nt_0t_1t_0t_1t_2t_1t_0t_3t_1t_0 = Nt_1t_0t_3t_2t_0t_1t_0t_2 \in [01231013]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_1t_0t_1t_3N$.

Moreover, $Nt_0t_1t_0t_1t_2t_1t_0t_3t_1t_1 = Nt_0t_1t_0t_1t_2t_1t_0t_3 \in [01012103]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_1t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_0t_3t_1t_3$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_0t_3t_1t_2$.

16. [010121032] Same as 6.

17. Now, we consider coset $N^{(010121031)}$.
However, $t_0t_1t_0t_1t_2t_1t_0t_3t_1 = (1,3)(2,0)(t_3t_0t_2t_0t_1t_1t_0t_2) = (1,3)(2,0)(t_0t_1t_2t_1t_3t_1t_0t_2)^{(1,2)}$ $\in Nt_0t_1t_2t_1t_3t_1t_0t_2$. Therefore, t_1 takes $Nt_0t_1t_0t_1t_2t_1t_0t_3t_1$ go to $Nt_0t_1t_2t_1t_3t_1t_0t_2$.

18. Now, we consider coset $N^{(010121032)}$.
However, $t_0t_1t_0t_1t_2t_1t_0t_3t_2 = t_0t_1t_0t_1t_2t_1t_3t_0$.
Therefore, t_2 takes $Nt_0t_1t_0t_1t_2t_1t_0t_3t_2$ go to itself $Nt_0t_1t_0t_1t_2t_1t_3t_0$.

19. Now, we consider the cosets stabilizer $N^{(010121303)}$.
$N^{(010121303)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_1t_3t_0t_3N$ is at most: $\frac{|N|}{|N^{(010121303)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010121303)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_1t_3t_0t_3t_0$, $Nt_0t_1t_0t_1t_2t_1t_3t_0t_3t_1$, $Nt_0t_1t_0t_1t_2t_1t_3t_0t_3t_2$ and $Nt_0t_1t_0t_1t_2t_1t_3t_0t_3t_3$ belong? We know, $t_0t_1t_0t_1t_2t_1t_3t_0t_3 = (1,2,3)(t_0t_1t_2t_1t_2t_0t_3t_2t_1)^{(1,2,0)}$.
$Nt_0t_1t_0t_1t_2t_1t_3t_0t_3 = Nt_1t_2t_0t_2t_0t_1t_3t_0t_2$.
$Nt_0t_1t_0t_1t_2t_1t_3t_0t_3t_2 = Nt_1t_2t_0t_2t_0t_1t_3t_0 \in [01212032]$, then one symmetric generator

goes back to $Nt_0t_1t_2t_1t_2t_0t_3t_2N$.

We know, $t_0t_1t_0t_1t_2t_1t_3t_0t_3 = (1,3,0)(t_0t_1t_2t_1t_2t_3t_0t_2t_1)^{(2,3)}$.

$Nt_0t_1t_0t_1t_2t_1t_3t_0t_3 = Nt_0t_1t_3t_1t_3t_2t_0t_3t_1$.

$Nt_0t_1t_0t_1t_2t_1t_3t_0t_3t_3 = Nt_0t_1t_3t_1t_3t_2t_0t_3 \in [01212302]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_2t_3t_0t_2N$.

Moreover, $Nt_0t_1t_0t_1t_2t_1t_3t_0t_3t_3 = Nt_0t_1t_0t_1t_2t_1t_3t_0 \in [01012130]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_1t_3t_0N$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_3t_0t_3t_0$.

20. Now, we consider the cosets stabilizer $N^{(010121310)}$.

$N^{(010121310)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_1t_3t_1t_0N$ is at most: $\frac{|N|}{|N^{(010121310)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010121310)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_1t_3t_1t_0t_0$, $Nt_0t_1t_0t_1t_2t_1t_3t_1t_0t_1$, $Nt_0t_1t_0t_1t_2t_1t_3t_1t_0t_2$ and $Nt_0t_1t_0t_1t_2t_1t_3t_1t_0t_3$ belong? We know, $t_0t_1t_0t_1t_2t_1t_3t_1t_0 = (2,0,3)(t_0t_1t_0t_2t_0t_3t_0t_1t_2)^{(1,0,2)}$.

$Nt_0t_1t_0t_1t_2t_1t_3t_1t_0 = Nt_2t_0t_2t_1t_2t_3t_2t_0t_1$.

$Nt_0t_1t_0t_1t_2t_1t_3t_1t_0t_1 = Nt_2t_0t_2t_1t_2t_3t_2t_0 \in [01020301]$, then one symmetric generator goes back to $Nt_0t_1t_0t_2t_0t_3t_0t_1N$.

Moreover, $Nt_0t_1t_0t_1t_2t_1t_3t_1t_0t_0 = Nt_0t_1t_0t_1t_2t_1t_3t_1 \in [01012131]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_1t_3t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_3t_1t_0t_3$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_3t_1t_0t_2$.

21. [0101211312] Same as 11.

22. Now, we consider coset $N^{(010121313)}$.

However, $t_0t_1t_0t_1t_2t_1t_3t_1t_3 = (t_3t_1t_2t_3t_2t_3t_0t_1) = (t_0t_1t_2t_0t_2t_0t_3t_1)^{(3,0)} \in Nt_0t_1t_2t_0t_2t_0t_3t_1$. Therefore, t_3 takes $Nt_0t_1t_0t_1t_2t_1t_3t_1t_3$ go to $Nt_0t_1t_2t_0t_2t_0t_3t_1$.

23. Now, we consider the cosets stabilizer $N^{(010121320)}$.

$N^{(010121320)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_1t_3t_2t_0N$ is at most: $\frac{|N|}{|N^{(010121320)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010121320)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_1t_3t_2t_0t_0$, $Nt_0t_1t_0t_1t_2t_1t_3t_2t_0t_1$, $Nt_0t_1t_0t_1t_2t_1t_3t_2t_0t_2$ and $Nt_0t_1t_0t_1t_2t_1t_3t_2t_0t_3$ belong?
However, $Nt_0t_1t_0t_1t_2t_1t_3t_2t_0t_0 = Nt_0t_1t_0t_1t_2t_1t_3t_2 \in [01012132]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_1t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_3t_2t_0t_1$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_3t_2t_0t_3$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_3t_2t_0t_2$.

24. Now, we consider the cosets stabilizer $N^{(010121321)}$.
$N^{(010121321)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_1t_3t_2t_1N$ is at most: $\frac{|N|}{|N^{(010121321)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010121321)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_1t_3t_2t_1t_0$, $Nt_0t_1t_0t_1t_2t_1t_3t_2t_1t_1$, $Nt_0t_1t_0t_1t_2t_1t_3t_2t_1t_2$ and $Nt_0t_1t_0t_1t_2t_1t_3t_2t_1t_3$ belong? We know, $t_0t_1t_0t_1t_2t_1t_3t_2t_1 = (1,3,0)(t_0t_1t_2t_0t_1t_3t_2t_3t_2)^{(2,0)}$.
$Nt_0t_1t_0t_1t_2t_1t_3t_2t_1 = Nt_2t_1t_0t_2t_1t_3t_0t_3t_0$.
$Nt_0t_1t_0t_1t_2t_1t_3t_2t_1t_0 = Nt_2t_1t_0t_2t_1t_3t_0t_3 \in [01201323]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_1t_3t_2t_3N$.
We know, $t_0t_1t_0t_1t_2t_1t_3t_2t_1 = (1,2,3)(t_0t_1t_2t_1t_0t_1t_3t_0t_1)^{(1,2)}$.
$Nt_0t_1t_0t_1t_2t_1t_3t_2t_1 = Nt_0t_2t_1t_2t_0t_2t_3t_0t_2$.
$Nt_0t_1t_0t_1t_2t_1t_3t_2t_1t_2 = Nt_0t_2t_1t_2t_0t_2t_3t_0 \in [01210130]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_0t_1t_3t_0N$.
Moreover, $Nt_0t_1t_0t_1t_2t_1t_3t_2t_1t_1 = Nt_0t_1t_0t_1t_2t_1t_3t_2 \in [01012132]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_1t_3t_2N$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_1t_3t_2t_1t_3$.

25. [010121323] Same as 6.

26. Now, we consider the cosets stabilizer $N^{(010123020)}$.
$N^{(010123020)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_3t_0t_2t_0N$ is at most: $\frac{|N|}{|N^{(010123020)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010123020)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a

representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_0t_2t_0t_0$, $Nt_0t_1t_0t_1t_2t_3t_0t_2t_0t_1$, $Nt_0t_1t_0t_1t_2t_3t_0t_2t_0t_2$ and $Nt_0t_1t_0t_1t_2t_3t_0t_2t_0t_3$ belong?
We know, $t_0t_1t_0t_1t_2t_3t_0t_2t_0 = (1,3,0)(t_0t_1t_0t_2t_0t_1t_0t_3t_0)^{(1,0)(3,2)}$.
$Nt_0t_1t_0t_1t_2t_3t_0t_2t_0 = Nt_1t_0t_1t_3t_1t_0t_1t_2t_1$.
$Nt_0t_1t_0t_1t_2t_3t_0t_2t_0t_1 = Nt_1t_0t_1t_3t_1t_0t_1t_2 \in [01020103]$, then one symmetric generator goes back to $Nt_0t_1t_0t_2t_0t_1t_0t_3N$.
Moreover, $Nt_0t_1t_0t_1t_2t_3t_0t_2t_0t_0 = Nt_0t_1t_0t_1t_2t_3t_0t_2 \in [01012302]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_3t_0t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_0t_2t_0t_3$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_0t_2t_0t_2$.

27. Now, we consider the cosets stabilizer $N^{(010123021)}$.
$N^{(010123021)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_3t_0t_2t_1N$ is at most: $\frac{|N|}{|N^{(010123021)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010123021)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_0t_2t_1t_0$, $Nt_0t_1t_0t_1t_2t_3t_0t_2t_1t_1$, $Nt_0t_1t_0t_1t_2t_3t_0t_2t_1t_2$ and $Nt_0t_1t_0t_1t_2t_3t_0t_2t_1t_3$ belong?
However, $Nt_0t_1t_0t_1t_2t_3t_0t_2t_1t_1 = Nt_0t_1t_0t_1t_2t_3t_0t_2 \in [01012302]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_3t_0t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_0t_2t_1t_0$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_0t_2t_1t_3$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_0t_2t_1t_2$.

28. Now, we consider the cosets stabilizer $N^{(010123023)}$.
$N^{(010123023)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_3t_0t_2t_3N$ is at most: $\frac{|N|}{|N^{(010123023)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010123023)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_0t_2t_3t_0$, $Nt_0t_1t_0t_1t_2t_3t_0t_2t_3t_1$, $Nt_0t_1t_0t_1t_2t_3t_0t_2t_3t_2$ and $Nt_0t_1t_0t_1t_2t_3t_0t_2t_3t_3$ belong?
We know, $t_0t_1t_0t_1t_2t_3t_0t_2t_3 = (1,3)(2,0)(t_0t_1t_2t_0t_1t_0t_3t_0t_1)^{(1,2,3,0)}$.
$Nt_0t_1t_0t_1t_2t_3t_0t_2t_3 = Nt_1t_3t_2t_1t_2t_1t_0t_1t_2$.
$Nt_0t_1t_0t_1t_2t_3t_0t_2t_3t_2 = Nt_1t_3t_2t_1t_2t_1t_0t_1 \in [01201030]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_1t_0t_3t_0N$.
Moreover, $Nt_0t_1t_0t_1t_2t_3t_0t_2t_3t_3 = Nt_0t_1t_0t_1t_2t_3t_0t_2 \in [01012302]$, then one symmetric

generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_3t_0t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_0t_2t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_0t_2t_3t_1$.

29. Now, we consider the cosets stabilizer $N^{(010123030)}$.
We know, $t_0t_1t_0t_1t_2t_3t_0t_3t_0 = t_2t_1t_2t_1t_0t_3t_2t_3t_2$ then
$N^{(010123030)} = \{e, (0,2)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_3t_0t_3t_0N$ is at most: $\frac{|N|}{|N^{(010123030)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(010123030)}$ on $\{0,1,2,3\}$ are $\{0,2\}$, $\{1\}$ and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_0t_3t_0t_0$, $Nt_0t_1t_0t_1t_2t_3t_0t_3t_0t_1$ $Nt_0t_1t_0t_1t_2t_3t_0t_3t_0t_3$ and belong?
However, $Nt_0t_1t_0t_1t_2t_3t_0t_3t_0t_0 = Nt_0t_1t_0t_1t_2t_3t_0t_3 \in [01012303]$, then two symmetric generators go back to the double coset $Nt_0t_1t_0t_1t_2t_3t_0t_3N$,
We know, $t_0t_1t_0t_1t_2t_3t_0t_3t_0 = (t_0t_1t_2t_3t_2t_3t_2t_0t_1)^{(1,3,2,0)}$.
$Nt_0t_1t_0t_1t_2t_3t_0t_3t_0 = Nt_1t_3t_0t_2t_0t_2t_0t_1t_3$.
$Nt_0t_1t_0t_1t_2t_3t_0t_3t_0t_3 = Nt_1t_3t_0t_2t_0t_2t_0t_1 \in [01232320]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_2t_3t_2t_0N$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_0t_3t_0t_1$.

30. Now, we consider coset $N^{(010123031)}$.
However, $t_0t_1t_0t_1t_2t_3t_0t_3t_1 = (1,0)(2,3)(t_3t_2t_0t_2t_1t_2t_1t_3) = (1,0)(2,3)(t_0t_1t_2t_1t_3t_1t_3t_0)^{(1,2,0,3)}$ $\in Nt_0t_1t_2t_1t_3t_1t_3t_0$. Therefore, t_1 takes $Nt_0t_1t_0t_1t_2t_3t_0t_3t_1$ go to $Nt_0t_1t_2t_1t_3t_1t_3t_0$.

31. Now, we consider coset $N^{(010123032)}$.
However, $t_0t_1t_0t_1t_2t_3t_0t_3t_2 = (t_2t_3t_0t_3t_2t_1t_3t_0) = (t_0t_1t_2t_1t_0t_1t_3t_2)^{(3,1)(0,2)}$ $\in Nt_0t_1t_2t_1t_0t_1t_3t_2$. Therefore, t_2 takes $Nt_0t_1t_0t_1t_2t_3t_0t_3t_2$ go to $Nt_0t_1t_2t_1t_0t_1t_3t_2$.

32. Now, we consider coset $N^{(010123101)}$.
However, $t_0t_1t_0t_1t_2t_3t_1t_0t_1 = (t_0t_2t_0t_3t_2t_1t_0t_2) = (t_0t_1t_0t_2t_1t_3t_0t_1)^{(3,1)(0,2)}$ $\in Nt_0t_1t_0t_2t_1t_3t_0t_1$. Therefore, t_1 takes $Nt_0t_1t_0t_1t_2t_3t_1t_0t_1$ go to $Nt_0t_1t_0t_2t_1t_3t_0t_1$.

33. [010123120] Same as 7.

34. Now, we consider coset $N^{(010123123)}$.

However, $t_0t_1t_0t_1t_2t_3t_1t_2t_3 = (1,3,2)(t_0t_3t_0t_2t_1t_3t_2t_1) = (1,3,2)(t_0t_1t_0t_2t_3t_1t_2t_3)^{(3,1)}$ $\in Nt_0t_1t_0t_2t_3t_1t_2t_3$. Therefore, t_3 takes $Nt_0t_1t_0t_1t_2t_3t_1t_2t_3$ go to $Nt_0t_1t_0t_2t_3t_1t_2t_3$.

35. Now, we consider the cosets stabilizer $N^{(010123130)}$.

$N^{(010123130)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_3t_1t_3t_0N$ is at most: $\frac{|N|}{|N^{(010123130)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010123130)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_1t_3t_0t_0$, $Nt_0t_1t_0t_1t_2t_3t_1t_3t_0t_1$, $Nt_0t_1t_0t_1t_2t_3t_1t_3t_0t_2$ and $Nt_0t_1t_0t_1t_2t_3t_1t_3t_0t_3$ belong?

We know, $t_0t_1t_0t_1t_2t_3t_1t_3t_0 = (1,3,2,)(t_0t_1t_0t_2t_1t_0t_3t_1t_0)^{(1,2,3,0)}$.

$Nt_0t_1t_0t_1t_2t_3t_1t_3t_0 = Nt_1t_2t_1t_3t_2t_1t_0t_2t_1$.

$Nt_0t_1t_0t_1t_2t_3t_1t_3t_0t_2 = Nt_1t_2t_1t_3t_2t_1t_0t_2 \in [01021031]$, then one symmetric generator goes back to $Nt_0t_1t_0t_2t_1t_0t_3t_1N$.

Moreover, $Nt_0t_1t_0t_1t_2t_3t_1t_3t_0t_0 = Nt_0t_1t_0t_1t_2t_3t_1t_3 \in [01012313]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_3t_1t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_1t_3t_0t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_1t_3t_0t_3$.

36. Now, we consider coset $N^{(010123131)}$.

However, $t_0t_1t_0t_1t_2t_3t_1t_3t_1 = (t_3t_1t_2t_3t_2t_3t_0t_3) = (t_0t_1t_2t_0t_2t_0t_3t_0)^{(3,0)}$ $\in Nt_0t_1t_2t_0t_2t_0t_3t_0$. Therefore, t_1 takes $Nt_0t_1t_0t_1t_2t_3t_1t_3t_1$ go to $Nt_0t_1t_2t_0t_2t_0t_3t_0$.

37. Now, we consider coset $N^{(010123132)}$.

However, $t_0t_1t_0t_1t_2t_3t_1t_3t_2 = (1,3,0)(t_2t_0t_2t_3t_1t_2t_0t_1) = (1,3,0)(t_0t_1t_0t_2t_3t_0t_1t_3)^{(1,0,2,3)}$ $\in Nt_0t_1t_0t_2t_3t_0t_1t_3$. Therefore, t_2 takes $Nt_0t_1t_0t_1t_2t_3t_1t_3t_2$ go to $Nt_0t_1t_0t_2t_3t_0t_1t_3$.

38. Now, we consider coset $N^{(010123201)}$.

However, $t_0t_1t_0t_1t_2t_3t_2t_0t_1 = (1,3,0)(t_3t_2t_3t_1t_0t_1t_3t_0) = (1,3,0)(t_0t_1t_0t_2t_3t_2t_0t_3)^{(1,2)(0,3)}$ $\in Nt_0t_1t_0t_2t_3t_2t_0t_3$. Therefore, t_1 takes $Nt_0t_1t_0t_1t_2t_3t_2t_0t_1$ go to $Nt_0t_1t_0t_2t_3t_2t_0t_3$.

39. Now, we consider the cosets stabilizer $N^{(010123202)}$.
$N^{(010123202)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_3t_2t_0t_2N$ is at most: $\frac{|N|}{|N^{(010123202)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010123202)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_2t_0t_2t_0$, $Nt_0t_1t_0t_1t_2t_3t_2t_0t_2t_1$, $Nt_0t_1t_0t_1t_2t_3t_2t_0t_2t_2$ and $Nt_0t_1t_0t_1t_2t_3t_2t_0t_2t_3$ belong?
We know, $t_0t_1t_0t_1t_2t_3t_2t_0t_2 = (1,3,2,)(t_0t_1t_2t_1t_3t_1t_3t_0t_2)^{(1,0,2)}$.
$Nt_0t_1t_0t_1t_2t_3t_2t_0t_2 = Nt_2t_0t_1t_0t_3t_0t_3t_2t_1$.
$Nt_0t_1t_0t_1t_2t_3t_2t_0t_2t_1 = Nt_2t_0t_1t_0t_3t_0t_3t_2 \in$ [01213130], then one symmetric generator goes back to $Nt_0t_1t_2t_1t_3t_1t_3t_0N$.
Moreover, $Nt_0t_1t_0t_1t_2t_3t_2t_0t_2t_2 = Nt_0t_1t_0t_1t_2t_3t_2t_0 \in$ [01012320], then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_3t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_2t_0t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_2t_0t_2t_3$.

40. Now, we consider the cosets stabilizer $N^{(010123203)}$.
$N^{(010123203)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_3t_2t_0t_3N$ is at most: $\frac{|N|}{|N^{(010123203)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010123203)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_2t_0t_3t_0$, $Nt_0t_1t_0t_1t_2t_3t_2t_0t_3t_1$, $Nt_0t_1t_0t_1t_2t_3t_2t_0t_3t_2$ and $Nt_0t_1t_0t_1t_2t_3t_2t_0t_3t_3$ belong?
We know, $t_0t_1t_0t_1t_2t_3t_2t_0t_3 = (t_0t_1t_0t_2t_0t_2t_3t_0t_2)^{(1,2,0)}$.
$Nt_0t_1t_0t_1t_2t_3t_2t_0t_3 = Nt_1t_2t_1t_0t_1t_0t_3t_1t_0$.
$Nt_0t_1t_0t_1t_2t_3t_2t_0t_3t_0 = Nt_1t_2t_1t_0t_1t_0t_3t_1 \in$ [01020230], then one symmetric generator goes back to $Nt_0t_1t_0t_2t_0t_2t_3t_0N$.
We know, $t_0t_1t_0t_1t_2t_3t_2t_0t_3 = (1,3,2)(t_0t_1t_0t_2t_1t_0t_3t_1t_2)^{(2,1,0)}$.
$Nt_0t_1t_0t_1t_2t_3t_2t_0t_3 = Nt_2t_0t_2t_1t_0t_2t_3t_0t_1$.
$Nt_0t_1t_0t_1t_2t_3t_2t_0t_3t_1 = Nt_2t_0t_2t_1t_0t_2t_3t_0 \in$ [01021031], then one symmetric generator goes back to $Nt_0t_1t_0t_2t_1t_0t_3t_1N$.
We know, $t_0t_1t_0t_1t_2t_3t_2t_0t_3 = (0,2,3)(t_0t_1t_2t_1t_2t_3t_1t_2t_0)^{(0,2,3)}$.
$Nt_0t_1t_0t_1t_2t_3t_2t_0t_3 = Nt_2t_1t_3t_1t_3t_0t_1t_3t_2$.
$Nt_0t_1t_0t_1t_2t_3t_2t_0t_3t_2 = Nt_2t_1t_3t_1t_3t_0t_1t_3 \in$ [01212312], then one symmetric generator goes back to $Nt_0t_1t_2t_1t_2t_3t_1t_2N$.

Moreover, $Nt_0t_1t_0t_1t_2t_3t_2t_0t_3t_3 = Nt_0t_1t_0t_1t_2t_3t_2t_0 \in [01012320]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_3t_2t_0N$.

41. Now, we consider coset $N^{(010123210)}$.
However, $t_0t_1t_0t_1t_2t_3t_2t_1t_0 = (1,0,3)(t_0t_2t_3t_0t_1t_0t_3t_1) = (1,3,0)(t_0t_1t_2t_0t_3t_0t_3t_2)^{(1,2,3)}$ $\in Nt_0t_1t_2t_0t_3t_0t_3t_2$. Therefore, t_0 takes $Nt_0t_1t_0t_1t_2t_3t_2t_1t_0$ go to $Nt_0t_1t_2t_0t_3t_0t_3t_2$.

42. Now, we consider the cosets stabilizer $N^{(010123212)}$.
$N^{(010123212)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_3t_2t_1t_2N$ is at most: $\frac{|N|}{|N^{(010123212)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010123212)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_2t_1t_2t_0$, $Nt_0t_1t_0t_1t_2t_3t_2t_1t_2t_1$, $Nt_0t_1t_0t_1t_2t_3t_2t_1t_2t_2$ and $Nt_0t_1t_0t_1t_2t_3t_2t_1t_2t_3$ belong?
We know, $t_0t_1t_0t_1t_2t_3t_2t_1t_2 = (t_0t_1t_2t_1t_3t_1t_2t_0t_2)^{(1,2,3)}$.
$Nt_0t_1t_0t_1t_2t_3t_2t_1t_2 = Nt_0t_2t_3t_2t_1t_2t_3t_0t_3$.
$Nt_0t_1t_0t_1t_2t_3t_2t_1t_2t_3 = Nt_0t_2t_3t_2t_1t_2t_3t_0 \in [01213120]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_3t_1t_2t_0N$.
We know, $t_0t_1t_0t_1t_2t_3t_2t_1t_2 = (1,3)(0,2)(t_0t_1t_0t_2t_0t_2t_3t_1t_0)^{(1,0)}$.
$Nt_0t_1t_0t_1t_2t_3t_2t_1t_2 = Nt_1t_0t_1t_2t_1t_2t_3t_0t_1$.
$Nt_0t_1t_0t_1t_2t_3t_2t_1t_2t_1 = Nt_1t_0t_1t_2t_1t_2t_3t_0 \in [01020231]$, then one symmetric generator goes back to $Nt_0t_1t_0t_2t_0t_2t_3t_1N$.
Moreover, $Nt_0t_1t_0t_1t_2t_3t_2t_1t_2t_2 = Nt_0t_1t_0t_1t_2t_3t_2t_1 \in [01012321]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_3t_2t_1N$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_2t_1t_2t_0$.

43. Now, we consider the cosets stabilizer $N^{(010123231)}$.
We know, $t_0t_1t_0t_1t_2t_3t_2t_3t_1 = t_3t_1t_3t_1t_2t_0t_2t_0t_1$ then
$N^{(010123231)} = \{e,(0,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_3t_2t_3t_1N$ is at most: $\frac{|N|}{|N^{(010123231)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(010123231)}$ on $\{0,1,2,3\}$ are $\{0,3\}$, $\{1\}$ and $\{2\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_2t_3t_1t_0$, $Nt_0t_1t_0t_1t_2t_3t_2t_3t_1t_1$ $Nt_0t_1t_0t_1t_2t_3t_2t_3t_1t_2$ and belong?

However, $Nt_0t_1t_0t_1t_2t_3t_2t_3t_1t_1 = Nt_0t_1t_0t_1t_2t_3t_2t_3 \in [01012323]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_3t_2t_3N$,
We know, $t_0t_1t_0t_1t_2t_3t_2t_3t_1 = (t_0t_1t_2t_1t_2t_1t_3t_0t_3)^{(1,3,2,0)}$.
$Nt_0t_1t_0t_1t_2t_3t_2t_3t_1 = Nt_1t_3t_0t_3t_0t_3t_2t_1t_2$.
$Nt_0t_1t_0t_1t_2t_3t_2t_3t_1t_2 = Nt_1t_3t_0t_3t_0t_3t_2t_1 \in [01212130]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_2t_1t_3t_0N$, and two symmetric generator go to $Nt_0t_1t_0t_1t_2t_3t_2t_3t_1t_3$.

44. Now, we consider the cosets stabilizer $N^{(010123232)}$.
We know, $t_0t_1t_0t_1t_2t_3t_2t_3t_2 = t_0t_1t_0t_1t_3t_2t_3t_2t_3$
$= t_2t_1t_2t_1t_0t_3t_0t_3t_0 = t_3t_1t_3t_1t_0t_2t_0t_2t_0 = t_2t_1t_2t_1t_3t_0t_3t_0t_3 = t_3t_1t_3t_1t_2t_0t_2t_0t_2$ then $N^{(010123232)} = \{e, (2,3), (0,3), (0,2), (0,3,2), (0,2,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_3t_2t_3t_2N$ is at most: $\frac{|N|}{|N^{(010123232)}|} = \frac{24}{6} = 4$

The orbit of $N^{(010123232)}$ on $\{0,1,2,3\}$ are $\{1\}$, and $\{0,2,3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_2t_3t_2t_0$, $Nt_0t_1t_0t_1t_2t_3t_2t_3t_2t_1$, and belong?
However, $Nt_0t_1t_0t_1t_2t_3t_2t_3t_2t_2 = Nt_0t_1t_0t_1t_2t_3t_2t_3 \in [01012323]$, then three symmetric generators go back to the double coset $Nt_0t_1t_0t_1t_2t_3t_2t_3N$, and one symmetric generator goes to $Nt_0t_1t_0t_1t_2t_3t_2t_3t_2t_1$.

45. [010201030] Same as 26.

46. Now, we consider coset $N^{(010201230)}$.
However, $t_0t_1t_0t_2t_0t_1t_2t_3t_0 = (1,2,0)(t_0t_3t_2t_3t_0t_1t_0t_1) = (1,2,0)(t_0t_1t_2t_1t_0t_3t_0t_3)^{(1,3)}$ $\in Nt_0t_1t_2t_1t_0t_3t_0t_3$. Therefore, t_0 takes $Nt_0t_1t_0t_2t_0t_1t_2t_3t_0$ go to $Nt_0t_1t_2t_1t_0t_3t_0t_3$.

47. Now, we consider coset $N^{(010201231)}$.
However, $t_0t_1t_0t_2t_0t_1t_2t_3t_1 = (1,2,0)(t_0t_3t_2t_3t_1t_0t_1t_0) = (1,2,0)(t_0t_1t_2t_1t_3t_0t_3t_0)^{(1,3)}$ $\in Nt_0t_1t_2t_1t_3t_0t_3t_0$. Therefore, t_1 takes $Nt_0t_1t_0t_2t_0t_1t_2t_3t_1$ go to $Nt_0t_1t_2t_1t_3t_0t_3t_0$.

48. Now, we consider the cosets stabilizer $N^{(010201232)}$.
$N^{(010201232)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_1t_2t_3t_2N$ is at most: $\frac{|N|}{|N^{(010201232)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010201232)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_1t_2t_3t_2t_0$, $Nt_0t_1t_0t_2t_0t_1t_2t_3t_2t_1$, $Nt_0t_1t_0t_2t_0t_1t_2t_3t_2t_2$ and $Nt_0t_1t_0t_2t_0t_1t_2t_3t_2t_3$ belong?

We know, $t_0t_1t_0t_2t_0t_1t_2t_3t_2 = (1,2)(3,0)(t_0t_1t_2t_0t_3t_2t_0t_1t_3)^{(0,3)}$.

$Nt_0t_1t_0t_2t_0t_1t_2t_3t_2 = Nt_3t_1t_2t_3t_0t_2t_3t_1t_0$.

$Nt_0t_1t_0t_2t_0t_1t_2t_3t_2t_0 = Nt_3t_1t_2t_3t_0t_2t_3t_1 \in [01203201]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_3t_2t_0t_1N$.

Moreover, $Nt_0t_1t_0t_2t_0t_1t_2t_3t_2t_2 = Nt_0t_1t_0t_2t_0t_1t_2t_3 \in [01020123]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_1t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_2t_3t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_2t_3t_2t_3$.

49. Now, we consider the cosets stabilizer $N^{(010201302)}$.

$N^{(010201302)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_1t_3t_0t_2N$ is at most: $\frac{|N|}{|N^{(010201302)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010201302)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_1t_3t_0t_2t_0$, $Nt_0t_1t_0t_2t_0t_1t_3t_0t_2t_1$, $Nt_0t_1t_0t_2t_0t_1t_3t_0t_2t_2$ and $Nt_0t_1t_0t_2t_0t_1t_3t_0t_2t_3$ belong?

We know, $t_0t_1t_0t_2t_0t_1t_3t_0t_2 = (1,2,0)(t_0t_1t_0t_2t_0t_2t_3t_0t_2)^{(1,3)}$.

$Nt_0t_1t_0t_2t_0t_1t_3t_0t_2 = Nt_0t_3t_0t_2t_0t_2t_1t_2t_0$.

$Nt_0t_1t_0t_2t_0t_1t_3t_0t_2t_0 = Nt_0t_3t_0t_2t_0t_2t_1t_2 \in [01020230]$, then one symmetric generator goes back to $Nt_0t_1t_0t_2t_0t_2t_3t_0N$.

Moreover, $Nt_0t_1t_0t_2t_0t_1t_3t_0t_2t_2 = Nt_0t_1t_0t_2t_0t_1t_3t_0 \in [01020130]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_1t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_3t_0t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_3t_0t_2t_3$.

50. Now, we consider the cosets stabilizer $N^{(010201303)}$.

$N^{(010201303)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_1t_3t_0t_3N$ is at most: $\frac{|N|}{|N^{(010201303)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010201303)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_1t_3t_0t_3t_0$, $Nt_0t_1t_0t_2t_0t_1t_3t_0t_3t_1$, $Nt_0t_1t_0t_2t_0t_1t_3t_0t_3t_2$ and $Nt_0t_1t_0t_2t_0t_1t_3t_0t_3t_3$ belong?

However, $Nt_0t_1t_0t_2t_0t_1t_3t_0t_3t_3 = Nt_0t_1t_0t_2t_0t_1t_3t_0 \in [01020130]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_1t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_3t_0t_3t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_3t_0t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_3t_0t_3t_2$.

51. Now, we consider coset $N^{(010201312)}$.
However, $t_0t_1t_0t_2t_0t_1t_3t_1t_2 = (2,0,3)(t_3t_2t_0t_2t_1t_3t_1t_0) = (2,0,3)(t_0t_1t_2t_1t_3t_0t_3t_2)^{(1,2,0,3)}$ $\in Nt_0t_1t_2t_1t_3t_0t_3t_2$. Therefore, t_2 takes $Nt_0t_1t_0t_2t_0t_1t_3t_1t_2$ go to $Nt_0t_1t_2t_1t_3t_0t_3t_2$.

52. Now, we consider the cosets stabilizer $N^{(010201313)}$.
$N^{(010201313)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_1t_3t_1t_3N$ is at most: $\frac{|N|}{|N^{(010201313)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010201313)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_1t_3t_1t_3t_0$, $Nt_0t_1t_0t_2t_0t_1t_3t_1t_3t_1$, $Nt_0t_1t_0t_2t_0t_1t_3t_1t_3t_2$ and $Nt_0t_1t_0t_2t_0t_1t_3t_1t_3t_3$ belong?
We know, $t_0t_1t_0t_2t_0t_1t_3t_1t_3 = (t_0t_1t_2t_3t_0t_2t_1t_0t_1)^{(1,2,3)}$.
$Nt_0t_1t_0t_2t_0t_1t_3t_1t_3 = Nt_0t_2t_3t_1t_0t_3t_2t_0t_2$.
$Nt_0t_1t_0t_2t_0t_1t_3t_1t_3t_2 = Nt_0t_2t_3t_1t_0t_3t_2t_0 \in [01230210]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_0t_2t_1t_0N$.
Moreover, $Nt_0t_1t_0t_2t_0t_1t_3t_1t_3t_3 = Nt_0t_1t_0t_2t_0t_1t_3t_1 \in [01020131]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_1t_3t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_3t_1t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_1t_3t_1t_3t_0$.

53. Now, we consider coset $N^{(010201323)}$.
However, $t_0t_1t_0t_2t_0t_1t_3t_2t_3 = (2,0,1)(t_0t_2t_3t_0t_1t_0t_1t_0) = (2,0,1)(t_0t_1t_2t_0t_3t_0t_3t_0)^{(1,2,3)}$ $\in Nt_0t_1t_2t_0t_3t_0t_3t_0$. Therefore, t_3 takes $Nt_0t_1t_0t_2t_0t_1t_3t_2t_3$ go to $Nt_0t_1t_2t_0t_3t_0t_3t_0$.

54. Now, we consider the cosets stabilizer $N^{(010202130)}$.
$N^{(010202130)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_2t_1t_3t_0N$ is at most: $\frac{|N|}{|N^{(010202130)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010202130)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_2t_1t_3t_0t_0$,

$Nt_0t_1t_0t_2t_0t_2t_1t_3t_0t_1$, $Nt_0t_1t_0t_2t_0t_2t_1t_3t_0t_2$ and $Nt_0t_1t_0t_2t_0t_2t_1t_3t_0t_3$ belong?
We know, $t_0t_1t_0t_2t_0t_2t_1t_3t_0 = (1,0)(2,3)(t_0t_1t_0t_2t_3t_1t_2t_0t_3)^{(1,0)(2,3)}$.
$Nt_0t_1t_0t_2t_0t_2t_1t_3t_0 = Nt_1t_0t_1t_3t_2t_0t_3t_1t_2$.
$Nt_0t_1t_0t_2t_0t_2t_1t_3t_0t_2 = Nt_1t_0t_1t_3t_2t_0t_3t_1 \in [01023120]$, then one symmetric generator goes back to $Nt_0t_1t_0t_2t_3t_1t_2t_0N$.
Moreover, $Nt_0t_1t_0t_2t_0t_2t_1t_3t_0t_0 = Nt_0t_1t_0t_2t_0t_2t_1t_3 \in [01020213]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_2t_1t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_1t_3t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_1t_3t_0t_3$.

55. Now, we consider the cosets stabilizer $N^{(010202131)}$.
$N^{(010202131)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_2t_1t_3t_1N$ is at most: $\frac{|N|}{|N^{(010202131)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010202131)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_2t_1t_3t_1t_0$, $Nt_0t_1t_0t_2t_0t_2t_1t_3t_1t_1$, $Nt_0t_1t_0t_2t_0t_2t_1t_3t_1t_2$ and $Nt_0t_1t_0t_2t_0t_2t_1t_3t_1t_3$ belong?
However, $Nt_0t_1t_0t_2t_0t_2t_1t_3t_1t_1 = Nt_0t_1t_0t_2t_0t_2t_1t_3 \in [01020213]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_2t_1t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_1t_3t_1t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_1t_3t_1t_3$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_1t_3t_1t_2$.

56. Now, we consider the cosets stabilizer $N^{(010202132)}$.
$N^{(010202132)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_2t_1t_3t_2N$ is at most: $\frac{|N|}{|N^{(010202132)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010202132)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_2t_1t_3t_2t_0$, $Nt_0t_1t_0t_2t_0t_2t_1t_3t_2t_1$, $Nt_0t_1t_0t_2t_0t_2t_1t_3t_2t_2$ and $Nt_0t_1t_0t_2t_0t_2t_1t_3t_2t_3$ belong?
However, $Nt_0t_1t_0t_2t_0t_2t_1t_3t_2t_2 = Nt_0t_1t_0t_2t_0t_2t_1t_3 \in [01020213]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_2t_1t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_1t_3t_2t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_1t_3t_2t_3$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_2t_1t_3t_2t_1$.

57. Now, we consider coset $N^{(010202301)}$.
However, $t_0t_1t_0t_2t_0t_2t_3t_0t_1 = (t_3t_0t_3t_0t_2t_3t_1t_3) = (t_0t_1t_0t_1t_2t_0t_3t_0)^{(1,0,3)}$
$\in Nt_0t_1t_0t_1t_2t_0t_3t_0$. Therefore, t_1 takes $Nt_0t_1t_0t_2t_0t_2t_3t_0t_1$ go to $Nt_0t_1t_0t_1t_2t_0t_3t_0$.

58. [0102023020] Same as 40.

59. Now, we consider coset $N^{(010202303)}$.
However, $t_0t_1t_0t_2t_0t_2t_3t_0t_3 = t_0t_1t_2t_3t_2t_3t_2t_0$.
Therefore, t_3 takes $Nt_0t_1t_0t_2t_0t_2t_3t_0t_3$ go to $Nt_0t_1t_2t_3t_2t_3t_2t_0$.

60. [010123212] Same as 42.

