POSTPRINT VERSION. The final version is published here :
Merabti, H., Massicotte, D. et Zhu, W. P. (2019, 10-12 Dec. 2019). Electrodynamic Loudspeaker Linearization using a Low Complexity pth-Order Inverse Nonlinear Filter. Communication présentée 2019 IEEE International Symposium on Signal Processing and Information Technology (ISSPIT), Ajman. doi: 10.1109/ISSPIT47144.2019.9001831

# Electrodynamic Loudspeaker Linearization using a Low Complexity pth-Order Inverse Nonlinear Filter

Hocine Merabti[1], Daniel Massicotte[1], Wei-Ping Zhu[2]

[1] Dept. of Electrical and Computer Engineering, Université du Québec à Trois-Rivières, Québec, Canada
E-mail: {hocine.merabti, daniel.massicotte}@uqtr.ca
[2] Dept. of Electrical and Computer Engineering, Concordia University, Québec, Canada
E-mail: weiping@ece.concordia.ca

***Abstract*—Nonlinear distortions are very challenging to tackle in electromechanical loudspeakers. They are observed in large signals mode, where high amplitude stimulus drives different components of the transducer to operate in their nonlinear region, resulting in harmonic and intermodulation distortions in the reproduced sounds. Many linearization schemes have been proposed to address this problem, they operate by pre-distorting the input signal before exciting the loudspeaker, in the aim of radiating distortion-free sound waves. In this work, we are interested in the performance evaluation of a low computational complexity feedforward linearization structure which is based on the pth order inverse of a one-dimension Volterra model of the driver. The scheme is designed to compensate for the 2nd and 3rd harmonic distortions. We will study the effect of varying the input voltage amplitude on the harmonic distortions reduction performance. A lumped-parameters model with parameters of a real driver will be used for the evaluation.**

***Keywords— nonlinear systems, loudspeaker linearization, black box modeling, Volterra models***

## I. Introduction

In the recent years, the consumer audio industry is experiencing a rapid growth thanks to the introduction of innovative and enhanced products like smart speakers, earbuds, earphones and headphones. The quality of the reproduced sounds is a core characteristic of a good audio system, where pure and undistorted sound playback is highly sought after. Electrodynamic speakers are widely used in these products, where the operation principle is to convert electrical signals into acoustical waves via mechanical vibrations.

To achieve maximum loudness with minimum design size and costs, the electrodynamic system is driven near maximum excursion, where different parts of the driver exhibit a nonlinear behavioral. This nonlinear mode introduces nonlinear distortions in the reproduced sound, which degrades the perceived sound quality [1].

Many works have been proposed to compensate for nonlinear distortions by introducing a predistortion system before driving the transducer. They are either based on a parametric model of the electromechanical driver using lumped-parameters modeling [2]-[4], or black box modeling [5]. In the lumped-parameters approach, prior known information about the physical structure is exploited. Predistortion systems based on the lumped-parameters model require electrical and/or mechanical sensors in order to measure and estimate the system state variables needed to compute instantaneous value of the nonlinear parameters. Also, the linearization performance depends strongly on the accuracy of the lumped-parameters model and measured parameters, which is sometimes challenging especially for small drivers.

In the black box approach, the predistortion system is based on the inversion of generic models like the Volterra and Hammerstein models [6]-[7], and neural networks [8]. The generic model does not require any prior knowledge about the internal structure of the driver. However, this nice property is usually traded with a much greater number of parameters to estimate, and difficulty to provide physical interpretations of the model parameters. Volterra models are generic models widely used to model nonlinear systems (e.g. [9]). They can model linear responses and high order nonlinearities with memory. This interesting feature was the motivation behind many works for electromechanical transducers modeling and linearization [10]-[13].

A major limitation of using conventional Volterra filters is the computational and implementation complexity which increases significantly when the model order is above 2. Given that electrodynamic drivers exhibit 3rd order harmonic distortions, using a linearization scheme based on a full Volterra model is not feasible practically. Recently, a simplified 'One-Dimension' Volterra model has been proposed to reduce the nonlinear distortions caused by frequency and amplitude modulation in parametric speakers [11],[14]. The idea is to use only the diagonal parameters of the conventional Volterra model, which significantly reduces the computational complexity of the linearization scheme while slightly affecting the performance.

