A discrete Adomian decomposition method for the discrete nonlinear Schrödinger equation

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Abstract

We present a discrete Adomian decomposition method to approximate the theoretical solution of discrete nonlinear cubic Schrödinger equations. The method is examined for plane waves and for x-independent solutions in the case of semi–discrete Schrödinger equations.

Index Terms

Adomian decomposition method; discrete nonlinear Schrödinger equations; finite differences; solitons; plane waves

I. Introduction

In this work we want to describe a discrete version of the well–known Adomian decomposition method (ADM) applied to nonlinear cubic Schrödinger equations (NLS). The ADM was introduced by Adomian [4], [5] in the early 1980s to solve nonlinear ordinary and partial differential equation. This method avoids artificial boundary conditions, linearization and yields an efficient numerical solution with high accuracy.

The NLS equation is a typical dispersive nonlinear partial differential equation that plays a key role in a variety of areas in mathematical physics. It describes the spatio–temporal evolution of the complex field $u = u(x,t) \in \mathbb{C}$ and has the general form

$$i\partial_t u + \partial_x^2 u + q|u|^2 u = 0, \quad x \in \mathbb{R},\ t > 0, \tag{1a}$$

$$u(x,0) = f(x), \tag{1b}$$

where the parameter $q \in \mathbb{R}$ corresponds to a focusing $(q > 0)$ or defocusing $(q < 0)$ effect of the nonlinearity.

Equation (1a) describes many problems in physics. The fields of application varies from optics [16], propagation of the electric field in optical fibers [14], self–focusing and collapse of Langmuir waves in plasma physics [26] to modelling deep water waves and freak waves (so–called rogue waves) in the ocean [19].

It is well known that (1a) is completely S–integrable (in the sense of Calogero [7]) with the inverse scattering method (ISM) [25] and a single soliton solution is given by

$$u(x,t) = \Bigl(\frac{2a}{q}\Bigr)^{1/2} \exp\Bigl[i\bigl(\frac{c}{2}x - \theta t\bigr)\Bigr] \operatorname{sech}\bigl[a^{1/2}(x - ct)\bigr], \tag{2}$$

with $\theta = c^2/4 - a$. For fixed t the function u in (2) decays exponentially as $|x| \to \infty$. It travels with the envelope speed c and its amplitude is governed by the parameter $a \in \mathbb{R}$.

Then an N–soliton solution for $q \neq 0$ is given by [20]

$$u(x,t) = \Bigl(\frac{2a}{q}\Bigr)^{1/2} \sum_{p=1}^{N} \exp\Bigl[i\bigl(\frac{c_p}{2}x_p - \theta_p t\bigr)\Bigr] \operatorname{sech}\bigl[a^{1/2}(x_p - c_p t)\bigr], \tag{3}$$

with $\theta_p = c_p^2/4 - a$, the position x_p of the p–soliton and c_p its velocity.

Finally, a particular simple form of solutions to the Schrödinger equation (1a) are the plane wave solutions

$$u(x,t) = \exp\Bigl[i\bigl(\kappa x - \omega t\bigr)\Bigr], \quad x \in \mathbb{R}, \quad t > 0, \tag{4}$$

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where κ is the wave number and ω denotes the frequency. Substituting (4) into the NLS (1a) yields the dispersion relation

$$\kappa^2 - \omega = q. \tag{5}$$

Since (1a) is S–integrable it is a Hamiltonian system with an infinite number of conserved quantities. Here we will only present the two most important quantities. First the L^2-norm (mass, number of particles) is conserved:

$$N = \frac{2}{q}\int_{-\infty}^{\infty} |u(x,t)|^2\,dx = \text{const}\,. \tag{6}$$

Note that this conservation property (6) has an important meaning in physical applications. It can be interpreted as the conservation of the power of the beam in nonlinear optics and in Bose–Einstein condensation it denotes the conservation of the number of atoms in the condensate.

Another conserved quantity is the Hamiltonian

$$H = \int_{-\infty}^{\infty}\Big[|\partial_x u(x,t)|^2 + \frac{q}{2}|u(x,t)|^4\Big]\,dx = \text{const}\,. \tag{7}$$

with $i\partial_t u = \partial_{u^*}H = \{H,u\}$, where the standard Poisson brackets have been used and * denotes complex conjugation.