61. Now, we consider coset $N^{(010202312)}$.
However, $t_0t_1t_0t_2t_0t_2t_3t_1t_2 = (2,0,3)(t_0t_1t_0t_2t_0t_3t_1t_3) = (2,0,3)(t_0t_1t_0t_2t_0t_3t_1t_3)$
$\in Nt_0t_1t_0t_2t_0t_3t_1t_3$. Therefore, t_2 takes $Nt_0t_1t_0t_2t_0t_2t_3t_1t_2$ go to $Nt_0t_1t_0t_2t_0t_3t_1t_3$.

62. Now, we consider the cosets stabilizer $N^{(010202313)}$.
$N^{(010202313)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_2t_3t_1t_3N$ is at most: $\frac{|N|}{|N^{(010202313)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010202313)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_2t_3t_1t_3t_0$, $Nt_0t_1t_0t_2t_0t_2t_3t_1t_3t_1$, $Nt_0t_1t_0t_2t_0t_2t_3t_1t_3t_2$ and $Nt_0t_1t_0t_2t_0t_2t_3t_1t_3t_3$ belong?
We know, $t_0t_1t_0t_2t_0t_2t_3t_1t_3 = (1,3,2)(t_0t_1t_0t_2t_3t_2t_3t_1t_3)^{(1,2,3,0)}$.
$Nt_0t_1t_0t_2t_0t_2t_3t_1t_3 = Nt_1t_2t_1t_3t_0t_3t_0t_2t_0$.
$Nt_0t_1t_0t_2t_0t_2t_3t_1t_3t_0 = Nt_1t_2t_1t_3t_0t_3t_0t_2 \in$ [01023231], then one symmetric generator goes back to $Nt_0t_1t_2t_1t_3t_1t_2t_0N$.

We know, $t_0t_1t_0t_2t_0t_2t_3t_1t_3 = (t_0t_1t_2t_0t_1t_3t_0t_2t_3)^{(1,3,2)}$.
$Nt_0t_1t_0t_2t_0t_2t_3t_1t_3 = Nt_0t_3t_1t_0t_3t_2t_0t_1t_2$.
$Nt_0t_1t_0t_2t_0t_2t_3t_1t_3t_1 = Nt_0t_3t_1t_0t_3t_2t_0t_1 \in$ [01201302], then one symmetric generator goes back to $Nt_0t_1t_2t_0t_1t_3t_0t_2N$.

We know, $t_0t_1t_0t_2t_0t_2t_3t_1t_3 = (t_0t_1t_2t_1t_3t_0t_3t_0t_2)^{(1,2)}$.
$Nt_0t_1t_0t_2t_0t_2t_3t_1t_3 = Nt_0t_2t_1t_2t_3t_0t_3t_0t_1$.
$Nt_0t_1t_0t_2t_0t_2t_3t_1t_3t_1 = Nt_0t_2t_1t_2t_3t_0t_3t_0 \in [01213030]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_3t_0t_3t_0N$.

Moreover, $Nt_0t_1t_0t_2t_0t_2t_3t_1t_3t_3 = Nt_0t_1t_0t_2t_0t_2t_3t_1 \in [01020231]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_2t_3t_1N$.

63. [010202320] Same as 49.

64. Now, we consider coset $N^{(010202321)}$.
However, $t_0t_1t_0t_2t_0t_2t_3t_2t_1 = (1,2)(0,3)(t_1t_2t_1t_0t_3t_2t_3t_2) = (1,2)(0,3)(t_0t_1t_0t_2t_3t_1t_3t_1)^{(1,2,0)}$ $\in Nt_0t_1t_0t_2t_3t_1t_3t_1$. Therefore, t_1 takes $Nt_0t_1t_0t_2t_0t_2t_3t_1t_2$ go to $Nt_0t_1t_0t_2t_3t_1t_3t_1$.

65. Now, we consider coset $N^{(010202323)}$.
However, $t_0t_1t_0t_2t_0t_2t_3t_2t_3 = t_0t_1t_2t_1t_2t_0t_3t_1$.
Therefore, t_3 takes $Nt_0t_1t_0t_2t_0t_2t_3t_2t_3$ go to $Nt_0t_1t_2t_1t_2t_0t_3t_1$.

66. Now, we consider the cosets stabilizer $N^{(010203010)}$.
$N^{(010203010)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0N$ is at most: $\frac{|N|}{|N^{(010203010)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010203010)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_0$, $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_1$, $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_2$ and $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_3$ belong?
We know, $t_0t_1t_0t_2t_0t_3t_0t_1t_0 = (1,3,2)(t_0t_1t_2t_1t_3t_1t_0t_1t_2)^{(1,2)(0,3)}$.
$Nt_0t_1t_0t_2t_0t_3t_0t_1t_0 = Nt_3t_2t_1t_2t_0t_2t_3t_2t_1$.
$Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_1 = Nt_3t_2t_1t_2t_0t_2t_3t_2 \in [01213101]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_3t_1t_0t_1N$.
Moreover, $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_0 = Nt_0t_1t_0t_2t_0t_3t_0t_1 \in [01020301]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_3t_0t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_3$.

67. [010203012] Same as 20.

68. Now, we consider the cosets stabilizer $N^{(010203013)}$.
$N^{(010203013)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_3t_0t_1t_3N$ is at most: $\frac{|N|}{|N^{(010203013)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010203013)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_3t_0t_1t_3t_0$, $Nt_0t_1t_0t_2t_0t_3t_0t_1t_3t_1$, $Nt_0t_1t_0t_2t_0t_3t_0t_1t_3t_2$ and $Nt_0t_1t_0t_2t_0t_3t_0t_1t_3t_3$ belong?
We know, $t_0t_1t_0t_2t_0t_3t_0t_1t_3 = (1,0,2)(t_0t_1t_2t_0t_2t_1t_3t_0t_1)^{(0,2)}$.
$Nt_0t_1t_0t_2t_0t_3t_0t_1t_3 = Nt_2t_1t_0t_2t_0t_1t_3t_2t_1$.
$Nt_0t_1t_0t_2t_0t_3t_0t_1t_3t_1 = Nt_2t_1t_0t_2t_0t_1t_3t_2 \in [01202130]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_2t_1t_3t_0N$.
Moreover, $Nt_0t_1t_0t_2t_0t_3t_0t_1t_3t_3 = Nt_0t_1t_0t_2t_0t_3t_0t_1 \in [01020301]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_3t_0t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_0t_1t_3t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_0t_1t_3t_0$.

69. Now, we consider the cosets stabilizer $N^{(010203020)}$.
$N^{(010203020)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_3t_0t_2t_0N$ is at most: $\frac{|N|}{|N^{(010203020)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010203020)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_3t_0t_2t_0t_0$, $Nt_0t_1t_0t_2t_0t_3t_0t_2t_0t_1$, $Nt_0t_1t_0t_2t_0t_3t_0t_2t_0t_2$ and $Nt_0t_1t_0t_2t_0t_3t_0t_2t_0t_3$ belong?
We know, $t_0t_1t_0t_2t_0t_3t_0t_2t_0 = (0,3,2)(t_0t_1t_2t_0t_2t_3t_2t_0t_2)^{(3,0)}$.
$Nt_0t_1t_0t_2t_0t_3t_0t_2t_0 = Nt_2t_1t_0t_2t_0t_1t_3t_2t_1$.
$Nt_0t_1t_0t_2t_0t_3t_0t_2t_0t_1 = Nt_2t_1t_0t_2t_0t_1t_3t_2 \in [01202320]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_2t_3t_2t_0N$.
We know, $t_0t_1t_0t_2t_0t_3t_0t_2t_0 = (2,3,0)(t_0t_1t_2t_3t_2t_0t_2t_3t_2)^{(0,3,2)}$.
$Nt_0t_1t_0t_2t_0t_3t_0t_2t_0 = Nt_2t_1t_3t_0t_3t_2t_3t_0t_3$.
$Nt_0t_1t_0t_2t_0t_3t_0t_2t_0t_3 = Nt_2t_1t_3t_0t_3t_2t_3t_0 \in [01232023]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_2t_0t_2t_3N$.
Moreover, $Nt_0t_1t_0t_2t_0t_3t_0t_2t_0t_0 = Nt_0t_1t_0t_2t_0t_3t_0t_2 \in [01020302]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_3t_0t_2N$, and one symmetric genera-

tor goes to $Nt_0t_1t_0t_2t_0t_3t_0t_2t_0t_1$.

70. Now, we consider the cosets stabilizer $N^{(010203021)}$.
$N^{(010203021)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_3t_0t_2t_1N$ is at most: $\frac{|N|}{|N^{(010203021)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010203021)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_3t_0t_2t_1t_0$, $Nt_0t_1t_0t_2t_0t_3t_0t_2t_1t_1$, $Nt_0t_1t_0t_2t_0t_3t_0t_2t_1t_2$ and $Nt_0t_1t_0t_2t_0t_3t_0t_2t_1t_3$ belong?
We know, $t_0t_1t_0t_2t_0t_3t_0t_2t_1 = (2,1,3)(t_0t_1t_2t_3t_1t_3t_0t_3t_2)^{(1,2,3,0)}$.
$Nt_0t_1t_0t_2t_0t_3t_0t_2t_1 = Nt_1t_2t_3t_0t_2t_0t_1t_0t_3$.
$Nt_0t_1t_0t_2t_0t_3t_0t_2t_1t_3 = Nt_1t_2t_3t_0t_2t_0t_1t_0 \in [01231303]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_1t_3t_0t_3N$.
Moreover, $Nt_0t_1t_0t_2t_0t_3t_0t_2t_1t_1 = Nt_0t_1t_0t_2t_0t_3t_0t_2 \in [01020302]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_3t_0t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_0t_2t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_0t_2t_1t_0$.

71. Now, we consider coset $N^{(010203030)}$.
However, $t_0t_1t_0t_2t_0t_3t_0t_3t_0 = t_0t_1t_2t_0t_3t_2t_3t_2$.
Therefore, t_0 takes $Nt_0t_1t_0t_2t_0t_3t_0t_3t_0$ go to $Nt_0t_1t_2t_0t_3t_2t_3t_2$.

72. Now, we consider coset $N^{(010203031)}$.
However, $t_0t_1t_0t_2t_0t_3t_0t_3t_1 = (t_1t_0t_3t_1t_2t_1t_2t_0) = (t_0t_1t_2t_0t_3t_0t_3t_1)^{(1,0)(2,3)}$
$\in Nt_0t_1t_2t_0t_3t_0t_3t_1$. Therefore, t_1 takes $Nt_0t_1t_0t_2t_0t_3t_0t_3t_1$ go to $Nt_0t_1t_2t_0t_3t_0t_3t_1$.

73. Now, we consider the cosets stabilizer $N^{(010203032)}$.
$N^{(010203032)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_3t_0t_3t_2N$ is at most: $\frac{|N|}{|N^{(010203032)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010203032)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_3t_0t_3t_2t_0$, $Nt_0t_1t_0t_2t_0t_3t_0t_3t_2t_1$, $Nt_0t_1t_0t_2t_0t_3t_0t_3t_2t_2$ and $Nt_0t_1t_0t_2t_0t_3t_0t_3t_2t_3$ belong?
We know, $t_0t_1t_0t_2t_0t_3t_0t_3t_2 = (2,3,0)(t_0t_1t_2t_0t_2t_0t_3t_2t_3)^{(2,0)}$.
$Nt_0t_1t_0t_2t_0t_3t_0t_3t_2 = Nt_2t_1t_0t_2t_3t_2t_3t_0t_3$.

$Nt_0t_1t_0t_2t_0t_3t_0t_3t_2t_3 = Nt_2t_1t_0t_2t_3t_2t_3t_0 \in [01203032]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_2t_0t_3t_2N$.

Moreover, $Nt_0t_1t_0t_2t_0t_3t_0t_3t_2t_2 = Nt_0t_1t_0t_2t_0t_3t_0t_3 \in [01020303]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_3t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_0t_3t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_0t_3t_2t_0$.

74. Now, we consider coset $N^{(010203130)}$.

However, $t_0t_1t_0t_2t_0t_3t_0t_3t_1 = (1,2,3)(t_0t_1t_0t_3t_0t_2t_1t_2) = (1,2,3)(t_0t_1t_0t_2t_0t_3t_1t_3)^{(1,0)(2,3)}$ $\in Nt_0t_1t_0t_2t_0t_3t_1t_3$. Therefore, t_0 takes $Nt_0t_1t_0t_2t_0t_3t_1t_3t_0$ go to $Nt_0t_1t_0t_2t_0t_3t_1t_3$.

75. Now, we consider the cosets stabilizer $N^{(010203131)}$.

$N^{(010203131)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_3t_1t_3t_1N$ is at most: $\frac{|N|}{|N^{(010203131)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010203131)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_3t_1t_3t_1t_0$, $Nt_0t_1t_0t_2t_0t_3t_1t_3t_1t_1$, $Nt_0t_1t_0t_2t_0t_3t_1t_3t_1t_2$ and $Nt_0t_1t_0t_2t_0t_3t_1t_3t_1t_3$ belong?

We know, $t_0t_1t_0t_2t_0t_3t_1t_3t_1 = (2,3,0)(t_0t_1t_2t_3t_2t_3t_0t_3t_1)^{(1,2,3)}$.

$Nt_0t_1t_0t_2t_0t_3t_1t_3t_1 = Nt_0t_2t_3t_1t_3t_1t_0t_1t_2$.

$Nt_0t_1t_0t_2t_0t_3t_1t_3t_1t_2 = Nt_0t_2t_3t_1t_3t_1t_0t_1 \in [01232303]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_2t_3t_0t_3N$.

Moreover, $Nt_0t_1t_0t_2t_0t_3t_1t_3t_1t_1 = Nt_0t_1t_0t_2t_0t_3t_1t_3 \in [01020313]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_3t_1t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_1t_3t_1t_3$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_1t_3t_1t_0$.

76. Now, we consider coset $N^{(010203132)}$.

However, $t_0t_1t_0t_2t_0t_3t_1t_3t_2 = (2,3,0)t_0t_1t_0t_2t_0t_2t_3t_1$.

Therefore, t_0 takes $Nt_0t_1t_0t_2t_0t_3t_1t_3t_2$ go to $Nt_0t_1t_0t_2t_0t_2t_3t_1$.

77. Now, we consider coset $N^{(010203203)}$.

However, $t_0t_1t_0t_2t_0t_3t_2t_0t_3 = (0,2,3)(t_2t_1t_2t_0t_2t_3t_0t_2) = (0,2,3)(t_0t_1t_0t_2t_0t_3t_2t_0)^{(2,0)}$ $\in Nt_0t_1t_0t_2t_0t_3t_2t_0$. Therefore, t_3 takes $Nt_0t_1t_0t_2t_0t_3t_2t_0t_3$ go to itself $Nt_0t_1t_0t_2t_0t_3t_2t_0$.

78. Now, we consider the cosets stabilizer $N^{(010203210)}$.
$N^{(010203210)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_3t_2t_1t_0N$ is at most: $\frac{|N|}{|N^{(010203210)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010203210)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_3t_2t_1t_0t_0$, $Nt_0t_1t_0t_2t_0t_3t_2t_1t_0t_1$, $Nt_0t_1t_0t_2t_0t_3t_2t_1t_0t_2$ and $Nt_0t_1t_0t_2t_0t_3t_2t_1t_0t_3$ belong?
We know, $t_0t_1t_0t_2t_0t_3t_2t_1t_0 = (1,3)(0,2)(t_0t_1t_0t_2t_1t_2t_3t_0t_1)^{(3,1)(2,0)}$.
$Nt_0t_1t_0t_2t_0t_3t_2t_1t_0 = Nt_2t_3t_2t_0t_2t_0t_1t_3t_2$.
$Nt_0t_1t_0t_2t_0t_3t_2t_1t_0t_2 = Nt_2t_3t_2t_0t_2t_0t_1t_3 \in [01021230]$, then one symmetric generator goes back to $Nt_0t_1t_0t_2t_1t_2t_3t_0N$.
We know, $t_0t_1t_0t_2t_0t_3t_2t_1t_0 = (1,3,0)(t_0t_1t_2t_0t_2t_3t_2t_0t_3)^{(0,3,1,2)}$.
$Nt_0t_1t_0t_2t_0t_3t_2t_1t_0 = Nt_3t_2t_0t_3t_0t_1t_0t_3t_1$.
$Nt_0t_1t_0t_2t_0t_3t_2t_1t_0t_1 = Nt_3t_2t_0t_3t_0t_1t_0t_3 \in [01202320]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_2t_3t_2t_0N$.
Moreover, $Nt_0t_1t_0t_2t_0t_3t_2t_1t_0t_0 = Nt_0t_1t_0t_2t_0t_3t_2t_1 \in [01020321]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_3t_2t_1N$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_2t_1t_0t_2$.

79. Now, we consider the cosets stabilizer $N^{(010203212)}$.
$N^{(010203212)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_3t_2t_1t_2N$ is at most: $\frac{|N|}{|N^{(010203212)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010203212)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_3t_2t_1t_2t_0$, $Nt_0t_1t_0t_2t_0t_3t_2t_1t_2t_1$, $Nt_0t_1t_0t_2t_0t_3t_2t_1t_2t_2$ and $Nt_0t_1t_0t_2t_0t_3t_2t_1t_2t_3$ belong?
We know, $t_0t_1t_0t_2t_0t_3t_2t_1t_2 = (1,3,0)(t_0t_1t_2t_1t_0t_3t_1t_3t_2)^{(1,2,0)}$.
$Nt_0t_1t_0t_2t_0t_3t_2t_1t_2 = Nt_1t_2t_0t_2t_1t_3t_2t_3t_0$.
$Nt_0t_1t_0t_2t_0t_3t_2t_1t_2t_0 = Nt_1t_2t_0t_2t_1t_3t_2t_3 \in [01210313]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_0t_3t_1t_3N$.
Moreover, $Nt_0t_1t_0t_2t_0t_3t_2t_1t_2t_2 = Nt_0t_1t_0t_2t_0t_3t_2t_1 \in [01020321]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_3t_2t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_2t_1t_2t_3$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_2t_1t_2t_1$.

80. Now, we consider the cosets stabilizer $N^{(010203213)}$.
$N^{(010203213)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_3t_2t_1t_3N$ is at most: $\frac{|N|}{|N^{(010203213)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010203213)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_3t_2t_1t_3t_0$, $Nt_0t_1t_0t_2t_0t_3t_2t_1t_3t_1$, $Nt_0t_1t_0t_2t_0t_3t_2t_1t_3t_2$ and $Nt_0t_1t_0t_2t_0t_3t_2t_1t_3t_3$ belong?
We know, $t_0t_1t_0t_2t_0t_3t_2t_1t_3 = (1,3)(0,2)(t_0t_1t_2t_0t_1t_3t_0t_2t_1)^{(1,2,3)}$.
$Nt_0t_1t_0t_2t_0t_3t_2t_1t_3 = Nt_0t_2t_3t_0t_2t_1t_0t_3t_2$.
$Nt_0t_1t_0t_2t_0t_3t_2t_1t_3t_2 = Nt_0t_2t_3t_0t_2t_1t_0t_3 \in [01201302]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_1t_3t_0t_2N$.
We know, $t_0t_1t_0t_2t_0t_3t_2t_1t_3 = (t_0t_1t_2t_0t_2t_0t_3t_1t_2)^{(2,1,0)}$.
$Nt_0t_1t_0t_2t_0t_3t_2t_1t_3 = Nt_2t_0t_1t_2t_1t_2t_3t_0t_1$.
$Nt_0t_1t_0t_2t_0t_3t_2t_1t_3t_1 = Nt_2t_0t_1t_2t_1t_2t_3t_0 \in [01202031]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_2t_0t_3t_1N$.
Moreover, $Nt_0t_1t_0t_2t_0t_3t_2t_1t_3t_3 = Nt_0t_1t_0t_2t_0t_3t_2t_1 \in [01020321]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_3t_2t_1N$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_2t_1t_3t_0$.

81. Now, we consider coset $N^{(010210123)}$.
However, $t_0t_1t_0t_2t_1t_0t_1t_2t_3 = (0,1,2)(t_3t_0t_3t_1t_2t_1t_2t_1) = (0,1,2)(t_0t_1t_0t_2t_3t_2t_3t_2)^{(1,0,3,2)}$ $\in Nt_0t_1t_0t_2t_3t_2t_3t_2$. Therefore, t_3 takes $Nt_0t_1t_0t_2t_1t_0t_1t_2t_3$ go to $Nt_0t_1t_0t_2t_3t_2t_3t_2$.

82. [010210310] Same as 35.

83. [010210312] Same as 40.

84. Now, we consider coset $N^{(010210313)}$.
However, $t_0t_1t_0t_2t_1t_0t_3t_1t_3 = t_0t_1t_0t_2t_1t_0t_3t_1$.
Therefore, t_3 takes $Nt_0t_1t_0t_2t_1t_0t_3t_1t_3$ go to $Nt_0t_1t_0t_2t_1t_0t_3t_1$.

85. [010212301] Same as 78.

86. Now, we consider the cosets stabilizer $N^{(010212303)}$.

$N^{(010212303)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_1t_2t_3t_0t_3N$ is at most: $\frac{|N|}{|N^{(010212303)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010212303)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_2t_3t_0t_3t_0$, $Nt_0t_1t_0t_2t_1t_2t_3t_0t_3t_1$, $Nt_0t_1t_0t_2t_1t_2t_3t_0t_3t_2$ and $Nt_0t_1t_0t_2t_1t_2t_3t_0t_3t_3$ belong?

We know, $t_0t_1t_0t_2t_1t_2t_3t_0t_3 = (t_0t_1t_2t_0t_2t_0t_3t_0t_2)^{(3,2,0)}$.

$Nt_0t_1t_0t_2t_1t_2t_3t_0t_3 = Nt_3t_1t_0t_3t_0t_3t_2t_3t_0$.

$Nt_0t_1t_0t_2t_1t_2t_3t_0t_3t_0 = Nt_3t_1t_0t_3t_0t_3t_2t_3 \in [01202030]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_2t_0t_3t_0N$.

Moreover, $Nt_0t_1t_0t_2t_1t_2t_3t_0t_3t_3 = Nt_0t_1t_0t_2t_1t_2t_3t_0 \in [01021230]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_1t_2t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_2t_3t_0t_3t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_2t_3t_0t_3t_1$.

87. Now, we consider the cosets stabilizer $N^{(010212313)}$.

We know, $t_0t_1t_0t_2t_1t_2t_3t_1t_3 = t_0t_1t_0t_3t_1t_3t_2t_1t_2$
$= t_2t_1t_2t_0t_1t_0t_3t_1t_3 = t_3t_1t_3t_2t_1t_2t_0t_1t_0 = t_2t_1t_2t_3t_1t_3t_0t_1t_0 = t_3t_1t_3t_0t_1t_0t_2t_1t_2$ then $N^{(010212313)} = \{e,(2,3),(0,3),(0,2),(0,3,2),(0,2,3)\}$, then the number of the single cosets in the double coset
$Nt_0t_1t_0t_2t_1t_2t_3t_1t_3N$ is at most: $\frac{|N|}{|N^{(010212313)}|} = \frac{24}{6} = 4$

The orbit of $N^{(010212313)}$ on $\{0,1,2,3\}$ are $\{1\}$, and $\{0,2,3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_2t_3t_1t_3t_3$, $Nt_0t_1t_0t_2t_1t_2t_3t_1t_3t_1$, and belong?

However, $Nt_0t_1t_0t_2t_1t_2t_3t_1t_3t_3 = Nt_0t_1t_0t_2t_1t_2t_3t_1 \in [01021231]$, then three symmetric generators go back to the double coset $Nt_0t_1t_0t_2t_1t_2t_3t_1N$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_2t_3t_1t_3t_1$.

88. Now, we consider the cosets stabilizer $N^{(010213010)}$.

$N^{(010213010)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_1t_3t_0t_1t_0N$ is at most: $\frac{|N|}{|N^{(010213010)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010213010)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_3t_0t_1t_0t_0$,

$Nt_0t_1t_0t_2t_1t_3t_0t_1t_0t_1$, $Nt_0t_1t_0t_2t_1t_3t_0t_1t_0t_2$ and $Nt_0t_1t_0t_2t_1t_3t_0t_1t_0t_3$ belong?
We know, $t_0t_1t_0t_2t_1t_3t_0t_1t_0 = (0,1)(2,3)(t_0t_1t_2t_3t_2t_0t_2t_1t_3)^{(1,3)}$.
$Nt_0t_1t_0t_2t_1t_3t_0t_1t_0 = Nt_0t_3t_2t_1t_2t_0t_2t_3t_1$.
$Nt_0t_1t_0t_2t_1t_3t_0t_1t_0t_0 = Nt_0t_3t_2t_1t_2t_0t_2t_3 \in [01232021]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_2t_0t_2t_1N$.
Moreover, $Nt_0t_1t_0t_2t_1t_3t_0t_1t_0t_3 = Nt_0t_1t_0t_2t_1t_3t_0t_1 \in [01021301]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_1t_3t_0t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_0t_1t_0t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_0t_1t_0t_1$.

89. Now, we consider coset $N^{(010213012)}$.
However, $t_0t_1t_0t_2t_1t_3t_0t_1t_2 = (0,2)(1,3)(t_3t_2t_0t_2t_0t_1t_0t_3) = (0,2)(1,3)(t_0t_1t_2t_1t_2t_3t_2t_0)^{(1,2,0,3)}$ $\in Nt_0t_1t_2t_1t_2t_3t_2t_0$. Therefore, t_2 takes $Nt_0t_1t_0t_2t_1t_3t_0t_1t_2$ go to $Nt_0t_1t_2t_1t_2t_3t_2t_0$.

90. Now, we consider coset $N^{(010213013)}$.
However, $t_0t_1t_0t_2t_1t_3t_0t_1t_3 = (1,0,2)(t_1t_2t_1t_2t_0t_3t_2t_1) = (1,0,2)(t_0t_1t_0t_1t_2t_3t_1t_0)^{(1,2,0)}$ $\in Nt_0t_1t_0t_1t_2t_3t_1t_0$. Therefore, t_3 takes $Nt_0t_1t_0t_2t_1t_3t_0t_1t_3$ go to $Nt_0t_1t_0t_1t_2t_3t_1t_0$.

91. Now, we consider the cosets stabilizer $N^{(010213101)}$.
$N^{(010213101)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_1t_3t_1t_0t_1N$ is at most: $\frac{|N|}{|N^{(010213101)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010213101)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_3t_1t_0t_1t_0$, $Nt_0t_1t_0t_2t_1t_3t_1t_0t_1t_1$, $Nt_0t_1t_0t_2t_1t_3t_1t_0t_1t_2$ and $Nt_0t_1t_0t_2t_1t_3t_1t_0t_1t_3$ belong?
However, $Nt_0t_1t_0t_2t_1t_3t_1t_0t_1t_1 = Nt_0t_1t_0t_2t_1t_3t_1t_0 \in [01021310]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_1t_3t_1t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_1t_0t_1t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_1t_0t_1t_3$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_1t_0t_1t_2$.

92. Now, we consider coset $N^{(010213120)}$.
However, $t_0t_1t_0t_2t_1t_3t_1t_2t_0 = (1,3,2)(t_0t_1t_0t_3t_1t_2t_1t_3)$
$= (1,3,2)(t_0t_1t_0t_2t_1t_3t_1t_2)^{(2,3)} \in Nt_0t_1t_0t_2t_1t_3t_1t_2$. Therefore, t_0 takes $Nt_0t_1t_0t_2t_1t_3t_1t_2t_0$ go to $Nt_0t_1t_0t_2t_1t_3t_1t_2$.

93. Now, we consider the cosets stabilizer $N^{(010213121)}$.

$N^{(010213121)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_1t_3t_1t_2t_1N$ is at most: $\frac{|N|}{|N^{(010213121)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010213121)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_3t_1t_2t_1t_0$, $Nt_0t_1t_0t_2t_1t_3t_1t_2t_1t_1$, $Nt_0t_1t_0t_2t_1t_3t_1t_2t_1t_2$ and $Nt_0t_1t_0t_2t_1t_3t_1t_2t_1t_3$ belong?

We know, $t_0t_1t_0t_2t_1t_3t_1t_2t_1 = (1,2,3)(t_0t_1t_2t_1t_3t_1t_2t_0t_1)^{(1,3,2)}$.

$Nt_0t_1t_0t_2t_1t_3t_1t_2t_1 = Nt_0t_3t_1t_3t_2t_3t_1t_0t_3$.

$Nt_0t_1t_0t_2t_1t_3t_1t_2t_1t_3 = Nt_0t_3t_1t_3t_2t_3t_1t_0 \in [01213120]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_3t_1t_2t_0N$.

Moreover, $Nt_0t_1t_0t_2t_1t_3t_1t_2t_1t_1 = Nt_0t_1t_0t_2t_1t_3t_1t_2 \in [01021312]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_1t_3t_1t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_1t_2t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_1t_2t_1t_0$.

94. Now, we consider the cosets stabilizer $N^{(010213123)}$.

$N^{(010213123)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_1t_3t_1t_2t_3N$ is at most: $\frac{|N|}{|N^{(010213123)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010213123)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_3t_1t_2t_3t_0$, $Nt_0t_1t_0t_2t_1t_3t_1t_2t_3t_1$, $Nt_0t_1t_0t_2t_1t_3t_1t_2t_3t_2$ and $Nt_0t_1t_0t_2t_1t_3t_1t_2t_3t_3$ belong?

We know, $t_0t_1t_0t_2t_1t_3t_1t_2t_3 = (t_0t_1t_0t_2t_3t_1t_2t_1t_3)^{(3,2)}$.

$Nt_0t_1t_0t_2t_1t_3t_1t_2t_3 = Nt_0t_1t_0t_3t_2t_1t_3t_1t_2$.

$Nt_0t_1t_0t_2t_1t_3t_1t_2t_3t_2 = Nt_0t_1t_0t_3t_2t_1t_3t_1 \in [01023121]$, then one symmetric generator goes back to $Nt_0t_1t_0t_2t_3t_1t_2t_1N$.

Moreover, $Nt_0t_1t_0t_2t_1t_3t_1t_2t_3t_3 = Nt_0t_1t_0t_2t_1t_3t_1t_2 \in [01021312]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_1t_3t_1t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_1t_2t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_1t_2t_3t_0$.

95. Now, we consider coset $N^{(010213130)}$.

However, $t_0t_1t_0t_2t_1t_3t_1t_3t_0 = (t_1t_0t_3t_0t_2t_0t_2t_1) = (t_0t_1t_2t_1t_3t_1t_3t_0)^{(1,0)(2,3)}$ $\in Nt_0t_1t_2t_1t_3t_1t_3t_0$. Therefore, t_0 takes $Nt_0t_1t_0t_2t_1t_3t_1t_3t_0$ go to $Nt_0t_1t_2t_1t_3t_1t_3t_0$.

96. Now, we consider coset $N^{(010213131)}$.
However, $t_0t_1t_0t_2t_1t_3t_1t_3t_1 = t_0t_1t_0t_2t_1t_3t_1t_3$.
Therefore, t_1 takes $Nt_0t_1t_0t_2t_1t_3t_1t_3t_1$ go to itself $Nt_0t_1t_0t_2t_1t_3t_1t_3$.

97. Now, we consider coset $N^{(010213132)}$.
However, $t_0t_1t_0t_2t_1t_3t_1t_3t_2 = (1,3,0)(t_2t_3t_0t_2t_3t_1t_3t_0) = (1,3,0)(t_0t_1t_2t_0t_1t_3t_0t_2)^{(1,3)(2,0)}$ $\in Nt_0t_1t_2t_0t_1t_3t_0t_2$. Therefore, t_2 takes $Nt_0t_1t_0t_2t_1t_3t_1t_3t_2$ go to $Nt_0t_1t_2t_0t_1t_3t_0t_2$.

98. Now, we consider coset $N^{(010213210)}$.
However, $t_0t_1t_0t_2t_1t_3t_2t_1t_0 = (1,0,3)(t_2t_0t_1t_0t_1t_2t_3t_2) = (1,0,3)(t_0t_1t_2t_1t_2t_0t_3t_0)^{(1,0,2)}$ $\in Nt_0t_1t_2t_1t_2t_0t_3t_0$. Therefore, t_0 takes $Nt_0t_1t_0t_2t_1t_3t_2t_1t_0$ go to $Nt_0t_1t_2t_1t_2t_0t_3t_0$.

99. Now, we consider the cosets stabilizer $N^{(010213212)}$.
$N^{(010213212)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_1t_3t_2t_1t_2N$ is at most: $\frac{|N|}{|N^{(010213212)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010213212)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_3t_2t_1t_2t_0$, $Nt_0t_1t_0t_2t_1t_3t_2t_1t_2t_1$, $Nt_0t_1t_0t_2t_1t_3t_2t_1t_2t_2$ and $Nt_0t_1t_0t_2t_1t_3t_2t_1t_2t_3$ belong?
We know, $t_0t_1t_0t_2t_1t_3t_2t_1t_2 = (0,1)(2,3)(t_0t_1t_2t_0t_2t_1t_2t_3t_0)^{(1,2)}$.
$Nt_0t_1t_0t_2t_1t_3t_2t_1t_2 = Nt_0t_2t_1t_0t_1t_2t_1t_3t_0$.
$Nt_0t_1t_0t_2t_1t_3t_2t_1t_2t_0 = Nt_0t_2t_1t_0t_1t_2t_1t_3 \in [01202123]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_2t_1t_2t_3N$.

Moreover, $Nt_0t_1t_0t_2t_1t_3t_2t_1t_2t_2 = Nt_0t_1t_0t_2t_1t_3t_2t_1 \in [01021321]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_1t_3t_2t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_2t_1t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_1t_3t_2t_1t_2t_3$.

100. Now, we consider coset $N^{(010213213)}$.
However, $t_0t_1t_0t_2t_1t_3t_2t_1t_3 = t_0t_1t_0t_2t_1t_3t_2t_1$.
Therefore, t_3 takes $Nt_0t_1t_0t_2t_1t_3t_2t_1t_3$ go to itself $Nt_0t_1t_0t_2t_1t_3t_2t_1$.

101. Now, we consider coset $N^{(010230130)}$.
However, $t_0t_1t_0t_2t_3t_0t_1t_3t_0 = (1,2,3)(t_1t_3t_1t_3t_0t_2t_3t_2) = (1,2,3)(t_0t_1t_0t_1t_2t_3t_1t_3)^{(1,3,2,0)}$ $\in Nt_0t_1t_0t_1t_2t_3t_1t_3$. Therefore, t_0 takes $Nt_0t_1t_0t_2t_3t_0t_1t_3t_0$ go to $Nt_0t_1t_0t_1t_2t_3t_1t_3$.

102. Now, we consider coset $N^{(010230131)}$.
However, $t_0t_1t_0t_2t_3t_0t_1t_3t_1 = (1,0,2)(t_1t_0t_2t_1t_2t_0t_3t_2) = (1,0,2)(t_0t_1t_2t_0t_2t_1t_3t_2)^{(1,0)}$ $\in Nt_0t_1t_2t_0t_2t_1t_3t_2$. Therefore, t_1 takes $Nt_0t_1t_0t_2t_3t_0t_1t_3t_1$ go to $Nt_0t_1t_2t_0t_2t_1t_3t_2$.

103. Now, we consider the cosets stabilizer $N^{(010230132)}$.
$N^{(010230132)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_0t_1t_3t_2N$ is at most: $\frac{|N|}{|N^{(010230132)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010230132)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_0t_1t_3t_2t_0$, $Nt_0t_1t_0t_2t_3t_0t_1t_3t_2t_1$, $Nt_0t_1t_0t_2t_3t_0t_1t_3t_2t_2$ and $Nt_0t_1t_0t_2t_3t_0t_1t_3t_2t_3$ belong?
We know, $t_0t_1t_0t_2t_3t_0t_1t_3t_2 = (1,2)(0,3)(t_0t_1t_2t_0t_3t_2t_1t_0t_2)^{(1,2,0)}$.
$Nt_0t_1t_0t_2t_3t_0t_1t_3t_2 = Nt_1t_2t_0t_1t_3t_0t_2t_1t_0$.
$Nt_0t_1t_0t_2t_3t_0t_1t_3t_2t_0 = Nt_1t_2t_0t_1t_3t_0t_2t_1 \in [01203210]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_3t_2t_1t_0N$.
We know, $t_0t_1t_0t_2t_3t_0t_1t_3t_2 = (t_0t_1t_2t_0t_3t_0t_3t_1t_2)^{(1,2)}$.
$Nt_0t_1t_0t_2t_3t_0t_1t_3t_2 = Nt_0t_2t_3t_0t_2t_1t_0t_3t_2$.
$Nt_0t_1t_0t_2t_3t_0t_1t_3t_2t_1 = Nt_0t_2t_3t_0t_2t_1t_0t_3 \in [01203031]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_3t_0t_3t_1N$.
Moreover, $Nt_0t_1t_0t_2t_3t_0t_1t_3t_2t_2 = Nt_0t_1t_0t_2t_3t_0t_1t_3 \in [01023013]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_0t_1t_3N$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_1t_3t_2t_3$.

104. Now, we consider the cosets stabilizer $N^{(010230301)}$.
$N^{(010230301)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_0t_3t_0t_1N$ is at most: $\frac{|N|}{|N^{(010230301)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010230301)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_0t_3t_0t_1t_0$, $Nt_0t_1t_0t_2t_3t_0t_3t_0t_1t_1$, $Nt_0t_1t_0t_2t_3t_0t_3t_0t_1t_2$ and $Nt_0t_1t_0t_2t_3t_0t_3t_0t_1t_3$ belong?

We know, $t_0t_1t_0t_2t_3t_0t_3t_0t_1 = (t_0t_1t_2t_1t_0t_3t_0t_3t_1)^{(1,3,2,0)}$.
$Nt_0t_1t_0t_2t_3t_0t_3t_0t_1 = Nt_1t_3t_0t_3t_1t_2t_1t_2t_3$.
$Nt_0t_1t_0t_2t_3t_0t_3t_0t_1t_3 = Nt_1t_3t_0t_3t_1t_2t_1t_2 \in [01210303]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_0t_3t_0t_3N$.

Moreover, $Nt_0t_1t_0t_2t_3t_0t_3t_0t_1t_1 = Nt_0t_1t_0t_2t_3t_0t_3t_0 \in [01023030]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_0t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_3t_0t_1t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_3t_0t_1t_2$.

105. Now, we consider coset $N^{(010230302)}$.
However, $t_0t_1t_0t_2t_3t_0t_3t_0t_2 = t_0t_1t_2t_0t_2t_3t_2t_3$.
Therefore, t_2 takes $Nt_0t_1t_0t_2t_3t_0t_3t_0t_2$ go to $Nt_0t_1t_2t_0t_2t_3t_2t_3$.

106. Now, we consider coset $N^{(010230303)}$.
However, $t_0t_1t_0t_2t_3t_0t_3t_0t_3 = t_0t_1t_2t_0t_3t_2t_3t_2$.
Therefore, t_2 takes $Nt_0t_1t_0t_2t_3t_0t_3t_0t_3$ go to $Nt_0t_1t_2t_0t_3t_2t_3t_2$.