In this work, we are interested in evaluating the linearization performance of the 3rd order nonlinear inverse filter in the context of electrodynamic loudspeakers. The simplified Volterra model parameters are identified based on a lumped-parameters model of a practically measured 1.5 inch mini-speaker. We will perform a harmonic distortions analysis for different excitation amplitudes. We expose the following points: Section II presents generalities on nonlinear modeling and the structure of the evaluated nonlinear inverse filter. The general lumped-parameter model used to model the electromechanical transducer behavior is presented in Section III. In Section IV, results obtained from the simulation model are presented. Finally, we expose our conclusion in Section V.

## II. Nonlinear Modeling and Inverse Filter

### *A. The Volterra Model and its Simplified Version*

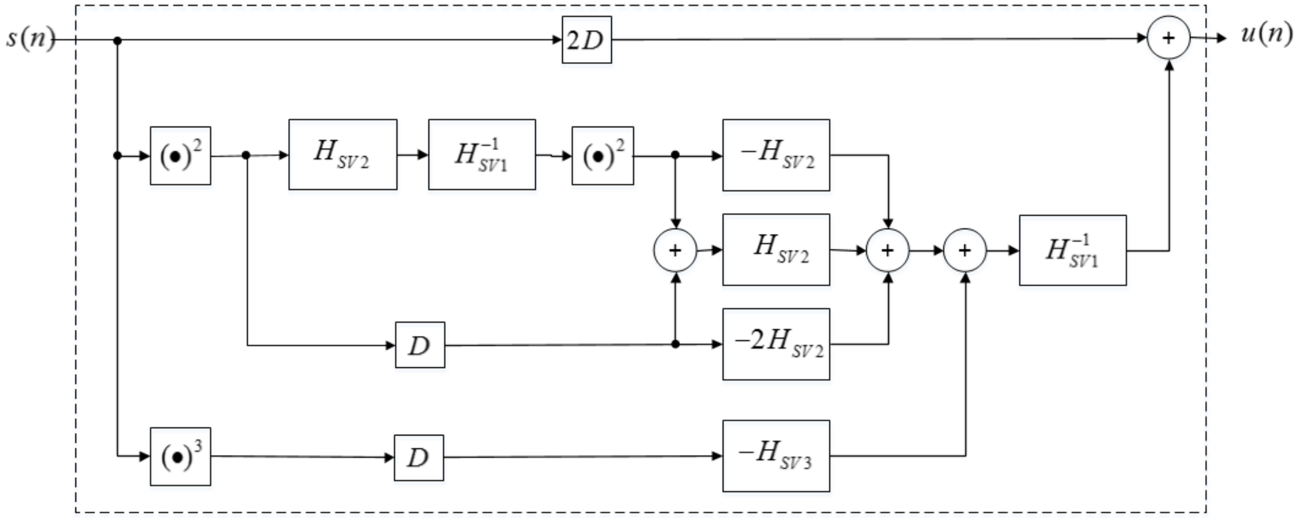


Fig. 1 Architecture of the 3rd order predistortion system.

We model the linear and nonlinear behavior of the electromechanical driver using a third order truncated Volterra model, describing the relationship between the digitized output pressure signal $y(n)$ and the digitized input signal $u(n)$ of the driver. The standard truncated 3rd order Volterra model is expressed as [9]

$$\begin{aligned} y(n) &= \sum_{m_1=0}^{N_1-1} h_{V1}(m_1)u(n-m_1) \\ &+ \sum_{m_1=0}^{N_2-1}\sum_{m_2=0}^{N_2-1} h_{V2}(m_1,m_2)u(n-m_1)u(n-m_2) \\ &+ \sum_{m_1=0}^{N_3-1}\sum_{m_2=0}^{N_3-1}\sum_{m_3=0}^{N_3-1} h_{V3}(m_1,m_2,m_3)u(n-m_1)u(n-m_2)u(n-m_3) \end{aligned} \quad (1)$$

Where $h_{V1}(m_1)$ are the linear kernels, $h_{V2}(m_1,m_2)$ the second order kernels, $h_{V3}(m_1,m_2,m_3)$ the third order kernels, and $N_1$, $N_2$ , $N_3$ are their truncated lengths, respectively.