II. The Adomian Decomposition Method

In this Section we shall sketch the ADM for partial differential equations applied to the NLS equation. To this end, we consider (1a) written in operator form as

$$L_t u = i\partial_x^2 u + iqF(u), \quad x\in\mathbb{R},\ t>0 \tag{8}$$

with the notation $L_t = \partial_t$ and the cubic nonlinear term as $F(u) = |u|^2 u$. Then the inverse operator of L_t is defined by the definite integral

$$[L_t^{-1}v](t) = \int_0^t v(\tau)\,d\tau, \quad t>0. \tag{9}$$

Now applying formally the inverse operator L_t^{-1} to (8) subjext to the initial condition (1b) yields the formal solution to (1)

$$u(x,t) = f(x) + iL_t^{-1}\partial_x^2 u + iqL_t^{-1}F(u), \quad x\in\mathbb{R},\ t>0. \tag{10}$$

The Adomian decomposition method [5] assumes a solution of the series form $u(x,t) = \sum_{l=0}^{\infty} u_l(x,t)$, where the components $u_l(x,t)$ are going to be determined recurrently. The nonlinear term $F(u)$ in (10) is decomposed into an infinite series of polynomials of the form $F(u) = \sum_{l=0}^{\infty} A_l(u)$, where the A_l are the so–called Adomian polynomials. Substituting these decomposition series into (10) gives

$$\sum_{l=0}^{\infty} u_l(x,t) = f(x) + i\sum_{l=0}^{\infty} L_t^{-1}\partial_x^2 u_l(x,t) + iq\sum_{l=0}^{\infty} L_t^{-1}A_l. \tag{11}$$

According to [5] $u_0(x,t)$ is identified with the initial data $f(x)$ and the following recurrence is proposed:

$$u_0(x,t) = f(x), \tag{12a}$$

$$u_{l+1}(x,t) = iL_t^{-1}\partial_x^2 u_l(x,t) + iqL_t^{-1}A_l; \quad l = 0,1,2,\dots. \tag{12b}$$

It remains to determine the Adomian polynomials A_l. They are defined by

$$A_l = \frac{1}{l!}\frac{\mathrm{d}^l}{\mathrm{d}\lambda^l}\Big[F\big(\sum_{p=0}^{\infty}\lambda^p u_p\big)\Big]_{\lambda=0}, \text{ for } l = 0,1,2,\dots \tag{13}$$

and constructed for all classes of nonlinearity according to algorithms given either by Adomian [5] or alternatively by Wazwaz [24]. To do so, we set $F(u) = u^2\bar{u}$ and obtain in a straight forward calculation

$$A_0 = u_0^2\bar{u}_0, \tag{14a}$$

$$A_1 = 2u_0u_1\bar{u}_0 + u_0^2\bar{u}_1, \tag{14b}$$

$$A_2 = 2u_0u_2\bar{u}_0 + u_1^2\bar{u}_0 + 2u_0u_1\bar{u}_1 + u_0^2\bar{u}_2, \tag{14c}$$

$$A_3 = 2(u_0u_3\bar{u}_0 + u_1u_2\bar{u}_0 + u_0u_2\bar{u}_1 + u_0u_1\bar{u}_2) + u_1^2\bar{u}_1 + u_0^2\bar{u}_3, \tag{14d}$$

$$\dots.$$

The polynomials A_l, $l\geq 4$ can be computed in a similar manner.

Let us finally note that the convergence of this method was established in [8], [9] using a fixed point theorem. Since in practice not all terms (12) of the series $u(x,t) = \sum_{l=0}^{\infty} u_l(x,t)$ can be calculated we use a finite sum $U_L(x,t) = \sum_{l=0}^{L} u_l(x,t)$ to approximate the solution.

III. Discrete nonlinear Schrödinger equations

Discrete nonlinear Schrödinger equations (DNLS) are omnipresent [15] in applied sciences, e.g. describing the propagation of electromagnetic waves in glass fibers, one–dimensional arrays of coupled optical waveguides [11] and light–induced photonic crystal lattices [10]. Moreover, they are used to describe Bose–Einstein condensates in optical lattices [23] and they are an established model for optical pulse propagation in various doped fibers [12], [13].

In this section we will consider the two most common discrete versions of the cubic NLS equation (1a) that arise from different spatial discretizations. These discrete nonlinear Schrödinger equations (DNLS) are also called lattice NLS equations and we refer the reader to [22, Chapter 5.2.2] for a concise discussion on this topic.

