Brief Announcement: Cooperative Guarding in Polygons with Holes

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Abstract

We study the Cooperative Guarding problem for polygons with holes in a mobile multi-agents setting. Given a set of agents, initially deployed at a point in a polygon with n vertices and h holes, we require the agents to collaboratively explore and position themselves in such a way that every point in the polygon is visible to at least one agent and that the set of agents are visibly connected. We study the problem under two models of computation, one in which the agents can compute exact distances and angles between two points in its visibility, and one in which agents can only compare distances and angles. In the stronger model, we provide a deterministic $O(n)$ round algorithm to compute such a cooperative guard set while not requiring more than $\frac{n+h}{2}$ agents and $O(\log n)$ bits of persistent memory per agent. In the weaker model, we provide an $O(n^4)$ round algorithm, that does not require more than $\frac{n+2h}{2}$ agents.

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1 Introduction

The *Art Gallery Problem* is a classical computational geometry problem which asks for the minimum number of guards required to completely guard the interior of a given art gallery. The art gallery is modelled as a simple polygon and guards are points on or inside the polygon. A set of guards is said to guard the art gallery, if every point in the art gallery is visible to at least one guard. This problem was first posed by Klee in 1973 and since then the problem has been of interest to researchers. The problem and its many variations have been well-studied over the years. Chvátal [4] was the first to show that $\lfloor \frac{n}{3} \rfloor$ guards are sufficient and sometimes necessary to guard a polygon with n vertices. Fisk [5] proved the same result via an elegant coloring argument, which also leads to an $O(n)$ algorithm.

We study a variant of the classical Art Gallery Problem known as the *Cooperative Guards* problem in polygons with holes from a distributed multi-agents perspective. The *Cooperative Guards* problem is similar to the *Art Gallery Problem* with the additional constraint that the visibility graph of the guards, i.e., the graph with guards as vertices and edges between guards that can see each other, should form a single connected component. This implies that if the guards can communicate through line of sight, then any two guards can communicate with each other (either directly or indirectly through intermediate guards). The *Cooperative Guards* problem was first introduced by Liaw *et al.* in 1993 [6], in which they show that this variant of the *Art Gallery Problem* is also NP-hard.

The pursuit of solving the *Cooperative Guards* problem for us is primarily motivated by the search for distributed multi-agent exploration algorithms for agents deployed in the polygon. One of the easiest ways to explore the polygon is by maintaining connectivity through line of sight and having the agents cooperatively send exploratory agents and expand the area of the polygon that they can collectively see. Such an algorithm would require the agents to be visibly connected and if at the end they are collectively able to see the entire polygon, then they form a solution to the *Cooperative Guards* problem. This lead us to first tackle the *Cooperative Guards* problem in the centralized setting and adapt it to the distributed multi-agent model. Once the agents are positioned to cooperatively guard the polygon, they have sufficient information to solve several computational geometry problems on the polygon by executing a distributed algorithm on their visibility graph. Hence our algorithm can also serve as an initial pre-processing step for distributed multi-agents to solve other problems on the polygon, for example they can compute the number of vertices of the polygon, diameter of the polygon etc. Our focus throughout this paper is to find time optimal algorithms for computing a set of cooperative guards on the polygon (not necessarily minimum). Analogous to Fisk's [5] coloring argument, we provide an efficient algorithm that computes a set of visibly connected guards using no more than $\frac{n+h-2}{2}$ guards.

2 Our Contributions

We first present a centralized algorithm for the Cooperative Guards problem in polygonal region with n vertices and h holes, that does not require more than $\frac{n+2h-2}{2}$ guards and runs in $O(n \log n)$ time. We then use an observation from Zylinski [8] to reduce the number of guards to $\frac{n+h-2}{2}$. The reduction uses the method of Sachs and Souvaine [3] to reduce the problem to a simple polygon with $n + h$ vertices and no holes.

We propose two distributed models of computation, one in which the agents can perceive exact distances (which we call *depth perception*) and hence can compute co-ordinates (with respect to some common reference frame) to map out the polygon, and another in which the agents receive only a combinatorial view of their visibility. In the former model we assume that the agents are positioned in an $[0, n^c] \times [0, n^c]$ grid so that their coordinates can be evaluated within $O(\log n)$ bits.

We present a distributed algorithm (in the stronger *depth perception* model) that runs in $O(n)$ rounds and does not require more than $\lfloor \frac{n+h-2}{2} \rfloor$ agents. Our distributed algorithm emulates the centralized algorithms presented except that the polygon is not known in advance. We simultaneously explore and incrementally construct a triangulation.

Finally, we present a distributed algorithm for the weaker *proximity perception* model, which takes $O(n^4)$ rounds, but requires $\frac{n+2h-2}{2}$ agents. These results improve upon the works by Obermeyer, Ganguli and Buffon [7] (which requires $\frac{n+2h-2}{2}$ guards in the worst case and $O(n^2)$ communication rounds) and Ashok et al. [1] (which requires $O(n)$ rounds but works only for polygons without holes). A summary of our results is presented in Table 1. The full version [2] of the paper contains all the details.

3 Open Problems

We discuss about further improvements and some closely related open problems below.

Table 1 A summary of our results.

Model & Algorithm	Rounds	Broadcasts	Persistent Memory	Guards
Depth Perception (Large memory)	$O(n)$	$O(n)$	$O(n \log n)$	$(n+h-2)/2$
Depth Perception (Small memory)	$O(n)$	$O(n \log n)$	$O(\log n)$	$(n+2h-2)/2$
Depth Perception (Improving guards)	$O(n)$	$O(n(h+\log n))$	$O(\log n)$	$(n+h-2)/2$
Proximity Perception	$O(n^4)$	$O(n^4)$	$O(\log n)$	$(n+2h-2)/2$

▶ **Open Problem 1.** *How many co-operative guards are required for a polygon with n vertices and h holes? What if guards are restricted to be on the vertices of the polygon?*

Our construction requires at most $\frac{n+h-2}{2}$ co-operative guards, however we have not been able to construct polygons requiring so many guards. For $h \leq 1$, the problem has been solved by Zylinski [8], but for $h \geq 2$, the problem remains open. We have not been able to provide a better upper bound than $\frac{n+2h-2}{2}$ guards when the guards are restricted to the vertices of the polygon as well as provide examples of polygons that need more than $\frac{n+h-2}{2}$ such guards.

▶ **Open Problem 2.** *Is there a more efficient centralized algorithm to compute a cooperative guard set with $\frac{n+h-2}{2}$ guards?*

The centralized algorithm proposed runs in $O(n^2)$ time, whereas we are able to construct a cooperative guard set using no more than $\frac{n+2h-2}{2}$ agents in $O(n \log n)$ time. Is it possible to bridge the gap in the running time?

▶ **Open Problem 3.** *Is it possible to construct $O(poly(D))$ time algorithms where D is the hop diameter of the polygon, when agents have limited persistent memory?*

The hop diameter of the polygon is the minimum number of straight line segments required to connect any two vertices of the polygon. In simple polygons, it is easy to construct such algorithms, however in polygons with holes, the problem appears to be more challenging.

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