A simplified version could be used, where only the diagonal kernels are nonzero. Several names have been given for this simplified representation like the One-dimension Volterra [10], the Generalized Hammerstein Model (GHM), [7] and Hammerstein Group Model (HGM) [15].

The simplified model is expressed as

$$\begin{aligned} y(n) &= \sum_{m_1=0}^{N_1-1} h_{SV1}(m_1)u(n-m_1) \\ &+ \sum_{m_2=0}^{N_2-1} h_{SV2}(m_2)u(n-m_2)^2 \\ &+ \sum_{m_3=0}^{N_3-1} h_{SV3}(m_3)u(n-m_3)^3 \end{aligned} \quad (2)$$

It is obvious from (1) and (2) that the computational and memory complexity is significantly reduced, going from $O(n^3)$ down to $O(n)$.

## B. Identifying the Simplified Volterra Parameters

A simple and fast way to identify the model parameters is to use a logarithmic swept-sine as a stimulus [16]. This method is widely used to perform nonlinear systems analysis using a single chirp. The magnitude plot of the obtained responses is the generally used information. Recently, it was shown that the phase plots of the higher order responses obtained using the method in [16] is incorrect [17]. The authors have proposed an improved method named Synchronous Swept-Sine which guarantees that both the magnitude and phase responses are accurate. Considering that the design of the nonlinear inverse filter requires both amplitude and phase responses to be accurate, we will implement the synchronous swept-sine method which is described by the following equations

The stimulus is defined as

$$u(t) = \sin\left[2\pi f_1 L \exp\left(\frac{t}{L}\right)\right] \text{, with } L = \frac{1}{f_1} round\left[\frac{f_1}{\ln\left(\frac{f_2}{f_1}\right)}\tilde{T}\right] \quad (3)$$

where $\tilde{T}$ is the approximate duration of the excitation, $f_1$ and $f_2$ are the starting and ending frequencies of the sweep, respectively.

The total impulse response $h(n)$ containing the linear and higher order impulse responses concatenated in a single vector is obtained by multiplying the system output $Y(f)$ with an inverse filter defined as

$$\tilde{U}(f) = 2\sqrt{\frac{f}{L}}\exp\left\{-j2\pi f L\left[1-\ln\left(\frac{f}{f_1}\right)\right]+j\frac{\pi}{4}\right\} \quad (4)$$

The time delay between the linear response and the $N^{th}$ order nonlinear response needed to separate the impulse responses is given by

$$\Delta t = L\ln(N) \quad (5)$$

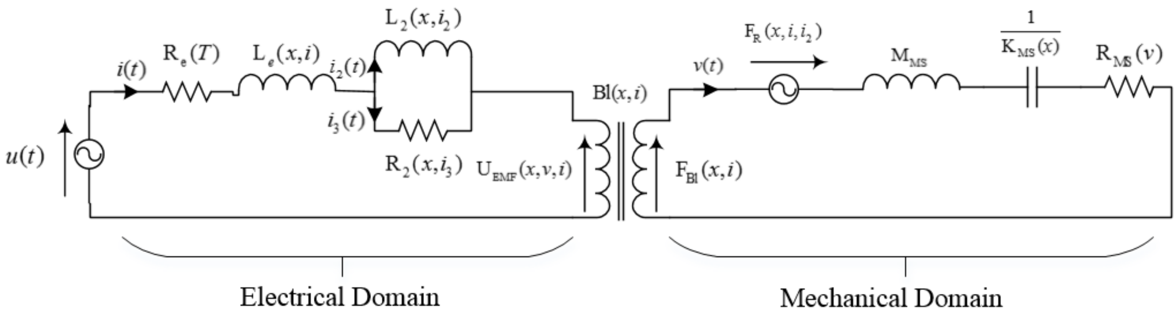


Fig. 2 Electromechanical transducer equivalent lumped-parameters model.