A. The standard discrete NLS

If one applies the standard spatial discretization to (1a) and replaces $F(u) = |u|^2 u$ with a diagonal discretization $F_D(u_j) = |u_j|^2 u_j$, we obtain the usual DNLS equation:

$$i\partial_t u_j + D_h^2 u_j + q|u_j|^2 u_j = 0, \quad j \in \mathbb{Z},\ t > 0, \tag{15a}$$

$$u_j(0) = f_j, \quad j \in \mathbb{Z}, \tag{15b}$$

with $u_j = u_j(t)$, $h = \Delta x$ and $D_h^2 u_j = (u_{j+1} - 2u_j + u_{j-1})/h^2$ denotes the standard second order difference quotient. The parameter $\varepsilon := h^{-2}$ is called (discrete) dispersion and the parameter q is called anharmonicity, since equation (15a) with $\varepsilon = 0$ describes a set of uncoupled anharmonic oscillators.

The DNLS equation (15a) has a discrete conserved number (mass, total excitation norm, power in nonlinear optics)

$$N_D = \frac{2}{q} \sum_{j\in\mathbb{Z}} |u_j|^2 \tag{16}$$

and the discrete Hamiltonian

$$H_D = -\sum_{j\in\mathbb{Z}} \Big[u_j^*(u_{j+1} + u_{j-1}) - 2|u_j|^2 + \frac{q}{2}|u_j|^4 \Big], \tag{17}$$

where * denotes the complex conjugate.

However, the standard DNLS equation (15a) is not an exactly integrable DNLS (if the spatial grid consists of more than 2 points) and thus less amenable to mathematical analysis. We can only give particular discrete plane wave solutions to the DNLS equation (15a) of the form

$$u_j(t) = \exp\Big[i\big(j\kappa h - \omega t\big) \Big], \quad j \in \mathbb{Z},\ t > 0. \tag{18}$$

Inserting (18) into the DNLS (15a) yields the discrete dispersion relation

$$\frac{4}{h^2} \sin^2\Big(\frac{\kappa h}{2}\Big) - \omega = q, \tag{19}$$

which is obviously consistent with the contnuous relation (5). Hence we will turn in the sequel to an integrable discrete NLS equation.

B. The Ablowitz–Ladik equation

After a discretization in space by replacing the cubic nonlinearity $F(u) = |u|^2 u$ in (1a) with an off–diagonal discretization $F_{AL}(u_j) = |u_j|^2 (u_{j+1} + u_{j-1})/2$ and keeping the time variable continuous we obtain the Ablowitz–Ladik (AL) equation [1], [2]:

$$i\partial_t u_j + D_h^2 u_j + q|u_j|^2 \frac{u_{j+1} + u_{j-1}}{2} = 0, \quad j \in \mathbb{Z},\ t > 0, \tag{20a}$$

$$u_j(0) = f_j, \quad j \in \mathbb{Z}. \tag{20b}$$

Note that one term in (20a) can be removed through the transformation

$$u_j(t) = v_j(t) \exp(-i2t), \quad t > 0,$$

and equation (20a) reduces to the normalized form

$$i\partial_t v_j + \frac{v_{j+1} + v_{j-1}}{h^2} + q|v_j|^2 \frac{v_{j+1} + v_{j-1}}{2} = 0, \quad j \in \mathbb{Z},\ t > 0. \tag{21}$$

The AL equation has a conserved number

$$N_{AL} = \frac{2}{q} \sum_{j\in\mathbb{Z}} \log\big(1 + \frac{q}{2}|u_j|^2\big) \tag{22}$$

and the Hamiltonian

$$H_{AL} = -\sum_{j\in\mathbb{Z}}\Big[u_j^*(u_{j+1}+u_{j-1}) - \frac{4}{q}\log\big(1+\frac{q}{2}|u_j|^2\big)\Big]. \tag{23}$$

The nonlinear differential–difference equation (20a) is the most famous integrable DNLS equation. As the AL equation (20a) is integrable it is possible to give exact travelling–wave solutions on the real line $j \in \mathbb{Z}$, including (cf. [3], [21], [22])

$$v_j(t) = A\exp\big[i(\omega t + \alpha j + v_0)\big]\,\mathrm{cn}\big[\beta(j - vt); k\big], \quad t > 0, \tag{24}$$

where $\mathrm{cn}[\cdot; k]$ is a Jacobi elliptic function of modulus k. For the case $h = 1$, $q = 2$ the parameters in (24) can be written as