107. Now, we consider the cosets stabilizer $N^{(010230312)}$.
$N^{(010230312)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_0t_3t_1t_2N$ is at most: $\frac{|N|}{|N^{(010230312)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010230312)}$ on $\{0, 1, 2, 3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_0t_3t_1t_2t_0$, $Nt_0t_1t_0t_2t_3t_0t_3t_1t_2t_1$, $Nt_0t_1t_0t_2t_3t_0t_3t_1t_2t_2$ and $Nt_0t_1t_0t_2t_3t_0t_3t_1t_2t_3$ belong?
We know, $t_0t_1t_0t_2t_3t_0t_3t_1t_2 = (2, 0, 3)(t_0t_1t_2t_0t_1t_3t_0t_1t_0)^{(3,0)}$.
$Nt_0t_1t_0t_2t_3t_0t_3t_1t_2 = Nt_3t_1t_2t_3t_1t_0t_3t_1t_3$.
$Nt_0t_1t_0t_2t_3t_0t_3t_1t_2t_3 = Nt_3t_1t_2t_3t_1t_0t_3t_1 \in [01201301]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_1t_3t_0t_1N$.
Moreover, $Nt_0t_1t_0t_2t_3t_0t_3t_1t_2t_2 = Nt_0t_1t_0t_2t_3t_0t_3t_1 \in [01023031]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_0t_3t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_3t_1t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_3t_1t_2t_1$.

108. Now, we consider the cosets stabilizer $N^{(010230313)}$.
$N^{(010230313)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_0t_3t_1t_3N$ is at most: $\frac{|N|}{|N^{(010230313)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010230313)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_0t_3t_1t_3t_0$, $Nt_0t_1t_0t_2t_3t_0t_3t_1t_3t_1$, $Nt_0t_1t_0t_2t_3t_0t_3t_1t_3t_2$ and $Nt_0t_1t_0t_2t_3t_0t_3t_1t_3t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_0t_3t_1t_3t_3 = Nt_0t_1t_0t_2t_3t_0t_3t_1 \in [01023031]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_0t_3t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_3t_1t_3t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_3t_1t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_3t_1t_3t_2$.

109. Now, we consider the cosets stabilizer $N^{(010230321)}$.
$N^{(010230321)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_0t_3t_2t_1N$ is at most: $\frac{|N|}{|N^{(010230321)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010230321)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_0t_3t_2t_1t_0$, $Nt_0t_1t_0t_2t_3t_0t_3t_2t_1t_1$, $Nt_0t_1t_0t_2t_3t_0t_3t_2t_1t_2$ and $Nt_0t_1t_0t_2t_3t_0t_3t_2t_1t_3$ belong?
We know, $t_0t_1t_0t_2t_3t_0t_3t_2t_1 = (1,0)(2,3)(t_0t_1t_2t_3t_1t_3t_2t_0t_1)^{(1,3)(2,0)}$.
$Nt_0t_1t_0t_2t_3t_0t_3t_2t_1 = Nt_2t_3t_0t_1t_3t_1t_0t_2t_3$.
$Nt_0t_1t_0t_2t_3t_0t_3t_2t_1t_3 = Nt_2t_3t_0t_1t_3t_1t_0t_2 \in [01231320]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_1t_3t_2t_0N$.
We know, $t_0t_1t_0t_2t_3t_0t_3t_2t_1 = (t_0t_1t_2t_0t_3t_0t_2t_3t_1)^{(1,2)}$.
$Nt_0t_1t_0t_2t_3t_0t_3t_2t_1 = Nt_0t_2t_1t_0t_3t_0t_1t_3t_2$.
$Nt_0t_1t_0t_2t_3t_0t_3t_2t_1t_2 = Nt_0t_2t_1t_0t_3t_0t_1t_3 \in [01203023]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_3t_0t_2t_3N$.

Moreover, $Nt_0t_1t_0t_2t_3t_0t_3t_2t_1t_1 = Nt_0t_1t_0t_2t_3t_0t_3t_2 \in [01023032]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_0t_3t_2N$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_3t_2t_1t_1$.

110. Now, we consider the cosets stabilizer $N^{(010230323)}$.
$N^{(010230323)} = \{e\}$, then the number of the single cosets in the double coset

$Nt_0t_1t_0t_2t_3t_0t_3t_2t_3N$ is at most: $\frac{|N|}{|N^{(010230323)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010230323)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_0t_3t_2t_3t_0$, $Nt_0t_1t_0t_2t_3t_0t_3t_2t_3t_1$, $Nt_0t_1t_0t_2t_3t_0t_3t_2t_3t_2$ and $Nt_0t_1t_0t_2t_3t_0t_3t_2t_3t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_0t_3t_2t_3t_3 = Nt_0t_1t_0t_2t_3t_0t_3t_2 \in [01023032]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_0t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_3t_2t_3t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_3t_2t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_0t_3t_2t_3t_2$.

111. Now, we consider the cosets stabilizer $N^{(010231201)}$.
$N^{(010231201)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_1t_2t_0t_1N$ is at most: $\frac{|N|}{|N^{(010231201)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010231201)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_1t_2t_0t_1t_0$, $Nt_0t_1t_0t_2t_3t_1t_2t_0t_1t_1$, $Nt_0t_1t_0t_2t_3t_1t_2t_0t_1t_2$ and $Nt_0t_1t_0t_2t_3t_1t_2t_0t_1t_3$ belong?
We know, $t_0t_1t_0t_2t_3t_1t_2t_0t_1 = (1,2)(0,3)(t_0t_1t_2t_3t_1t_3t_0t_3t_1)^{(1,2,3)}$.
$Nt_0t_1t_0t_2t_3t_1t_2t_0t_1 = Nt_0t_2t_3t_1t_2t_1t_0t_1t_2$.
$Nt_0t_1t_0t_2t_3t_1t_2t_0t_1t_2 = Nt_0t_2t_3t_1t_2t_1t_0t_1 \in [01231303]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_1t_3t_0t_3N$.
We know, $t_0t_1t_0t_2t_3t_1t_2t_0t_1 = (t_0t_1t_2t_3t_2t_1t_2t_0t_1)^{(1,0,2)}$.
$Nt_0t_1t_0t_2t_3t_1t_2t_0t_1 = Nt_2t_0t_1t_3t_1t_0t_1t_2t_0$.
$Nt_0t_1t_0t_2t_3t_1t_2t_0t_1t_0 = Nt_2t_0t_1t_3t_1t_0t_1t_2 \in [01232120]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_2t_1t_2t_0N$.

Moreover, $Nt_0t_1t_0t_2t_3t_1t_2t_0t_1t_1 = Nt_0t_1t_0t_2t_3t_1t_2t_0 \in [01023120]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_1t_2t_0N$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_2t_0t_1t_3$.

112. Now, we consider the cosets stabilizer $N^{(010231202)}$.
$N^{(010231202)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_1t_2t_0t_2N$ is at most: $\frac{|N|}{|N^{(010231202)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010231202)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a

representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_1t_2t_0t_2t_0$, $Nt_0t_1t_0t_2t_3t_1t_2t_0t_2t_1$, $Nt_0t_1t_0t_2t_3t_1t_2t_0t_2t_2$ and $Nt_0t_1t_0t_2t_3t_1t_2t_0t_2t_3$ belong?
We know, $t_0t_1t_0t_2t_3t_1t_2t_0t_2 = (1,2)(0,3)(t_0t_1t_2t_0t_1t_3t_1t_3t_1)^{(1,0,2,3)}$.
$Nt_0t_1t_0t_2t_3t_1t_2t_0t_2 = Nt_3t_0t_1t_3t_0t_2t_0t_2t_0$.
$Nt_0t_1t_0t_2t_3t_1t_2t_0t_2t_0 = Nt_3t_0t_1t_3t_0t_2t_0t_2 \in [01201313]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_1t_3t_1t_3N$.
Moreover, $Nt_0t_1t_0t_2t_3t_1t_2t_0t_2t_2 = Nt_0t_1t_0t_2t_3t_1t_2t_0 \in [01023120]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_1t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_2t_0t_2t_3$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_2t_0t_2t_1$.

113. [010231203] Same as 54.

114. Now, we consider the cosets stabilizer $N^{(010231212)}$.
$N^{(010231212)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_1t_2t_1t_2N$ is at most: $\frac{|N|}{|N^{(010231212)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010231212)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_1t_2t_1t_2t_0$, $Nt_0t_1t_0t_2t_3t_1t_2t_1t_2t_1$, $Nt_0t_1t_0t_2t_3t_1t_2t_1t_2t_2$ and $Nt_0t_1t_0t_2t_3t_1t_2t_1t_2t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_1t_2t_1t_2t_2 = Nt_0t_1t_0t_2t_3t_1t_2t_1 \in [01023121]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_1t_2t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_2t_1t_2t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_2t_1t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_2t_1t_2t_3$.

115. [010231213] Same as 94.

116. Now, we consider the cosets stabilizer $N^{(010231230)}$.
$N^{(010231230)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_1t_2t_3t_0N$ is at most: $\frac{|N|}{|N^{(010231230)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010231230)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_1t_2t_3t_0t_0$, $Nt_0t_1t_0t_2t_3t_1t_2t_3t_0t_1$, $Nt_0t_1t_0t_2t_3t_1t_2t_3t_0t_2$ and $Nt_0t_1t_0t_2t_3t_1t_2t_3t_0t_3$ belong?

We know, $t_0t_1t_0t_2t_3t_1t_2t_3t_0 = (1,2)(0,3)(t_0t_1t_2t_1t_0t_3t_1t_3t_0)^{(1,0,2,3)}$.
$Nt_0t_1t_0t_2t_3t_1t_2t_3t_0 = Nt_3t_0t_1t_0t_3t_2t_0t_2t_3$.
$Nt_0t_1t_0t_2t_3t_1t_2t_3t_0t_3 = Nt_3t_0t_1t_0t_3t_2t_0t_2 \in [01210313]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_0t_3t_1t_3N$.
Moreover, $Nt_0t_1t_0t_2t_3t_1t_2t_3t_0t_0 = Nt_0t_1t_0t_2t_3t_1t_2t_3 \in [01023120]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_1t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_2t_3t_0t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_2t_3t_0t_1$.

117. Now, we consider coset $N^{(010231231)}$.
However, $t_0t_1t_0t_2t_3t_1t_2t_3t_1 = (1,3,2)(t_0t_3t_0t_3t_2t_1t_3t_2)$
$= (1,3,2)(t_0t_1t_0t_1t_2t_3t_1t_2)^{(1,3)} \in Nt_0t_1t_0t_1t_2t_3t_1t_2$. Therefore, t_1 takes $Nt_0t_1t_0t_2t_3t_1t_2t_3t_1$ go to $Nt_0t_1t_0t_1t_2t_3t_1t_2$.

118. Now, we consider the cosets stabilizer $N^{(010231301)}$.
$N^{(010231301)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_1t_3t_0t_1N$ is at most: $\frac{|N|}{|N^{(010231301)}|} = \frac{24}{1} = 24$.
The orbit of $N^{(010231301)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_1t_3t_0t_1t_0$, $Nt_0t_1t_0t_2t_3t_1t_3t_0t_1t_1$, $Nt_0t_1t_0t_2t_3t_1t_3t_0t_1t_2$ and $Nt_0t_1t_0t_2t_3t_1t_3t_0t_1t_3$ belong?
We know, $t_0t_1t_0t_2t_3t_1t_3t_0t_1 = (1,0,3)(t_0t_1t_2t_3t_1t_3t_2t_3t_0)^{(1,2,0,3)}$.
$Nt_0t_1t_0t_2t_3t_1t_3t_0t_1 = Nt_3t_2t_0t_1t_2t_1t_0t_1t_3$.
$Nt_0t_1t_0t_2t_3t_1t_3t_0t_1t_3 = Nt_3t_2t_0t_1t_2t_1t_0t_1 \in [01231323]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_1t_3t_2t_3N$.
Moreover, $Nt_0t_1t_0t_2t_3t_1t_3t_0t_1t_1 = Nt_0t_1t_0t_2t_3t_1t_3t_0 \in [01023130]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_1t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_3t_0t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_3t_0t_1t_0$.

119. Now, we consider coset $N^{(010231302)}$.
However, $t_0t_1t_0t_2t_3t_1t_3t_0t_2 = (1,0,2)(t_1t_2t_0t_1t_3t_0t_2t_0)$
$= (1,0,2)(t_0t_1t_2t_0t_3t_2t_1t_2)^{(1,2,0)} \in Nt_0t_1t_2t_0t_3t_2t_1t_2$. Therefore, t_2 takes $Nt_0t_1t_0t_2t_3t_1t_3t_0t_2$ go to $Nt_0t_1t_2t_0t_3t_2t_1t_2$.

120. Now, we consider coset $N^{(010231303)}$.
However, $t_0t_1t_0t_2t_3t_1t_3t_0t_3 = (1,0)(3,2)(t_0t_3t_2t_1t_2t_0t_2t_0) = (1,0)(3,2)(t_0t_1t_2t_3t_2t_0t_2t_0)^{(1,3)}$ $\in Nt_0t_1t_2t_3t_2t_0t_2t_0$. Therefore, t_3 takes $Nt_0t_1t_0t_2t_3t_1t_3t_0t_3$ go to $Nt_0t_1t_2t_3t_2t_0t_2t_0$.

121. Now, we consider the cosets stabilizer $N^{(010231310)}$.
$N^{(010231310)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_1t_3t_1t_0N$ is at most: $\frac{|N|}{|N^{(010231310)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010231310)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_1t_3t_1t_0t_0$, $Nt_0t_1t_0t_2t_3t_1t_3t_1t_0t_1$, $Nt_0t_1t_0t_2t_3t_1t_3t_1t_0t_2$ and $Nt_0t_1t_0t_2t_3t_1t_3t_1t_0t_3$ belong?
We know, $t_0t_1t_0t_2t_3t_1t_3t_1t_0 = (1,3,2,0)(t_0t_1t_2t_3t_1t_0t_3t_2t_3)^{(1,3,2)}$.
$Nt_0t_1t_0t_2t_3t_1t_3t_1t_0 = Nt_0t_3t_1t_2t_3t_0t_2t_1t_2$.
$Nt_0t_1t_0t_2t_3t_1t_3t_1t_0t_2 = Nt_0t_3t_1t_2t_3t_0t_2t_1 \in [01231032]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_1t_0t_3t_2N$.
Moreover, $Nt_0t_1t_0t_2t_3t_1t_3t_1t_0t_0 = Nt_0t_1t_0t_2t_3t_1t_3t_1 \in [01023130]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_1t_3t_1N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_3t_1t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_3t_1t_0t_3$.

122. Now, we consider coset $N^{(010231312)}$.
However, $t_0t_1t_0t_2t_3t_1t_3t_1t_2 = (1,3)(0,2)(t_1t_2t_1t_0t_1t_0t_3t_0) = (1,3)(0,2)(t_0t_1t_0t_2t_0t_2t_3t_2)^{(1,2,0)}$ $\in Nt_0t_1t_0t_2t_0t_2t_3t_2$. Therefore, t_2 takes $Nt_0t_1t_0t_2t_3t_1t_3t_1t_2$ go to $Nt_0t_1t_0t_2t_0t_2t_3t_2$.

123. Now, we consider the cosets stabilizer $N^{(010231313)}$.
$N^{(010231313)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_1t_3t_1t_3N$ is at most: $\frac{|N|}{|N^{(010231313)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010231313)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_1t_3t_1t_3t_0$, $Nt_0t_1t_0t_2t_3t_1t_3t_1t_3t_1$, $Nt_0t_1t_0t_2t_3t_1t_3t_1t_3t_2$ and $Nt_0t_1t_0t_2t_3t_1t_3t_1t_3t_3$ belong?
We know, $t_0t_1t_0t_2t_3t_1t_3t_1t_3 = (t_0t_1t_2t_0t_3t_0t_3t_0t_1)^{(1,0)(2,3)}$.
$Nt_0t_1t_0t_2t_3t_1t_3t_1t_3 = Nt_1t_0t_3t_1t_2t_1t_2t_1t_0$.
$Nt_0t_1t_0t_2t_3t_1t_3t_1t_3t_0 = Nt_1t_0t_3t_1t_2t_1t_2t_1 \in [01203030]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_3t_0t_3t_0N$.

We know, $t_0t_1t_0t_2t_3t_1t_3t_1t_3 = (t_0t_1t_0t_2t_1t_3t_1t_3t_1)$.

$Nt_0t_1t_0t_2t_3t_1t_3t_1t_3 = Nt_0t_1t_0t_2t_1t_3t_1t_3t_1$.

$Nt_0t_1t_0t_2t_3t_1t_3t_1t_3t_1 = Nt_0t_1t_0t_2t_1t_3t_1t_3 \in [01021313]$, then one symmetric generator goes back to $Nt_0t_1t_0t_2t_1t_3t_1t_3N$.

Moreover, $Nt_0t_1t_0t_2t_3t_1t_3t_1t_3t_3 = Nt_0t_1t_0t_2t_3t_1t_3t_1 \in [01023131]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_1t_3t_1N$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_1t_3t_1t_3t_2$.

124. Now, we consider coset $N^{(010232032)}$.

However, $t_0t_1t_0t_2t_3t_2t_0t_3t_2 = (0,3,2)(t_1t_2t_1t_0t_1t_0t_3t_0) = (0,3,2)(t_0t_1t_0t_1t_2t_3t_2t_0)^{(1,2)(3,0)}$
$\in Nt_0t_1t_0t_1t_2t_3t_2t_0$. Therefore, t_2 takes $Nt_0t_1t_0t_2t_3t_2t_0t_3t_2$ go to $Nt_0t_1t_0t_1t_2t_3t_2t_0$.

125. Now, we consider coset $N^{(010232120)}$.

However, $t_0t_1t_0t_2t_3t_2t_1t_2t_0 = (0,3,2)(t_2t_1t_3t_2t_0t_2t_0t_1) = (0,3,2)(t_0t_1t_2t_0t_3t_0t_3t_1)^{(2,3,0)}$
$\in Nt_0t_1t_2t_0t_3t_0t_3t_1$. Therefore, t_0 takes $Nt_0t_1t_0t_2t_3t_2t_1t_2t_0$ go to $Nt_0t_1t_2t_0t_3t_0t_3t_1$.

126. Now, we consider the cosets stabilizer $N^{(010232121)}$.

$N^{(010232121)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_2t_1t_2t_1N$ is at most: $\frac{|N|}{|N^{(010232121)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010232121)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_2t_1t_2t_1t_0$, $Nt_0t_1t_0t_2t_3t_2t_1t_2t_1t_1$, $Nt_0t_1t_0t_2t_3t_2t_1t_2t_1t_2$ and $Nt_0t_1t_0t_2t_3t_2t_1t_2t_1t_3$ belong?

We know, $t_0t_1t_0t_2t_3t_2t_1t_2t_1 = (2,1,0)(t_0t_1t_2t_3t_1t_0t_1t_0t_2)^{(1,2,0)}$.

$Nt_0t_1t_0t_2t_3t_2t_1t_2t_1 = Nt_1t_2t_0t_3t_2t_1t_2t_1t_0$.

$Nt_0t_1t_0t_2t_3t_2t_1t_2t_1t_0 = Nt_1t_2t_0t_3t_2t_1t_2t_1 \in [01231010]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_1t_0t_1t_0N$.

Moreover, $Nt_0t_1t_0t_2t_3t_2t_1t_2t_1t_1 = Nt_0t_1t_0t_2t_3t_2t_1t_2 \in [01023212]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_2t_1t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_1t_2t_1t_2$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_1t_2t_1t_3$.

127. Now, we consider the cosets stabilizer $N^{(010232123)}$.

$N^{(010232123)} = \{e\}$, then the number of the single cosets in the double coset

$Nt_0t_1t_0t_2t_3t_2t_1t_2t_3N$ is at most: $\frac{|N|}{|N^{(010232123)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010232123)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_2t_1t_2t_3t_0$, $Nt_0t_1t_0t_2t_3t_2t_1t_2t_3t_1$, $Nt_0t_1t_0t_2t_3t_2t_1t_2t_3t_2$ and $Nt_0t_1t_0t_2t_3t_2t_1t_2t_3t_3$ belong?
However, $Nt_0t_1t_0t_2t_3t_2t_1t_2t_3t_3 = Nt_0t_1t_0t_2t_3t_2t_1t_2 \in [01023212]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_2t_1t_2N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_1t_2t_3t_0$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_1t_2t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_1t_2t_3t_2$.

128. Now, we consider coset $N^{(010232301)}$.
However, $t_0t_1t_0t_2t_3t_2t_3t_0t_1 = (1,2,3)(t_1t_3t_0t_2t_3t_0t_1t_3) = (1,2,3)(t_0t_1t_2t_3t_1t_2t_0t_1)^{(1,3,2,0)}$ $\in Nt_0t_1t_2t_3t_1t_2t_0t_1$. Therefore, t_1 takes $Nt_0t_1t_0t_2t_3t_2t_3t_0t_1$ go to $Nt_0t_1t_2t_3t_1t_2t_0t_1$.

129. Now, we consider coset $N^{(010232302)}$.
However, $t_0t_1t_0t_2t_3t_2t_3t_0t_2 = (t_0t_1t_3t_0t_3t_0t_2t_0)$
$= (t_0t_1t_2t_0t_2t_0t_3t_0)^{(3,2)} \in Nt_0t_1t_2t_0t_2t_0t_3t_0$.
Therefore, t_2 takes $Nt_0t_1t_0t_2t_3t_2t_3t_0t_2$ go to $Nt_0t_1t_2t_0t_2t_0t_3t_0$.

130. Now, we consider the cosets stabilizer $N^{(010232303)}$.
$N^{(010232303)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_2t_3t_0t_3N$ is at most: $\frac{|N|}{|N^{(010232303)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(010232303)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_2t_3t_0t_3t_0$, $Nt_0t_1t_0t_2t_3t_2t_3t_0t_3t_1$, $Nt_0t_1t_0t_2t_3t_2t_3t_0t_3t_2$ and $Nt_0t_1t_0t_2t_3t_2t_3t_0t_3t_3$ belong?
We know, $t_0t_1t_0t_2t_3t_2t_3t_0t_3 = (2,0,3)(t_0t_1t_2t_3t_2t_0t_2t_0t_3)^{(3,2,0)}$.
$Nt_0t_1t_0t_2t_3t_2t_3t_0t_3 = Nt_3t_1t_0t_2t_0t_3t_0t_3t_2$.
$Nt_0t_1t_0t_2t_3t_2t_3t_0t_3t_2 = Nt_3t_1t_0t_2t_0t_3t_0t_3 \in [01232020]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_2t_0t_2t_0N$.
Moreover, $Nt_0t_1t_0t_2t_3t_2t_3t_0t_3t_3 = Nt_0t_1t_0t_2t_3t_2t_3t_0 \in [01023230]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_2t_3t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_3t_0t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_3t_0t_3t_1$.

131. Now, we consider coset $N^{(010232313)}$.
However, $t_0t_1t_0t_2t_3t_2t_3t_1t_3 = t_0t_1t_0t_2t_3t_2t_3t_1$.
Therefore, t_3 takes $Nt_0t_1t_0t_2t_3t_2t_3t_1t_3$ go to itself $Nt_0t_1t_0t_2t_3t_2t_3t_1$.

132. Now, we consider coset $N^{(010232320)}$.
However, $t_0t_1t_0t_2t_3t_2t_3t_2t_0 = (1,3,2)(t_1t_2t_1t_3t_2t_1t_2t_3)$
$= (1,3,2)(t_0t_1t_0t_2t_1t_0t_1t_2)^{(1,2,3,0)} \in Nt_0t_1t_0t_2t_1t_0t_1t_2$.
Therefore, t_0 takes $Nt_0t_1t_0t_2t_3t_2t_3t_2t_0$ go to $Nt_0t_1t_0t_2t_1t_0t_1t_2$.

133. Now, we consider the cosets stabilizer $N^{(010232321)}$.
We know, $t_0t_1t_0t_2t_3t_2t_3t_2t_1 = t_0t_1t_0t_3t_2t_3t_2t_3t_1$ then
$N^{(010232321)} = \{e, (2,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_2t_3t_2t_1N$ is at most: $\frac{|N|}{|N^{(010232321)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(010232321)}$ on $\{0,1,2,3\}$ are $\{2,3\}$, $\{0\}$ and $\{1\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_2t_3t_2t_1t_0$,
$Nt_0t_1t_0t_2t_3t_2t_3t_2t_1t_1$
$Nt_0t_1t_0t_2t_3t_2t_3t_2t_1t_2$ and belong?
However, $Nt_0t_1t_0t_2t_3t_2t_3t_2t_1t_1 = Nt_0t_1t_0t_2t_3t_2t_3t_2 \in$ [01023232], then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_3t_2t_3t_2N$,
We know, $t_0t_1t_0t_2t_3t_2t_3t_2t_1 = (t_0t_1t_2t_1t_2t_0t_3t_0t_3)^{(1,2,0)}$.
$Nt_0t_1t_0t_2t_3t_2t_3t_2t_1 = Nt_1t_2t_0t_2t_1t_0t_3t_1t_3$.
$Nt_0t_1t_0t_2t_3t_2t_3t_2t_1t_3 = Nt_1t_2t_0t_2t_1t_0t_3t_1 \in$ [01212130], then two symmetric generators go back to $Nt_0t_1t_2t_1t_2t_0t_3t_0N$. and one symmetric generator goes to $Nt_0t_1t_0t_2t_3t_2t_3t_2t_1t_0$.

134. Now, we consider the cosets stabilizer $N^{(012010131)}$.
$N^{(012010131)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_1t_0t_1t_3t_1N$ is at most: $\frac{|N|}{|N^{(012010131)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012010131)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_1t_0t_1t_3t_1t_0$, $Nt_0t_1t_2t_0t_1t_0t_1t_3t_1t_1$, $Nt_0t_1t_2t_0t_1t_0t_1t_3t_1t_2$ and $Nt_0t_1t_2t_0t_1t_0t_1t_3t_1t_3$ belong?
We know, $t_0t_1t_2t_0t_1t_0t_1t_3t_1 = (2,3,0)(t_0t_1t_2t_0t_1t_3t_0t_3t_2)^{(2,3)}$.
$Nt_0t_1t_2t_0t_1t_0t_1t_3t_1 = Nt_0t_1t_3t_0t_1t_2t_0t_2t_3$.

$Nt_0t_1t_2t_0t_1t_0t_1t_3t_1t_3 = Nt_0t_1t_3t_0t_1t_2t_0t_2 \in [01201303]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_1t_3t_0t_3N$.

We know, $t_0t_1t_2t_0t_1t_0t_1t_3t_1 = (t_0t_1t_2t_0t_2t_3t_1t_3t_1)^{(1,0,2,3)}$.

$Nt_0t_1t_2t_0t_1t_0t_1t_3t_1 = Nt_2t_0t_3t_2t_3t_1t_0t_1t_0$.

$Nt_0t_1t_2t_0t_1t_0t_1t_3t_1t_0 = Nt_2t_0t_3t_2t_3t_1t_0t_1 \in [01202313]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_2t_3t_1t_3N$.

We know, $t_0t_1t_2t_0t_1t_0t_1t_3t_1 = (0,2)(1,3)(t_0t_1t_2t_3t_1t_0t_3t_1t_3)$.

$Nt_0t_1t_2t_0t_1t_0t_1t_3t_1 = Nt_0t_3t_1t_2t_3t_0t_2t_3t_2$.

$Nt_0t_1t_2t_0t_1t_0t_1t_3t_1t_2 = Nt_0t_3t_1t_2t_3t_0t_2t_3 \in [01231031]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_1t_0t_3t_1N$.

Moreover, $Nt_0t_1t_2t_0t_1t_0t_1t_3t_1t_1 = Nt_0t_1t_2t_0t_1t_0t_1t_3 \in [01201013]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_1t_0t_1t_3N$.

135. Now, we consider the cosets stabilizer $N^{(012010132)}$.

$N^{(012010132)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_1t_0t_1t_3t_2N$ is at most: $\frac{|N|}{|N^{(012010132)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012010132)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_1t_0t_1t_3t_2t_0$, $Nt_0t_1t_2t_0t_1t_0t_1t_3t_2t_1$, $Nt_0t_1t_2t_0t_1t_0t_1t_3t_2t_2$ and $Nt_0t_1t_2t_0t_1t_0t_1t_3t_2t_3$ belong?

We know, $t_0t_1t_2t_0t_1t_0t_1t_3t_2 = (0,1)(2,3)(t_0t_1t_2t_3t_0t_2t_1t_0t_2)^{(0,2,3,1)}$.

$Nt_0t_1t_2t_0t_1t_0t_1t_3t_2 = Nt_2t_0t_3t_1t_2t_3t_0t_2t_3$.

$Nt_0t_1t_2t_0t_1t_0t_1t_3t_2t_3 = Nt_2t_0t_3t_1t_2t_3t_0t_2 \in [01230210]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_0t_2t_1t_0N$.

Moreover, $Nt_0t_1t_2t_0t_1t_0t_1t_3t_2t_2 = Nt_0t_1t_2t_0t_1t_0t_1t_3 \in [01201013]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_1t_0t_1t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_0t_1t_3t_2t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_0t_1t_3t_2t_1$.

136. [012010301] Same as 28.

137. [012013010] Same as 107.

138. Now, we consider the cosets stabilizer $N^{(012013012)}$.
$N^{(012013012)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_1t_3t_0t_1t_2N$ is at most: $\frac{|N|}{|N^{(012013012)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012013012)}$ on $\{0, 1, 2, 3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_1t_3t_0t_1t_2t_0$, $Nt_0t_1t_2t_0t_1t_3t_0t_1t_2t_1$, $Nt_0t_1t_2t_0t_1t_3t_0t_1t_2t_2$ and $Nt_0t_1t_2t_0t_1t_3t_0t_1t_2t_3$ belong?
We know, $t_0t_1t_2t_0t_1t_3t_0t_1t_2 = (t_0t_1t_2t_3t_1t_2t_0t_1t_2)^{(0,2,3)}$.
$Nt_0t_1t_2t_0t_1t_3t_0t_1t_2 = Nt_3t_1t_0t_2t_1t_0t_3t_1t_0$.
$Nt_0t_1t_2t_0t_1t_3t_0t_1t_2t_0 = Nt_3t_1t_0t_2t_1t_0t_3t_1 \in [01231201]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_1t_2t_0t_1N$.

Moreover, $Nt_0t_1t_2t_0t_1t_3t_0t_1t_2t_2 = Nt_0t_1t_2t_0t_1t_3t_0t_1 \in [01201301]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_1t_3t_0t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_0t_1t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_0t_1t_2t_3$.

139. Now, we consider coset $N^{(012013013)}$.
However, $t_0t_1t_2t_0t_1t_3t_0t_1t_3 = (1, 0, 3)(t_3t_0t_2t_1t_0t_3t_1t_0)$
$= (1, 0, 3)(t_0t_1t_2t_3t_1t_0t_3t_1)^{(1,0,3)} \in Nt_0t_1t_2t_3t_1t_0t_3t_1$.
Therefore, t_3 takes $Nt_0t_1t_2t_0t_1t_3t_0t_1t_3$ go to $Nt_0t_1t_2t_3t_1t_0t_3t_1$.

140. Now, we consider coset $N^{(012013020)}$.
However, $t_0t_1t_2t_0t_1t_3t_0t_2t_0 = (1, 3, 2)(t_2t_3t_2t_0t_3t_1t_3t_1)$
$= (1, 3, 2)(t_0t_1t_0t_2t_1t_3t_1t_3)^{(1,3)(2,0)} \in Nt_0t_1t_0t_2t_1t_3t_1t_3$.
Therefore, t_0 takes $Nt_0t_1t_2t_0t_1t_3t_0t_2t_0$ go to $Nt_0t_1t_0t_2t_1t_3t_1t_3$.

141. [012013021] Same as 80.

142. [012013023] Same as 62.

143. Now, we consider the cosets stabilizer $N^{(012013030)}$.
$N^{(012013030)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_1t_3t_0t_3t_0N$ is at most: $\frac{|N|}{|N^{(012013030)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012013030)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_1t_3t_0t_3t_0t_0$, $Nt_0t_1t_2t_0t_1t_3t_0t_3t_0t_1$, $Nt_0t_1t_2t_0t_1t_3t_0t_3t_0t_2$ and $Nt_0t_1t_2t_0t_1t_3t_0t_3t_0t_3$ belong?
We know, $t_0t_1t_2t_0t_1t_3t_0t_3t_0 = (t_0t_1t_2t_3t_0t_2t_0t_2t_1)^{(1,2,3)}$.
$Nt_0t_1t_2t_0t_1t_3t_0t_3t_0 = Nt_0t_2t_3t_1t_0t_3t_0t_3t_2$.
$Nt_0t_1t_2t_0t_1t_3t_0t_3t_0t_2 = Nt_0t_2t_3t_1t_0t_3t_0t_3 \in [01230202]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_0t_2t_0t_2N$.
Moreover, $Nt_0t_1t_2t_0t_1t_3t_0t_3t_0t_0 = Nt_0t_1t_2t_0t_1t_3t_0t_3 \in [01201303]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_1t_3t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_0t_3t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_0t_3t_0t_3$.

144. Now, we consider coset $N^{(012013031)}$.
However, $t_0t_1t_2t_0t_1t_3t_0t_3t_1 = (2,3,0)(t_0t_1t_3t_1t_2t_0t_2t_0)$
$= (2,3,0)(t_0t_1t_2t_1t_3t_0t_3t_0)^{(2,3)} \in Nt_0t_1t_2t_1t_3t_0t_3t_0$.
Therefore, t_1 takes $Nt_0t_1t_2t_0t_1t_3t_0t_3t_1$ go to $Nt_0t_1t_2t_1t_3t_0t_3t_0$.

145. [012013031] Same as 134.

146. Now, we consider the cosets stabilizer $N^{(012013130)}$.
$N^{(012013130)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_1t_3t_1t_3t_0N$ is at most: $\frac{|N|}{|N^{(012013130)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012013130)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_1t_3t_1t_3t_0t_0$, $Nt_0t_1t_2t_0t_1t_3t_1t_3t_0t_1$, $Nt_0t_1t_2t_0t_1t_3t_1t_3t_0t_2$ and $Nt_0t_1t_2t_0t_1t_3t_1t_3t_0t_3$ belong?
However, $Nt_0t_1t_2t_0t_1t_3t_1t_3t_0t_0 = Nt_0t_1t_2t_0t_1t_3t_1t_3 \in [01201313]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_1t_3t_1t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_1t_3t_0t_1$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_1t_3t_0t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_1t_3t_0t_2$.

147. [012013131] Same as 112.

148. Now, we consider coset $N^{(012013132)}$.

However, $t_0t_1t_2t_0t_1t_3t_1t_3t_2 = (1,0,3)(t_2t_3t_1t_0t_1t_2t_1t_3)$
$= (1,0,3)(t_0t_1t_2t_3t_2t_0t_2t_1)^{(2,3)} \in Nt_0t_1t_2t_3t_2t_0t_2t_1$. Therefore, t_2 takes $Nt_0t_1t_2t_0t_1t_3t_1t_3t_2$ go to $Nt_0t_1t_2t_3t_2t_0t_2t_1$.

149. Now, we consider the cosets stabilizer $N^{(012013230)}$.

$N^{(012013230)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_1t_3t_2t_3t_0N$ is at most: $\frac{|N|}{|N^{(012013230)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012013230)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_1t_3t_2t_3t_0t_0$, $Nt_0t_1t_2t_0t_1t_3t_2t_3t_0t_1$, $Nt_0t_1t_2t_0t_1t_3t_2t_3t_0t_2$ and $Nt_0t_1t_2t_0t_1t_3t_2t_3t_0t_3$ belong?

We know, $t_0t_1t_2t_0t_1t_3t_2t_3t_0 = (2,0,3)(t_0t_1t_2t_3t_2t_1t_2t_0t_2)^{(1,0,3)}$.

$Nt_0t_1t_2t_0t_1t_3t_2t_3t_0 = Nt_3t_0t_2t_1t_2t_0t_2t_0t_2$.

$Nt_0t_1t_2t_0t_1t_3t_2t_3t_0t_2 = Nt_3t_0t_2t_1t_2t_0t_2t_0 \in [01232120]$, then one symmetric generator goes back to $Nt_0t_1t_2t_3t_2t_1t_2t_0N$.

Moreover, $Nt_0t_1t_2t_0t_1t_3t_2t_3t_0t_0 = Nt_0t_1t_2t_0t_1t_3t_2t_3 \in [01201323]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_1t_3t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_2t_3t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_1t_3t_2t_3t_0t_3$.

150. [012013232] Same as 24.

151. Now, we consider coset $N^{(012020301)}$.

However, $t_0t_1t_2t_0t_2t_0t_3t_0t_1 = (t_3t_1t_3t_1t_2t_0t_1t_0) = (t_0t_1t_0t_1t_2t_3t_1t_3)^{(0,3)} \in Nt_0t_1t_0t_1t_2t_3t_1t_3$. Therefore, t_1 takes $Nt_0t_1t_2t_0t_2t_0t_3t_0t_1$ go to $Nt_0t_1t_0t_1t_2t_3t_1t_3$.

152. [012020302] Same as 86.

153. Now, we consider coset $N^{(012020303)}$.

However, $t_0t_1t_2t_0t_2t_0t_3t_0t_3 = (t_0t_1t_0t_3t_2t_3t_2t_0) = (t_0t_1t_0t_2t_3t_2t_3t_0)^{(2,3)} \in Nt_0t_1t_0t_2t_3t_2t_3t_0$. Therefore, t_3 takes $Nt_0t_1t_2t_0t_2t_0t_3t_0t_3$ go to $Nt_0t_1t_0t_2t_3t_2t_3t_0$.

154. Now, we consider coset $N^{(012020310)}$.
However, $t_0t_1t_2t_0t_2t_0t_3t_1t_2 = (t_3t_1t_3t_1t_2t_1t_0t_1) = (t_0t_1t_0t_1t_2t_1t_3t_1)^{(0,3)} \in Nt_0t_1t_0t_1t_2t_1t_3t_1$.
Therefore, t_0 takes $Nt_0t_1t_2t_0t_2t_0t_3t_1t_2$ go to $Nt_0t_1t_0t_1t_2t_1t_3t_1$.
155. [012020312] Same as 80.

156. [012021230] Same as 99.