The obtained time windowed linear impulse response $h_{SS1}(n)$, second order impulse response $h_{SS2}(n)$ and the third order impulse response $h_{SS3}(n)$ can be mapped to the diagonal Volterra model truncated to the 3rd order scheme as following [18], [7]

$$\begin{cases} H_{SV1} = H_{SS1} + 3H_{SS3} \\ H_{SV2} = 2jH_{SS2} \\ H_{SV3} = -4H_{SS3} \end{cases} \quad (6)$$

where $j = \sqrt{-1}$ is the imaginary number.

### C. Model Inversion

We ideally aim to find the parameters of a system that when connected in tandem with the Volterra model of the loudspeaker produces near to zero *p*th-order harmonics. This can be achieved by using a feedforward structure based on the theory of *p*th-order inverses of nonlinear systems [19]. Nevertheless, the inverse is valid for a finite range of input amplitudes [20], and the difference between the existing theoretical error bound and the true error is too large to be used to determine the maximum input voltage that guarantees the validity of the inverse. For this raison, we will evaluate the performance of the method for different input amplitudes.

Fig. 1 shows the structure of the predistortion system that is connected in tandem with the loudspeaker. The delay blocks are inserted to satisfy a causal system.

## III. Loudspeaker Modeling

The transducer lumped-parameters model implemented in the simulation platform is presented in Fig. 2 [1], [21]-[23]. The driver is considered working in piston mode in the linear displacement on the x-axis. Three dominant nonlinearities are included in the model, namely: the lossy voice coil inductance which is modeled by an L2R2 ($R_e, L_e, R_2, L_2$) structure to account for losses due to Eddy currents, the force factor $Bl(x,i)$, the suspension stiffness $K_{MS}(x)$, and the mechanical resistance $R_{MS}(v)$. The model considers nonlinearities due to voice coil displacement $x(t)$, velocity $v(t)$, and current $i(t)$, $i_2(t)$. $M_{MS}$ is the mechanical mass. $U_{EMF}(x,v,i)$ is the back electromotive force, $F_{Bl}(x,i)$ is the driving force, and $F_R(x,i)$ is the reluctance force.

After Using Kirchhoff's circuit laws, we obtain a system of nonlinear ordinary differential equations. Solving this system yields the instantaneous state variables. The predicted generated sound pressure level at distance $d$ is expressed as

$$p(t) = \frac{\rho S_D}{2\pi d}\frac{dv(t)}{dt} \quad (SPL) \quad (7)$$

where $S_D$ is the driver radiator surface, $d$ is the distance between the radiator and the observing point, and $\rho$ is the air density.

## IV. Simulation Results

The theoretical performance of the proposed approach is evaluated using the Matlab/Simulink tools. We use the lumped-parameters model of Fig. 2 to emulate a real electromechanical driver behavioral operating in piston mode. The corresponding system of nonlinear differential equations is solved using Simulink. We will study the case of a 1.5 inch mini-speaker from *Dayton Audio*, referenced *"CE40P-8 1-1/2"*. The Thiele/Small (linear) parameters, and nonlinear parameters of the driver fitted to the 4th order polynomial were measured using the Klippel equipment, they are listed in Table I. The method performance will be assessed by evaluating the relative 2nd and 3rd order harmonic distortions (HD) and the total harmonic distortions (THD) expressed as

Table I Lumped-parameters of the *Dayton CE40P-8 1-1/2*.

| **Parameters** | **Values** |
|---|---|
| Re | 7.27 ohm |
| R2 | 0.46 ohm |
| Le (0) | 0.0773 mh |
| Le (x) | {-5.1e-3, 1.25, 1.72e+2, -5.96e+4} |
| L2(0) | 0.0027 mh |
| M | 0.543 g |
| Kms (0) | 1111 N/m |
| Kms (x) | {-5.6e+4, 2.8e+8, 1.11e+11, 5.7e+13} |
| Bl (0) | 1.84 T.m |
| Bl (x) | {-4.27e+2, 2.5e+5, 3.3e+7,9.9e+9} |
| Rms(0) | 0.212 Kg/s |
| Fs | 265 Hz |