$$A = \frac{k\,\mathrm{sn}[\beta; k]}{\mathrm{dn}[\beta; k]}, \qquad \omega = \frac{2\,\mathrm{cn}[\beta; k]\cos\alpha}{\mathrm{dn}^2[\beta; k]}, \qquad v = \frac{2\,\mathrm{sn}[\beta; k]\sin\alpha}{\beta\,\mathrm{dn}[\beta; k]},$$

where $-\pi \le \alpha \le \pi$, $\beta > 0$, $0 < k < 1$ are free parameters. In the limiting case $k \to 1$ (hyperbolic limit) we get for the Jacobi elliptic (sn, cn, dn) functions [17]:

$$\lim_{k\to1}\mathrm{sn}[\beta; k] = \tanh\beta, \quad \lim_{k\to1}\mathrm{cn}[\beta; k] = \lim_{k\to1}\mathrm{dn}[\beta; k] = \mathrm{sech}\,\beta,$$

and obtain the discrete soliton solution of (21)

$$v_j(t) = \sinh\beta\exp\big[-i(\omega t + \alpha j + v_0)\big]\,\mathrm{sech}\big[\beta(j - vt)\big], \quad t > 0, \tag{25}$$

with

$$\omega = -2\cosh\beta\cos\alpha, \qquad v = -\frac{2}{\beta}\sinh\beta\sin\alpha,$$

that can travel at any velocity. It can be easily seen that the discrete soliton (25) is a fairly obvious discrete version of the continuous soliton solution (2).

There exist discrete plane wave solutions to the Ablowitz–Ladik equation (20a) of the form (18). Inserting (18) into the AL equation (20a) gives the discrete dispersion relation

$$\frac{4}{h^2}\sin^2\Big(\frac{\kappa h}{2}\Big) - \omega = q\cos(\kappa h). \tag{26}$$

The main interest in the AL equation arises from mathematics (in contrast to the standard DNLS equation); only a few physical models [18] can be described by an AL–type equation.

IV. The semi–discrete Adomian Decomposition Method

The analogue discrete steps to the continuous ADM of Section II are simply the formal solution to the DNLS equation (15) or the AL equation (20):

$$u_j(t) = f_j + iL_t^{-1}D_h^2 u_j + iqL_t^{-1}F_{D,AL}(u_j), \quad j \in \mathbb{Z},\ t > 0, \tag{27}$$

and the assumption that there exists a solution of the series form $u_j(t) = \sum_{l=0}^{\infty} u_{j,l}(t)$. The nonlinear term $F_{D,AL}(u_j)$ in (27) is decomposed into an infinite series of discrete Adomian polynomials $F_{D,AL}(u_j) = \sum_{l=0}^{\infty} A_l(u_j)$. Substituting these decompositions into (27) gives

$$\sum_{l=0}^{\infty} u_{j,l}(t) = f_j + i\sum_{l=0}^{\infty} L_t^{-1}D_h^2 u_j(t) + iq\sum_{l=0}^{\infty} L_t^{-1}A_l. \tag{28}$$

Again, $u_{j,0}(t)$ is identified with the initial data f_j and the following recurrence is proposed to determine the solution components $u_{j,l}(t)$:

$$u_{j,0}(t) = f_j, \tag{29a}$$

$$u_{j,l+1}(t) = iL_t^{-1}D_h^2 u_{j,l}(t) + iqL_t^{-1}A_l, \quad l = 0, 1, 2, \ldots \tag{29b}$$

For the standard DNLS equation the Adomian polynomials are the same as (14), but for the AL equation we write $F_{AL}(u_j) = |u_j|^2(u_{j+1}+u_{j-1})/2$ and obtain analogously to (14)

$$A_0 = u_{j,0}\frac{u_{j+1,0}+u_{j-1,0}}{2}\bar{u}_{j,0}, \tag{30a}$$

$$A_1 = \big[u_{j,0}\frac{u_{j+1,1}+u_{j-1,1}}{2} + \frac{u_{j+1,0}+u_{j-1,0}}{2}u_{j,1}\big]\bar{u}_{j,0} + u_{j,0}\frac{u_{j+1,0}+u_{j-1,0}}{2}\bar{u}_{j,1}, \tag{30b}$$

$$\begin{aligned} A_2 = &\big[u_{j,0}\frac{u_{j+1,2}+u_{j-1,2}}{2} + \frac{u_{j+1,0}+u_{j-1,0}}{2}u_{j,2}\big]\bar{u}_{j,0} + u_{j,1}\frac{u_{j+1,1}+u_{j-1,1}}{2}\bar{u}_{j,0} \\ &+ \big[u_{j,0}\frac{u_{j+1,1}+u_{j-1,1}}{2} + \frac{u_{j+1,0}+u_{j-1,0}}{2}u_{j,1}\big]\bar{u}_{j,1} + u_{j,0}\frac{u_{j+1,0}+u_{j-1,0}}{2}\bar{u}_{j,2}, \end{aligned} \tag{30c}$$