157. Now, we consider the cosets stabilizer $N^{(012021231)}$.
$N^{(012021231)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_2t_1t_2t_3t_1N$ is at most: $\frac{|N|}{|N^{(012021231)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012021231)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_2t_1t_2t_3t_1t_0$, $Nt_0t_1t_2t_0t_2t_1t_2t_3t_1t_1$, $Nt_0t_1t_2t_0t_2t_1t_2t_3t_1t_2$ and $Nt_0t_1t_2t_0t_2t_1t_2t_3t_1t_3$ belong?
We know, $t_0t_1t_2t_0t_2t_1t_2t_3t_1 = (2,3,1)(t_0t_1t_2t_0t_1t_0t_3t_0t_3)^{(2,0)}$.
$Nt_0t_1t_2t_0t_2t_1t_2t_3t_1 = Nt_2t_1t_0t_1t_2t_1t_3t_2t_3$.
$Nt_0t_1t_2t_0t_2t_1t_2t_3t_1t_3 = Nt_2t_1t_0t_1t_2t_1t_3t_2 \in [01201030]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_1t_0t_3t_0N$.
We know, $t_0t_1t_2t_0t_2t_1t_2t_3t_1 = (0,3)(1,2)(t_0t_1t_2t_1t_3t_1t_2t_0t_3)^{(0,2,3)}$.
$Nt_0t_1t_2t_0t_2t_1t_2t_3t_1 = Nt_2t_1t_3t_1t_0t_1t_3t_2t_0$.
$Nt_0t_1t_2t_0t_2t_1t_2t_3t_1t_0 = Nt_2t_1t_3t_1t_0t_1t_3t_2 \in [01213120]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_3t_1t_2t_0N$.
Moreover, $Nt_0t_1t_2t_0t_2t_1t_2t_3t_1t_1 = Nt_0t_1t_2t_0t_2t_1t_2t_3 \in [01202123]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_2t_1t_2t_3N$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_1t_2t_3t_1t_2$.

158. [012021301] Same as 68.

159. Now, we consider coset $N^{(012021303)}$.
However, $t_0t_1t_2t_0t_2t_1t_3t_0t_3 = (2,0,3)(t_3t_2t_1t_3t_0t_1t_2t_1)$
$= (2,0,3)(t_0t_1t_2t_0t_3t_2t_1t_2)^{(1,2)(0,3)} \in Nt_0t_1t_2t_0t_3t_2t_1t_2$.
Therefore, t_3 takes $Nt_0t_1t_2t_0t_2t_1t_3t_0t_3$ go to $Nt_0t_1t_2t_0t_3t_2t_1t_2$.

160. Now, we consider coset $N^{(012021320)}$.
However, $t_0t_1t_2t_0t_2t_1t_3t_2t_0 = (1,0,2)(t_1t_0t_1t_2t_3t_1t_0t_3)$
$= (1,0,2)(t_0t_1t_0t_2t_3t_0t_1t_3)^{(1,0)} \in Nt_0t_1t_0t_2t_3t_0t_1t_3$. Therefore, t_0 takes $Nt_0t_1t_2t_0t_2t_1t_3t_2t_0$ go to $Nt_0t_1t_0t_2t_3t_0t_1t_3$.

161. Now, we consider the cosets stabilizer $N^{(012021323)}$.
$N^{(012021323)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_2t_1t_3t_2t_3N$ is at most: $\frac{|N|}{|N^{(012021323)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012021323)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_2t_1t_3t_2t_3t_0$, $Nt_0t_1t_2t_0t_2t_1t_3t_2t_3t_1$, $Nt_0t_1t_2t_0t_2t_1t_3t_2t_3t_2$ and $Nt_0t_1t_2t_0t_2t_1t_3t_2t_3t_3$ belong?
We know, $t_0t_1t_2t_0t_2t_1t_3t_2t_3 = (2,1,3)(t_0t_1t_2t_1t_3t_1t_0t_1t_3)^{(1,2,0,3)}$.
$Nt_0t_1t_2t_0t_2t_1t_3t_2t_3 = Nt_3t_2t_0t_2t_1t_2t_3t_2t_1$.
$Nt_0t_1t_2t_0t_2t_1t_3t_2t_3t_1 = Nt_3t_2t_0t_2t_1t_2t_3t_2 \in [01213101]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_3t_1t_0t_1N$.
Moreover, $Nt_0t_1t_2t_0t_2t_1t_3t_2t_3t_3 = Nt_0t_1t_2t_0t_2t_1t_3t_2 \in [01202132]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_2t_1t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_1t_3t_2t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_1t_3t_2t_3t_2$.

162. Now, we consider the cosets stabilizer $N^{(012023101)}$.
$N^{(012023101)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_2t_3t_1t_0t_1N$ is at most: $\frac{|N|}{|N^{(012023101)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012023101)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_2t_3t_1t_0t_1t_0$, $Nt_0t_1t_2t_0t_2t_3t_1t_0t_1t_1$, $Nt_0t_1t_2t_0t_2t_3t_1t_0t_1t_2$ and $Nt_0t_1t_2t_0t_2t_3t_1t_0t_1t_3$ belong?
However, $Nt_0t_1t_2t_0t_2t_3t_1t_0t_1t_1 = Nt_0t_1t_2t_0t_2t_3t_1t_0 \in [01202310]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_2t_3t_1t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_1t_0t_1t_0$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_1t_0t_1t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_1t_0t_1t_2$.

163. Now, we consider coset $N^{(012023130)}$.
However, $t_0t_1t_2t_0t_2t_3t_1t_3t_0 = (1,3,2)(t_1t_3t_2t_3t_2t_0t_2t_3)$

$= (1,3,2)(t_0t_1t_2t_1t_2t_3t_2t_1)^{(1,3,0)} \in Nt_0t_1t_2t_1t_2t_3t_2t_1$.
Therefore, t_0 takes $Nt_0t_1t_2t_0t_2t_3t_1t_3t_0$ go to $Nt_0t_1t_2t_1t_2t_3t_2t_1$.

164. [012023131] Same as 134.

165. Now, we consider coset $N^{(012023132)}$.
However, $t_0t_1t_2t_0t_2t_3t_1t_3t_2 = (1,0,3)(t_3t_1t_2t_3t_2t_0t_1t_0)$
$= (1,0,3)(t_0t_1t_2t_0t_2t_3t_1t_3)^{(3,0)} \in Nt_0t_1t_2t_0t_2t_3t_1t_3$.
Therefore, t_2 takes $Nt_0t_1t_2t_0t_2t_3t_1t_3t_2$ go to $Nt_0t_1t_2t_0t_2t_3t_1t_3$.

166. Now, we consider the cosets stabilizer $N^{(012023201)}$.
$N^{(012023201)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_2t_3t_2t_0t_1N$ is at most: $\frac{|N|}{|N^{(012023201)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012023201)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_2t_3t_2t_0t_1t_0$, $Nt_0t_1t_2t_0t_2t_3t_2t_0t_1t_1$, $Nt_0t_1t_2t_0t_2t_3t_2t_0t_1t_2$ and $Nt_0t_1t_2t_0t_2t_3t_2t_0t_1t_3$ belong?
We know, $t_0t_1t_2t_0t_2t_3t_2t_0t_1 = (t_0t_1t_2t_0t_3t_2t_1t_0t_1)^{(2,1,0)}$.
$Nt_0t_1t_2t_0t_2t_3t_2t_0t_1 = Nt_2t_0t_1t_2t_3t_1t_0t_2t_0$.
$Nt_0t_1t_2t_0t_2t_3t_2t_0t_1t_0 = Nt_2t_0t_1t_2t_3t_1t_0t_2 \in [01203210]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_3t_2t_1t_0N$.
We know, $t_0t_1t_2t_0t_2t_3t_2t_0t_1 = (0,3)(1,2)(t_0t_1t_2t_1t_0t_1t_3t_0t_2)^{(0,2,3)}$.
$Nt_0t_1t_2t_0t_2t_3t_2t_0t_1 = Nt_2t_1t_3t_1t_2t_1t_0t_2t_3$.
$Nt_0t_1t_2t_0t_2t_3t_2t_0t_1t_3 = Nt_2t_1t_3t_1t_2t_1t_0t_2 \in [01210130]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_0t_1t_3t_0N$.
Moreover, $Nt_0t_1t_2t_0t_2t_3t_2t_0t_1t_1 = Nt_0t_1t_2t_0t_2t_3t_2t_0 \in [01202320]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_2t_3t_2t_0N$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_2t_0t_1t_2$.

167. [012023202] Same as 69.

168. [012023203] Same as 78.

169. Now, we consider the cosets stabilizer $N^{(012023230)}$.
$N^{(012023230)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_2t_3t_2t_3t_0N$ is at most: $\frac{|N|}{|N^{(012023230)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012023230)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_2t_3t_2t_3t_0t_0$, $Nt_0t_1t_2t_0t_2t_3t_2t_3t_0t_1$, $Nt_0t_1t_2t_0t_2t_3t_2t_3t_0t_2$ and $Nt_0t_1t_2t_0t_2t_3t_2t_3t_0t_3$ belong?
We know, $t_0t_1t_2t_0t_2t_3t_2t_3t_0 = (3,1,2)(t_0t_1t_2t_1t_2t_0t_3t_1t_0)^{(0,3)}$.
$Nt_0t_1t_2t_0t_2t_3t_2t_3t_0 = Nt_3t_1t_2t_1t_2t_3t_0t_1t_3$.
$Nt_0t_1t_2t_0t_2t_3t_2t_3t_0t_3 = Nt_3t_1t_2t_1t_2t_3t_0t_1 \in [01212031]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_2t_0t_3t_1N$.
Moreover, $Nt_0t_1t_2t_0t_2t_3t_2t_3t_0t_0 = Nt_0t_1t_2t_0t_2t_3t_2t_3 \in [01202323]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_2t_3t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_2t_3t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_2t_3t_0t_2$.

170. Now, we consider the cosets stabilizer $N^{(012023231)}$.
$N^{(012023231)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_2t_3t_2t_3t_1N$ is at most: $\frac{|N|}{|N^{(012023231)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012023231)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_2t_3t_2t_3t_1t_0$, $Nt_0t_1t_2t_0t_2t_3t_2t_3t_1t_1$, $Nt_0t_1t_2t_0t_2t_3t_2t_3t_1t_2$ and $Nt_0t_1t_2t_0t_2t_3t_2t_3t_1t_3$ belong?
However, $Nt_0t_1t_2t_0t_2t_3t_2t_3t_1t_1 = Nt_0t_1t_2t_0t_2t_3t_2t_3 \in [01202323]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_2t_3t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_2t_3t_1t_0$, one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_2t_3t_1t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_2t_3t_2t_3t_1t_3$.

171. Now, we consider coset $N^{(012023232)}$.
However, $t_0t_1t_2t_0t_2t_3t_2t_3t_2 = t_0t_1t_0t_2t_3t_0t_3t_0$.
Therefore, t_2 takes $Nt_0t_1t_2t_0t_2t_3t_2t_3t_2$ go to itself $Nt_0t_1t_0t_2t_3t_0t_3t_0$.

172. Now, we consider coset $N^{(012030230)}$.
However, $t_0t_1t_2t_0t_3t_0t_2t_3t_0 = (2,0,3)(t_0t_3t_0t_3t_1t_2t_1t_3)$
$= (2,0,3)(t_0t_1t_0t_1t_2t_3t_2t_1)^{(1,3,2)} \in Nt_0t_1t_0t_1t_2t_3t_2t_1$.

Therefore, t_0 takes $Nt_0t_1t_2t_0t_3t_0t_2t_3t_0$ go to $Nt_0t_1t_0t_1t_2t_3t_2t_1$.

173. [012030301] Same as 123.

174. Now, we consider coset $N^{(012030302)}$.
However, $t_0t_1t_2t_0t_3t_0t_3t_0t_2 = (1,3,0)(t_0t_3t_0t_1t_0t_3t_2t_1)$
$= (1,3,0)(t_0t_1t_0t_2t_0t_1t_3t_2)^{(1,3,2)} \in Nt_0t_1t_0t_2t_0t_1t_3t_2$.
Therefore, t_2 takes $Nt_0t_1t_2t_0t_3t_0t_3t_0t_2$ go to $Nt_0t_1t_0t_2t_0t_1t_3t_2$.

175. Now, we consider coset $N^{(012030310)}$.
However, $t_0t_1t_2t_0t_3t_0t_3t_1t_0 = (t_1t_0t_1t_3t_1t_2t_1t_2) = (t_0t_1t_0t_2t_0t_3t_0t_3)^{(1,0)(3,2)}$
$\in Nt_0t_1t_0t_2t_0t_3t_0t_3$. Therefore, t_0 takes $Nt_0t_1t_2t_0t_3t_0t_3t_1t_0$ go to $Nt_0t_1t_0t_2t_0t_3t_0t_3$.

176. [012030312] Same as 103.

177. Now, we consider coset $N^{(012030313)}$.
However, $t_0t_1t_2t_0t_3t_0t_3t_1t_3 = (2,3,0)(t_3t_1t_3t_0t_2t_0t_1t_0)$
$= (2,3,0)(t_0t_1t_0t_2t_3t_2t_1t_2)^{(2,0,3)} \in Nt_0t_1t_0t_2t_3t_2t_1t_2$.
Therefore, t_3 takes $Nt_0t_1t_2t_0t_3t_0t_3t_1t_3$ go to $Nt_0t_1t_0t_2t_3t_2t_1t_2$.

178. Now, we consider coset $N^{(012030320)}$.
However, $t_0t_1t_2t_0t_3t_0t_3t_2t_0 = (t_0t_1t_3t_2t_3t_2t_0t_2) = (t_0t_1t_2t_3t_2t_3t_0t_3)^{(2,3)}$
$\in Nt_0t_1t_2t_3t_2t_3t_0t_3$. Therefore, t_0 takes $Nt_0t_1t_2t_0t_3t_0t_3t_2t_0$ go to $Nt_0t_1t_2t_3t_2t_3t_0t_3$.

179. Now, we consider coset $N^{(012030321)}$.
However, $t_0t_1t_2t_0t_3t_0t_3t_2t_1 = (1,0,3)(t_0t_2t_3t_1t_2t_3t_0t_2)$
$= (1,0,3)(t_0t_1t_2t_3t_1t_2t_0t_1)^{(1,2,3)} \in Nt_0t_1t_2t_3t_1t_2t_0t_1$.
Therefore, t_1 takes $Nt_0t_1t_2t_0t_3t_0t_3t_2t_1$ go to $Nt_0t_1t_2t_3t_1t_2t_0t_1$.

180. [012030323] Same as 73.

181. [012032010] Same as 7.

182. Now, we consider coset $N^{(012032012)}$.
However, $t_0t_1t_2t_0t_3t_2t_0t_1t_2 = (1,3,0)(t_3t_2t_1t_0t_1t_0t_2t_3)$
$= (1,3,0)(t_0t_1t_2t_3t_2t_3t_1t_0)^{(1,2)(0,3)} \in Nt_0t_1t_2t_3t_2t_3t_1t_0$.
Therefore, t_2 takes $Nt_0t_1t_2t_0t_3t_2t_0t_1t_2$ go to $Nt_0t_1t_2t_3t_2t_3t_1t_0$.

183. [012032013] Same as 48.

184. Now, we consider the cosets stabilizer $N^{(012032023)}$.
We know, $t_0t_1t_2t_0t_3t_2t_0t_2t_0 = t_0t_1t_0t_3t_2t_3t_2t_3t_1$ then
$N^{(012032023)} = \{e, (0,1)(2,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_3t_2t_0t_2t_0N$ is at most: $\frac{|N|}{|N^{(012032023)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(012032023)}$ on $\{0,1,2,3\}$ are $\{2,3\}$, and $\{0,1\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_3t_2t_0t_2t_0t_0$, and $Nt_0t_1t_2t_0t_3t_2t_0t_2t_0t_2$ belong?
However, $Nt_0t_1t_2t_0t_3t_2t_0t_2t_0t_0 = Nt_0t_1t_2t_0t_3t_2t_0t_2 \in$ [01203202], then two symmetric generators go back to the double coset $Nt_0t_1t_2t_0t_3t_2t_0t_2N$, and two symmetric generator go to the double coset $Nt_0t_1t_2t_0t_3t_2t_0t_2t_0t_2$.

185. Now, we consider the cosets stabilizer $N^{(012032023)}$.
$N^{(012032023)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_3t_2t_0t_2t_3N$ is at most: $\frac{|N|}{|N^{(012032023)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012032023)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_3t_2t_0t_2t_3t_0$, $Nt_0t_1t_2t_0t_3t_2t_0t_2t_3t_1$, $Nt_0t_1t_2t_0t_3t_2t_0t_2t_3t_2$ and $Nt_0t_1t_2t_0t_3t_2t_0t_2t_3t_3$ belong?
We know, $t_0t_1t_2t_0t_3t_2t_0t_2t_3 = (0,2)(1,3)(t_0t_1t_0t_2t_1t_0t_3t_1t_3)^{(0,1)(2,3)}$.
$Nt_0t_1t_2t_0t_3t_2t_0t_2t_3 = Nt_1t_0t_1t_3t_0t_1t_2t_0t_2$.
$Nt_0t_1t_2t_0t_3t_2t_0t_2t_3t_3 = Nt_1t_0t_1t_3t_0t_1t_2t_0 \in$ [01030120], then one symmetric generator goes back to $Nt_0t_1t_0t_2t_1t_0t_3t_1N$.
Moreover, $Nt_0t_1t_2t_0t_3t_2t_0t_2t_3t_3 = Nt_0t_1t_2t_0t_3t_2t_0t_2 \in$ [01203202], then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_0t_3t_2t_0t_2N$, one symmetric generator

goes to $Nt_0t_1t_2t_0t_3t_2t_0t_2t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_0t_3t_2t_0t_2t_3t_0$.

186. [012032023] Same as 166.

187. [012032102] Same as 103.

188. Now, we consider coset $N^{(012032120)}$.
However, $t_0t_1t_2t_0t_3t_2t_1t_2t_0 = (1,0,3)(t_3t_2t_1t_3t_1t_2t_0t_3)$
$= (1,0,3)(t_0t_1t_2t_0t_2t_1t_3t_0)^{(1,2)(0,3)} \in Nt_0t_1t_2t_0t_2t_1t_3t_0$.
Therefore, t_0 takes $Nt_0t_1t_2t_0t_3t_2t_1t_2t_0$ go to $Nt_0t_1t_2t_0t_2t_1t_3t_0$.

189. Now, we consider coset $N^{(012032121)}$.
However, $t_0t_1t_2t_0t_3t_2t_1t_2t_1 = (1,2,0)(t_3t_2t_1t_3t_1t_2t_0t_3)$
$= (1,2,0)(t_0t_1t_0t_2t_3t_1t_3t_0)^{(1,0,2)} \in Nt_0t_1t_0t_2t_3t_1t_3t_0$.
Therefore, t_1 takes $Nt_0t_1t_2t_0t_3t_2t_1t_2t_1$ go to $Nt_0t_1t_0t_2t_3t_1t_3t_0$.

190. Now, we consider coset $N^{(012032123)}$.
However, $t_0t_1t_2t_0t_3t_2t_1t_2t_3 = (1,3,2)(t_3t_2t_0t_1t_2t_3t_2t_1)$
$= (1,3,2)(t_0t_1t_2t_3t_1t_0t_1t_3)^{(1,2,0,3)} \in Nt_0t_1t_2t_3t_1t_0t_1t_3$.
Therefore, t_3 takes $Nt_0t_1t_2t_0t_3t_2t_1t_2t_3$ go to $Nt_0t_1t_2t_3t_1t_0t_1t_3$.

191. Now, we consider coset $N^{(012032320)}$.
However, $t_0t_1t_2t_0t_3t_2t_3t_2t_0 = t_0t_1t_0t_2t_0t_3t_0t_3$.
Therefore, t_0 takes $Nt_0t_1t_2t_0t_3t_2t_3t_2t_0$ go to itself $Nt_0t_1t_0t_2t_0t_3t_0t_3$.

192. Now, we consider coset $N^{(012032323)}$.
However, $t_0t_1t_2t_0t_3t_2t_3t_2t_3 = t_0t_1t_0t_2t_3t_0t_3t_0$.
Therefore, t_0 takes $Nt_0t_1t_2t_0t_3t_2t_3t_2t_3$ go to itself $Nt_0t_1t_0t_2t_3t_0t_3t_0$.

193. [012101301] Same as 24.

194. [012101302] Same as 166.

195. [012101303] Same as 57.

196. [012101320] Same as 12.

197. [012101321] Same as 1.

198. Now, we consider the cosets stabilizer $N^{(012101323)}$.
$N^{(012101323)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_0t_1t_3t_2t_3N$ is at most: $\frac{|N|}{|N^{(012101323)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012101323)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_0t_1t_3t_2t_3t_0$, $Nt_0t_1t_2t_1t_0t_1t_3t_2t_3t_1$, $Nt_0t_1t_2t_1t_0t_1t_3t_2t_3t_2$ and $Nt_0t_1t_2t_1t_0t_1t_3t_2t_3t_3$ belong?
However, $Nt_0t_1t_2t_1t_0t_1t_3t_2t_3t_3 = Nt_0t_1t_2t_1t_0t_1t_3t_2 \in [01210132]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_0t_1t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_1t_3t_2t_3t_0$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_1t_3t_2t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_1t_3t_2t_3t_2$.

199. Now, we consider coset $N^{(012103030)}$.
However, $t_0t_1t_2t_1t_0t_3t_0t_3t_0 = (2,3,0)(t_0t_3t_0t_2t_0t_3t_2t_1)$
$= (2,3,0)(t_0t_1t_0t_2t_0t_1t_2t_3)^{(1,3)} \in Nt_0t_1t_0t_2t_0t_1t_2t_3$.
Therefore, t_3 takes $Nt_0t_1t_2t_1t_0t_3t_0t_3t_0$ go to $Nt_0t_1t_0t_2t_0t_1t_2t_3$.

200. [012103031] Same as 2.

201. [012103120] Same as 116.

202. Now, we consider coset $N^{(012103121)}$.
However, $t_0t_1t_2t_1t_0t_3t_1t_2t_1 = (2,0,3)(t_3t_2t_3t_2t_1t_0t_3t_0)$
$= (2,0,3)(t_0t_1t_0t_1t_2t_3t_0t_3)^{(1,2)(3,0)} \in Nt_0t_1t_0t_1t_2t_3t_0t_3$.
Therefore, t_1 takes $Nt_0t_1t_2t_1t_0t_3t_1t_2t_1$ go to $Nt_0t_1t_0t_1t_2t_3t_0t_3$.

203. Now, we consider coset $N^{(012103123)}$.
However, $t_0t_1t_2t_1t_0t_3t_1t_2t_3 = (1,0,2)(t_1t_0t_2t_0t_1t_3t_0t_2)$
$= (1,0,2)(t_0t_1t_2t_1t_0t_3t_1t_2)^{(1,0)} \in Nt_0t_1t_2t_1t_0t_3t_1t_2$.
Therefore, t_3 takes $Nt_0t_1t_2t_1t_0t_3t_1t_2t_3$ go to $Nt_0t_1t_2t_1t_0t_3t_1t_2$.

204. [012103130] Same as 79.

205. Now, we consider the cosets stabilizer $N^{(012103131)}$.
$N^{(012103131)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_0t_3t_1t_3t_1N$ is at most: $\frac{|N|}{|N^{(012103131)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012103131)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_0t_3t_1t_3t_1t_0$, $Nt_0t_1t_2t_1t_0t_3t_1t_3t_1t_1$, $Nt_0t_1t_2t_1t_0t_3t_1t_3t_1t_2$ and $Nt_0t_1t_2t_1t_0t_3t_1t_3t_1t_3$ belong?
However, $Nt_0t_1t_2t_1t_0t_3t_1t_3t_1t_1 = Nt_0t_1t_2t_1t_0t_3t_1t_3 \in [01210313]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_0t_3t_1t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_3t_1t_3t_1t_0$, one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_3t_1t_3t_1t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_0t_3t_1t_3t_1t_3$.

206. [012103132] Same as 79.

207. Now, we consider coset $N^{(012120301)}$.
However, $t_0t_1t_2t_1t_2t_0t_3t_0t_1 = (1,2,3)(t_1t_2t_1t_0t_2t_3t_0t_2)$
$= (1,2,3)(t_0t_1t_0t_2t_1t_3t_2t_1)^{(1,2,0)} \in Nt_0t_1t_0t_2t_1t_3t_2t_1$.
Therefore, t_1 takes $Nt_0t_1t_2t_1t_2t_0t_3t_0t_1$ go to $Nt_0t_1t_0t_2t_1t_3t_2t_1$.

208. [012120302] Same as 13.

209. [012120303] Same as 133.

210. [012120310] Same as 169.

211. Now, we consider coset $N^{(012120313)}$.
However, $t_0t_1t_2t_1t_2t_0t_3t_1t_3 = (1,2)(0,3)(t_0t_1t_0t_2t_0t_2t_3t_2) \in Nt_0t_1t_0t_2t_0t_2t_3t_2$.
Therefore, t_3 takes $Nt_0t_1t_2t_1t_2t_0t_3t_1t_3$ go to $Nt_0t_1t_0t_2t_0t_2t_3t_2$.

212. Now, we consider the cosets stabilizer $N^{(012120320)}$.
$N^{(012120320)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_2t_0t_3t_2t_0N$ is at most: $\frac{|N|}{|N^{(012120320)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012120320)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_2t_0t_3t_2t_0t_0$, $Nt_0t_1t_2t_1t_2t_0t_3t_2t_0t_1$, $Nt_0t_1t_2t_1t_2t_0t_3t_2t_0t_2$ and $Nt_0t_1t_2t_1t_2t_0t_3t_2t_0t_3$ belong?
We know, $t_0t_1t_2t_1t_2t_0t_3t_2t_0 = (1,2,3)(t_0t_1t_2t_1t_3t_1t_0t_2t_1)^{(2,1)}$.
$Nt_0t_1t_2t_1t_2t_0t_3t_2t_0 = Nt_0t_2t_1t_2t_3t_2t_0t_1t_2$.
$Nt_0t_1t_2t_1t_2t_0t_3t_2t_0t_2 = Nt_0t_2t_1t_2t_3t_2t_0t_1 \in [01213102]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_3t_1t_0t_2N$.
Moreover, $Nt_0t_1t_2t_1t_2t_0t_3t_2t_0t_0 = Nt_0t_1t_2t_1t_2t_0t_3t_2 \in [01212032]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_2t_0t_3t_2N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_0t_3t_2t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_0t_3t_2t_0t_3$.

213. [012120321] Same as 19.

214. Now, we consider coset $N^{(012120323)}$.
However, $t_0t_1t_2t_1t_2t_0t_3t_2t_3 = (1,2,3)(t_1t_0t_3t_2t_1t_0t_2t_1)$
$= (1,2,3)(t_0t_1t_2t_3t_1t_0t_3t_1)^{(1,0)(2,3)} \in Nt_0t_1t_2t_3t_1t_0t_3t_1$.
Therefore, t_3 takes $Nt_0t_1t_2t_1t_2t_0t_3t_2t_3$ go to $Nt_0t_1t_2t_3t_1t_0t_3t_1$.

215. [012121301] Same as 2.

216. [012121303] Same as 43.

217. Now, we consider coset $N^{(012123020)}$.
However, $t_0t_1t_2t_1t_2t_3t_0t_2t_0 = (t_2t_3t_0t_3t_0t_1t_0t_2)$
$= (t_0t_1t_2t_1t_2t_3t_2t_0)^{(1,3)(2,0)} \in Nt_0t_1t_2t_1t_2t_3t_2t_0$.

Therefore, t_0 takes $Nt_0t_1t_2t_1t_2t_3t_0t_2t_0$ go to $Nt_0t_1t_2t_1t_2t_3t_2t_0$.

218. [012123021] Same as 19.

219. Now, we consider coset $N^{(012123023)}$.
However, $t_0t_1t_2t_1t_2t_3t_0t_2t_3 = (1,3,2)(t_3t_1t_0t_1t_2t_1t_3t_0)$
$= (1,3,2)(t_0t_1t_2t_1t_3t_1t_0t_2)^{(2,0,3)} \in Nt_0t_1t_2t_1t_3t_1t_0t_2$.
Therefore, t_3 takes $Nt_0t_1t_2t_1t_2t_3t_0t_2t_3$ go to $Nt_0t_1t_2t_1t_3t_1t_0t_2$.

220. [012123120] Same as 40.

221. Now, we consider coset $N^{(012123123)}$.
However, $t_0t_1t_2t_1t_2t_3t_1t_2t_3 = (1,3,2)(t_0t_2t_3t_1t_2t_1t_3t_1)$
$= (1,3,2)(t_0t_1t_2t_3t_1t_3t_2t_3)^{(1,2,3)} \in Nt_0t_1t_2t_3t_1t_3t_2t_3$.
Therefore, t_3 takes $Nt_0t_1t_2t_1t_2t_3t_1t_2t_3$ go to $Nt_0t_1t_2t_3t_1t_3t_2t_3$.

222. Now, we consider coset $N^{(012123201)}$.
However, $t_0t_1t_2t_1t_2t_3t_2t_0t_1 = (1,2)(0,3)(t_2t_3t_2t_1t_3t_0t_2t_3)$
$= (1,2)(3,0)(t_0t_1t_0t_2t_1t_3t_0t_1)^{(1,3,0,2)} \in Nt_0t_1t_0t_2t_1t_3t_0t_1$.
Therefore, t_1 takes $Nt_0t_1t_2t_1t_2t_3t_2t_0t_1$ go to $Nt_0t_1t_0t_2t_1t_3t_0t_1$.

223. Now, we consider coset $N^{(012123202)}$.
However, $t_0t_1t_2t_1t_2t_3t_2t_0t_2 = (t_2t_3t_0t_3t_0t_1t_2t_0) = (t_0t_1t_2t_1t_2t_3t_0t_2)^{(1,3)(0,2)}$
$\in Nt_0t_1t_2t_1t_2t_3t_0t_2$. Therefore, t_2 takes $Nt_0t_1t_2t_1t_2t_3t_2t_0t_2$ go to $Nt_0t_1t_2t_1t_2t_3t_0t_2$.

224. Now, we consider the cosets stabilizer $N^{(012123203)}$.
$N^{(012123203)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_2t_3t_2t_0t_3N$ is at most: $\frac{|N|}{|N^{(012123203)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012123203)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_2t_3t_2t_0t_3t_0$, $Nt_0t_1t_2t_1t_2t_3t_2t_0t_3t_1$, $Nt_0t_1t_2t_1t_2t_3t_2t_0t_3t_2$ and $Nt_0t_1t_2t_1t_2t_3t_2t_0t_3t_3$ belong?

However, $Nt_0t_1t_2t_1t_2t_3t_2t_0t_3t_3 = Nt_0t_1t_2t_1t_2t_3t_2t_0 \in [01212320]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_2t_3t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_2t_0t_3t_0$, one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_2t_0t_3t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_1t_2t_3t_2t_0t_3t_2$.

225. Now, we consider the cosets stabilizer $N^{(012123210)}$.
We know, $t_0t_1t_2t_1t_2t_3t_2t_1t_0 = t_1t_0t_3t_0t_3t_2t_3t_0t_1$ then
$N^{(012123210)} = \{e, (0,1)(2,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_2t_3t_2t_1t_0N$ is at most: $\frac{|N|}{|N^{(012123210)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(012123210)}$ on $\{0,1,2,3\}$ are $\{2,3\}$, and $\{0,1\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_2t_3t_2t_1t_0t_0$, and $Nt_0t_1t_2t_1t_2t_3t_2t_1t_0t_2$ belong?
However, $Nt_0t_1t_2t_1t_2t_3t_2t_1t_0t_0 = Nt_0t_1t_2t_1t_2t_3t_2t_1 \in [0121231]$, then two symmetric generators go back to the double coset $Nt_0t_1t_2t_1t_2t_3t_2t_1N$, and two symmetric generator go to the double coset $Nt_0t_1t_2t_1t_2t_3t_2t_1t_0t_2$.

226. Now, we consider coset $N^{(012123213)}$.
However, $t_0t_1t_2t_1t_2t_3t_2t_1t_3 = (1,0,2)(t_3t_0t_2t_3t_2t_1t_0t_1)$
$= (1,0,2)(t_0t_1t_2t_0t_2t_3t_1t_3)^{(1,0,3)} \in Nt_0t_1t_2t_0t_2t_3t_1t_3$.
Therefore, t_3 takes $Nt_0t_1t_2t_1t_2t_3t_2t_1t_3$ go to $Nt_0t_1t_2t_0t_2t_3t_1t_3$.

227. Now, we consider coset $N^{(012130301)}$.
However, $t_0t_1t_2t_1t_3t_0t_3t_0t_1 = (2,3,0)(t_0t_1t_3t_0t_1t_2t_0t_2)$
$= (2,3,0)(t_0t_1t_2t_0t_1t_3t_0t_3)^{(2,3)} \in Nt_0t_1t_2t_0t_1t_3t_0t_3$.
Therefore, t_1 takes $Nt_0t_1t_2t_1t_3t_0t_3t_0t_1$ go to $Nt_0t_1t_2t_0t_1t_3t_0t_3$.

228. [012130302] Same as 62.

229. Now, we consider coset $N^{(012130303)}$.
However, $t_0t_1t_2t_1t_3t_0t_3t_0t_3 = (2,3,0)(t_0t_3t_0t_2t_0t_3t_0t_1)$
$= (2,3,0)(t_0t_1t_0t_2t_0t_1t_2t_3)^{(1,3)} \in Nt_0t_1t_0t_2t_0t_1t_2t_3$.
Therefore, t_3 takes $Nt_0t_1t_2t_1t_3t_0t_3t_0t_3$ go to $Nt_0t_1t_0t_2t_0t_1t_2t_3$.

230. Now, we consider coset $N^{(012130320)}$.
However, $t_0t_1t_2t_1t_3t_0t_3t_2t_0 = (2,3,0)(t_0t_2t_0t_2t_1t_2t_0t_3)$
$= (2,3,0)(t_0t_1t_0t_1t_2t_1t_0t_3)^{(1,2)} \in Nt_0t_1t_0t_1t_2t_1t_0t_3$.
Therefore, t_0 takes $Nt_0t_1t_2t_1t_3t_0t_3t_2t_0$ go to $Nt_0t_1t_0t_1t_2t_1t_0t_3$.

231. Now, we consider coset $N^{(012130321)}$.
However, $t_0t_1t_2t_1t_3t_0t_3t_2t_1 = (1,0,2)(t_2t_3t_2t_1t_2t_3t_0t_3)$
$= (1,0,2)(t_0t_1t_0t_2t_0t_1t_3t_1)^{(1,3,0,2)} \in Nt_0t_1t_0t_2t_0t_1t_3t_1$.
Therefore, t_1 takes $Nt_0t_1t_2t_1t_3t_0t_3t_2t_1$ go to $Nt_0t_1t_0t_2t_0t_1t_3t_1$.

232. Now, we consider coset $N^{(012130323)}$.
However, $t_0t_1t_2t_1t_3t_0t_3t_2t_3 = (1,0,3)(t_2t_0t_3t_1t_3t_1t_2t_1)$
$= (1,0,3)(t_0t_1t_2t_3t_2t_3t_0t_3)^{(1,0,3,2)} \in Nt_0t_1t_2t_3t_2t_3t_0t_3$.
Therefore, t_3 takes $Nt_0t_1t_2t_1t_3t_0t_3t_2t_3$ go to $Nt_0t_1t_2t_3t_2t_3t_0t_3$.

233. Now, we consider coset $N^{(012130323)}$.
However, $t_0t_1t_2t_1t_3t_0t_3t_2t_3 = (t_0t_1t_3t_2t_0t_3t_0t_3) = (t_0t_1t_2t_3t_0t_2t_0t_2)^{(3,2)}$
$\in Nt_0t_1t_2t_3t_0t_2t_0t_2$. Therefore, t_3 takes $Nt_0t_1t_2t_1t_3t_0t_3t_2t_3$ go to $Nt_0t_1t_2t_3t_0t_2t_0t_2$.

234. [012302021] Same as 143.

235. Now, we consider coset $N^{(012132023)}$.
However, $t_0t_1t_2t_1t_3t_2t_0t_2t_3 = (t_0t_1t_3t_2t_3t_0t_3t_0) = (t_0t_1t_2t_3t_2t_0t_2t_0)^{(3,2)}$
$\in Nt_0t_1t_2t_3t_2t_0t_2t_0$. Therefore, t_3 takes $Nt_0t_1t_2t_1t_3t_2t_0t_2t_3$ go to $Nt_0t_1t_2t_3t_2t_0t_2t_0$.

236. [012302101] Same as 52.

237. [012302102] Same as 135.

238. Now, we consider the cosets stabilizer $N^{(012302103)}$.
We know, $t_0t_1t_2t_3t_0t_2t_1t_0t_3 = t_3t_0t_2t_1t_3t_2t_0t_3t_1 = t_1t_3t_2t_0t_1t_2t_3t_1t_0$ then

$N^{(012302103)} = \{e, (0,3,1), (0,1,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_0t_2t_1t_0t_3N$ is at most: $\frac{|N|}{|N^{(012302103)}|} = \frac{24}{2} = 8$

The orbit of $N^{(012302103)}$ on $\{0,1,2,3\}$ are $\{2\}$, and $\{0,1,2\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_0t_2t_1t_0t_3t_3$, and $Nt_0t_1t_2t_3t_0t_2t_1t_0t_3t_2$ belong?

However, $Nt_0t_1t_2t_3t_0t_2t_1t_0t_3t_3 = Nt_0t_1t_2t_3t_0t_2t_1t_0 \in [01230210]$, then three symmetric generators go back to the double coset $Nt_0t_1t_2t_3t_0t_2t_1t_0N$, and two symmetric generator go to the double coset $Nt_0t_1t_2t_3t_0t_2t_1t_0t_3t_2$.

239. [012310102] Same as 126.

240. Now, we consider coset $N^{(0121310130)}$.

However, $t_0t_1t_2t_3t_1t_0t_1t_3t_0 = (1,0,3)(t_2t_3t_1t_2t_0t_1t_3t_1)$
$= (1,0,3)(t_0t_1t_2t_0t_3t_2t_1t_2)^{(1,3,0,2)} \in Nt_0t_1t_2t_0t_3t_2t_1t_2$.
Therefore, t_0 takes $Nt_0t_1t_2t_3t_1t_0t_1t_3t_0$ go to $Nt_0t_1t_2t_0t_3t_2t_1t_2$.

241.[012310131] Same as 15.

242. Now, we consider coset $N^{(0121310132)}$.

However, $t_0t_1t_2t_3t_1t_0t_1t_3t_2 = (1,0,3)(t_3t_1t_2t_0t_1t_3t_1t_0)$
$= (1,0,3)(t_0t_1t_2t_3t_1t_0t_1t_3)^{(3,0)} \in Nt_0t_1t_2t_3t_1t_0t_1t_3$.
Therefore, t_2 takes $Nt_0t_1t_2t_3t_1t_0t_1t_3t_2$ go to $Nt_0t_1t_2t_3t_1t_0t_1t_3$.

243. Now, we consider coset $N^{(0121310310)}$.

However, $t_0t_1t_2t_3t_1t_0t_3t_1t_0 = (1,3,0)(t_1t_3t_2t_1t_3t_0t_1t_3)$
$= (1,3,0)(t_0t_1t_2t_0t_1t_3t_0t_1)^{(1,3,0)} \in Nt_0t_1t_2t_0t_1t_3t_0t_1$.
Therefore, t_0 takes $Nt_0t_1t_2t_3t_1t_0t_3t_1t_0$ go to $Nt_0t_1t_2t_0t_1t_3t_0t_1$.

244. Now, we consider coset $N^{(0121310312)}$.

However, $t_0t_1t_2t_3t_1t_0t_3t_1t_2 = (2,3,0)(t_1t_0t_3t_0t_3t_1t_2t_3)$
$= (2,3,0)(t_0t_1t_2t_1t_2t_0t_3t_2)^{(1,0)(2,3)} \in Nt_0t_1t_2t_1t_2t_0t_3t_2$.
Therefore, t_2 takes $Nt_0t_1t_2t_3t_1t_0t_3t_1t_2$ go to $Nt_0t_1t_2t_1t_2t_0t_3t_2$.