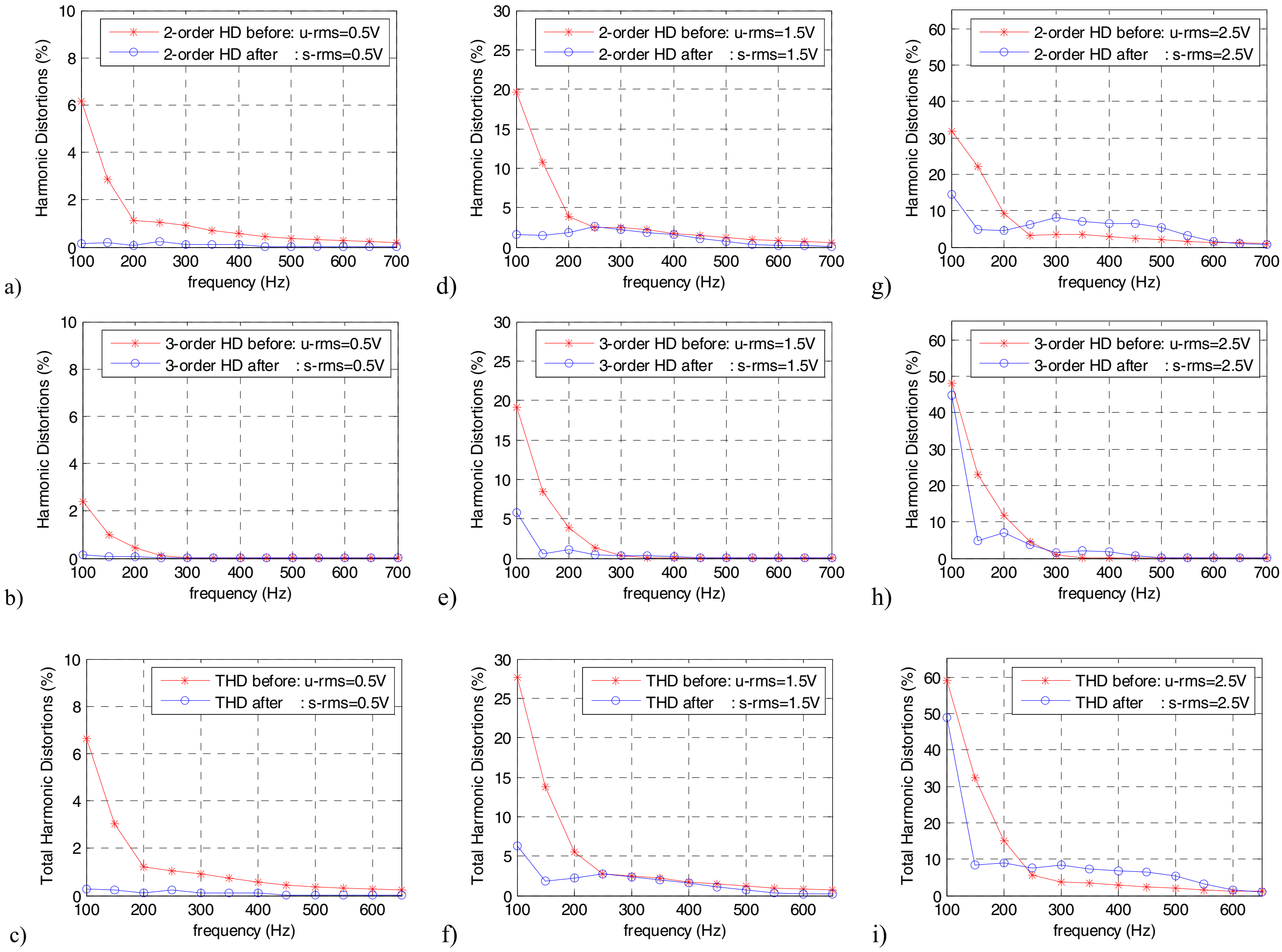


Fig. 3 Harmonic distortions analysis for the 3rd order predistortion system for different stimulus amplitude levels.