$$\begin{aligned} A_3 = &\big[u_{j,0}\frac{u_{j+1,3}+u_{j-1,3}}{2} + \frac{u_{j+1,0}+u_{j-1,0}}{2}u_{j,3}\big]\bar{u}_{j,0} \\ &+ \big[u_{j,1}\frac{u_{j+1,2}+u_{j-1,2}}{2} + \frac{u_{j+1,1}+u_{j-1,1}}{2}u_{j,2}\big]\bar{u}_{j,0} \\ &+ \big[u_{j,0}\frac{u_{j+1,2}+u_{j-1,2}}{2} + \frac{u_{j+1,0}+u_{j-1,0}}{2}u_{j,2}\big]\bar{u}_{j,1} \\ &+ \big[u_{j,0}\frac{u_{j+1,1}+u_{j-1,1}}{2} + \frac{u_{j+1,0}+u_{j-1,0}}{2}u_{j,1}\big]\bar{u}_{j,2} \\ &+ u_{j,1}\frac{u_{j+1,1}+u_{j-1,1}}{2}\bar{u}_{j,1} + u_{j,0}\frac{u_{j+1,0}+u_{j-1,0}}{2}\bar{u}_{j,3}, \end{aligned} \tag{30d}$$

$$\ldots.$$

The polynomials A_l, $l \geq 4$ can be computed analogously in a tedious calculation. The calculation to obtain the Adomian polynomials for the AL equation (30) was performed using `Mathematica`.

First we consider the simple example of a plane wave solution (18) to the DNLS equation (15a). We obtain by the Adomian decomposition technique (29) with the Adomian polynomials (14) and the semi–discrete dispersion relation $\omega = (4/h^2)\sin^2\big(\kappa h/2\big) - q$ given in (19):

$$u_{j,0}(t) = f_j = e^{ij\kappa h}, \tag{31a}$$

$$\begin{aligned} u_{j,1}(t) &= iL_t^{-1}D_h^2 u_{j,0}(t) + iqL_t^{-1}A_0 = iL_t^{-1}D_h^2 e^{ij\kappa h} + iqL_t^{-1}[u_{j,0}^2\bar{u}_{j,0}] \\ &= -i\frac{4}{h^2}\sin^2\Big(\frac{\kappa h}{2}\Big)te^{ij\kappa h} + iqte^{ij\kappa h} = -i\omega te^{ij\kappa h}, \end{aligned} \tag{31b}$$

$$\begin{aligned} u_{j,2}(t) &= iL_t^{-1}D_h^2 u_{j,1}(t) + iqL_t^{-1}A_1 \\ &= iL_t^{-1}D_h^2[-i\omega te^{ij\kappa h}] + iqL_t^{-1}[2u_{j,0}u_{j,1}\bar{u}_{j,0} + u_{j,0}^2\bar{u}_{j,1}] \\ &= -\frac{1}{2}\omega\frac{4}{h^2}\sin^2\Big(\frac{\kappa h}{2}\Big)t^2e^{ij\kappa h} + \frac{1}{2}q\omega t^2e^{i\kappa x} = -\frac{1}{2}\omega^2t^2e^{i\kappa x}, \end{aligned} \tag{31c}$$

$$\begin{aligned} u_{j,3}(t) &= iL_t^{-1}D_h^2 u_{j,2}(t) + iqL_t^{-1}A_2 \\ &= iL_t^{-1}D_h^2[-\frac{1}{2}\omega^2t^2e^{ij\kappa h}] \\ &\quad + iqL_t^{-1}[2u_{j,0}u_{j,2}\bar{u}_{j,0} + u_{j,1}^2\bar{u}_{j,0} + 2u_{j,0}u_{j,1}\bar{u}_{j,1} + u_{j,0}^2\bar{u}_{j,2}] \\ &= \frac{i}{6}\omega^3t^3e^{ij\kappa h}. \end{aligned} \tag{31d}$$

Finally summing up the iterates yields

$$\begin{aligned} u_j(t) &= \sum_{l=0}^{\infty} u_{j,l}(t) = e^{ij\kappa h}\Big\{1 - i\omega t - \frac{1}{2}\omega^2t^2 + \frac{i}{6}\omega^3t^3 + \ldots\Big\} \\ &= e^{ij\kappa h}e^{-i\omega t} = e^{i(j\kappa h-\omega t)}. \end{aligned}$$

Secondly, we want to compute the x–independent solution $u_j(t) = f\exp(iq|f|^2t)$. In this special case the DNLS equation (15) and the AL equation (20) coincide and it is fairly easy to see that the ADM yields exactly the desired solution.