245. [012310313] Same as 134.

246. Now, we consider the cosets stabilizer $N^{(012310321)}$.
We know, $t_0t_1t_2t_3t_1t_0t_3t_2t_1 = t_3t_0t_2t_1t_0t_3t_1t_2t_0 = t_1t_3t_2t_0t_3t_1t_0t_2t_3$ then $N^{(012310321)} = \{e, (0,3,1), (0,1,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_1t_0t_3t_2t_1N$ is at most: $\frac{|N|}{|N^{(012310321)}|} = \frac{24}{3} = 8$

The orbit of $N^{(012310321)}$ on $\{0,1,2,3\}$ are $\{2\}$, and $\{0,1,2\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_1t_0t_3t_2t_1t_1$, and $Nt_0t_1t_2t_3t_1t_0t_3t_2t_1t_2$ belong?
However, $Nt_0t_1t_2t_3t_1t_0t_3t_2t_1t_1 = Nt_0t_1t_2t_3t_1t_0t_3t_2 \in [01231032]$, then three symmetric generators go back to the double coset $Nt_0t_1t_2t_3t_1t_0t_3t_2N$, and one symmetric generator goes to the double coset $Nt_0t_1t_2t_3t_1t_0t_3t_2t_1t_2$.

247. [02310323] Same as 121.

248. Now, we consider coset $N^{(012312010)}$.
However, $t_0t_1t_2t_3t_1t_2t_0t_1t_0 = (1,3,0)(t_2t_0t_2t_3t_1t_3t_1t_2)$
$= (1,3,0)(t_0t_1t_0t_2t_3t_2t_3t_0)^{(1,0,2,3)} \in Nt_0t_1t_0t_2t_3t_2t_3t_0$.
Therefore, t_0 takes $Nt_0t_1t_2t_3t_1t_2t_0t_1t_0$ go to $Nt_0t_1t_0t_2t_3t_2t_3t_0$.

249.[012312012] Same as 138.

250. Now, we consider coset $N^{(012312013)}$.
However, $t_0t_1t_2t_3t_1t_2t_0t_1t_3 = (2,0,3)(t_0t_3t_1t_0t_2t_0t_2t_1)$
$= (2,0,3)(t_0t_1t_2t_0t_3t_0t_3t_2)^{(1,3,2)} \in Nt_0t_1t_2t_0t_3t_0t_3t_2$.
Therefore, t_3 takes $Nt_0t_1t_2t_3t_1t_2t_0t_1t_3$ go to $Nt_0t_1t_2t_0t_3t_0t_3t_2$.

251. Now, we consider the cosets stabilizer $N^{(012313030)}$.
$N^{(012313030)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_1t_3t_0t_3t_0N$ is at most: $\frac{|N|}{|N^{(012313030)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012313030)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a

representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_1t_3t_0t_3t_0t_0$, $Nt_0t_1t_2t_3t_1t_3t_0t_3t_0t_1$, $Nt_0t_1t_2t_3t_1t_3t_0t_3t_0t_2$ and $Nt_0t_1t_2t_3t_1t_3t_0t_3t_0t_3$ belong?
However, $Nt_0t_1t_2t_3t_1t_3t_0t_3t_0t_0 = Nt_0t_1t_2t_3t_1t_3t_0t_3 \in [01231303]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_1t_3t_0t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_3t_0t_3t_0t_3$, one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_3t_0t_3t_0t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_1t_3t_0t_3t_0t_2$.

252. [012313031] Same as 111.

253 . [012313032] Same as 70.

254. [012313301] Same as 109.

255. [012313230] Same as 118.

256. Now, we consider coset $N^{(012313231)}$.
However, $t_0t_1t_2t_3t_1t_3t_2t_3t_1 = (1,2,3)(t_1t_3t_1t_3t_1t_0t_2t_0)$
$= (1,2,3)(t_0t_1t_0t_1t_0t_2t_3t_2)^{(1,3,2,0)} \in Nt_0t_1t_0t_1t_0t_2t_3t_2$.
Therefore, t_1 takes $Nt_0t_1t_2t_3t_1t_3t_2t_3t_1$ go to $Nt_0t_1t_0t_1t_0t_2t_3t_2$.

257. Now, we consider coset $N^{(012313232)}$.
However, $t_0t_1t_2t_3t_1t_3t_2t_3t_2 = (1,2,3)(t_0t_3t_1t_3t_1t_2t_3t_1)$
$= (1,2,3)(t_0t_1t_2t_1t_2t_3t_1t_2)^{(1,3,2)} \in Nt_0t_1t_2t_1t_2t_3t_1t_2$.
Therefore, t_2 takes $Nt_0t_1t_2t_3t_1t_3t_2t_3t_2$ go to $Nt_0t_1t_2t_1t_2t_3t_1t_2$.

258. Now, we consider coset $N^{(012320201)}$.
However, $t_0t_1t_2t_3t_2t_0t_2t_0t_1 = (1,2)(0,3)(t_0t_3t_0t_2t_1t_3t_1t_0)$
$= (1,2)(0,3)(t_0t_1t_0t_2t_3t_1t_3t_0)^{(1,3)} \in Nt_0t_1t_0t_2t_3t_1t_3t_0$.
Therefore, t_1 takes $Nt_0t_1t_2t_3t_2t_0t_2t_0t_1$ go to $Nt_0t_1t_0t_2t_3t_1t_3t_0$.

259. Now, we consider coset $N^{(012320202)}$.
However, $t_0t_1t_2t_3t_2t_0t_2t_0t_2 = (t_0t_1t_3t_2t_0t_3t_0t_3)$

$= (t_0t_1t_2t_3t_0t_2t_0t_2)^{(2,3)} \in Nt_0t_1t_2t_3t_0t_2t_0t_2$.

Therefore, t_2 takes $Nt_0t_1t_2t_3t_2t_0t_2t_0t_2$ go to $Nt_0t_1t_2t_3t_0t_2t_0t_2$.

260. [012320203] Same as 130.

261. Now, we consider coset $N^{(012320210)}$.

However, $t_0t_1t_2t_3t_2t_0t_2t_1t_0 = (1,3,2)(t_3t_2t_0t_3t_2t_1t_2t_1)$

$= (1,3,2)(t_0t_1t_2t_0t_1t_3t_1t_3)^{(1,2,0,3)} \in Nt_0t_1t_2t_0t_1t_3t_1t_3$.

Therefore, t_0 takes $Nt_0t_1t_2t_3t_2t_0t_2t_1t_0$ go to $Nt_0t_1t_2t_0t_1t_3t_1t_3$.

262. Now, we consider the cosets stabilizer $N^{(012320212)}$.

$N^{(012320212)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_2t_0t_2t_1t_2N$ is at most: $\frac{|N|}{|N^{(012320212)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012320212)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_2t_0t_2t_1t_2t_0$, $Nt_0t_1t_2t_3t_2t_0t_2t_1t_2t_1$, $Nt_0t_1t_2t_3t_2t_0t_2t_1t_2t_2$ and $Nt_0t_1t_2t_3t_2t_0t_2t_1t_2t_3$ belong?

We know, $t_0t_1t_2t_3t_2t_0t_2t_1t_2 = (1,3,2)(t_0t_1t_2t_1t_3t_1t_0t_1t_0)^{(3,1,0)}$.

$Nt_0t_1t_2t_3t_2t_0t_2t_1t_2 = Nt_3t_0t_2t_0t_1t_0t_3t_0t_3$.

$Nt_0t_1t_2t_3t_2t_0t_2t_1t_2t_3 = N_3t_0t_2t_0t_1t_0t_3t_0 \in [01213101]$, then one symmetric generator goes back to $Nt_0t_1t_2t_1t_3t_1t_0t_1N$.

Moreover, $Nt_0t_1t_2t_3t_2t_0t_2t_1t_2t_2 = Nt_0t_1t_2t_3t_2t_0t_2t_1 \in [01232021]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_2t_0t_2t_1N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_0t_2t_1t_2t_1$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_0t_2t_1t_2t_0$.

263. [0123201213] Same as 88.

264. [012320230] Same as 185.

265. Now, we consider the cosets stabilizer $N^{(012320231)}$.

$N^{(012320231)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_2t_0t_2t_3t_1N$ is at most: $\frac{|N|}{|N^{(012320231)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012320231)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a

representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_2t_0t_2t_3t_1t_0$, $Nt_0t_1t_2t_3t_2t_0t_2t_3t_1t_1$, $Nt_0t_1t_2t_3t_2t_0t_2t_3t_1t_2$ and $Nt_0t_1t_2t_3t_2t_0t_2t_3t_1t_3$ belong?
However, $Nt_0t_1t_2t_3t_2t_0t_2t_3t_1t_1 = Nt_0t_1t_2t_3t_2t_0t_2t_3 \in [01232023]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_2t_0t_2t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_0t_2t_3t_1t_3$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_0t_2t_3t_1t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_0t_2t_3t_1t_2$.

266. [012320232] Same as 69.

267. [012321201] Same as 111.

268. [012321202] Same as 149.

269. Now, we consider the cosets stabilizer $N^{(012321203)}$.
$N^{(012321203)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_2t_1t_2t_0t_3N$ is at most: $\frac{|N|}{|N^{(012321203)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012321203)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_2t_1t_2t_0t_3t_0$, $Nt_0t_1t_2t_3t_2t_1t_2t_0t_3t_1$, $Nt_0t_1t_2t_3t_2t_1t_2t_0t_3t_2$ and $Nt_0t_1t_2t_3t_2t_1t_2t_0t_3t_3$ belong?
However, $Nt_0t_1t_2t_3t_2t_1t_2t_0t_3t_3 = Nt_0t_1t_2t_3t_2t_1t_2t_0 \in [01232120]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_2t_1t_2t_0N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_1t_2t_0t_3t_1$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_1t_2t_0t_3t_0$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_1t_2t_0t_3t_2$.

270. Now, we consider coset $N^{(012323030)}$.
However, $t_0t_1t_2t_3t_2t_3t_0t_3t_0 = (t_0t_1t_3t_0t_2t_0t_2t_3) = (t_0t_1t_2t_0t_3t_0t_3t_2)^{(2,3)} \in Nt_0t_1t_2t_0t_3t_0t_3t_2$.
Therefore, t_0 takes $Nt_0t_1t_2t_3t_2t_3t_0t_3t_0$ go to $Nt_0t_1t_2t_0t_3t_0t_3t_2$.

271. [012323031] Same as 75.

272. Now, we consider coset $N^{(012323032)}$.
However, $t_0t_1t_2t_3t_2t_3t_0t_3t_2 = (1,3,2)(t_1t_3t_0t_3t_2t_1t_2t_0)$

$= (1,3,2)(t_0t_1t_2t_1t_3t_0t_3t_2)^{(1,3,2,0)} \in Nt_0t_1t_2t_1t_3t_0t_3t_2$.
Therefore, t_2 takes $Nt_0t_1t_2t_3t_2t_3t_0t_3t_2$ go to $Nt_0t_1t_2t_1t_3t_0t_3t_2$.

273. Now, we consider coset $N^{(012323101)}$.
However, $t_0t_1t_2t_3t_2t_3t_1t_0t_1 = (2,3,0)(t_3t_2t_1t_3t_0t_1t_3t_2)$
$= (2,3,0)(t_0t_1t_2t_0t_3t_2t_0t_1)^{(1,2)(3,0)} \in Nt_0t_1t_2t_0t_3t_2t_0t_1$.
Therefore, t_1 takes $Nt_0t_1t_2t_3t_2t_3t_1t_0t_1$ go to $Nt_0t_1t_2t_0t_3t_2t_0t_1$.

274. Now, we consider the cosets stabilizer $N^{(012323130)}$.
$N^{(012323130)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_2t_3t_1t_3t_0N$ is at most: $\frac{|N|}{|N^{(012323130)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(012323130)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_3t_2t_3t_1t_3t_0t_0$, $Nt_0t_1t_2t_3t_2t_3t_1t_3t_0t_1$, $Nt_0t_1t_2t_3t_2t_3t_1t_3t_0t_2$ and $Nt_0t_1t_2t_3t_2t_3t_1t_3t_0t_3$ belong?
However, $Nt_0t_1t_2t_3t_2t_3t_1t_3t_0t_0 = Nt_0t_1t_2t_3t_2t_3t_1t_3 \in [01232313]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_3t_2t_3t_1t_3N$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_3t_1t_3t_0t_1$, one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_3t_1t_3t_0t_3$, and one symmetric generator goes to $Nt_0t_1t_2t_3t_2t_3t_1t_3t_0t_2$.

275. Now, we consider coset $N^{(012323131)}$.
However, $t_0t_1t_2t_3t_2t_3t_1t_3t_1 = (1,3,2)(t_0t_1t_3t_2t_3t_2t_1t_2)$
$= (1,3,2)(t_0t_1t_2t_3t_2t_3t_1t_3)^{(2,3)} \in Nt_0t_1t_2t_3t_2t_3t_1t_3$.
Therefore, t_1 takes $Nt_0t_1t_2t_3t_2t_3t_1t_3t_1$ go to $Nt_0t_1t_2t_3t_2t_3t_1t_3$.

276. Now, we consider coset $N^{(012323202)}$.
However, $t_0t_1t_2t_3t_2t_3t_2t_0t_2 = (t_0t_1t_0t_3t_0t_3t_2t_0)$
$= (t_0t_1t_0t_2t_0t_2t_3t_0)^{(3,2)} \in Nt_0t_1t_0t_2t_0t_2t_3t_0$.
Therefore, t_2 takes $Nt_0t_1t_2t_3t_2t_3t_2t_0t_2$ go to $Nt_0t_1t_0t_2t_0t_2t_3t_0$.

277. [012323201] Same as 29.

278. [012131010] Same as 262.

279. [012121012] Same as 66.

280. [012131013] Same as 161.

281. Now, we consider coset $N^{(012131020)}$.
However, $t_0t_1t_2t_1t_3t_1t_0t_2t_0 = (1,3,0)(t_2t_1t_3t_1t_3t_0t_2t_3)$
$= (1,3,0)(t_0t_1t_2t_1t_2t_3t_0t_2)^{(2,3,0)} \in Nt_0t_1t_2t_1t_2t_3t_0t_2$.
Therefore, t_0 takes $Nt_0t_1t_2t_1t_3t_1t_0t_2t_0$ go to $Nt_0t_1t_2t_1t_2t_3t_0t_2$.

282 . [012131021] Same as 212.

283. Now, we consider coset $N^{(012131023)}$.
However, $t_0t_1t_2t_1t_3t_1t_0t_2t_3 = (1,2)(3,0)(t_1t_3t_1t_3t_2t_3t_0t_1)$
$= (1,2)(3,0)(t_0t_1t_0t_1t_2t_1t_3t_0)^{(1,3,0)} \in Nt_0t_1t_0t_1t_2t_1t_3t_0$.
Therefore, t_3 takes $Nt_0t_1t_2t_1t_3t_1t_0t_2t_3$ go to $Nt_0t_1t_0t_1t_2t_1t_3t_0$.

284. [012131201] Same as 93.

285. [012131202] Same as 42.

286. [012131203] Same as 157.

287. Now, we consider the cosets stabilizer $N^{(0121312\overline{1}0)}$.
We know, $t_0t_1t_2t_1t_3t_1t_2t_1t_0 = t_0t_2t_3t_2t_1t_2t_3t_2t_0 = t_0t_3t_1t_3t_2t_3t_1t_3t_0$ then $N^{(012131210)} = \{e, (2,3,1), (2,1,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_1t_3t_1t_2t_1t_0N$ is at most: $\frac{|N|}{|N^{(012131210)}|} = \frac{24}{2} = 8$

The orbit of $N^{(012131210)}$ on $\{0,1,2,3\}$ are $\{0\}$, and $\{3,1,2\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_1t_3t_1t_2t_1t_0t_0$, and $Nt_0t_1t_2t_1t_3t_1t_2t_1t_0t_1$ belong?

However, $Nt_0t_1t_2t_1t_3t_1t_2t_1t_0t_0 = Nt_0t_1t_2t_1t_3t_1t_2t_1 \in$ [01213121], then one symmetric generator goes back to the double coset $Nt_0t_1t_2t_1t_3t_1t_2t_1N$, and three symmetric generators go to the double coset $Nt_0t_1t_2t_1t_3t_1t_2t_1t_0t_1$.

288. Now, we consider coset $N^{(012131301)}$.
However, $t_0t_1t_2t_1t_3t_1t_3t_0t_1 = (t_1t_0t_1t_3t_0t_2t_1t_2) = (t_0t_1t_0t_2t_1t_3t_1t_3)^{(1,0)(3,2)}$
$\in Nt_0t_1t_0t_2t_1t_3t_1t_3$. Therefore, t_1 takes $Nt_0t_1t_2t_1t_3t_1t_3t_0t_1$ go to $Nt_0t_1t_0t_2t_1t_3t_1t_3$.

289. [012131302] Same as 39.

290. Now, we consider coset $N^{(012131303)}$.
$t_0t_1t_2t_1t_3t_1t_3t_0t_3 = (1,0)(3,2)(t_2t_3t_2t_3t_1t_0t_2t_0) = (1,2)(3,0)(t_0t_1t_0t_1t_2t_3t_0t_3)^{(1,3,0,2)}$
$\in Nt_0t_1t_0t_1t_2t_3t_0t_3$. Therefore, t_3 takes $Nt_0t_1t_2t_1t_3t_1t_3t_0t_3$ go to $Nt_0t_1t_0t_1t_2t_3t_0t_3$.

291. [012030231] Same as 109.

Length 10.

1. Now, we consider coset $N^{(0101023021)}$.
However, $t_0t_1t_0t_1t_0t_2t_3t_0t_2t_1 = (0,1)(2,3)(t_3t_1t_3t_1t_0t_1t_2t_0t_3)$
$= (0,1)(2,3)(t_0t_1t_0t_1t_2t_1t_3t_2t_0)^{(2,0,3)} \in Nt_0t_1t_0t_1t_2t_1t_3t_2t_0$.
Therefore, t_1 takes $Nt_0t_1t_0t_1t_0t_2t_3t_0t_2t_1$ go to $Nt_0t_1t_0t_1t_2t_1t_3t_2t_0$.

2. Now, we consider coset $N^{(0101023023)}$.
However, $t_0t_1t_0t_1t_0t_2t_3t_0t_2t_3 = (2,0,3)(t_3t_1t_3t_1t_2t_0t_3t_2t_0)$
$= (2,0,3)(t_0t_1t_0t_1t_2t_3t_0t_2t_3)^{(0,3)} \in Nt_0t_1t_0t_1t_2t_3t_0t_2t_3$.
Therefore, t_3 takes $Nt_0t_1t_0t_1t_0t_2t_3t_0t_2t_3$ go to $Nt_0t_1t_0t_1t_2t_3t_0t_2t_3$.

3. Now, we consider coset $N^{(0101023031)}$.
However, $t_0t_1t_0t_1t_0t_2t_3t_0t_3t_1 = (0,1)(2,3)(t_0t_3t_2t_0t_2t_1t_2t_1t_3)$
$= (0,1)(2,3)(t_0t_1t_2t_0t_2t_3t_2t_3t_1)^{(1,3)} \in Nt_0t_1t_2t_0t_2t_3t_2t_3t_1$.
Therefore, t_1 takes $Nt_0t_1t_0t_1t_0t_2t_3t_0t_2t_3$ go to $Nt_0t_1t_2t_0t_2t_3t_2t_3t_1$.

4. Now, we consider coset $N^{(0101201301)}$.

However, $t_0t_1t_0t_1t_2t_0t_1t_3t_0t_1 = (1,0,2)(t_1t_0t_2t_0t_2t_3t_2t_1t_3)$
$= (1,0,2)(t_0t_1t_2t_1t_2t_3t_2t_0t_3)^{(0,1)} \in Nt_0t_1t_2t_1t_2t_3t_2t_0t_3$.
Therefore, t_1 takes $Nt_0t_1t_0t_1t_2t_0t_1t_3t_0t_1$ go to $Nt_0t_1t_2t_1t_2t_3t_2t_0t_3$.

5.

Now, we consider coset $N^{(0101201321)}$.
However, $t_0t_1t_0t_1t_2t_0t_1t_3t_2t_1 = (1,3,2)(t_1t_0t_2t_0t_2t_3t_2t_1t_3)$
$= (1,3,2)(t_0t_1t_0t_2t_3t_1t_3t_1t_3)^{(2,1)} \in Nt_0t_1t_0t_2t_3t_1t_3t_1t_3$.
Therefore, t_1 takes $Nt_0t_1t_0t_1t_2t_0t_1t_3t_2t_1$ go to $Nt_0t_1t_0t_2t_3t_1t_3t_1t_3$.

7. Now, we consider coset $N^{(0101203020)}$.

However, $t_0t_1t_0t_1t_2t_0t_3t_0t_2t_0 = (2,0,3)(t_2t_1t_2t_1t_3t_0t_2t_3t_2)$
$= (2,0,3)(t_0t_1t_0t_1t_2t_3t_0t_2t_0)^{(2,3,0)} \in Nt_0t_1t_0t_1t_2t_3t_0t_2t_0$.
Therefore, t_0 takes $Nt_0t_1t_0t_1t_2t_0t_3t_0t_2t_0$ go to $Nt_0t_1t_0t_1t_2t_3t_0t_2t_0$.

8. Now, we consider coset $N^{(0101203021)}$.

However, $t_0t_1t_0t_1t_2t_0t_3t_0t_2t_1 = t_0t_1t_0t_1t_2t_0t_3t_0t_2 \in Nt_0t_1t_0t_1t_2t_0t_3t_0t_2$.
Therefore, t_1 takes $Nt_0t_1t_0t_1t_2t_0t_3t_0t_2t_1$ go back to itself $Nt_0t_1t_0t_1t_2t_0t_3t_0t_2$

9. Now, we consider coset $N^{(0101203023)}$.

However, $t_0t_1t_0t_1t_2t_0t_3t_0t_2t_3 = t_0t_1t_0t_1t_2t_0t_3t_0t_2 \in Nt_0t_1t_0t_1t_2t_0t_3t_0t_2$.
Therefore, t_3 takes $Nt_0t_1t_0t_1t_2t_0t_3t_0t_2t_3$ go back to itself $Nt_0t_1t_0t_1t_2t_0t_3t_0t_2$.

10. Now, we consider coset $N^{(0101203201)}$.

However, $t_0t_1t_0t_1t_2t_0t_3t_2t_0t_1 = (0,1,3)(t_1t_2t_1t_0t_1t_0t_2t_3t_2)$
$= (0,1,3)(t_0t_1t_0t_2t_0t_2t_1t_3t_1)^{(1,2,0)} \in Nt_0t_1t_0t_2t_0t_2t_1t_3t_1$.
Therefore, t_1 takes $Nt_0t_1t_0t_1t_2t_0t_3t_2t_0t_1$ go to $Nt_0t_1t_0t_2t_0t_2t_1t_3t_1$.

11. Now, we consider coset $N^{(0101203203)}$.

However, $t_0t_1t_0t_1t_2t_0t_3t_2t_0t_3 = (0,2,3)(t_2t_1t_2t_1t_0t_2t_3t_0t_2)$

$= (0,2,3)(t_0t_1t_0t_1t_2t_0t_3t_2t_0)^{(2,0)} \in Nt_0t_1t_0t_1t_2t_0t_3t_2t_0$.

Therefore, t_3 takes $Nt_0t_1t_0t_1t_2t_0t_3t_2t_0t_3$ go to $Nt_0t_1t_0t_1t_2t_0t_3t_2t_0$.

12. Now, we consider coset $N^{(0101203210)}$.

However, $t_0t_1t_0t_1t_2t_0t_3t_2t_1t_0 = (1,3)(2,0)(t_3t_0t_2t_0t_2t_3t_1t_2t_3)$

$= (1,3)(2,0)(t_0t_1t_2t_1t_2t_0t_3t_2t_0)^{(1,0,3)} \in Nt_0t_1t_2t_1t_2t_0t_3t_2t_0$.

Therefore, t_0 takes $Nt_0t_1t_0t_1t_2t_0t_3t_2t_1t_0$ go to $Nt_0t_1t_2t_1t_2t_0t_3t_2t_0$.

13. Now, we consider coset $N^{(0101203212)}$.

However, $t_0t_1t_0t_1t_2t_0t_3t_2t_1t_2 = (1,3,2)(t_0t_2t_0t_1t_2t_3t_2t_1t_2)$

$= (1,3,2)(t_0t_1t_0t_2t_1t_3t_1t_2t_3)^{(1,2)} \in Nt_0t_1t_0t_2t_1t_3t_1t_2t_3$.

Therefore, t_2 takes $Nt_0t_1t_0t_1t_2t_0t_3t_2t_1t_2$ go to $Nt_0t_1t_0t_2t_1t_3t_1t_2t_3$.

14. Now, we consider coset $N^{(0101210312)}$.

However, $t_0t_1t_0t_1t_2t_1t_0t_3t_1t_2 = (1,0)(3,2)(t_0t_3t_1t_3t_1t_2t_1t_3t_0)$

$= (1,0)(3,2)(t_0t_1t_2t_1t_2t_3t_2t_1t_0)^{(1,3,2)} \in Nt_0t_1t_2t_1t_2t_3t_2t_1t_0$.

Therefore, t_2 takes $Nt_0t_1t_0t_1t_2t_1t_0t_3t_1t_2$ go to $Nt_0t_1t_2t_1t_2t_3t_2t_1t_0$.

15. Now, we consider coset $N^{(0101213103)}$.

However, $t_0t_1t_0t_1t_2t_1t_3t_1t_0t_3 = (t_1t_2t_1t_3t_1t_3t_2t_0t_2)$

$= (t_0t_1t_0t_2t_0t_2t_1t_3t_1)^{(1,2,3,0)} \in Nt_0t_1t_0t_2t_0t_2t_1t_3t_1$.

Therefore, t_3 takes $Nt_0t_1t_0t_1t_2t_1t_3t_1t_0t_3$ go to $Nt_0t_1t_0t_2t_0t_2t_1t_3t_1$.

16. Now, we consider coset $N^{(0101213120)}$.

However, $t_0t_1t_0t_1t_2t_1t_3t_1t_2t_0 = (0,3)(1,2)(t_0t_2t_0t_1t_0t_1t_2t_3t_1)$

$= (0,3)(1,2)(t_0t_1t_0t_2t_0t_2t_1t_3t_2)^{(1,2)} \in Nt_0t_1t_0t_2t_0t_2t_1t_3t_2$.

Therefore, t_0 takes $Nt_0t_1t_0t_1t_2t_1t_3t_1t_2t_0$ go to $Nt_0t_1t_0t_2t_0t_2t_1t_3t_2$.

17. Now, we consider coset $N^{(0101213121)}$.

However, $t_0t_1t_0t_1t_2t_1t_3t_1t_2t_1 = (1,3)(0,2)(t_3t_1t_2t_0t_2t_1t_2t_3t_0)$

$= (1,3)(0,2)(t_0t_1t_2t_3t_2t_1t_2t_0t_3)^{(3,0)} \in Nt_0t_1t_2t_3t_2t_1t_2t_0t_3$.

Therefore, t_1 takes $Nt_0t_1t_0t_1t_2t_1t_3t_1t_2t_1$ go to $Nt_0t_1t_2t_3t_2t_1t_2t_0t_3$.

18. Now, we consider coset $N^{(0101213201)}$.
However, $t_0t_1t_0t_1t_2t_1t_3t_2t_0t_1 = (1,2)(0,3)(t_2t_1t_2t_1t_2t_3t_0t_2t_3)$
$= (1,2)(0,3)(t_0t_1t_0t_1t_0t_2t_3t_0t_2)^{(2,3,0)} \in Nt_0t_1t_0t_1t_0t_2t_3t_0t_2$.
Therefore, t_1 takes $Nt_0t_1t_0t_1t_2t_1t_3t_2t_0t_1$ go to $Nt_0t_1t_0t_1t_0t_2t_3t_0t_2$.

19. Now, we consider coset $N^{(0101210313)}$.
However, $t_0t_1t_0t_1t_2t_1t_0t_3t_1t_3 = (2,0,3)(t_0t_2t_0t_1t_0t_1t_2t_3t_0)$
$= (2,0,3)(t_0t_1t_0t_2t_0t_2t_1t_3t_0)^{(1,2)} \in Nt_0t_1t_0t_2t_0t_2t_1t_3t_0$.
Therefore, t_3 takes $Nt_0t_1t_0t_1t_2t_1t_0t_3t_1t_3$ go to $Nt_0t_1t_0t_2t_0t_2t_1t_3t_0$.

20. Now, we consider coset $N^{(0101213030)}$.
However, $t_0t_1t_0t_1t_2t_1t_3t_0t_3t_0 = (t_3t_1t_2t_3t_2t_0t_1t_3t_1)$
$= (t_0t_1t_2t_0t_2t_3t_1t_0t_1)^{(0,3)} \in Nt_0t_1t_2t_0t_2t_3t_1t_0t_1$.
Therefore, t_0 takes $Nt_0t_1t_0t_1t_2t_1t_3t_0t_3t_0$ go to $Nt_0t_1t_2t_0t_2t_3t_1t_0t_1$.

21. Now, we consider coset $N^{(0101213202)}$.
However, $t_0t_1t_0t_1t_2t_1t_3t_2t_0t_2 = (1,3)(0,2)(t_1t_2t_1t_0t_1t_3t_0t_2t_0)$
$= (1,3)(0,2)(t_0t_1t_0t_2t_0t_3t_2t_1t_2)^{(1,2,0)} \in Nt_0t_1t_0t_2t_0t_3t_2t_1t_2$.
Therefore, t_2 takes $Nt_0t_1t_0t_1t_2t_1t_3t_2t_0t_2$ go to $Nt_0t_1t_0t_2t_0t_3t_2t_1t_2$.

22. Now, we consider coset $N^{(0101213203)}$.
However, $t_0t_1t_0t_1t_2t_1t_3t_2t_0t_3 = (2,3,0)(t_1t_2t_1t_0t_1t_3t_0t_2t_0)$
$= (2,3,0)(t_0t_1t_2t_0t_1t_3t_0t_1t_2)^{(1,3,0)} \in Nt_0t_1t_2t_0t_1t_3t_0t_1t_2$.
Therefore, t_3 takes $Nt_0t_1t_0t_1t_2t_1t_3t_2t_0t_3$ go to $Nt_0t_1t_2t_0t_1t_3t_0t_1t_2$.

23. Now, we consider coset $N^{(0101213213)}$.
However, $t_0t_1t_0t_1t_2t_1t_3t_2t_0t_3 = (1,2,3)(t_0t_2t_0t_2t_1t_2t_3t_1t_2)$
$= (1,2,3)(t_0t_1t_2t_0t_1t_3t_0t_1t_2)^{(1,2)} \in Nt_0t_1t_0t_1t_2t_1t_3t_2t_1$.
Therefore, t_3 takes $Nt_0t_1t_0t_1t_2t_1t_3t_2t_1t_3$ go to $Nt_0t_1t_0t_1t_2t_1t_3t_2t_1$.

24. Now, we consider coset $N^{(0101230202)}$.
However, $t_0t_1t_0t_1t_2t_3t_0t_2t_0t_2 = (2,0,3)(t_0t_1t_2t_3t_2t_0t_2t_3t_1)$
$= (2,0,3)(t_0t_1t_2t_3t_2t_0t_2t_3t_1) \in Nt_0t_1t_2t_3t_2t_0t_2t_3t_1$.
Therefore, t_2 takes $Nt_0t_1t_0t_1t_2t_3t_0t_2t_0t_2$ go to $Nt_0t_1t_2t_3t_2t_0t_2t_3t_1$.

25. Now, we consider coset $N^{(0101230203)}$.
However, $t_0t_1t_0t_1t_2t_3t_0t_2t_0t_3 = (2,3,0)(t_3t_1t_3t_1t_0t_3t_2t_3t_0)$
$= (2,3,0)(t_0t_1t_0t_1t_2t_0t_3t_0t_2)^{(2,0,3)} \in Nt_0t_1t_0t_1t_2t_0t_3t_0t_2$.
Therefore, t_3 takes $Nt_0t_1t_0t_1t_2t_3t_0t_2t_0t_3$ go to $Nt_0t_1t_0t_1t_2t_0t_3t_0t_2$.

26. Now, we consider coset $N^{(0101230212)}$.
However, $t_0t_1t_0t_1t_2t_3t_0t_2t_1t_2 = (0,1)(2,3)(t_3t_2t_0t_1t_2t_1t_3t_1t_3)$
$= (0,1)(2,3)(t_0t_1t_2t_3t_1t_3t_0t_3t_0)^{(1,2,0,3)} \in Nt_0t_1t_2t_3t_1t_3t_0t_3t_0$.
Therefore, t_2 takes $Nt_0t_1t_0t_1t_2t_3t_0t_2t_1t_2$ go to $Nt_0t_1t_2t_3t_1t_3t_0t_3t_0$.

27. Now, we consider coset $N^{(0101230213)}$.
However, $t_0t_1t_0t_1t_2t_3t_0t_2t_1t_3 = (2,0,3)(t_0t_3t_0t_1t_0t_1t_3t_2t_0)$
$= (2,0,3)(t_0t_1t_0t_2t_0t_2t_1t_3t_0)^{(1,3,2)} \in Nt_0t_1t_0t_2t_0t_2t_1t_3t_0$.
Therefore, t_3 takes $Nt_0t_1t_0t_1t_2t_3t_0t_2t_1t_3$ go to $Nt_0t_1t_0t_2t_0t_2t_1t_3t_0$.

28. Now, we consider coset $N^{(0101230230)}$.
However, $t_0t_1t_0t_1t_2t_3t_0t_2t_3t_0 = (2,0,3)(t_3t_1t_3t_1t_3t_2t_0t_3t_2)$
$= (2,0,3)(t_0t_1t_0t_1t_0t_2t_3t_0t_2)^{(0,3)} \in Nt_0t_1t_0t_1t_0t_2t_3t_0t_2$.
Therefore, t_0 takes $Nt_0t_1t_0t_1t_2t_3t_0t_2t_3t_0$ go to $Nt_0t_1t_0t_1t_0t_2t_3t_0t_2$.

29. Now, we consider coset $N^{(0101230231)}$.
However, $t_0t_1t_0t_1t_2t_3t_0t_2t_3t_1 = (1,0,2)(t_1t_3t_0t_3t_1t_2t_3t_2t_3)$
$= (1,0,2)(t_0t_1t_2t_1t_0t_3t_1t_3t_1)^{(0,3)} \in Nt_0t_1t_2t_1t_0t_3t_1t_3t_1$.
Therefore, t_1 takes $Nt_0t_1t_0t_1t_2t_3t_0t_2t_3t_1$ go to $Nt_0t_1t_2t_1t_0t_3t_1t_3t_1$.

30. Now, we consider coset $N^{(0101231303)}$.
However, $t_0t_1t_0t_1t_2t_3t_1t_3t_0t_3 = (0,1)(2,3)(t_0t_1t_3t_2t_1t_2t_0t_2t_0)$

$= (0,1)(2,3)(t_0t_1t_2t_3t_1t_3t_0t_3t_0)^{(2,3)} \in Nt_0t_1t_2t_3t_1t_3t_0t_3t_0$.
Therefore, t_3 takes $Nt_0t_1t_0t_1t_2t_3t_1t_3t_0t_3$ go to $Nt_0t_1t_2t_3t_1t_3t_0t_3t_0$.

31. Now, we consider coset $N^{(0101232020)}$.
However, $t_0t_1t_0t_1t_2t_3t_2t_0t_2t_0 = (t_2t_1t_2t_0t_3t_1t_3t_2t_1)$
$= (t_0t_1t_0t_2t_3t_1t_3t_0t_1)^{(2,0)} \in Nt_0t_1t_0t_2t_3t_1t_3t_0t_1$.
Therefore, t_0 takes $Nt_0t_1t_0t_1t_2t_3t_2t_0t_2t_0$ go to $Nt_0t_1t_0t_2t_3t_1t_3t_0t_1$.

32. Now, we consider coset $N^{(0101232023)}$.
However, $t_0t_1t_0t_1t_2t_3t_2t_0t_2t_3 = (1,0,2)(t_3t_0t_1t_0t_3t_0t_2t_1t_2)$
$= (1,0,2)(t_0t_1t_2t_1t_0t_1t_3t_2t_3)^{(1,0,3,2)} \in Nt_0t_1t_2t_1t_0t_1t_3t_2t_3$.
Therefore, t_3 takes $Nt_0t_1t_0t_1t_2t_3t_2t_0t_2t_3$ go to $Nt_0t_1t_2t_1t_0t_1t_3t_2t_3$.

33. Now, we consider coset $N^{(0101232120)}$.
However, $t_0t_1t_0t_1t_2t_3t_2t_1t_2t_0 = (1,3,0)(t_3t_2t_3t_0t_2t_1t_3t_2t_3)$
$= (1,3,0)(t_0t_1t_0t_2t_1t_3t_0t_1t_0)^{(1,2,0,3)} \in Nt_0t_1t_0t_2t_1t_3t_0t_1t_0$.
Therefore, t_0 takes $Nt_0t_1t_0t_1t_2t_3t_2t_1t_2t_0$ go to $Nt_0t_1t_0t_2t_1t_3t_0t_1t_0$.

34. Now, we consider coset $N^{(0101232313)}$.
However, $t_0t_1t_0t_1t_2t_3t_2t_3t_1t_3 = (1,3,0)(t_3t_2t_3t_1t_0t_3t_0t_1t_0)$
$= (1,3,0)(t_0t_1t_0t_2t_3t_0t_3t_2t_3)^{(1,2,0,3)} \in Nt_0t_1t_0t_2t_3t_0t_3t_2t_3$.
Therefore, t_3 takes $Nt_0t_1t_0t_1t_2t_3t_2t_3t_1t_3$ go to $Nt_0t_1t_0t_2t_3t_0t_3t_2t_3$.

35. Now, we consider coset $N^{(0101232321)}$.
However, $t_0t_1t_0t_1t_2t_3t_2t_3t_2t_1 = (t_0t_1t_0t_2t_1t_2t_3t_1t_3)$
$= t_0t_1t_0t_2t_1t_2t_3t_1t_3 \in Nt_0t_1t_0t_2t_1t_2t_3t_1t_3$.
Therefore, t_1 takes $Nt_0t_1t_0t_1t_2t_3t_2t_3t_2t_1$ go to $Nt_0t_1t_0t_2t_1t_2t_3t_1t_3$.

36. Now, we consider coset $N^{(0102012321)}$.
However, $t_0t_1t_0t_2t_0t_1t_2t_3t_2t_1 = (1,2,0)(t_0t_2t_0t_3t_0t_3t_2t_1t_2)$
$= (1,2,0)(t_0t_1t_0t_2t_0t_2t_1t_3t_1)^{(1,2,3)} \in Nt_0t_1t_0t_2t_0t_2t_1t_3t_1$.
Therefore, t_1 takes $Nt_0t_1t_0t_2t_0t_1t_2t_3t_2t_1$ go to $Nt_0t_1t_0t_2t_0t_2t_1t_3t_1$.