$$HD2(\%) = \frac{V_2}{V_1}\times 100\,,\; HD3(\%) = \frac{V_3}{V_1}\times 100 \qquad (8)$$

$$THD(\%) = \frac{\sqrt{V_2^2 + V_3^2 + V_4^2 + V_5^2 + V_6^2}}{V_1}\times 100$$

where $V_n$ is the amplitude of the $n^{th}$ order harmonic.

We first measure the linear and higher order impulse responses of the driver using the synchronous swept sine method presented in Section II. A sweep duration of 1 second is used with a starting and ending frequencies of 50 Hz and 20 KHz, respectively. A sampling frequency of 48 KHz was used. The sound pressure level is measured at 1 m from the radiator and the standard air density of 1.225 kg/m3 was used. In order to evaluate the impact of the stimulus amplitude on the performance of the linearization scheme, we drive the loudspeaker with different voltage levels.

To obtain the inverse of the linear response, we use an approach used in adaptive equalization [9]. This ensures that the phase information of the inverse filter is preserved, while guarantying that all the filter coefficients are pure real. The inverse filter is an FIR filter with 2403 taps. A system delay of $D = 1201$ samples was used.

The left column of Fig. 3 shows the linearization performance for the input voltage of 0.5V. We can observe that for this loudspeaker the distortions are more important below 200-250 Hz. The linearization scheme performs very well with harmonic distortions below 0.2% over the entire frequency range. In the middle column, harmonic distortion plots for an input voltage of 1.5V are reported. We can see that the harmonic distortions prior linearization have significantly increased over the previous case. After linearization, the distortions are considerably reduced below 250 Hz. Above 250 Hz, almost no improvement is observed. This is not critical since the distortions are already low and decreasing with frequency. In the last case corresponding to an input voltage of 2.5V, the linearization performance below 250 Hz is in continuous degradation compared to the two previous cases, while getting worse than the original distortions for frequencies above 250 Hz.

## V. Conclusion

In this paper, we have evaluated the performance of a low implementation complexity loudspeaker linearization scheme based on inverting a simplified Volterra model. The method can effectively compensate for 2nd and 3rd order harmonic distortions when the input stimulus amplitude generates small to moderate harmonic distortions at the output. The results also concord with the theory concerning the validity of the inverse filter for only a limited range of input amplitudes. In future work, we will perform experimental measurements to validate the simulation results.

## ACKNOWLEDGMENT

This work was supported in part by the Regroupement stratégique en microsystèmes du Québec (ReSMiQ), Mitacs, Simaudio Ltd, and the Natural Sciences and Engineering Research Council (NSERC) of Canada.

## REFERENCES

[1] W. Klippel, "Tutorial: Loudspeaker nonlinearities. Causes, parameters, symptoms," *J. Audio Eng. Soc.* 54, 907–939, 2006.

[2] W. Klippel, " The Mirror Filter - A New Basis for Reducing Nonlinear Distortion and Equalizing Response in Woofer Systems," *J. Audio Eng. Soc.*, vol. 40, pp. 675 – 691, 1992.

[3] W. Klippel, "Adaptive Nonlinear Control of Loudspeaker Systems," *J. Audio Eng. Soc.* vol. 46, pp. 939 – 954, 1998.

[4] S. S. Payal, V. J. Mathews, A. Iyer, R. Lambert and J. Hutchings, "Equalization of excursion and current-dependent nonlinearities in loudspeakers," *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, Florence, 2014, pp. 6697-6701.

[5] Jonas Sjöberg, Qinghua Zhang, Lennart Ljung, Albert Benveniste, Bernard Delyon, Pierre-Yves Glorennec, Hakan Hjalmarsson, Anatoli Juditsky, "Nonlinear black-box modeling in system identification: a unified overview," *Automatica*, Vol. 31, no. 12, 1995.