Now we consider the AL equation (20a) and a plane wave solution (18). We get using the Adomian decomposition technique

(29) and the Adomian polynomials (30):

$$u_{j,0}(t) = f_j = e^{ij\kappa h}, \tag{32a}$$

$$\begin{aligned} u_{j,1}(t) &= iL_t^{-1}D_h^2 u_{j,0}(t) + iqL_t^{-1}A_0 \\ &= iL_t^{-1}D_h^2 e^{ij\kappa h} + iqL_t^{-1}[u_{j,0}\frac{u_{j+1,0}+u_{j-1,0}}{2}\bar{u}_{j,0}] \\ &= -i\frac{4}{h^2}\sin^2\Big(\frac{\kappa h}{2}\Big)te^{ij\kappa h} + iqt\cos(\kappa h) = -i\omega t e^{ij\kappa h}, \end{aligned} \tag{32b}$$

$$\begin{aligned} u_{j,2}(t) &= iL_t^{-1}D_h^2 u_{j,1}(t) + iqL_t^{-1}A_1 \\ &= iL_t^{-1}D_h^2[-i\omega t e^{ij\kappa h}] + iqL_t^{-1}\Big[u_{j,0}\frac{u_{j+1,0}+u_{j-1,0}}{2}\bar{u}_{j,1} \\ &\qquad + \big[u_{j,0}\frac{u_{j+1,1}+u_{j-1,1}}{2} + \frac{u_{j+1,0}+u_{j-1,0}}{2}u_{j,1}\big]\bar{u}_{j,0}\Big] \\ &= -\frac{1}{2}\omega\frac{4}{h^2}\sin^2\Big(\frac{\kappa h}{2}\Big)t^2 e^{ij\kappa h} + \frac{1}{2}q\cos(\kappa h)\omega t^2 e^{i\kappa x} = -\frac{1}{2}\omega^2 t^2 e^{i\kappa x}, \end{aligned} \tag{32c}$$

$$\begin{aligned} u_{j,3}(t) &= iL_t^{-1}D_h^2 u_{j,2}(t) + iqL_t^{-1}A_2 \\ &= iL_t^{-1}D_h^2[-\frac{1}{2}\omega^2 t^2 e^{ij\kappa h}] + iqL_t^{-1}\Big[u_{j,1}\frac{u_{j+1,1}+u_{j-1,1}}{2}\bar{u}_{j,0} \\ &\qquad + \big[u_{j,0}\frac{u_{j+1,2}+u_{j-1,2}}{2} + \frac{u_{j+1,0}+u_{j-1,0}}{2}u_{j,2}\big]\bar{u}_{j,0} \\ &\qquad + \big[u_{j,0}\frac{u_{j+1,1}+u_{j-1,1}}{2} + \frac{u_{j+1,0}+u_{j-1,0}}{2}u_{j,1}\big]\bar{u}_{j,1} \\ &\qquad + u_{j,0}\frac{u_{j+1,0}+u_{j-1,0}}{2}\bar{u}_{j,2}\Big] \end{aligned} \tag{32d}$$

$$= \frac{i}{6}\omega^3 t^3 e^{ij\kappa h}, \tag{32e}$$

where $\omega = (4/h^2)\sin^2\big(\kappa h/2\big) - q\cos(\kappa h)$ is given in (26). Finally summing up the iterates yields, again, the exact plane wave solution

$$u_j(t) = \sum_{l=0}^{\infty} u_{j,l}(t) = e^{ij\kappa h}e^{-i\omega t} = e^{i(j\kappa h - \omega t)}.$$

Conclusions

In this work we have shown how the well–known Adomian decomposition technique can be adapted in order to be used to the semi-DNLS equations. We applied our findings to (semi)–discrete nonlinear Schrödinger equations. Authors has worked further this task and managed to extend this work. A much more detailed version of this paper including Mathematica programs, treatment of more semi-DNLS equations and the application of the Adomian method for fully discrete Schrödinger equations as well, can be found in [6].

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