37. Now, we consider coset $N^{(0102012323)}$.
However, $t_0t_1t_0t_2t_0t_1t_2t_3t_2t_3 = (1,2,0)(t_2t_0t_2t_1t_3t_0t_3t_0t_2)$
$= (1,2,0)(t_0t_1t_0t_2t_3t_1t_3t_1t_0)^{(1,0,2)} \in Nt_0t_1t_0t_2t_3t_1t_3t_1t_0$.
Therefore, t_3 takes $Nt_0t_1t_0t_2t_0t_1t_2t_3t_2t_3$ go to $Nt_0t_1t_0t_2t_3t_1t_3t_1t_0$.

38. Now, we consider coset $N^{(0102013021)}$.
However, $t_0t_1t_0t_2t_0t_1t_3t_0t_2t_1 = (1,0)(2,3)(t_3t_1t_3t_2t_1t_2t_0t_3t_0)$
$= (1,0)(2,3)(t_0t_1t_0t_2t_1t_2t_3t_0t_3)^{(0,3)} \in Nt_0t_1t_0t_2t_1t_2t_3t_0t_3$.
Therefore, t_1 takes $Nt_0t_1t_0t_2t_0t_1t_3t_0t_2t_1$ go to $Nt_0t_1t_0t_2t_1t_2t_3t_0t_3$.

39. Now, we consider coset $N^{(0102013023)}$.
However, $t_0t_1t_0t_2t_0t_1t_3t_0t_2t_3 = (t_0t_1t_0t_2t_0t_1t_3t_0t_2)$
$= t_0t_1t_0t_2t_0t_1t_3t_0t_2 \in Nt_0t_1t_0t_2t_0t_1t_3t_0t_2$.
Therefore, t_3 takes $Nt_0t_1t_0t_2t_0t_1t_3t_0t_2t_3$ go to itself $Nt_0t_1t_0t_2t_0t_1t_3t_0t_2$.

40. Now, we consider coset $N^{(0102013030)}$.
However, $t_0t_1t_0t_2t_0t_1t_3t_0t_3t_0 = (2,0,3)(t_2t_0t_2t_1t_2t_3t_2t_1t_2)$
$= (2,0,3)(t_0t_1t_0t_2t_0t_3t_0t_2t_0)^{(1,0,2)} \in Nt_0t_1t_0t_2t_0t_3t_0t_2t_0$.
Therefore, t_0 takes $Nt_0t_1t_0t_2t_0t_1t_3t_0t_3t_0$ go to $Nt_0t_1t_0t_2t_0t_3t_0t_2t_0$.

41. Now, we consider coset $N^{(0102013031)}$.
However, $t_0t_1t_0t_2t_0t_1t_3t_0t_3t_1 = (1,3,0)(t_3t_1t_2t_0t_2t_0t_1t_0t_3)$
$= (1,3,0)(t_0t_1t_2t_3t_2t_3t_1t_3t_0)^{(0,3)} \in Nt_0t_1t_2t_3t_2t_3t_1t_3t_0$.
Therefore, t_1 takes $Nt_0t_1t_0t_2t_0t_1t_3t_0t_3t_1$ go to $Nt_0t_1t_2t_3t_2t_3t_1t_3t_0$.

42. Now, we consider coset $N^{(0102013130)}$.
However, $t_0t_1t_0t_2t_0t_1t_3t_1t_3t_0 = (1,2)(0,3)(t_2t_0t_1t_2t_0t_3t_0t_3t_2)$
$= (1,2)(0,3)(t_0t_1t_2t_0t_1t_3t_1t_3t_0)^{(1,2,0,3)} \in Nt_0t_1t_2t_0t_1t_3t_1t_3t_0$.
Therefore, t_0 takes $Nt_0t_1t_0t_2t_0t_1t_3t_1t_3t_0$ go to $Nt_0t_1t_2t_0t_1t_3t_1t_3t_0$.

43. Now, we consider coset $N^{(0102013131)}$.
However, $t_0t_1t_0t_2t_0t_1t_3t_1t_3t_1 = (1,2,0)(t_3t_0t_1t_3t_1t_2t_0t_3t_0)$
$= (1,2,0)(t_0t_1t_2t_0t_2t_3t_1t_0t_1)^{(1,0,3,2)} \in Nt_0t_1t_2t_0t_2t_3t_1t_0t_1$.
Therefore, t_1 takes $Nt_0t_1t_0t_2t_0t_1t_3t_1t_3t_1$ go to $Nt_0t_1t_2t_0t_2t_3t_1t_0t_1$.

44. Now, we consider coset $N^{(0102021301)}$.
However, $t_0t_1t_0t_2t_0t_2t_1t_3t_0t_1 = (1,0,3)(t_0t_2t_0t_2t_3t_1t_0t_3t_2)$
$= (1,0,3)(t_0t_1t_0t_1t_2t_3t_0t_2t_1)^{(1,2,3)} \in Nt_0t_1t_0t_1t_2t_3t_0t_2t_1$.
Therefore, t_1 takes $Nt_0t_1t_0t_2t_0t_2t_1t_3t_0t_1$ go to $Nt_0t_1t_0t_1t_2t_3t_0t_2t_1$.

45. Now, we consider coset $N^{(0102021303)}$.
However, $t_0t_1t_0t_2t_0t_2t_1t_3t_0t_3 = (1,3,0)(t_0t_2t_0t_2t_1t_2t_0t_3t_2)$
$= (1,0,3)(t_0t_1t_0t_1t_2t_1t_0t_3t_1)^{(1,2)} \in Nt_0t_1t_0t_1t_2t_1t_0t_3t_1$.
Therefore, t_3 takes $Nt_0t_1t_0t_2t_0t_2t_1t_3t_0t_3$ go to $Nt_0t_1t_0t_1t_2t_1t_0t_3t_1$.

46. Now, we consider coset $N^{(0102021310)}$.
However, $t_0t_1t_0t_2t_0t_2t_1t_3t_1t_0 = (2,3,0)(t_2t_0t_2t_0t_1t_2t_3t_1t_2)$
$= (2,3,0)(t_0t_1t_0t_1t_2t_0t_3t_2t_0)^{(1,0,2)} \in Nt_0t_1t_0t_1t_2t_0t_3t_2t_0$.
Therefore, t_0 takes $Nt_0t_1t_0t_2t_0t_2t_1t_3t_1t_0$ go to $Nt_0t_1t_0t_1t_2t_0t_3t_2t_0$.

47. Now, we consider coset $N^{(0102021312)}$.
However, $t_0t_1t_0t_2t_0t_2t_1t_3t_1t_2 = (t_3t_0t_3t_0t_1t_0t_2t_0t_3)$
$= (t_0t_1t_0t_1t_2t_1t_3t_1t_0)^{(1,0,3,2)} \in Nt_0t_1t_0t_1t_2t_1t_3t_1t_0$.
Therefore, t_2 takes $Nt_0t_1t_0t_2t_0t_2t_1t_3t_1t_2$ go to $Nt_0t_1t_0t_1t_2t_1t_3t_1t_0$.

48. Now, we consider coset $N^{(0102021313)}$.
However, $t_0t_1t_0t_2t_0t_2t_1t_3t_1t_3 = (1,3,0)(t_0t_3t_0t_1t_0t_3t_1t_2t_1)$
$= (1,3,0)(t_0t_1t_0t_2t_0t_1t_2t_3t_2)^{(1,3,2)} \in Nt_0t_1t_0t_2t_0t_1t_2t_3t_2$.
Therefore, t_3 takes $Nt_0t_1t_0t_2t_0t_2t_1t_3t_1t_3$ go to $Nt_0t_1t_0t_2t_0t_1t_2t_3t_2$.

49. Now, we consider coset $N^{(0102021320)}$.
However, $t_0t_1t_0t_2t_0t_2t_1t_3t_2t_0 = (1,3,0)(t_0t_2t_0t_2t_1t_0t_3t_2t_3)$

$= (1,3,0)(t_0t_1t_0t_1t_2t_0t_3t_1t_3)^{(1,2)} \in Nt_0t_1t_0t_1t_2t_0t_3t_1t_3$.

Therefore, t_0 takes $Nt_0t_1t_0t_2t_0t_2t_1t_3t_2t_0$ go to $Nt_0t_1t_0t_1t_2t_0t_3t_1t_3$.

50. Now, we consider coset $N^{(0102021323)}$.

However, $t_0t_1t_0t_2t_0t_2t_1t_3t_2t_3 = (1,2,0)(t_0t_3t_2t_0t_3t_0t_3t_1t_2)$

$= (1,2,0)(t_0t_1t_2t_0t_1t_0t_1t_3t_2)^{(1,3)} \in Nt_0t_1t_2t_0t_1t_0t_1t_3t_2$.

Therefore, t_3 takes $Nt_0t_1t_0t_2t_0t_2t_1t_3t_2t_3$ go to $Nt_0t_1t_2t_0t_1t_0t_1t_3t_2$.

51. Now, we consider coset $N^{(0102030130)}$.

However, $t_0t_1t_0t_2t_0t_3t_0t_1t_3t_0 = (1,0,3)(t_2t_0t_2t_0t_3t_2t_1t_2t_3)$

$= (1,0,3)(t_0t_1t_0t_1t_2t_0t_3t_0t_2)^{(1,0,2,3)} \in Nt_0t_1t_0t_1t_2t_0t_3t_0t_2$.

Therefore, t_0 takes $Nt_0t_1t_0t_2t_0t_3t_0t_1t_3t_0$ go to $Nt_0t_1t_0t_1t_2t_0t_3t_0t_2$.

52. Now, we consider coset $N^{(0102030132)}$.

However, $t_0t_1t_0t_2t_0t_3t_0t_1t_3t_2 = (1,0)(2,3)(t_3t_0t_3t_2t_3t_1t_2t_0t_3)$

$= (1,0)(2,3)(t_0t_1t_0t_2t_0t_3t_2t_1t_0)^{(1,0,3)} \in Nt_0t_1t_0t_2t_0t_3t_2t_1t_0$.

Therefore, t_2 takes $Nt_0t_1t_0t_2t_0t_3t_0t_1t_3t_2$ go to $Nt_0t_1t_0t_2t_0t_3t_2t_1t_0$.

53. Now, we consider coset $N^{(0102030201)}$.

However, $t_0t_1t_0t_2t_0t_3t_0t_2t_0t_1 = (1,0,3)(t_1t_2t_1t_0t_1t_2t_3t_1t_3)$

$= (1,0,3)(t_0t_1t_0t_2t_0t_1t_3t_0t_3)^{(1,2,0)} \in Nt_0t_1t_0t_2t_0t_1t_3t_0t_3$.

Therefore, t_1 takes $Nt_0t_1t_0t_2t_0t_3t_0t_2t_0t_1$ go to $Nt_0t_1t_0t_2t_0t_1t_3t_0t_3$.

54. Now, we consider coset $N^{(0102030210)}$.

However, $t_0t_1t_0t_2t_0t_3t_0t_2t_1t_0 = (1,3,2)(t_3t_1t_2t_0t_2t_0t_1t_0t_3)$

$= (1,3,2)(t_0t_1t_2t_3t_2t_3t_1t_3t_0)^{(3,0)} \in Nt_0t_1t_2t_3t_2t_3t_1t_3t_0$.

Therefore, t_1 takes $Nt_0t_1t_0t_2t_0t_3t_0t_2t_1t_0$ go to $Nt_0t_1t_2t_3t_2t_3t_1t_3t_0$.

55. Now, we consider coset $N^{(0102030212)}$.

However, $t_0t_1t_0t_2t_0t_3t_0t_2t_1t_2 = (1,0)(3,2)(t_3t_0t_1t_0t_1t_3t_2t_1t_3)$

$= (1,0)(3,2)(t_0t_1t_2t_1t_2t_0t_3t_2t_0)^{(1,0,3,2)} \in Nt_0t_1t_2t_1t_2t_0t_3t_2t_0$.

Therefore, t_1 takes $Nt_0t_1t_0t_2t_0t_3t_0t_2t_1t_2$ go to $Nt_0t_1t_2t_1t_2t_0t_3t_2t_0$.

56. Now, we consider coset $N^{(0102030320)}$.
However, $t_0t_1t_0t_2t_0t_3t_0t_3t_2t_0 = (t_0t_1t_0t_3t_2t_3t_2t_0t_2)$
$= (t_0t_1t_0t_2t_3t_2t_3t_0t_3)^{(3,2)} \in Nt_0t_1t_0t_2t_3t_2t_3t_0t_3$.
Therefore, t_0 takes $Nt_0t_1t_0t_2t_0t_3t_0t_3t_2t_0$ go to $Nt_0t_1t_0t_2t_3t_2t_3t_0t_3$.

57. Now, we consider coset $N^{(0102030321)}$.
However, $t_0t_1t_0t_2t_0t_3t_0t_3t_2t_1 = (1,3,0)(t_2t_3t_0t_2t_3t_1t_0t_1t_2)$
$= (1,3,0)(t_0t_1t_2t_0t_1t_3t_2t_3t_0)^{(3,1)(0,2)} \in Nt_0t_1t_2t_0t_1t_3t_2t_3t_0$.
Therefore, t_1 takes $Nt_0t_1t_0t_2t_0t_3t_0t_3t_2t_1$ go to $Nt_0t_1t_2t_0t_1t_3t_2t_3t_0$.

58. Now, we consider coset $N^{(0102031310)}$.
However, $t_0t_1t_0t_2t_0t_3t_1t_3t_1t_0 = (3,2,0)(t_1t_0t_2t_1t_3t_2t_1t_2t_3)$
$= (3,2,0)(t_0t_1t_2t_0t_3t_2t_1t_2t_3)^{(0,1)} \in Nt_0t_1t_2t_0t_3t_2t_1t_2t_3$.
Therefore, t_0 takes $Nt_0t_1t_0t_2t_0t_3t_1t_3t_1t_0$ go to $Nt_0t_1t_2t_0t_3t_2t_1t_2t_3$.

59. Now, we consider coset $N^{(0102031313)}$.
However, $t_0t_1t_0t_2t_0t_3t_1t_3t_1t_3 = (1,2,0)(t_3t_0t_1t_3t_1t_3t_0t_3t_0)$
$= (1,2,0)(t_0t_1t_2t_0t_2t_3t_1t_0t_1)^{(1,0,2,3)} \in Nt_0t_1t_2t_0t_2t_3t_1t_0t_1$.
Therefore, t_3 takes $Nt_0t_1t_0t_2t_0t_3t_1t_3t_1t_3$ go to $Nt_0t_1t_2t_0t_2t_3t_1t_0t_1$.

60. Now, we consider coset $N^{(0102032102)}$.
However, $t_0t_1t_0t_2t_0t_3t_2t_1t_0t_2 = (1,3,0)(t_3t_1t_2t_0t_2t_1t_2t_3t_0)$
$= (1,3,0)(t_0t_1t_2t_3t_2t_1t_2t_0t_3)^{(0,3)} \in Nt_0t_1t_2t_3t_2t_1t_2t_0t_3$.
Therefore, t_2 takes $Nt_0t_1t_0t_2t_0t_3t_2t_1t_0t_2$ go to $Nt_0t_1t_2t_3t_2t_1t_2t_0t_3$.

61. Now, we consider coset $N^{(0102032121)}$.
However, $t_0t_1t_0t_2t_0t_3t_2t_1t_2t_1 = (1,3)(2,0)(t_2t_0t_2t_0t_1t_0t_3t_1t_2)$
$= (1,3)(2,0)(t_0t_1t_0t_1t_2t_1t_3t_2t_0)^{(1,0,2)} \in Nt_0t_1t_0t_1t_2t_1t_3t_2t_0$.
Therefore, t_1 takes $Nt_0t_1t_0t_2t_0t_3t_2t_1t_2t_1$ go to $Nt_0t_1t_0t_1t_2t_1t_3t_2t_0$.

62. Now, we consider coset $N^{(0102032130)}$.
However, $t_0t_1t_0t_2t_0t_3t_2t_1t_3t_0 = (1,3,0)(t_1t_2t_1t_0t_3t_1t_3t_0t_3)$
$= (1,3,0)(t_0t_1t_0t_2t_3t_0t_3t_2t_3)^{(1,2,0)} \in Nt_0t_1t_0t_2t_3t_0t_3t_2t_3$.
Therefore, t_0 takes $Nt_0t_1t_0t_2t_0t_3t_2t_1t_3t_0$ go to $Nt_0t_1t_0t_2t_3t_0t_3t_2t_3$.

63. Now, we consider coset $N^{(0102123031)}$.
However, $t_0t_1t_0t_2t_1t_2t_3t_0t_3t_1 = (1,3)(2,0)(t_3t_1t_3t_2t_3t_1t_0t_3t_2)$
$= (1,3)(2,0)(t_0t_1t_0t_2t_0t_1t_3t_0t_2)^{(3,0)} \in Nt_0t_1t_0t_2t_0t_1t_3t_0t_2$.
Therefore, t_1 takes $Nt_0t_1t_0t_2t_1t_2t_3t_0t_3t_1$ go to $Nt_0t_1t_0t_2t_0t_1t_3t_0t_2$.

64. Now, we consider coset $N^{(0102123032)}$.
However, $t_0t_1t_0t_2t_1t_2t_3t_0t_3t_2 = (2,3,0)(t_0t_1t_3t_0t_2t_3t_0t_3t_0)$
$= (2,3,0)(t_0t_1t_2t_0t_3t_2t_0t_2t_0)^{(3,2)} \in Nt_0t_1t_2t_0t_3t_2t_0t_2t_0$.
Therefore, t_2 takes $Nt_0t_1t_0t_2t_1t_2t_3t_0t_3t_2$ go to $Nt_0t_1t_2t_0t_3t_2t_0t_2t_0$.

65. Now, we consider coset $N^{(010212331)}$.
However, $t_0t_1t_0t_2t_1t_2t_3t_1t_3t_1 = (t_0t_1t_0t_1t_2t_3t_2t_3t_2)$
$= t_0t_1t_0t_1t_2t_3t_2t_3t_2 \in Nt_0t_1t_0t_1t_2t_3t_2t_3t_2$.
Therefore, t_1 takes $Nt_0t_1t_0t_2t_1t_2t_3t_1t_3t_1$ go to $Nt_0t_1t_0t_1t_2t_3t_2t_3t_2$.

66. Now, we consider coset $N^{(0102130102)}$.
However, $t_0t_1t_0t_2t_1t_3t_0t_1t_0t_2 = (2,0,3)(t_2t_3t_2t_3t_1t_0t_1t_3t_1)$
$= (2,0,3)(t_0t_1t_0t_1t_2t_3t_2t_1t_2)^{(1,3,0,2)} \in Nt_0t_1t_0t_1t_2t_3t_2t_1t_2$.
Therefore, t_2 takes $Nt_0t_1t_0t_2t_1t_3t_0t_1t_0t_2$ go to $Nt_0t_1t_0t_1t_2t_3t_2t_1t_2$.

67. Now, we consider coset $N^{(0102130103)}$.
However, $t_0t_1t_0t_2t_1t_3t_0t_1t_0t_3 = (1,2,3)(t_1t_3t_1t_0t_2t_3t_2t_3t_1)$
$= (1,2,3)(t_0t_1t_0t_2t_3t_1t_3t_1t_0)^{(1,3,0,2)} \in Nt_0t_1t_0t_2t_3t_1t_3t_1t_0$.
Therefore, t_3 takes $Nt_0t_1t_0t_2t_1t_3t_0t_1t_0t_3$ go to $Nt_0t_1t_0t_2t_3t_1t_3t_1t_0$.

68. Now, we consider coset $N^{(0102131010)}$.
However, $t_0t_1t_0t_2t_1t_3t_1t_0t_1t_0 = (1,0,2)(t_1t_2t_0t_1t_2t_3t_2t_3t_1)$

$= (1,0,2)(t_0t_1t_2t_0t_1t_3t_1t_3t_0)^{(1,2,0)} \in Nt_0t_1t_2t_0t_1t_3t_1t_3t_0$.

Therefore, t_0 takes $Nt_0t_1t_0t_2t_1t_3t_1t_0t_1t_0$ go to $Nt_0t_1t_2t_0t_1t_3t_1t_3t_0$.

69. Now, we consider coset $N^{(0102131012)}$.

However, $t_0t_1t_0t_2t_1t_3t_1t_0t_1t_2 = (2,0,3)(t_2t_0t_2t_1t_3t_2t_3t_2t_0)$

$= (2,0,3)(t_0t_1t_0t_2t_3t_0t_3t_0t_1)^{(1,0,2)} \in Nt_0t_1t_0t_2t_3t_0t_3t_0t_1$.

Therefore, t_2 takes $Nt_0t_1t_0t_2t_1t_3t_1t_0t_1t_2$ go to $Nt_0t_1t_0t_2t_3t_0t_3t_0t_1$.

70. Now, we consider coset $N^{(0102131013)}$.

However, $t_0t_1t_0t_2t_1t_3t_1t_0t_1t_3 = (2,0,3)(t_2t_1t_0t_3t_1t_3t_2t_3t_0)$

$= (2,0,3)(t_0t_1t_2t_3t_1t_3t_0t_3t_0)^{(0,2)} \in Nt_0t_1t_2t_3t_1t_3t_0t_3t_0$.

Therefore, t_3 takes $Nt_0t_1t_0t_2t_1t_3t_1t_0t_1t_3$ go to $Nt_0t_1t_2t_3t_1t_3t_0t_3t_0$.

71. Now, we consider coset $N^{(0102131210)}$.

However, $t_0t_1t_0t_2t_1t_3t_1t_2t_1t_0 = (1,3,2)(t_3t_1t_0t_1t_2t_1t_0t_1t_3)$

$= (1,3,2)(t_0t_1t_2t_1t_3t_1t_2t_1t_0)^{(0,3,2)} \in Nt_0t_1t_2t_1t_3t_1t_2t_1t_0$.

Therefore, t_0 takes $Nt_0t_1t_0t_2t_1t_3t_1t_2t_1t_0$ go to $Nt_0t_1t_2t_1t_3t_1t_2t_1t_0$.

72. Now, we consider coset $N^{(0102131212)}$.

However, $t_0t_1t_0t_2t_1t_3t_1t_2t_1t_2 = (1,0,2)(t_1t_2t_0t_1t_0t_3t_0t_3t_1)$

$= (1,0,2)(t_0t_1t_2t_0t_2t_3t_2t_3t_0)^{(0,1,2)} \in Nt_0t_1t_2t_0t_2t_3t_2t_3t_0$.

Therefore, t_2 takes $Nt_0t_1t_0t_2t_1t_3t_1t_2t_1t_2$ go to $Nt_0t_1t_2t_0t_2t_3t_2t_3t_0$.

73. Now, we consider coset $N^{(0102131231)}$.

However, $t_0t_1t_0t_2t_1t_3t_1t_2t_3t_1 = (1,3,2)(t_0t_2t_0t_2t_1t_0t_3t_1t_2)$

$= (1,3,2)(t_0t_1t_0t_1t_2t_0t_3t_2t_1)^{(1,2)} \in Nt_0t_1t_0t_1t_2t_0t_3t_2t_1$.

Therefore, t_1 takes $Nt_0t_1t_0t_2t_1t_3t_1t_2t_3t_1$ go to $Nt_0t_1t_0t_1t_2t_0t_3t_2t_1$.

74. Now, we consider coset $N^{(0102132121)}$.

However, $t_0t_1t_0t_2t_1t_3t_2t_1t_2t_1 = (1,0,2)(t_1t_2t_0t_1t_0t_3t_0t_3t_2)$

$= (1,0,2)(t_0t_1t_2t_0t_2t_3t_2t_3t_1)^{(1,2,0)} \in Nt_0t_1t_2t_0t_2t_3t_2t_3t_1$.

Therefore, t_1 takes $Nt_0t_1t_0t_2t_1t_3t_2t_1t_2t_1$ go to $Nt_0t_1t_2t_0t_2t_3t_2t_3t_1$.

75. Now, we consider coset $N^{(0102303010)}$.
However, $t_0t_1t_0t_2t_3t_0t_3t_0t_1t_0 = (1,0,3)(t_1t_2t_1t_0t_2t_3t_2t_1t_2)$
$= (1,0,3)(t_0t_1t_0t_2t_1t_3t_1t_0t_1)^{(1,2,0)} \in Nt_0t_1t_0t_2t_1t_3t_1t_0t_1$.
Therefore, t_0 takes $Nt_0t_1t_0t_2t_3t_0t_3t_0t_1t_0$ go to $Nt_0t_1t_0t_2t_1t_3t_1t_0t_1$.

76. Now, we consider coset $N^{(0102303012)}$.
However, $t_0t_1t_0t_2t_3t_0t_3t_0t_1t_2 = (1,0,3)(t_2t_3t_1t_0t_1t_2t_1t_3t_1)$
$= (1,0,3)(t_0t_1t_2t_3t_2t_0t_2t_1t_2)^{(1,3,0,2)} \in Nt_0t_1t_2t_3t_2t_0t_2t_1t_2$.
Therefore, t_2 takes $Nt_0t_1t_0t_2t_3t_0t_3t_0t_1t_2$ go to $Nt_0t_1t_2t_3t_2t_0t_2t_1t_2$.

77. Now, we consider coset $N^{(0102303230)}$.
However, $t_0t_1t_0t_2t_3t_0t_3t_2t_3t_0 = (2,3,0)(t_3t_2t_3t_2t_1t_0t_1t_0t_2)$
$= (2,3,0)(t_0t_1t_0t_1t_2t_3t_2t_3t_1)^{(1,2)(0,3)} \in Nt_0t_1t_0t_1t_2t_3t_2t_3t_1$.
Therefore, t_0 takes $Nt_0t_1t_0t_2t_3t_0t_3t_2t_3t_0$ go to $Nt_0t_1t_0t_1t_2t_3t_2t_3t_1$.

78. Now, we consider coset $N^{(0102303231)}$.
However, $t_0t_1t_0t_2t_3t_0t_3t_2t_3t_1 = (1,2,3)(t_3t_0t_3t_2t_1t_2t_0t_2t_0)$
$= (2,3,1)(t_0t_1t_0t_2t_3t_2t_1t_2t_1)^{(1,0,3)} \in Nt_0t_1t_0t_2t_3t_2t_1t_2t_1$.
Therefore, t_1 takes $Nt_0t_1t_0t_2t_3t_0t_3t_2t_3t_1$ go to $Nt_0t_1t_0t_2t_3t_2t_1t_2t_1$.

79. Now, we consider coset $N^{(0102303232)}$.
However, $t_0t_1t_0t_2t_3t_0t_3t_2t_3t_2 = (0,2,3)(t_2t_0t_2t_1t_2t_3t_1t_0t_3)$
$= (2,3,1)(t_0t_1t_0t_2t_0t_3t_2t_1t_3)^{(1,0,2)} \in Nt_0t_1t_0t_2t_0t_3t_2t_1t_3$.
Therefore, t_2 takes $Nt_0t_1t_0t_2t_3t_0t_3t_2t_3t_2$ go to $Nt_0t_1t_0t_2t_0t_3t_2t_1t_3$.

80. Now, we consider coset $N^{(0102312021)}$.
However, $t_0t_1t_0t_2t_3t_1t_2t_0t_2t_1 = (t_2t_0t_1t_0t_1t_3t_1t_2t_3)$
$= (t_0t_1t_2t_1t_2t_3t_2t_0t_3)^{(1,0,2)} \in Nt_0t_1t_2t_1t_2t_3t_2t_0t_3$.
Therefore, t_1 takes $Nt_0t_1t_0t_2t_3t_1t_2t_0t_2t_1$ go to $Nt_0t_1t_2t_1t_2t_3t_2t_0t_3$.

81. Now, we consider coset $N^{(0102312023)}$.
However, $t_0t_1t_0t_2t_3t_1t_2t_0t_2t_3 = (1,3,2)(t_1t_0t_2t_1t_3t_0t_3t_2t_0)$
$= (1,3,2)(t_0t_1t_2t_0t_2t_1t_2t_3t_1)^{(1,0)(3,2)} \in Nt_0t_1t_2t_0t_2t_1t_2t_3t_1$.
Therefore, t_3 takes $Nt_0t_1t_0t_2t_3t_1t_2t_0t_2t_3$ go to $Nt_0t_1t_2t_0t_2t_1t_2t_3t_1$.

82. Now, we consider coset $N^{(0102312302)}$.
However, $t_0t_1t_0t_2t_3t_1t_2t_3t_0t_2 = (1,0,2)(t_3t_2t_3t_1t_0t_1t_0t_3t_0)$
$= (1,0,2)(t_0t_1t_0t_2t_3t_2t_3t_0t_3)^{(1,2)(3,0)} \in Nt_0t_1t_0t_2t_3t_2t_3t_0t_3$.
Therefore, t_2 takes $Nt_0t_1t_0t_2t_3t_1t_2t_3t_0t_2$ go to $Nt_0t_1t_0t_2t_3t_2t_3t_0t_3$.

83. Now, we consider coset $N^{(0102313010)}$.
However, $t_0t_1t_0t_2t_3t_1t_3t_0t_1t_0 = (2,0,3)(t_2t_1t_3t_0t_3t_2t_3t_0t_1)$
$= (2,0,3)(t_0t_1t_2t_3t_2t_0t_2t_3t_1)^{(2,3,0)} \in Nt_0t_1t_2t_3t_2t_0t_2t_3t_1$.
Therefore, t_0 takes $Nt_0t_1t_0t_2t_3t_1t_3t_0t_1t_0$ go to $Nt_0t_1t_2t_3t_2t_0t_2t_3t_1$.

84. Now, we consider coset $N^{(0102313012)}$.
However, $t_0t_1t_0t_2t_3t_1t_3t_0t_1t_2 = (t_2t_1t_2t_1t_0t_3t_0t_2t_0)$
$= (t_0t_1t_0t_1t_2t_3t_2t_0t_2)^{(2,0)} \in Nt_0t_1t_0t_1t_2t_3t_2t_0t_2$.
Therefore, t_2 takes $Nt_0t_1t_0t_2t_3t_1t_3t_0t_1t_2$ go to $Nt_0t_1t_0t_1t_2t_3t_2t_0t_2$.

85. Now, we consider coset $N^{(0102313101)}$.
However, $t_0t_1t_0t_2t_3t_1t_3t_1t_0t_1 = (1,3,0)(t_2t_0t_2t_3t_0t_1t_2t_0t_2)$
$= (1,3,0)(t_0t_1t_0t_2t_1t_3t_0t_1t_0)^{(2,3,1,0)} \in Nt_0t_1t_0t_2t_1t_3t_0t_1t_0$.
Therefore, t_1 takes $Nt_0t_1t_0t_2t_3t_1t_3t_1t_0t_1$ go to $Nt_0t_1t_0t_2t_1t_3t_0t_1t_0$.

86. Now, we consider coset $N^{(0102313103)}$.
However, $t_0t_1t_0t_2t_3t_1t_3t_1t_0t_3 = (1,0,2)(t_1t_2t_1t_0t_1t_2t_0t_3t_0)$
$= (1,0,2)(t_0t_1t_0t_2t_0t_1t_2t_3t_2)^{(1,2,0)} \in Nt_0t_1t_0t_2t_0t_1t_2t_3t_2$.
Therefore, t_3 takes $Nt_0t_1t_0t_2t_3t_1t_3t_1t_0t_3$ go to $Nt_0t_1t_0t_2t_0t_1t_2t_3t_2$.

87. Now, we consider coset $N^{(0102321230)}$.
However, $t_0t_1t_0t_2t_3t_2t_1t_2t_3t_0 = (1,2,0)(t_3t_1t_0t_1t_3t_1t_2t_0t_2)$

$= (1,2,0)(t_0t_1t_2t_1t_0t_1t_3t_2t_3)^{(3,2,0)} \in Nt_0t_1t_2t_1t_0t_1t_3t_2t_3$.

Therefore, t_0 takes $Nt_0t_1t_0t_2t_3t_2t_1t_2t_3t_0$ go to $Nt_0t_1t_2t_1t_0t_1t_3t_2t_3$.

88. Now, we consider coset $N^{(0102321232)}$.

However, $t_0t_1t_0t_2t_3t_2t_1t_2t_3t_2 = (1,2,0)(t_2t_3t_1t_0t_1t_3t_1t_2t_0)$

$= (1,2,0)(t_0t_1t_2t_3t_2t_1t_2t_0t_3)^{(1,3,0,2)} \in Nt_0t_1t_2t_3t_2t_1t_2t_0t_3$.

Therefore, t_2 takes $Nt_0t_1t_0t_2t_3t_2t_1t_2t_3t_2$ go to $Nt_0t_1t_2t_3t_2t_1t_2t_0t_3$.

89. Now, we consider coset $N^{(0102323030)}$.

However, $t_0t_1t_0t_2t_3t_2t_3t_0t_3t_0 = (t_0t_1t_0t_3t_0t_2t_0t_2t_3)$

$= (t_0t_1t_0t_2t_0t_3t_0t_3t_2)^{(3,2)} \in Nt_0t_1t_0t_2t_0t_3t_0t_3t_2$.

Therefore, t_0 takes $Nt_0t_1t_0t_2t_3t_2t_3t_0t_3t_0$ go to $Nt_0t_1t_0t_2t_0t_3t_0t_3t_2$.

90. Now, we consider coset $N^{(0102323031)}$.

However, $t_0t_1t_0t_2t_3t_2t_3t_0t_3t_1 = (1,3,2)(t_3t_2t_3t_1t_0t_2t_1t_0t_3)$

$= (1,3,2)(t_0t_1t_0t_2t_3t_1t_2t_3t_0)^{(1,2)(3,0)} \in Nt_0t_1t_0t_2t_3t_1t_2t_3t_0$.

Therefore, t_1 takes $Nt_0t_1t_0t_2t_3t_2t_3t_0t_3t_1$ go to $Nt_0t_1t_0t_2t_3t_1t_2t_3t_0$.

91. Now, we consider coset $N^{(0120101320)}$.

However, $t_0t_1t_2t_0t_1t_0t_1t_3t_2t_0 = (1,3,0)(t_2t_3t_1t_2t_3t_0t_2t_3t_1)$

$= (1,3,0)(t_0t_1t_2t_0t_1t_3t_0t_1t_2)^{(1,3,0,2)} \in Nt_0t_1t_2t_0t_1t_3t_0t_1t_2$.

Therefore, t_0 takes $Nt_0t_1t_2t_0t_1t_0t_1t_3t_2t_0$ go to $Nt_0t_1t_2t_0t_1t_3t_0t_1t_2$.

92. Now, we consider coset $N^{(0120101321)}$.

However, $t_0t_1t_2t_0t_1t_0t_1t_3t_2t_1 = (2,3,0)(t_0t_3t_0t_2t_0t_2t_3t_1t_2)$

$= (2,3,0)(t_0t_1t_0t_2t_0t_2t_1t_3t_2)^{(1,3)} \in Nt_0t_1t_0t_2t_0t_2t_1t_3t_2$.

Therefore, t_1 takes $Nt_0t_1t_2t_0t_1t_0t_1t_3t_2t_1$ go to $Nt_0t_1t_0t_2t_0t_2t_1t_3t_2$.

93. Now, we consider coset $N^{(0120130121)}$.

However, $t_0t_1t_2t_0t_1t_3t_0t_1t_2t_1 = (2,3,1)(t_3t_0t_3t_0t_2t_0t_1t_2t_3)$

$= (2,3,1)(t_0t_1t_0t_1t_2t_1t_3t_2t_0)^{(1,0,3)} \in Nt_0t_1t_0t_1t_2t_1t_3t_2t_0$.

Therefore, t_1 takes $Nt_0t_1t_2t_0t_1t_3t_0t_1t_2t_1$ go to $Nt_0t_1t_0t_1t_2t_1t_3t_2t_0$.

94. Now, we consider coset $N^{(0120130123)}$.
However, $t_0t_1t_2t_0t_1t_3t_0t_1t_2t_3 = (2,3,1)(t_3t_2t_0t_3t_2t_3t_2t_1t_0)$
$= (2,3,1)(t_0t_1t_2t_0t_1t_0t_1t_3t_2)^{(1,2,0,3)} \in Nt_0t_1t_2t_0t_1t_0t_1t_3t_2$.
Therefore, t_3 takes $Nt_0t_1t_2t_0t_1t_3t_0t_1t_2t_3$ go to $Nt_0t_1t_2t_0t_1t_0t_1t_3t_2$.

95. Now, we consider coset $N^{(0120130301)}$.
However, $t_0t_1t_2t_0t_1t_3t_0t_3t_0t_1 = (0,2,1)(t_1t_0t_1t_3t_2t_0t_3t_0t_3)$
$= (0,2,1)(t_0t_1t_0t_2t_3t_1t_2t_1t_2)^{(1,0)(2,3)} \in Nt_0t_1t_0t_2t_3t_1t_2t_1t_2$.
Therefore, t_1 takes $Nt_0t_1t_2t_0t_1t_3t_0t_3t_0t_1$ go to $Nt_0t_1t_0t_2t_3t_1t_2t_1t_2$.

96. Now, we consider coset $N^{(0120130303)}$.
However, $t_0t_1t_2t_0t_1t_3t_0t_3t_0t_3 = (t_0t_1t_2t_1t_0t_3t_1t_3t_1)$
$= t_0t_1t_2t_1t_0t_3t_1t_3t_1 \in Nt_0t_1t_2t_1t_0t_3t_1t_3t_1$.
Therefore, t_3 takes $Nt_0t_1t_2t_0t_1t_3t_0t_3t_0t_3$ go to $Nt_0t_1t_2t_1t_0t_3t_1t_3t_1$.

97. Now, we consider coset $N^{(0120131301)}$.
However, $t_0t_1t_2t_0t_1t_3t_1t_3t_0t_1 = (1,3)(2,0)(t_1t_2t_1t_0t_1t_2t_3t_2t_3)$
$= (1,3)(2,0)(t_0t_1t_0t_2t_0t_1t_3t_1t_3)^{(1,2,0)} \in Nt_0t_1t_0t_2t_0t_1t_3t_1t_3$.
Therefore, t_1 takes $Nt_0t_1t_2t_0t_1t_3t_1t_3t_0t_1$ go to $Nt_0t_1t_0t_2t_0t_1t_3t_1t_3$.

98. Now, we consider coset $N^{(0120131302)}$.
However, $t_0t_1t_2t_0t_1t_3t_1t_3t_0t_2 = (1,2,0)(t_2t_0t_2t_1t_0t_3t_0t_2t_0)$
$= (1,2,0)(t_0t_1t_0t_2t_1t_3t_1t_0t_1)^{(1,0,2)} \in Nt_0t_1t_0t_2t_1t_3t_1t_0t_1$.
Therefore, t_2 takes $Nt_0t_1t_2t_0t_1t_3t_1t_3t_0t_2$ go to $Nt_0t_1t_0t_2t_1t_3t_1t_0t_1$.