[6] A. J. M. Kaizer, "Modeling of the Nonlinear Response of an Electrodynamic Loudspeaker by a Volterra Series Expansion," *J. Audio Eng. Soc.,* vol. 35, June 1987.

[7] Novak A, Simon L, Kadlec F, Lotton P. Nonlinear system identification using exponential swept-sine signal, " *IEEE Trans. Instrum. Meas.* 2010.

[8] P.R. Chang, C.G. Lin and B.F. Yeh, "Inverse Filtering of a Loudspeaker and Room Acoustics using Time-Delay Neural Networks," *J. Acoust. Soc. Am.*, vol. 95, pp. 3400-3408, June 1994.

[9] S. Haykin, *Adaptive Filter Theory*. New Jersey: Prentice Hall, 2001.

[10] X. Y. Gao and W. M. Snelgrove, "Adaptive linearization schemes for weakly nonlinear systems using adaptive linear and nonlinear FIR filters," *Proceedings of the 33rd Midwest Symposium on Circuits and Systems*, Calgary, Alta., 1990.

[11] Y. Mu, P. Ji, W. Ji, M. Wu and J. Yang, "Modeling and Compensation for the Distortion of Parametric Loudspeakers Using a One-Dimension Volterra Filter," *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, vol. 22, no. 12, pp. 2169-2181, Dec. 2014.

[12] K. Lashkari, "High quality sound from small loudspeakers using the exact inverse," *Conference Record of the Thirty-Eighth Asilomar Conference on Signals, Systems and Computers, 2004.*

[13] K. Lashkari, "A Novel Volterra-Wiener Model for Equalization of Loudspeaker Distortions," *IEEE Int. Conf. on Acoustics Speech and Signal Process.*, Toulouse, 2006.

[14] Y. Hatano, C. Shi and Y. Kajikawa, "Compensation for Nonlinear Distortion of the Frequency Modulation-Based Parametric Array Loudspeaker," in *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, vol. 25, no. 8, pp. 1709-1717, Aug. 2017.

[15] C. Hofmann, C. Huemmer and W. Kellermann, "Significance-aware Hammerstein group models for nonlinear acoustic echo cancellation," *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, Florence, 2014, pp. 5934-5938.

[16] Farina A., "Simultaneous measurement of impulse response and distortion with a swept-sine technique", *108th AES Convention*, Paris 18-22 February 2000.

[17] Novak, Antonin and Lotton, Pierrick and Simon, Laurent, "Synchronized Swept-Sine: Theory, Application, and Implementation," *J. Audio Eng. Soc,*2015, pp. 786-798.

[18] A. Farina, A. Bellini, and E. Armelloni, "Non-linear convolution: A new approach for the auralization of distorting systems," *110th AES convention*, Amsterdam, May 2001.

[19] M. Schetzen, "Theory of pth-order inverses of nonlinear systems," *IEEE Trans. on Circuits and Systems*, vol. 23, no. 5, pp. 285-291, 1976.

[20] Yang-Wang Fang, Li-Cheng Jiao, Xian-Da Zhang and Jin Pan, "On the convergence of Volterra filter equalizers using a pth-order inverse approach," in *IEEE Transactions on Signal Processing*, vol. 49, no. 8, pp. 1734-1744, Aug. 2001.

[21] R. H. Small, "Assessment of Nonlinearity in Loudspeakers Motors," in *IREECON Int. Convention Digest*, pp. 78-80, Aug. 1979.

[22] A. Dobrucki and C. Szmal,, "Nonlinear Distortions of Woofers in Fundamental Resonance Region," in *80th convention Audio Eng. Soc.,* Montreux, March 4-7, 1986, preprint 2344.

[23] W. Klippel, "Nonlinear Large-Signal Behavior of Electrodynamic Loudspeakers at Low Frequencies," *J. Audio Eng. Soc.* , vol. 40, pp. 483-496 (1992).