99. Now, we consider coset $N^{(0120132303)}$.
However, $t_0t_1t_2t_0t_1t_3t_2t_3t_0t_3 = (1,3,2)(t_2t_3t_2t_0t_2t_1t_2t_1t_0)$
$= (1,3,2)(t_0t_1t_0t_2t_0t_3t_0t_3t_2)^{(1,3)(0,2)} \in Nt_0t_1t_0t_2t_0t_3t_0t_3t_2$.
Therefore, t_3 takes $Nt_0t_1t_2t_0t_1t_3t_2t_3t_0t_3$ go to $Nt_0t_1t_0t_2t_0t_3t_0t_3t_2$.

100. Now, we consider coset $N^{(0120212312)}$.
However, $t_0t_1t_2t_0t_2t_1t_2t_3t_1t_2 = (0,3,2)(t_1t_0t_1t_3t_2t_0t_3t_1t_3)$
$= (0,3,2)(t_0t_1t_0t_2t_3t_1t_2t_0t_2)^{(1,0)(3,2)} \in Nt_0t_1t_0t_2t_3t_1t_2t_0t_2$.
Therefore, t_2 takes $Nt_0t_1t_2t_0t_2t_1t_2t_3t_1t_2$ go to $Nt_0t_1t_0t_2t_3t_1t_2t_0t_2$.

101. Now, we consider coset $N^{(0120213232)}$.
However, $t_0t_1t_2t_0t_2t_1t_3t_2t_3t_2 = (1,0,2)(t_1t_0t_2t_0t_1t_3t_0t_3t_0)$
$= (1,0,2)(t_0t_1t_2t_1t_0t_3t_1t_3t_1)^{(1,0)} \in Nt_0t_1t_2t_1t_0t_3t_1t_3t_1$.
Therefore, t_2 takes $Nt_0t_1t_2t_0t_2t_1t_3t_2t_3t_2$ go to $Nt_0t_1t_2t_1t_0t_3t_1t_3t_1$.

102. Now, we consider coset $N^{(0120231010)}$.
However, $t_0t_1t_2t_0t_2t_3t_1t_0t_1t_0 = (1,3,2)(t_1t_2t_1t_3t_1t_0t_2t_0t_2)$
$= (1,3,2)(t_0t_1t_0t_2t_0t_3t_1t_3t_1)^{(1,2,3,0)} \in Nt_0t_1t_0t_2t_0t_3t_1t_3t_1$.
Therefore, t_0 takes $Nt_0t_1t_2t_0t_2t_3t_1t_0t_1t_0$ go to $Nt_0t_1t_0t_2t_0t_3t_1t_3t_1$.

103. Now, we consider coset $N^{(0120231012)}$.
However, $t_0t_1t_2t_0t_2t_3t_1t_0t_1t_2 = (1,3,2)(t_1t_2t_1t_3t_1t_2t_0t_2t_0)$
$= (1,3,2)(t_0t_1t_0t_2t_0t_1t_3t_1t_3)^{(1,2,3,0)} \in Nt_0t_1t_0t_2t_0t_1t_3t_1t_3$.
Therefore, t_2 takes $Nt_0t_1t_2t_0t_2t_3t_1t_0t_1t_2$ go to $Nt_0t_1t_0t_2t_0t_1t_3t_1t_3$.

104. Now, we consider coset $N^{(0120231013)}$.
However, $t_0t_1t_2t_0t_2t_3t_1t_0t_1t_3 = (t_3t_1t_3t_1t_2t_1t_0t_3t_0)$
$= (t_0t_1t_0t_1t_2t_1t_3t_0t_3)^{(3,0)} \in Nt_0t_1t_0t_1t_2t_1t_3t_0t_3$.
Therefore, t_3 takes $Nt_0t_1t_2t_0t_2t_3t_1t_0t_1t_3$ go to $Nt_0t_1t_0t_1t_2t_1t_3t_0t_3$.

105. Now, we consider coset $N^{(0120232301)}$.
However, $t_0t_1t_2t_0t_2t_3t_2t_3t_0t_1 = (1,2,0)(t_2t_0t_2t_1t_0t_3t_0t_1t_0)$
$= (1,2,0)(t_0t_1t_0t_2t_1t_3t_1t_2t_1)^{(1,0,2)} \in Nt_0t_1t_0t_2t_1t_3t_1t_2t_1$.
Therefore, t_1 takes $Nt_0t_1t_2t_0t_2t_3t_2t_3t_0t_1$ go to $Nt_0t_1t_0t_2t_1t_3t_1t_2t_1$.

106. Now, we consider coset $N^{(0120232302)}$.
However, $t_0t_1t_2t_0t_2t_3t_2t_3t_0t_2 = (3,2,0)(t_3t_1t_2t_3t_2t_0t_2t_0t_3)$

$= (3,2,0)(t_0t_1t_2t_0t_2t_3t_2t_3t_0)^{(0,3)} \in Nt_0t_1t_2t_0t_2t_3t_2t_3t_0$.

Therefore, t_2 takes $Nt_0t_1t_2t_0t_2t_3t_2t_3t_0t_2$ go to $Nt_0t_1t_2t_0t_2t_3t_2t_3t_0$.

107. Now, we consider coset $N^{(0120232310)}$.

However, $t_0t_1t_2t_0t_2t_3t_2t_3t_1t_0 = (1,2,0)(t_2t_0t_2t_1t_0t_3t_1t_0t_1)$

$= (1,2,0)(t_0t_1t_0t_2t_1t_3t_2t_1t_2)^{(1,0,2)} \in Nt_0t_1t_0t_2t_1t_3t_2t_1t_2$.

Therefore, t_0 takes $Nt_0t_1t_2t_0t_2t_3t_2t_3t_1t_0$ go to $Nt_0t_1t_0t_2t_1t_3t_2t_1t_2$.

108. Now, we consider coset $N^{(0120232312)}$.

However, $t_0t_1t_2t_0t_2t_3t_2t_3t_1t_2 = (3,2,0)(t_1t_2t_1t_0t_3t_1t_3t_2t_0)$

$= (3,2,0)(t_0t_1t_0t_2t_3t_0t_3t_1t_2)^{(1,2,0)} \in Nt_0t_1t_0t_2t_3t_0t_3t_1t_2$.

Therefore, t_2 takes $Nt_0t_1t_2t_0t_2t_3t_2t_3t_1t_2$ go to $Nt_0t_1t_0t_2t_3t_0t_3t_1t_2$.

109. Now, we consider coset $N^{(0120232313)}$.

However, $t_0t_1t_2t_0t_2t_3t_2t_3t_1t_3 = (1,2)(3,0)(t_0t_3t_0t_3t_0t_2t_1t_0t_1)$

$= (1,2)(3,0)(t_0t_1t_0t_1t_0t_2t_3t_0t_3)^{(1,3)} \in Nt_0t_1t_0t_1t_0t_2t_3t_0t_3$.

Therefore, t_3 takes $Nt_0t_1t_2t_0t_2t_3t_2t_3t_1t_3$ go to $Nt_0t_1t_0t_1t_0t_2t_3t_0t_3$.

110. Now, we consider coset $N^{(0120320202)}$.

However, $t_0t_1t_2t_0t_3t_2t_0t_2t_0t_2 = (1,2,0)(t_1t_0t_1t_2t_0t_2t_3t_1t_3)$

$= (1,2,0)(t_0t_1t_0t_2t_1t_2t_3t_0t_3)^{(1,0)} \in Nt_0t_1t_0t_2t_1t_2t_3t_0t_3$.

Therefore, t_2 takes $Nt_0t_1t_2t_0t_3t_2t_0t_2t_0t_2$ go to $Nt_0t_1t_0t_2t_1t_2t_3t_0t_3$.

111. Now, we consider coset $N^{(0120320231)}$.

However, $t_0t_1t_2t_0t_3t_2t_0t_2t_3t_1 = (1,2,3)(t_1t_0t_1t_2t_1t_3t_0t_3t_0)$

$= (1,2,3)(t_0t_1t_0t_2t_0t_3t_1t_3t_1)^{(1,0)} \in Nt_0t_1t_0t_2t_0t_3t_1t_3t_1$.

Therefore, t_1 takes $Nt_0t_1t_2t_0t_3t_2t_0t_2t_3t_1$ go to $Nt_0t_1t_0t_2t_0t_3t_1t_3t_1$.

112. Now, we consider coset $N^{(01201013230)}$.

However, $t_0t_1t_2t_1t_0t_1t_3t_2t_3t_0 = (1,2,3)(t_1t_2t_1t_2t_3t_0t_3t_1t_3)$

$= (1,2,3)(t_0t_1t_0t_1t_2t_3t_2t_0t_2)^{(1,2,3,0)} \in Nt_0t_1t_0t_1t_2t_3t_2t_0t_2$.

Therefore, t_0 takes $Nt_0t_1t_2t_1t_0t_1t_3t_2t_3t_0$ go to $Nt_0t_1t_0t_1t_2t_3t_2t_0t_2$.

113. Now, we consider coset $N^{(01201013231)}$.
However, $t_0t_1t_2t_1t_0t_1t_3t_2t_3t_1 = (0,3,2)(t_3t_1t_2t_1t_3t_1t_0t_2t_0)$
$= (0,3,2)(t_0t_1t_2t_1t_0t_1t_3t_2t_3)^{(3,0)} \in Nt_0t_1t_2t_1t_0t_1t_3t_2t_3$.
Therefore, t_1 takes $Nt_0t_1t_2t_1t_0t_1t_3t_2t_3t_1$ go to $Nt_0t_1t_2t_1t_0t_1t_3t_2t_3$.

114. Now, we consider coset $N^{(01201013232)}$.
However, $t_0t_1t_2t_1t_0t_1t_3t_2t_3t_2 = (1,2,3)(t_2t_1t_2t_3t_0t_3t_1t_3t_0)$
$= (1,2,3)(t_0t_1t_0t_2t_3t_2t_1t_2t_3)^{(2,3,0)} \in Nt_0t_1t_0t_2t_3t_2t_1t_2t_3$.
Therefore, t_2 takes $Nt_0t_1t_2t_1t_0t_1t_3t_2t_3t_2$ go to $Nt_0t_1t_0t_2t_3t_2t_1t_2t_3$.

115. Now, we consider coset $N^{(01210313 10)}$.
However, $t_0t_1t_2t_1t_0t_3t_1t_3t_1t_0 = (0,3,2)(t_2t_0t_2t_0t_3t_1t_2t_3t_1)$
$= (0,3,2)(t_0t_1t_0t_1t_2t_3t_0t_2t_3)^{(1,0,2,3)} \in Nt_0t_1t_0t_1t_2t_3t_0t_2t_3$.
Therefore, t_0 takes $Nt_0t_1t_2t_1t_0t_3t_1t_3t_1t_0$ go to $Nt_0t_1t_0t_1t_2t_3t_0t_2t_3$.

116. Now, we consider coset $N^{(01210313 12)}$.
However, $t_0t_1t_2t_1t_0t_3t_1t_3t_1t_2 = (1,0,2)(t_1t_0t_2t_1t_2t_0t_3t_2t_3)$
$= (1,0,2)(t_0t_1t_2t_0t_2t_1t_3t_2t_3)^{(1,0)} \in Nt_0t_1t_2t_0t_2t_1t_3t_2t_3$.
Therefore, t_2 takes $Nt_0t_1t_2t_1t_0t_3t_1t_3t_1t_2$ go to $Nt_0t_1t_2t_0t_2t_1t_3t_2t_3$.

117. Now, we consider coset $N^{(01210313 13)}$.
However, $t_0t_1t_2t_1t_0t_3t_1t_3t_1t_3 = t_0t_1t_2t_0t_1t_3t_0t_3t_0$
$= t_0t_1t_2t_0t_1t_3t_0t_3t_0 \in Nt_0t_1t_2t_0t_1t_3t_0t_3t_0$.
Therefore, t_3 takes $Nt_0t_1t_2t_1t_0t_3t_1t_3t_1t_3$ go to $Nt_0t_1t_2t_0t_1t_3t_0t_3t_0$.

118. Now, we consider coset $N^{(01212320 30)}$.
However, $t_0t_1t_2t_1t_2t_3t_2t_0t_3t_0 = (2,0,3)(t_2t_3t_2t_3t_1t_2t_3t_0t_2)$
$= (2,0,3)(t_0t_1t_0t_1t_2t_0t_1t_3t_0)^{(1,3,0,2)} \in Nt_0t_1t_0t_1t_2t_0t_1t_3t_0$.
Therefore, t_0 takes $Nt_0t_1t_2t_1t_2t_3t_2t_0t_3t_0$ go to $Nt_0t_1t_0t_1t_2t_0t_1t_3t_0$.

119. Now, we consider coset $N^{(0121232031)}$.

However, $t_0t_1t_2t_1t_2t_3t_2t_0t_3t_1 = (1,0,3)(t_1t_2t_1t_3t_0t_1t_0t_2t_3)$

$= (1,0,3)(t_0t_1t_0t_2t_3t_0t_3t_1t_2)^{(1,2,3,0)} \in Nt_0t_1t_0t_2t_3t_0t_3t_1t_2$.

Therefore, t_1 takes $Nt_0t_1t_2t_1t_2t_3t_2t_0t_3t_1$ go to $Nt_0t_1t_0t_2t_3t_0t_3t_1t_2$.

120. Now, we consider coset $N^{(0121232032)}$.

However, $t_0t_1t_2t_1t_2t_3t_2t_0t_3t_2 = (t_1t_2t_1t_0t_3t_2t_0t_1t_0)$

$= (t_0t_1t_0t_2t_3t_1t_2t_0t_2)^{(1,2,0)} \in Nt_0t_1t_0t_2t_3t_1t_2t_0t_2$.

Therefore, t_2 takes $Nt_0t_1t_2t_1t_2t_3t_2t_0t_3t_2$ go to $Nt_0t_1t_0t_2t_3t_1t_2t_0t_2$.

121. Now, we consider coset $N^{(0121232102)}$.

However, $t_0t_1t_2t_1t_2t_3t_2t_1t_0t_2 = (t_1t_3t_1t_3t_2t_3t_1t_0t_3)$

$= (t_0t_1t_0t_1t_2t_1t_0t_3t_1)^{(1,3,0)} \in Nt_0t_1t_0t_1t_2t_1t_0t_3t_1$.

Therefore, t_2 takes $Nt_0t_1t_2t_1t_2t_3t_2t_1t_0t_2$ go to $Nt_0t_1t_0t_1t_2t_1t_0t_3t_1$.

122. Now, we consider coset $N^{(0123130301)}$.

However, $t_0t_1t_2t_3t_1t_3t_0t_3t_0t_1 = (0,1)(2,3)(t_2t_3t_2t_3t_1t_0t_2t_1t_3)$

$= (0,1)(2,3)(t_0t_1t_0t_1t_2t_3t_0t_2t_1)^{(1,3,0,2)} \in Nt_0t_1t_0t_1t_2t_3t_0t_2t_1$.

Therefore, t_1 takes $Nt_0t_1t_2t_3t_1t_3t_0t_3t_0t_1$ go to $Nt_0t_1t_0t_1t_2t_3t_0t_2t_1$.

123. Now, we consider coset $N^{(0123130302)}$.

However, $t_0t_1t_2t_3t_1t_3t_0t_3t_0t_2 = (0,1)(2,3)(t_0t_1t_0t_1t_3t_2t_1t_2t_0)$

$= (0,1)(2,3)(t_0t_1t_0t_1t_2t_3t_1t_3t_0)^{(3,2)} \in Nt_0t_1t_0t_1t_2t_3t_1t_3t_0$.

Therefore, t_2 takes $Nt_0t_1t_2t_3t_1t_3t_0t_3t_0t_2$ go to $Nt_0t_1t_0t_1t_2t_3t_1t_3t_0$.

124. Now, we consider coset $N^{(0123130303)}$.

However, $t_0t_1t_2t_3t_1t_3t_0t_3t_0t_3 = (0,3,2)(t_2t_1t_2t_0t_1t_3t_1t_2t_1)$

$= (0,3,2)(t_0t_1t_0t_2t_1t_3t_1t_0t_1)^{(0,2)} \in Nt_0t_1t_0t_2t_1t_3t_1t_0t_1$.

Therefore, t_3 takes $Nt_0t_1t_2t_3t_1t_3t_0t_3t_0t_3$ go to $Nt_0t_1t_0t_2t_1t_3t_1t_0t_1$.

125. Now, we consider coset $N^{(0123202120)}$.

However, $t_0t_1t_2t_3t_2t_0t_2t_1t_2t_0 = (1,3,2)(t_3t_2t_3t_0t_1t_3t_1t_3t_2)$

$= (1,3,2)(t_0t_1t_0t_2t_3t_0t_3t_0t_1)^{(1,2,0,3)} \in Nt_0t_1t_0t_2t_3t_0t_3t_0t_1$.
Therefore, t_0 takes $Nt_0t_1t_2t_3t_2t_0t_2t_1t_2t_0$ go to $Nt_0t_1t_0t_2t_3t_0t_3t_0t_1$.

126. Now, we consider coset $N^{(0123202310)}$.
However, $t_0t_1t_2t_3t_2t_0t_2t_3t_1t_0 = (2,0,3)(t_0t_1t_3t_2t_3t_0t_3t_2t_1)$
$= (2,0,3)(t_0t_1t_2t_3t_2t_0t_2t_3t_1)^{(2,3)} \in Nt_0t_1t_2t_3t_2t_0t_2t_3t_1$.
Therefore, t_0 takes $Nt_0t_1t_2t_3t_2t_0t_2t_3t_1t_0$ go to $Nt_0t_1t_2t_3t_2t_0t_2t_3t_1$.

127. Now, we consider coset $N^{(0123202312)}$.
However, $t_0t_1t_2t_3t_2t_0t_2t_3t_1t_2 = (2,3,0)(t_0t_1t_0t_1t_2t_3t_0t_2t_0)$
$= (2,3,0)(t_0t_1t_0t_1t_2t_3t_0t_2t_0) \in Nt_0t_1t_0t_1t_2t_3t_0t_2t_0$.
Therefore, t_2 takes $Nt_0t_1t_2t_3t_2t_0t_2t_3t_1t_2$ go to $Nt_0t_1t_0t_1t_2t_3t_0t_2t_0$.

128. Now, we consider coset $N^{(0123202313)}$.
However, $t_0t_1t_2t_3t_2t_0t_2t_3t_1t_3 = (2,3,0)(t_3t_1t_3t_0t_2t_1t_2t_3t_1)$
$= (2,3,0)(t_0t_1t_0t_2t_3t_1t_3t_0t_1)^{(2,0,3)} \in Nt_0t_1t_0t_2t_3t_1t_3t_0t_1$.
Therefore, t_3 takes $Nt_0t_1t_2t_3t_2t_0t_2t_3t_1t_3$ go to $Nt_0t_1t_0t_2t_3t_1t_3t_0t_1$.

129. Now, we consider coset $N^{(0123212030)}$.
However, $t_0t_1t_2t_3t_2t_1t_2t_0t_3t_0 = (2,3,0)(t_3t_2t_3t_0t_1t_0t_2t_0t_1)$
$= (2,3,0)(t_0t_1t_0t_2t_3t_2t_1t_2t_3)^{(1,2,0,3)} \in Nt_0t_1t_0t_2t_3t_2t_1t_2t_3$.
Therefore, t_0 takes $Nt_0t_1t_2t_3t_2t_1t_2t_0t_3t_0$ go to $Nt_0t_1t_0t_2t_3t_2t_1t_2t_3$.

130. Now, we consider coset $N^{(0123212031)}$.
However, $t_0t_1t_2t_3t_2t_1t_2t_0t_3t_1 = (2,3,1)(t_3t_1t_3t_1t_2t_3t_0t_1t_0)$
$= (2,3,1)(t_0t_1t_0t_1t_2t_0t_3t_1t_3)^{(0,3)} \in Nt_0t_1t_0t_1t_2t_0t_3t_1t_3$.
Therefore, t_1 takes $Nt_0t_1t_2t_3t_2t_1t_2t_0t_3t_1$ go to $Nt_0t_1t_0t_1t_2t_0t_3t_1t_3$.

131. Now, we consider coset $N^{(0123212032)}$.
However, $t_0t_1t_2t_3t_2t_1t_2t_0t_3t_2 = (3,0,1)(t_3t_1t_3t_2t_3t_0t_2t_1t_3)$
$= (3,0,1)(t_0t_1t_0t_2t_0t_3t_2t_1t_0)^{(0,3)} \in Nt_0t_1t_0t_2t_0t_3t_2t_1t_0$.
Therefore, t_2 takes $Nt_0t_1t_2t_3t_2t_1t_2t_0t_3t_2$ go to $Nt_0t_1t_0t_2t_0t_3t_2t_1t_0$.

132. Now, we consider coset $N^{(0123231301)}$.
However, $t_0t_1t_2t_3t_2t_3t_1t_3t_0t_1 = (3,0,1)(t_3t_1t_3t_2t_3t_1t_0t_3t_0)$
$= (3,0,1)(t_0t_1t_0t_2t_0t_1t_3t_0t_3)^{(0,3)} \in Nt_0t_1t_0t_2t_0t_1t_3t_0t_3$.
Therefore, t_1 takes $Nt_0t_1t_2t_3t_2t_3t_1t_3t_0t_1$ go to $Nt_0t_1t_0t_2t_0t_1t_3t_0t_3$.

133. Now, we consider coset $N^{(0123231302)}$.
However, $t_0t_1t_2t_3t_2t_3t_1t_3t_0t_2 = (3,2,1)(t_1t_0t_1t_2t_1t_3t_1t_0t_3)$
$= (3,2,1)(t_0t_1t_0t_2t_0t_3t_0t_1t_3)^{(0,1)} \in Nt_0t_1t_0t_2t_0t_3t_0t_1t_3$.
Therefore, t_2 takes $Nt_0t_1t_2t_3t_2t_3t_1t_3t_0t_2$ go to $Nt_0t_1t_0t_2t_0t_3t_0t_1t_3$.

134. Now, we consider coset $N^{(0123231303)}$.
However, $t_0t_1t_2t_3t_2t_3t_1t_3t_0t_3 = (1,2,0)(t_3t_1t_3t_2t_3t_0t_3t_2t_1)$
$= (1,2,0)(t_0t_1t_0t_2t_0t_3t_0t_2t_1)^{(0,3)} \in Nt_0t_1t_0t_2t_0t_3t_0t_2t_1$.
Therefore, t_3 takes $Nt_0t_1t_2t_3t_2t_3t_1t_3t_0t_3$ go to $Nt_0t_1t_0t_2t_0t_3t_0t_2t_1$.

135. Now, we consider coset $N^{(0121312101)}$.
However, $t_0t_1t_2t_1t_3t_1t_2t_1t_0t_1 = (1,0,3)(t_1t_3t_1t_2t_3t_0t_3t_2t_3)$
$= (1,0,3)(t_0t_1t_0t_2t_1t_3t_1t_2t_1)^{(0,1,3)} \in Nt_0t_1t_0t_2t_1t_3t_1t_2t_1$.
Therefore, t_2 takes $Nt_0t_1t_2t_1t_3t_1t_2t_1t_0t_1$ go to $Nt_0t_1t_0t_2t_1t_3t_1t_2t_1$.

136. Now, we consider coset $N^{(0102303120)}$.
However, $t_0t_1t_0t_2t_3t_0t_3t_1t_2t_0 = (2,3,0)(t_3t_0t_1t_0t_1t_2t_1t_3t_2)$
$= (2,3,0)(t_0t_1t_2t_1t_2t_3t_2t_0t_3)^{(1,0,3,2)} \in Nt_0t_1t_2t_1t_2t_3t_2t_0t_3$.
Therefore, t_0 takes $Nt_0t_1t_0t_2t_3t_0t_3t_1t_2t_0$ go to $Nt_0t_1t_2t_1t_2t_3t_2t_0t_3$.

137. Now, we consider coset $N^{(0102303121)}$.
However, $t_0t_1t_0t_2t_3t_0t_3t_1t_2t_1 = (1,3,2)(t_2t_0t_1t_2t_1t_3t_1t_3t_0)$
$= (1,3,2)(t_0t_1t_2t_0t_2t_3t_2t_3t_1)^{(1,0,2)} \in Nt_0t_1t_2t_0t_2t_3t_2t_3t_1$.
Therefore, t_1 takes $Nt_0t_1t_0t_2t_3t_0t_3t_1t_2t_1$ go to $Nt_0t_1t_2t_0t_2t_3t_2t_3t_1$.

138. Now, we consider coset $N^{(0102303131)}$.
However, $t_0t_1t_0t_2t_3t_0t_3t_1t_3t_1 = (1,0)(3,2)(t_0t_1t_0t_3t_2t_0t_2t_1t_2)$
$= (1,0)(3,2)(t_0t_1t_0t_2t_3t_0t_3t_1t_3)^{(3,2)} \in Nt_0t_1t_0t_2t_3t_0t_3t_1t_3$.
Therefore, t_1 takes $Nt_0t_1t_0t_2t_3t_0t_3t_1t_3t_1$ go to $Nt_0t_1t_0t_2t_3t_0t_3t_1t_3$.

139. Now, we consider coset $N^{(0102303130)}$.
However, $t_0t_1t_0t_2t_3t_0t_3t_1t_3t_0 = (t_0t_1t_0t_2t_3t_0t_3t_1t_3)$
$= t_0t_1t_0t_2t_3t_0t_3t_1t_3 \in Nt_0t_1t_0t_2t_3t_0t_3t_1t_3$.
Therefore, t_0 takes $Nt_0t_1t_0t_2t_3t_0t_3t_1t_3t_2$ go back to itself $Nt_0t_1t_0t_2t_3t_0t_3t_1t_3$.

140. Now, we consider coset $N^{(0102312120)}$.
However, $t_0t_1t_0t_2t_3t_1t_2t_1t_2t_0 = (1,0,3)(t_1t_0t_3t_1t_0t_2t_1t_2t_1)$
$= (1,0,3)(t_0t_1t_2t_0t_1t_3t_0t_3t_0)^{(0,1)(3,2)} \in Nt_0t_1t_2t_0t_1t_3t_0t_3t_0$.
Therefore, t_0 takes $Nt_0t_1t_0t_2t_3t_1t_2t_1t_2t_0$ go to $Nt_0t_1t_2t_0t_1t_3t_0t_3t_0$.

141. Now, we consider coset $N^{(0102312121)}$.
However, $t_0t_1t_0t_2t_3t_1t_2t_1t_2t_1 = (t_0t_1t_0t_3t_2t_1t_3t_1t_3)$
$= (t_0t_1t_0t_2t_3t_1t_2t_1t_2)^{(3,2)} \in Nt_0t_1t_0t_2t_3t_1t_2t_1t_2$.
Therefore, t_1 takes $Nt_0t_1t_0t_2t_3t_1t_2t_1t_2t_1$ go to $Nt_0t_1t_0t_2t_3t_1t_2t_1t_2$.

142. Now, we consider coset $N^{(0102312123)}$.
However, $t_0t_1t_0t_2t_3t_1t_2t_1t_2t_3 = (t_0t_1t_0t_3t_2t_3t_1t_3t_1)$
$= (t_0t_1t_0t_2t_3t_2t_1t_2t_1)^{(3,2)} \in Nt_0t_1t_0t_2t_3t_2t_1t_2t_1$.
Therefore, t_3 takes $Nt_0t_1t_0t_2t_3t_1t_2t_1t_2t_3$ go to $Nt_0t_1t_0t_2t_3t_2t_1t_2t_1$.

143. Now, we consider coset $N^{(0102313132)}$.
However, $t_0t_1t_0t_2t_3t_1t_3t_1t_3t_2 = (1,3,2)(t_0t_2t_0t_2t_1t_0t_2t_3t_1)$
$= (1,3,2)(t_0t_1t_0t_1t_2t_0t_1t_3t_2)^{(1,2)} \in Nt_0t_1t_0t_1t_2t_0t_1t_3t_2$.
Therefore, t_2 takes $Nt_0t_1t_0t_2t_3t_1t_3t_1t_3t_2$ go to $Nt_0t_1t_0t_1t_2t_0t_1t_3t_2$.

144. Now, we consider coset $N^{(0102321212)}$.
However, $t_0t_1t_0t_2t_3t_1t_2t_1t_2t_2 = (t_0t_1t_0t_3t_2t_1t_3t_1t_3)$

$= (t_0t_1t_0t_2t_3t_1t_2t_1t_2)^{(3,2)} \in Nt_0t_1t_0t_2t_3t_1t_2t_1t_2$.
Therefore, t_2 takes $Nt_0t_1t_0t_2t_3t_1t_2t_1t_2t_2$ go to $Nt_0t_1t_0t_2t_3t_1t_2t_1t_2$.

145. Now, we consider coset $N^{(0102321213)}$.
However, $t_0t_1t_0t_2t_3t_2t_1t_2t_1t_3 = (2,3,0)(t_1t_3t_1t_2t_0t_1t_0t_2t_0)$
$= (2,3,0)(t_0t_1t_0t_2t_3t_0t_3t_2t_3)^{(1,3,0)} \in Nt_0t_1t_0t_2t_3t_0t_3t_2t_3$.
Therefore, t_3 takes $Nt_0t_1t_0t_2t_3t_2t_1t_2t_1t_3$ go to $Nt_0t_1t_0t_2t_3t_0t_3t_2t_3$.

146. Now, we consider coset $N^{(01212032 01)}$.
However, $t_0t_1t_2t_1t_2t_0t_3t_2t_0t_1 = (1,2)(3,0)(t_1t_3t_1t_3t_2t_1t_0t_2t_3)$
$= (1,2)(3,0)(t_0t_1t_0t_1t_2t_0t_3t_2t_1)^{(1,3,0)} \in Nt_0t_1t_0t_1t_2t_0t_3t_2t_1$.
Therefore, t_1 takes $Nt_0t_1t_2t_1t_2t_0t_3t_2t_0t_1$ go to $Nt_0t_1t_0t_1t_2t_0t_3t_2t_1$.

147. Now, we consider coset $N^{(0121203203)}$.
However, $t_0t_1t_2t_1t_2t_0t_3t_2t_0t_3 = (1,2)(3,0)(t_1t_2t_1t_3t_1t_0t_1t_3t_2)$
$= (1,2)(3,0)(t_0t_1t_0t_2t_0t_3t_0t_2t_1)^{(1,2,3,0)} \in Nt_0t_1t_0t_2t_0t_3t_0t_2t_1$.
Therefore, t_2 takes $Nt_0t_1t_2t_1t_2t_0t_3t_2t_0t_3$ go to $Nt_0t_1t_0t_2t_0t_3t_0t_2t_1$.

148. Now, we consider the cosets stabilizer $N^{(0101023232)}$.
$N^{(0101023232)} = S_4$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_0t_2t_3t_2t_3t_2N$ is at most: $\frac{|N|}{|N^{(0101023232)}|} = \frac{24}{24} = 1$

The orbit of $N^{(0101023232)}$ on $\{0,1,2,3\}$ is $\{0,1,2,3\}$. Taking representatives from the orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_0t_2t_3t_2t_3t_2t_2$ belong? However, $Nt_0t_1t_0t_1t_0t_2t_3t_2t_3t_2t_2 = Nt_0t_1t_0t_1t_0t_2t_3t_2t_3 \in [010102323]$, then four symmetric generators go back to the double coset $Nt_0t_1t_0t_1t_0t_2t_3t_2t_3N$.

149. Now, we consider the cosets stabilizer $N^{(0101203010)}$.
$N^{(0101203010)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_0t_3t_0t_1t_0N$ is at most: $\frac{|N|}{|N^{(0101203010)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0101203010)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_0t_3t_0t_1t_0t_0$, $Nt_0t_1t_0t_1t_2t_0t_3t_0t_1t_0t_1$, $Nt_0t_1t_0t_1t_2t_0t_3t_0t_1t_0t_2$ and $Nt_0t_1t_0t_1t_2t_0t_3t_0t_1t_0t_3$ belong?

We know, $t_0t_1t_0t_1t_2t_0t_3t_0t_1t_0 = (1,0,2)(t_0t_1t_2t_0t_1t_3t_2t_3t_0t_1)$.
$Nt_0t_1t_0t_1t_2t_0t_3t_0t_1t_0 = Nt_0t_1t_2t_0t_1t_3t_2t_3t_0t_1$.
$Nt_0t_1t_0t_1t_2t_0t_3t_0t_1t_0t_1 = Nt_0t_1t_2t_0t_1t_3t_2t_3t_0 \in [012013230]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_1t_3t_2t_3t_0N$.
We know, $t_0t_1t_0t_1t_2t_0t_3t_0t_1t_0 = (1,3,0)(t_0t_1t_2t_0t_2t_1t_3t_2t_3t_0)^{(0,2,1)}$.
$Nt_0t_1t_0t_1t_2t_0t_3t_0t_1t_0 = Nt_2t_0t_1t_2t_1t_0t_3t_1t_3t_2$.
$Nt_0t_1t_0t_1t_2t_0t_3t_0t_1t_0t_2 = Nt_0t_1t_2t_0t_2t_1t_3t_2t_3 \in [012021323]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_2t_1t_3t_2t_3N$.
We know, $t_0t_1t_0t_1t_2t_0t_3t_0t_1t_0 = (2,3,0)(t_0t_1t_0t_2t_3t_0t_3t_1t_3t_2)^{(1,2,3)}$.
$Nt_0t_1t_0t_1t_2t_0t_3t_0t_1t_0 = Nt_0t_2t_0t_3t_1t_0t_1t_2t_1t_3$.
$Nt_0t_1t_0t_1t_2t_0t_3t_0t_1t_0t_3 = Nt_0t_1t_0t_2t_3t_0t_3t_1t_3 \in [010230323]$, then one symmetric generator goes back to $Nt_0t_1t_0t_2t_3t_0t_3t_1t_3N$.
Moreover, $Nt_0t_1t_0t_1t_2t_0t_3t_0t_1t_0t_0 = Nt_0t_1t_0t_1t_2t_0t_3t_0t_1 \in [010120301]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_0t_3t_0t_1N$.

150. Now, we consider the cosets stabilizer $N^{(0101230210)}$.
$N^{(0101230210)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_3t_0t_2t_1t_0N$ is at most: $\frac{|N|}{|N^{(0101230210)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0101230210)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_0t_2t_1t_0t_0$, $Nt_0t_1t_0t_1t_2t_3t_0t_2t_1t_0t_1$, $Nt_0t_1t_0t_1t_2t_3t_0t_2t_1t_0t_2$ and $Nt_0t_1t_0t_1t_2t_3t_0t_2t_1t_0t_3$ belong?
We know, $t_0t_1t_0t_1t_2t_3t_0t_2t_1t_0 = (1,3,0)(t_0t_1t_0t_2t_0t_1t_3t_0t_3t_2)^{(0,3,2,1)}$.
$Nt_0t_1t_0t_1t_2t_3t_0t_2t_1t_0 = Nt_3t_0t_3t_1t_3t_0t_2t_3t_2t_1$.
$Nt_0t_1t_0t_1t_2t_3t_0t_2t_1t_0t_1 = Nt_0t_1t_0t_2t_0t_1t_3t_0t_3 \in [010201303]$, then one symmetric generator goes back to $Nt_0t_1t_0t_2t_0t_1t_3t_0t_3N$.
We know, $t_0t_1t_0t_1t_2t_3t_0t_2t_1t_0 = (t_0t_1t_2t_0t_2t_3t_2t_0t_1t_2)^{(0,2,3)}$.
$Nt_0t_1t_0t_1t_2t_3t_0t_2t_1t_0 = Nt_2t_1t_3t_2t_3t_0t_3t_2t_1t_3$.
$Nt_0t_1t_0t_1t_2t_3t_0t_2t_1t_0t_3 = Nt_0t_1t_2t_0t_2t_3t_2t_0t_1 \in [012023201]$, then one symmetric generator goes back to $Nt_0t_1t_2t_0t_2t_3t_2t_0t_1N$.
We know, $t_0t_1t_0t_1t_2t_3t_0t_2t_1t_0 = (1,3)(2,0)(t_0t_1t_0t_2t_3t_0t_3t_1t_3t_0)^{(0,2,3)}$.
$Nt_0t_1t_0t_1t_2t_3t_0t_2t_1t_0 = Nt_2t_1t_2t_3t_0t_2t_0t_1t_0t_2$.
$Nt_0t_1t_0t_1t_2t_3t_0t_2t_1t_0t_2 = Nt_0t_1t_0t_2t_3t_0t_3t_1t_3 \in [010230323]$, then one symmetric genera-

tor goes back to $Nt_0t_1t_0t_2t_3t_0t_3t_1t_3N$.
Moreover, $Nt_0t_1t_0t_1t_2t_3t_0t_2t_1t_0t_0 = Nt_0t_1t_0t_1t_2t_3t_0t_2t_1 \in$ [010123021], then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_3t_0t_2t_1N$.

151. Now, we consider the cosets stabilizer $N^{(0102030103)}$.
$N^{(0102030103)} = \{e\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_3N$ is at most: $\frac{|N|}{|N^{(0102030103)}|} = \frac{24}{1} = 24$.

The orbit of $N^{(0102030103)}$ on $\{0,1,2,3\}$ are $\{0\}$, $\{1\}$, $\{2\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_3t_0$, $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_3t_1$, $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_3t_2$ and $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_3t_3$ belong? We know, $t_0t_1t_0t_2t_0t_3t_0t_1t_0t_3 = (1,2,3)(t_0t_1t_0t_2t_3t_1t_2t_3t_0t_1)^{(0,3,2,1)}$.
$Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_3 = Nt_3t_0t_3t_1t_2t_0t_1t_2t_3t_0$.
$Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_3t_0 = Nt_0t_1t_0t_2t_3t_1t_2t_3t_0 \in$ [010231230], then one symmetric generator goes back to $Nt_0t_1t_0t_2t_3t_1t_2t_3t_0N$.
We know, $t_0t_1t_0t_2t_0t_3t_0t_1t_0t_3 = (0,2)(t_0t_1t_0t_2t_1t_3t_1t_2t_3t_0)^{(0,2)(1,3)}$.
$Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_3 = Nt_2t_3t_1t_0t_3t_1t_3t_0t_1t_2$.
$Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_3t_2 = Nt_0t_1t_0t_2t_1t_3t_1t_2t_3 \in$ [010213123], then one symmetric generator goes back to $Nt_0t_1t_0t_2t_1t_3t_1t_2t_3N$.
We know, $t_0t_1t_0t_2t_0t_3t_0t_1t_0t_3 = (1,0,2)(t_0t_1t_0t_2t_3t_0t_3t_2t_1t_0)^{(0,1,2,3)}$.
$Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_3 = Nt_1t_2t_1t_3t_0t_1t_0t_3t_2t_1$.
$Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_3t_2 = Nt_0t_1t_0t_2t_3t_0t_3t_2t_1 \in$ [010230321], then one symmetric generator goes back to $Nt_0t_1t_0t_2t_3t_0t_3t_2t_1N$.
Moreover, $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_3t_3 = Nt_0t_1t_0t_2t_0t_3t_0t_1t_0 \in$ [010203010], then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0N$.

152. Now, we consider the cosets stabilizer $N^{(01012131 02)}$.
We know, $t_0t_1t_0t_1t_2t_1t_3t_1t_0t_2 = t_3t_2t_3t_2t_1t_2t_0t_2t_3t_1$ then
$N^{(0101213102)} = \{e, (2,1)(0,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_1t_3t_1t_0t_2N$ is at most: $\frac{|N|}{|N^{(0101213102)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(0101213102)}$ on $\{0,1,2,3\}$ are $\{0,3\}$, and $\{1,2\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_1t_3t_1t_0t_2t_0$, and $Nt_0t_1t_0t_1t_2t_1t_3t_1t_0t_2t_1$ belong?

We know, $t_0t_1t_0t_1t_2t_1t_3t_1t_0t_2 = (1,2)(0,3)(t_0t_1t_0t_2t_0t_2t_1t_3t_2t_1)^{(1,3,0,2)}$.
$Nt_0t_1t_0t_1t_2t_1t_3t_1t_0t_2 = Nt_2t_3t_2t_1t_2t_1t_3t_0t_1t_3$.
$Nt_0t_1t_0t_1t_2t_1t_3t_1t_0t_2t_3 = Nt_0t_1t_0t_2t_0t_2t_1t_3t_2 \in [010202132]$, then two symmetric generators go back to $Nt_0t_1t_0t_2t_0t_2t_1t_3t_2N$.
However, $Nt_0t_1t_0t_1t_2t_1t_3t_1t_0t_2t_2 = Nt_0t_1t_0t_1t_2t_1t_3t_1t_0 \in [010121310]$, then two symmetric generators go back to the double coset $Nt_0t_1t_0t_1t_2t_1t_3t_1t_0N$.

153. Now, we consider the cosets stabilizer $N^{(0101230301)}$.
We know, $t_0t_1t_0t_1t_2t_3t_0t_3t_0t_1 = t_2t_1t_2t_1t_0t_3t_2t_3t_2t_1$ then
$N^{(0101230301)} = \{e,(0,2)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_3t_0t_3t_0t_1N$ is at most: $\frac{|N|}{|N^{(0101230301)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(0101230301)}$ on $\{0,1,2,3\}$ are $\{0,2\}$, $\{1\}$, and $\{3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_0t_3t_0t_1t_0$, $Nt_0t_1t_0t_1t_2t_3t_0t_3t_0t_1t_1$, and $Nt_0t_1t_0t_1t_2t_3t_0t_3t_0t_1t_3$ belong?
We know, $t_0t_1t_0t_1t_2t_3t_0t_3t_0t_1 = (1,2)(0,3)(t_0t_1t_0t_2t_3t_0t_1t_3t_2t_3)^{(2,0,3)}$.
$Nt_0t_1t_0t_1t_2t_3t_0t_3t_0t_1 = Nt_3t_1t_3t_0t_2t_3t_1t_2t_0t_2$.
$Nt_0t_1t_0t_1t_2t_3t_0t_3t_0t_1t_2 = Nt_0t_1t_0t_2t_3t_0t_1t_3 \in [01023013]$, then two symmetric generators go back to $Nt_0t_1t_0t_2t_3t_0t_1t_3N$.
We know, $t_0t_1t_0t_1t_2t_3t_0t_3t_0t_1 = (t_0t_1t_0t_2t_3t_2t_3t_2t_1t_0)^{(0,3)}$.
$Nt_0t_1t_0t_1t_2t_3t_0t_3t_0t_1 = Nt_3t_1t_3t_2t_0t_2t_0t_2t_1t_3$.
$Nt_0t_1t_0t_1t_2t_3t_0t_3t_0t_1t_3 = Nt_0t_1t_0t_2t_3t_2t_3t_2t_1 \in [01023013]$, then one symmetric generator goes back to $Nt_0t_1t_0t_2t_3t_2t_3t_2t_1N$.
However, $Nt_0t_1t_0t_1t_2t_3t_0t_3t_0t_1t_1 = Nt_0t_1t_0t_1t_2t_3t_0t_3t_0 \in [010123030]$, then one symmetric generator goes back to the double coset $Nt_0t_1t_0t_1t_2t_3t_0t_3t_0N$.

154. Now, we consider the cosets stabilizer $N^{(0102132123)}$.
We know, $t_0t_1t_0t_2t_1t_3t_2t_1t_2t_3 = t_1t_0t_1t_3t_0t_2t_3t_0t_3t_2$ then
$N^{(0102132123)} = \{e,(0,1)(2,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_1t_3t_2t_1t_2t_3N$ is at most: $\frac{|N|}{|N^{(0101213102)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(0102132123)}$ on $\{0,1,2,3\}$ are $\{0,1\}$, and $\{2,3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_1t_3t_2t_1t_2t_3t_0$, and $Nt_0t_1t_0t_2t_1t_3t_2t_1t_2t_3t_2$ belong?

We know, $t_0t_1t_0t_2t_1t_3t_2t_1t_2t_3 = (1,3)(0,2)(t_0t_1t_0t_2t_3t_2t_1t_2t_3t_1)^{(1,0)(3,2)}$.
$Nt_0t_1t_0t_2t_1t_3t_2t_1t_2t_3 = Nt_1t_0t_1t_3t_2t_3t_0t_3t_2t_1$.
$Nt_0t_1t_0t_2t_1t_3t_2t_1t_2t_3t_1 = Nt_0t_1t_0t_2t_3t_2t_1t_2t_3 \in [010232123]$, then two symmetric generators go back to $Nt_0t_1t_0t_2t_3t_2t_1t_2t_3N$.
However, $Nt_0t_1t_0t_2t_1t_3t_2t_1t_2t_3t_2 = Nt_0t_1t_0t_2t_1t_3t_2t_1t_2 \in [010213212]$, then two symmetric generators go back to the double coset $Nt_0t_1t_0t_2t_1t_3t_2t_1t_2N$.

155. Now, we consider the cosets stabilizer $N^{(0101231302)}$.
$t_0t_1t_0t_1t_2t_3t_1t_3t_0t_2 = t_1t_0t_1t_0t_3t_2t_0t_2t_1t_3 = t_2t_3t_2t_3t_1t_0t_3t_0t_2t_1 = t_3t_2t_3t_2t_0t_1t_2t_1t_3t_0$ then $N^{(0101231302)} = \{e, (0,1)(2,3), (0,2,3,1), (0,3,1,2)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_1t_2t_3t_1t_3t_0t_2N$ is at most: $\frac{|N|}{|N^{(0101023323)}|} = \frac{24}{4} = 0$

The orbit of $N^{(0101231302)}$ on $\{0,1,2,3\}$ is $\{0,1,2,3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_1t_2t_3t_1t_3t_0t_2t_2$,
However, $Nt_0t_1t_0t_1t_2t_3t_1t_3t_0t_2t_2 = Nt_0t_1t_0t_1t_2t_3t_1t_3t_0 \in [010123030]$, then four symmetric generators go back to the double coset $Nt_0t_1t_0t_1t_2t_3t_1t_3t_0N$.

156. Now, we consider the cosets stabilizer $N^{(0102030102)}$.
We know, $t_0t_1t_0t_2t_0t_3t_0t_1t_0t_2 = t_0t_3t_0t_1t_0t_2t_0t_3t_0t_1 = t_0t_2t_0t_3t_0t_1t_0t_2t_0t_3$ then $N^{(0102030102)} = \{e, (3,2,1), (3,1,2)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_2N$ is at most: $\frac{|N|}{|N^{(0102030102)}|} = \frac{24}{3} = 8$

The orbit of $N^{(0102030102)}$ on $\{0,1,2,3\}$ are $\{3,1,2\}$ and $\{0\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_2t_2$ and $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_2t_0$ belong?
However, $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_2t_2 = Nt_0t_1t_0t_2t_0t_3t_0t_1t_0 \in [010203010]$, then three symmetric generators go back to the double coset $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0N$, one symmetric generator goes to $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_2t_0$.

157. Now, we consider the cosets stabilizer $N^{(0102032123)}$.
We know, $t_0t_1t_0t_2t_0t_3t_2t_1t_2t_3 = t_0t_3t_0t_1t_0t_2t_1t_3t_1t_2 = t_0t_2t_0t_3t_0t_1t_3t_2t_3t_1$ then $N^{(0102032123)} = \{e, (3,2,1), (3,1,2)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_3t_2t_1t_2t_3N$ is at most: $\frac{|N|}{|N^{(0102032123)}|} = \frac{24}{3} = 8$

The orbit of $N^{(0102032123)}$ on $\{0,1,2,3\}$ are $\{3,1,2\}$ and $\{0\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_0t_3t_2t_1t_2t_3t_0$ and $Nt_0t_1t_0t_2t_0t_3t_2t_1t_2t_3t_0$ belong?

However, $Nt_0t_1t_0t_2t_0t_3t_2t_1t_2t_3t_3 = Nt_0t_1t_0t_2t_0t_3t_2t_1t_2 \in [010203212]$, then three symmetric generators go back to the double coset $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0N$.

Moreover, $Nt_0t_1t_0t_2t_0t_3t_2t_1t_2t_3t_0 = Nt_0t_1t_0t_2t_0t_3t_2t_1t_2t_3$ one symmetric generator goes back to itself $Nt_0t_1t_0t_2t_0t_3t_2t_1t_2t_3t_0$.

158. Now, we consider the cosets stabilizer $N^{(0102312013)}$.

We know, $t_0t_1t_0t_2t_3t_1t_2t_0t_1t_3 = t_3t_0t_3t_2t_1t_0t_2t_3t_0t_1 = t_1t_3t_1t_2t_0t_3t_2t_1t_3t_0$ then
$N^{(0102312013)} = \{e,(3,0,1),(3,1,0)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_3t_1t_2t_0t_1t_3N$ is at most: $\frac{|N|}{|N^{(0102312013)}|} = \frac{24}{3} = 8$

The orbit of $N^{(0102312013)}$ on $\{0,1,2,3\}$ are $\{3,1,0\}$ and $\{2\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_0t_2t_3t_1t_2t_0t_1t_3t_3$ and $Nt_0t_1t_0t_2t_3t_1t_2t_0t_1t_3t_2$ belong?

However, $Nt_0t_1t_0t_2t_3t_1t_2t_0t_1t_3t_3 = Nt_0t_1t_0t_2t_3t_1t_2t_0t_1 \in [010231201]$, then three symmetric generators go back to the double coset $Nt_0t_1t_0t_2t_3t_1t_2t_0t_1N$.

Moreover, $Nt_0t_1t_0t_2t_3t_1t_2t_0t_1t_3t_2 = Nt_0t_1t_2t_3t_1t_0t_3t_2t_1$ one symmetric generator goes back to $N_0t_1t_2t_3t_1t_0t_3t_2t_1$.

159. Now, we consider the cosets stabilizer $N^{(0120131303)}$.

We know, $t_0t_1t_2t_0t_1t_3t_1t_3t_0t_3 = t_1t_0t_3t_1t_0t_2t_0t_2t_1t_2$ then
$N^{(0102030102)} = \{e,(0,1)(2,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_0t_1t_3t_1t_3t_0t_3N$ is at most: $\frac{|N|}{|N^{(0102030102)}|} = \frac{24}{2} = 12$.

The orbit of $N^{(0120131303)}$ on $\{0,1,2,3\}$ are $\{0,1\}$, and $\{2,3\}$. We take a representative from each orbit, and find to which the double cosets $Nt_0t_1t_2t_0t_1t_3t_1t_3t_0t_3t_0$, and $Nt_0t_1t_2t_0t_1t_3t_1t_3t_0t_3t_3$ belong?

We know, $t_0t_1t_2t_0t_1t_3t_1t_3t_0t_3 = (1,0)(2,3)(t_0t_1t_2t_3t_2t_0t_2t_1t_2t_1)$.

$Nt_0t_1t_2t_0t_1t_3t_1t_3t_0t_3 = Nt_0t_1t_2t_3t_2t_0t_2t_1t_2t_1$.

$Nt_0t_1t_2t_0t_1t_3t_1t_3t_0t_3t_1 = Nt_0t_1t_2t_3t_2t_0t_2t_1t_2 \in [012320212]$, then two symmetric generators go back to $Nt_0t_1t_2t_3t_2t_0t_2t_1t_2N$.

However, $Nt_0t_1t_2t_0t_1t_3t_1t_3t_0t_3t_3 = Nt_0t_1t_2t_0t_1t_3t_1t_3t_0 \in [010123030]$, then two symmetric generators go back to the double coset $Nt_0t_1t_2t_0t_1t_3t_1t_3t_0N$.

160. Now, we consider the cosets stabilizer $N^{(0123021032)}$. We know, $t_0t_1t_2t_3t_0t_2t_1t_0t_3t_2 = t_2t_0t_1t_3t_2t_1t_0t_2t_3t_1 = t_1t_2t_0t_3t_1t_0t_2t_1t_3t_0 = t_3t_0t_2t_1t_3t_2t_0t_3t_1t_2$ $= t_3t_2t_1t_0t_3t_1t_2t_3t_0t_1 = t_0t_2t_3t_1t_0t_3t_2t_0t_1t_3 = t_2t_3t_0t_1t_2t_0t_3t_2t_1t_0 = t_1t_3t_2t_0t_1t_2t_3t_1t_0t_2$ $= t_3t_1t_0t_2t_3t_0t_1t_3t_2t_0 = t_1t_0t_3t_2t_1t_3t_0t_1t_2t_3 = t_2t_1t_3t_0t_2t_3t_1t_2t_0t_3 = t_0t_3t_1t_2t_0t_1t_3t_0t_2t_1$ then $N^{(0102030102)} = \{e, (0,1)(2,3), (0,3)(1,2), (0,3)(2,1), (1,2,3), (1,3,2), (0,2,3), (0,3,2)$ $, (0,2,1), (0,1,2), (0,3,1), (0,1,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_2t_3t_0t_2t_1t_0t_3t_2N$ is at most: $\frac{|N|}{|N^{(0102030102)}|} = \frac{24}{12} = 2$.

The orbit of $N^{(0123021032)}$ on $\{0,1,2,3\}$ is $\{0,1,2,3\}$. We take a representative from each orbit, and find to which the double coset $Nt_0t_1t_2t_3t_0t_2t_1t_0t_3t_2t_2$ belong? However, $Nt_0t_1t_2t_3t_0t_2t_1t_0t_3t_2t_2 = Nt_0t_1t_2t_3t_0t_2t_1t_0t_3 \in [012302103]$, then four symmetric generators go back to the double coset $Nt_0t_1t_2t_3t_0t_2t_1t_0t_3N$.

Length 11

1. Now, we consider the cosets stabilizer $N^{(01020301020)}$. We know, $t_3t_2t_3t_1t_3t_0t_3t_2t_3t_1t_3 = t_1t_3t_1t_2t_1t_0t_1t_3t_1t_2t_1 = t_1t_0t_1t_3t_1t_2t_1t_0t_1t_3t_1 = t_3t_0t_3t_2t_3t_1t_3t_0t_3t_2t_3 =$ $t_0t_1t_0t_2t_0t_3t_0t_1t_0t_2t_0 = t_1t_2t_1t_0t_1t_3t_1t_2t_1t_0t_1 = t_2t_3t_2t_0t_2t_1t_2t_3t_2t_0t_2 = t_2t_0t_2t_1t_2t_3t_2t_0t_2t_1t_2 =$ $t_3t_1t_3t_0t_3t_2t_3t_1t_3t_0t_3 = t_0t_2t_0t_3t_0t_1t_0t_2t_0t_3t_0 = t_0t_3t_0t_1t_0t_2t_0t_3t_0t_1t_0 = t_2t_1t_2t_3t_2t_0t_2t_1t_2t_3t_2$ then $N^{(01020301020)} = \{e, (0,1)(2,3), (0,3)(1,2), (0,3)(2,1), (1,2,3), (1,3,2), (0,2,3), (0,3,2),$ $(0,2,1), (0,1,2), (0,3,1), (0,1,3)\}$, then the number of the single cosets in the double coset $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_2t_0N$ is at most: $\frac{|N|}{|N^{(0102030102)}|} = \frac{24}{12} = 2$

The orbit of $N^{(01020301020)}$ on $\{0,1,2,3\}$ is $\{0,1,2,3\}$. We take a representative from each orbit, and find to which the double coset $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_2t_0$ belong? However, $Nt_0t_1t_0t_2t_0t_3t_0t_1t_0t_2t_0t_0 = N_0t_1t_0t_2t_0t_3t_0t_1t_0t_2 \in [0102030102]$, then four symmetric generators go back to the double coset $N_0t_1t_0t_2t_0t_3t_0t_1t_0t_2N$.

Our double coset enumeration must be complete since the set of right coset is close under right multiplication by the symmetric generators.

The double coset enumeration shows that the index of $N \cong L_{12} : 2$ in G is at most:

$$\frac{|N|}{|N|} + \frac{|N|}{|N^{(0)}|} + \frac{|N|}{|N^{(01)}|} + \frac{|N|}{|N^{(010)}|} + \frac{|N|}{|N^{(012)}|} + \frac{|N|}{|N^{(0101)}|} + \frac{|N|}{|N^{(0102)}|} + \frac{|N|}{|N^{(0120)}|} + \frac{|N|}{|N^{(0121)}|} + \frac{|N|}{|N^{(0123)}|}$$
$$+ \frac{|N|}{|N^{(01010)}|} + \frac{|N|}{|N^{(01012)}|} + \frac{|N|}{|N^{(01020)}|} + \frac{|N|}{|N^{(01021)}|} + \frac{|N|}{|N^{(01023)}|} + \frac{|N|}{|N^{(01201)}|} + \frac{|N|}{|N^{(01202)}|} + \frac{|N|}{|N^{(01203)}|}$$

$$+\frac{|N|}{|N^{(01210)}|}+\frac{|N|}{|N^{(01212)}|}+\frac{|N|}{|N^{(01213)}|}+\frac{|N|}{|N^{(01230)}|}+\frac{|N|}{|N^{(01231)}|}+\frac{|N|}{|N^{(01232)}|}+\frac{|N|}{|N^{(010102)}|}+\frac{|N|}{|N^{(010120)}|}$$

$$+\frac{|N|}{|N^{(00121)}|}+\frac{|N|}{|N^{(010123)}|}+\frac{|N|}{|N^{(010201)}|}+\frac{|N|}{|N^{(010202)}|}+\frac{|N|}{|N^{(010203)}|}+\frac{|N|}{|N^{(010210)}|}+\frac{|N|}{|N^{(010212)}|}+\frac{|N|}{|N^{(010213)}|}$$

$$+\frac{|N|}{|N^{(010230)}|}+\frac{|N|}{|N^{(010231)}|}+\frac{|N|}{|N^{(010232)}|}+\frac{|N|}{|N^{(012010)}|}+\frac{|N|}{|N^{(012013)}|}+\frac{|N|}{|N^{(012020)}|}+\frac{|N|}{|N^{(012021)}|}+\frac{|N|}{|N^{(012023)}|}$$

$$+\frac{|N|}{|N^{(012030)}|}+\frac{|N|}{|N^{(012032)}|}+\frac{|N|}{|N^{(012101)}|}+\frac{|N|}{|N^{(012103)}|}+\frac{|N|}{|N^{(012120)}|}+\frac{|N|}{|N^{(012121)}|}+\frac{|N|}{|N^{(012123)}|}+\frac{|N|}{|N^{(012130)}|}$$

$$+\frac{|N|}{|N^{(012131)}|}+\frac{|N|}{|N^{(012132)}|}+\frac{|N|}{|N^{(012302)}|}+\frac{|N|}{|N^{(012310)}|}+\frac{|N|}{|N^{(012312)}|}+\frac{|N|}{|N^{(012313)}|}+\frac{|N|}{|N^{(012320)}|}+\frac{|N|}{|N^{(012321)}|}$$

$$+\frac{|N|}{|N^{(012323)}|}+\frac{|N|}{|N^{(0101023)}|}+\frac{|N|}{|N^{(0101201)}|}+\frac{|N|}{|N^{(0101202)}|}+\frac{|N|}{|N^{(0101203)}|}+\frac{|N|}{|N^{(0102010)}|}+\frac{|N|}{|N^{(0102012)}|}$$

$$+\frac{|N|}{|N^{(0102013)}|}+\frac{|N|}{|N^{(0102020)}|}+\frac{|N|}{|N^{(0102021)}|}+\frac{|N|}{|N^{(0102023)}|}+\frac{|N|}{|N^{(0102030)}|}+\frac{|N|}{|N^{(0102031)}|}+\frac{|N|}{|N^{(0102032)}|}$$

$$+\frac{|N|}{|N^{(0102101)}|}+\frac{|N|}{|N^{(0102103)}|}+\frac{|N|}{|N^{(0102123)}|}+\frac{|N|}{|N^{(0102130)}|}+\frac{|N|}{|N^{(0102131)}|}+\frac{|N|}{|N^{(0102132)}|}+\frac{|N|}{|N^{(0102301)}|}$$

$$+\frac{|N|}{|N^{(0102302)}|}+\frac{|N|}{|N^{(0102303)}|}+\frac{|N|}{|N^{(0102312)}|}+\frac{|N|}{|N^{(0102313)}|}+\frac{|N|}{|N^{(0102320)}|}+\frac{|N|}{|N^{(0102321)}|}+\frac{|N|}{|N^{(0102323)}|}$$

$$+\frac{|N|}{|N^{(0120101)}|}+\frac{|N|}{|N^{(0120103)}|}+\frac{|N|}{|N^{(0120130)}|}+\frac{|N|}{|N^{(0120131)}|}+\frac{|N|}{|N^{(0120132)}|}+\frac{|N|}{|N^{(0120203)}|}+\frac{|N|}{|N^{(0120212)}|}$$

$$+\frac{|N|}{|N^{(0120213)}|}+\frac{|N|}{|N^{(0120231)}|}+\frac{|N|}{|N^{(0120232)}|}+\frac{|N|}{|N^{(0120302)}|}+\frac{|N|}{|N^{(0120303)}|}+\frac{|N|}{|N^{(0120320)}|}+\frac{|N|}{|N^{(0120321)}|}$$

$$+\frac{|N|}{|N^{(0120323)}|}+\frac{|N|}{|N^{(0121012)}|}+\frac{|N|}{|N^{(0121013)}|}+\frac{|N|}{|N^{(0121030)}|}+\frac{|N|}{|N^{(0121031)}|}+\frac{|N|}{|N^{(0121203)}|}+\frac{|N|}{|N^{(0121213)}|}$$

$$+\frac{|N|}{|N^{(0121230)}|}+\frac{|N|}{|N^{(0121231)}|}+\frac{|N|}{|N^{(0121232)}|}+\frac{|N|}{|N^{(0121303)}|}+\frac{|N|}{|N^{(0121320)}|}+\frac{|N|}{|N^{(0121321)}|}+\frac{|N|}{|N^{(0121323)}|}$$

$$+\frac{|N|}{|N^{(0123020)}|}+\frac{|N|}{|N^{(0123021)}|}+\frac{|N|}{|N^{(0123023)}|}+\frac{|N|}{|N^{(0123101)}|}+\frac{|N|}{|N^{(0123103)}|}+\frac{|N|}{|N^{(0123120)}|}+\frac{|N|}{|N^{(0123130)}|}$$

$$+\frac{|N|}{|N^{(0123131)}|}+\frac{|N|}{|N^{(0123132)}|}+\frac{|N|}{|N^{(0123201)}|}+\frac{|N|}{|N^{(0123202)}|}+\frac{|N|}{|N^{(0123210)}|}+\frac{|N|}{|N^{(0123212)}|}+\frac{|N|}{|N^{(0101210)}|}$$

$$+\frac{|N|}{|N^{(0101213)}|}+\frac{|N|}{|N^{(0101230)}|}+\frac{|N|}{|N^{(0101231)}|}+\frac{|N|}{|N^{(0101232)}|}+\frac{|N|}{|N^{(01010230)}|}+\frac{|N|}{|N^{(01010232)}|}+\frac{|N|}{|N^{(01012013)}|}$$

$$+\frac{|N|}{|N^{(01012030)}|}+\frac{|N|}{|N^{(0123230)}|}+\frac{|N|}{|N^{(0123231)}|}+\frac{|N|}{|N^{(0123232)}|}+\frac{|N|}{|N^{(0121310)}|}+\frac{|N|}{|N^{(0121312)}|}+\frac{|N|}{|N^{(0121313)}|}$$

$$+\frac{|N|}{|N^{(01012031)}|}+\frac{|N|}{|N^{(01012032)}|}+\frac{|N|}{|N^{(01012103)}|}+\frac{|N|}{|N^{(01012130)}|}+\frac{|N|}{|N^{(01012131)}|}+\frac{|N|}{|N^{(01012132)}|}+\frac{|N|}{|N^{(01012302)}|}$$

$$+\frac{|N|}{|N^{(01012303)}|}+\frac{|N|}{|N^{(01012310)}|}+\frac{|N|}{|N^{(01012312)}|}+\frac{|N|}{|N^{(01012313)}|}+\frac{|N|}{|N^{(01012320)}|}+\frac{|N|}{|N^{(01012321)}|}+\frac{|N|}{|N^{(01012323)}|}$$

$$+\frac{|N|}{|N^{(01020103)}|}+\frac{|N|}{|N^{(01020123)}|}+\frac{|N|}{|N^{(01020130)}|}+\frac{|N|}{|N^{(01020131)}|}+\frac{|N|}{|N^{(01020132)}|}+\frac{|N|}{|N^{(01020213)}|}+\frac{|N|}{|N^{(01020230)}|}$$

$$+\frac{|N|}{|N^{(01020231)}|}+\frac{|N|}{|N^{(01020232)}|}+\frac{|N|}{|N^{(01020301)}|}+\frac{|N|}{|N^{(01020302)}|}+\frac{|N|}{|N^{(01020303)}|}+\frac{|N|}{|N^{(01020313)}|}+\frac{|N|}{|N^{(01020320)}|}$$

$$+\frac{|N|}{|N^{(01020321)}|}+\frac{|N|}{|N^{(01021230)}|}+\frac{|N|}{|N^{(01021231)}|}+\frac{|N|}{|N^{(01021012)}|}+\frac{|N|}{|N^{(01021031)}|}+\frac{|N|}{|N^{(01021301)}|}+\frac{|N|}{|N^{(01021310)}|}$$

$$+\frac{|N|}{|N^{(01021312)}|}+\frac{|N|}{|N^{(01021313)}|}+\frac{|N|}{|N^{(01021321)}|}+\frac{|N|}{|N^{(01021323)}|}+\frac{|N|}{|N^{(01023013)}|}+\frac{|N|}{|N^{(01023030)}|}+\frac{|N|}{|N^{(01023031)}|}$$

$$+\frac{|N|}{|N^{(01023032)}|}+\frac{|N|}{|N^{(01023120)}|}+\frac{|N|}{|N^{(01023121)}|}+\frac{|N|}{|N^{(01023123)}|}+\frac{|N|}{|N^{(01023130)}|}+\frac{|N|}{|N^{(01023131)}|}+\frac{|N|}{|N^{(01023203)}|}$$

$$+\frac{|N|}{|N^{(01023212)}|}+\frac{|N|}{|N^{(01023230)}|}+\frac{|N|}{|N^{(01023231)}|}+\frac{|N|}{|N^{(01023232)}|}+\frac{|N|}{|N^{(01201013)}|}+\frac{|N|}{|N^{(01201030)}|}+\frac{|N|}{|N^{(01201301)}|}$$

$$+\frac{|N|}{|N^{(01201302)}|}+\frac{|N|}{|N^{(01201303)}|}+\frac{|N|}{|N^{(01201313)}|}+\frac{|N|}{|N^{(01201323)}|}+\frac{|N|}{|N^{(01202030)}|}+\frac{|N|}{|N^{(01202031)}|}+\frac{|N|}{|N^{(01202123)}|}$$

$$+\frac{|N|}{|N^{(01202130)}|}+\frac{|N|}{|N^{(01202132)}|}+\frac{|N|}{|N^{(01202310)}|}+\frac{|N|}{|N^{(01202313)}|}+\frac{|N|}{|N^{(01202320)}|}+\frac{|N|}{|N^{(01202323)}|}+\frac{|N|}{|N^{(01203023)}|}$$

$$+\frac{|N|}{|N^{(01203030)}|}+\frac{|N|}{|N^{(01203031)}|}+\frac{|N|}{|N^{(01203032)}|}+\frac{|N|}{|N^{(01203201)}|}+\frac{|N|}{|N^{(01203202)}|}+\frac{|N|}{|N^{(01203210)}|}+\frac{|N|}{|N^{(01203212)}|}$$

$$+\frac{|N|}{|N^{(01203232)}|}+\frac{|N|}{|N^{(01210123)}|}+\frac{|N|}{|N^{(01210130)}|}+\frac{|N|}{|N^{(01210132)}|}+\frac{|N|}{|N^{(01210301)}|}+\frac{|N|}{|N^{(01210303)}|}+\frac{|N|}{|N^{(01210312)}|}$$

$$+\frac{|N|}{|N^{(01210313)}|}+\frac{|N|}{|N^{(01212030)}|}+\frac{|N|}{|N^{(01212031)}|}+\frac{|N|}{|N^{(01212032)}|}+\frac{|N|}{|N^{(01212130)}|}+\frac{|N|}{|N^{(01212302)}|}+\frac{|N|}{|N^{(01212312)}|}$$

$$+\frac{|N|}{|N^{(01212320)}|}+\frac{|N|}{|N^{(01212321)}|}+\frac{|N|}{|N^{(01213030)}|}+\frac{|N|}{|N^{(01213032)}|}+\frac{|N|}{|N^{(01230202)}|}+\frac{|N|}{|N^{(01230210)}|}+\frac{|N|}{|N^{(01231010)}|}$$

$$+\frac{|N|}{|N^{(01231031)}|}+\frac{|N|}{|N^{(01231013)}|}+\frac{|N|}{|N^{(01231032)}|}+\frac{|N|}{|N^{(01231201)}|}+\frac{|N|}{|N^{(01231303)}|}+\frac{|N|}{|N^{(01231320)}|}+\frac{|N|}{|N^{(01231323)}|}$$

$$+\frac{|N|}{|N^{(01232020)}|}+\frac{|N|}{|N^{(01232021)}|}+\frac{|N|}{|N^{(01232023)}|}+\frac{|N|}{|N^{(01232120)}|}+\frac{|N|}{|N^{(01232303)}|}+\frac{|N|}{|N^{(01232310)}|}+\frac{|N|}{|N^{(01232313)}|}$$
$$+\frac{|N|}{|N^{(01232320)}|}+\frac{|N|}{|N^{(01213101)}|}+\frac{|N|}{|N^{(01213102)}|}+\frac{|N|}{|N^{(01213120)}|}+\frac{|N|}{|N^{(01213121)}|}+\frac{|N|}{|N^{(01213130)}|}+\frac{|N|}{|N^{(010102302)}|}$$
$$+\frac{|N|}{|N^{(010102303)}|}+\frac{|N|}{|N^{(010102323)}|}+\frac{|N|}{|N^{(010120130)}|}+\frac{|N|}{|N^{(010120132)}|}+\frac{|N|}{|N^{(010120301)}|}+\frac{|N|}{|N^{(010120302)}|}$$
$$+\frac{|N|}{|N^{(010120313)}|}+\frac{|N|}{|N^{(010120320)}|}+\frac{|N|}{|N^{(010120321)}|}+\frac{|N|}{|N^{(010121031)}|}+\frac{|N|}{|N^{(010121303)}|}+\frac{|N|}{|N^{(010121310)}|}$$
$$+\frac{|N|}{|N^{(010121320)}|}+\frac{|N|}{|N^{(010121321)}|}+\frac{|N|}{|N^{(010123020)}|}+\frac{|N|}{|N^{(010123021)}|}+\frac{|N|}{|N^{(010123023)}|}+\frac{|N|}{|N^{(010123030)}|}$$
$$+\frac{|N|}{|N^{(010123130)}|}+\frac{|N|}{|N^{(010123202)}|}+\frac{|N|}{|N^{(010123203)}|}+\frac{|N|}{|N^{(010123212)}|}+\frac{|N|}{|N^{(010123231)}|}+\frac{|N|}{|N^{(010123232)}|}$$
$$+\frac{|N|}{|N^{(010201232)}|}+\frac{|N|}{|N^{(010201302)}|}+\frac{|N|}{|N^{(010201303)}|}+\frac{|N|}{|N^{(010201313)}|}+\frac{|N|}{|N^{(010202130)}|}+\frac{|N|}{|N^{(010202131)}|}$$
$$+\frac{|N|}{|N^{(010202132)}|}+\frac{|N|}{|N^{(010202313)}|}+\frac{|N|}{|N^{(010203010)}|}+\frac{|N|}{|N^{(010203013)}|}+\frac{|N|}{|N^{(010203020)}|}+\frac{|N|}{|N^{(010203021)}|}$$
$$+\frac{|N|}{|N^{(010203032)}|}+\frac{|N|}{|N^{(010203131)}|}+\frac{|N|}{|N^{(010203210)}|}+\frac{|N|}{|N^{(010203212)}|}+\frac{|N|}{|N^{(010203213)}|}+\frac{|N|}{|N^{(010212303)}|}$$
$$+\frac{|N|}{|N^{(010212313)}|}+\frac{|N|}{|N^{(010213010)}|}+\frac{|N|}{|N^{(010213101)}|}+\frac{|N|}{|N^{(010213121)}|}+\frac{|N|}{|N^{(010213123)}|}+\frac{|N|}{|N^{(010213212)}|}$$
$$+\frac{|N|}{|N^{(010230132)}|}+\frac{|N|}{|N^{(010230301)}|}+\frac{|N|}{|N^{(010230312)}|}+\frac{|N|}{|N^{(010230313)}|}+\frac{|N|}{|N^{(010230321)}|}+\frac{|N|}{|N^{(010230323)}|}$$
$$+\frac{|N|}{|N^{(010231201)}|}+\frac{|N|}{|N^{(010231202)}|}+\frac{|N|}{|N^{(010231212)}|}+\frac{|N|}{|N^{(010231230)}|}+\frac{|N|}{|N^{(010231301)}|}+\frac{|N|}{|N^{(010231310)}|}$$
$$+\frac{|N|}{|N^{(010231313)}|}+\frac{|N|}{|N^{(010232121)}|}+\frac{|N|}{|N^{(010232123)}|}+\frac{|N|}{|N^{(010232303)}|}+\frac{|N|}{|N^{(010232321)}|}+\frac{|N|}{|N^{(012010131)}|}$$
$$+\frac{|N|}{|N^{(012010132)}|}+\frac{|N|}{|N^{(012013012)}|}+\frac{|N|}{|N^{(012013030)}|}+\frac{|N|}{|N^{(012013130)}|}+\frac{|N|}{|N^{(012013230)}|}+\frac{|N|}{|N^{(012021231)}|}$$
$$+\frac{|N|}{|N^{(012021323)}|}+\frac{|N|}{|N^{(012023101)}|}+\frac{|N|}{|N^{(012023201)}|}+\frac{|N|}{|N^{(012023230)}|}+\frac{|N|}{|N^{(012023231)}|}+\frac{|N|}{|N^{(012032020)}|}$$
$$+\frac{|N|}{|N^{(012032023)}|}+\frac{|N|}{|N^{(012101323)}|}+\frac{|N|}{|N^{(012103131)}|}+\frac{|N|}{|N^{(012120320)}|}+\frac{|N|}{|N^{(012123203)}|}+\frac{|N|}{|N^{(012123210)}|}$$
$$+\frac{|N|}{|N^{(012302103)}|}+\frac{|N|}{|N^{(012310321)}|}+\frac{|N|}{|N^{(012313030)}|}+\frac{|N|}{|N^{(012320212)}|}+\frac{|N|}{|N^{(012320231)}|}+\frac{|N|}{|N^{(012321203)}|}$$
$$+\frac{|N|}{|N^{(012323130)}|}+\frac{|N|}{|N^{(012131210)}|}+\frac{|N|}{|N^{(0102030103)}|}+\frac{|N|}{|N^{(0102032123)}|}+\frac{|N|}{|N^{(0102132123)}|}+\frac{|N|}{|N^{(0102312013)}|}$$
$$+\frac{|N|}{|N^{(0120131303)}|}+\frac{|N|}{|N^{(0123021032)}|}+\frac{|N|}{|N^{(0101023232)}|}+\frac{|N|}{|N^{(0101203010)}|}+\frac{|N|}{|N^{(0101213102)}|}+\frac{|N|}{|N^{(0101230301)}|}$$
$$+\frac{|N|}{|N^{(0101230210)}|}+\frac{|N|}{|N^{(0101231302)}|}+\frac{|N|}{|N^{(0102030102)}|}+\frac{|N|}{|N^{(01020301020)}|}=7920.$$
$$\Longrightarrow |G|\leq 7920|N| = 7920\times 24 = 190080.$$

In order to show $|G| = 7920$, we consider G as a subgroup of S_{7920} acting on 7920 coset the we have found, and labeled as follows: (by magma). We find $f(x)$, $f(y)$ and $f(t_0)$ (by magma).

It readily checks that the order of $\langle x, y, t\rangle$, a subgroup of a symmetric group S_{7920} action on the 7920 right cosets of N in G, is 190080. Visibly $|x| = 4$ and $|y| = 2$, additionally $|xy| = 3$ and $[x, y]^2 = 2$, hence $\langle x, y\rangle \cong S_4$. If we conjugate t by S_4 we see that t has exactly 4 conjugates. We conclude that $\langle x, y, t\rangle$ is a homomorphic image of the progenitor $2^{*4} : S_4$.

Thus; if the original five relations hold in $\langle x, y, t\rangle$, then $\langle x, y, t\rangle$ is a homomorphic image of G and this will give $|G| \geq |\langle x, y, t\rangle| = 190080$.

The Homomorphism Image Of G

$\bar{G} = f(2^{*4} : S_4) = \langle f(x), f(y), f(t_0), f(t_1), f(t_2), f(t_3)\rangle$.

However;

$\{f(t_1), f(t_2), f(t_3)\} \in \{f(x), f(y), f(t_0)\}$.

So it is imply that: $\bar{G} = f(2^{*4} : S_4) = \langle f(x), f(y), f(t_0)\rangle$.

If the addition relation hold in $\langle f(x), f(y), f(t_0)\rangle$ then:

$$f\left(\frac{2^{*4} : S_4}{[(0,1,2,3)t_0]^{10} = 1}\right) = \langle f(x), f(y), f(t_0)\rangle.$$

The addition relation, namely $[(0,1,2,3)t_0]^{10} = 1$ hold in $\langle f(x), f(y), f(t_0)\rangle$ if $(0,2)(1,3)f(t_1)f(t_0)f(t_3)f(t_2)f(t_1)f(t_0)f(t_3)f(t_2)f(t_1)f(t_0)$ must equal to an identity.

By magma we know $(0,2)(1,3)f(t_1)f(t_0)f(t_3)f(t_2)f(t_1)f(t_0)f(t_3)f(t_2)f(t_1)f(t_0)$ is equal to an identity.

Thus the relation holds in $\bar{G}$.

$\Longrightarrow \bar{G}$ is a homomorphism image of G.

Now, the First Isomorphic Theorem $\left|G/Kerf\right| \cong \bar{G}$.

$\Longrightarrow |G| \geq |\bar{G}|$. It is easily verified that $\bar{G} = \langle f(x), f(y), f(t_0)\rangle = 190080$.

$|G| \geq 190080$.

But we have seen above $|G| \leq 190080$.

Hence; $|G| = 190080$.

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