LOW GROUP DELAY INTERPOLATION FILTER FOR DELTA-SIGMA CONVERTERS

Manouane Caza-Szoka and Daniel Massicotte

Université du Québec à Trois-Rivières, Department of Electrical and Computer Engineering,
3351, Boul. des Forges, Trois-Rivières, Québec, Canada
Laboratoire des Signaux et Systèmes Intégrés
{manouane.caza-szoka, daniel.massicotte}@uqtr.ca

ABSTRACT

This paper shows how a relaxation of the high frequency requirements can help reducing the latency in linear phase interpolation filter, with an audio production system perspective. The reduced need for attenuation is justified when the interpolation filter is followed by a noise-shaping Delta-Sigma loop and an analog filtering stage. This is done by using a non-constant error weight of the stop-band. In order to use the Parks-McClellan method for finite impulse response filter design from Matlab, the stop-band is divided and weighted logarithmically. Quantitative results are shown for different example filter design, limited to situations where the Parks-McClellan converges well. It has been found that the shorter the filter length needed to respect a given filter template, the more relative group delay reduction can be achieved by relaxing the high frequency requirement. For filter size of the order of 100, reduction of group delay of 30% can be expected. For sake of simplicity, the Delta-Sigma loop is discussed but not analysed here. The idea is demonstrated in the context of Digital-to-Analog converters (DAC) but by duality could be applicable also to Analog-to-Digital converters (ADC). The main performance metric used is a relative reduction of the impulse response group delay. The results are also presented as impulse responses and power spectrum examples. The presented approach may be generalised to complex and non-linear phase filters and does not prevent the use of polyphase structures.

Index Terms— DAC, ADC, Interpolation, Latency

1. INTRODUCTION

Low latency systems have received particularly high level of attention recently in a wide variety of domain, with very recent increased needs in communication [1] and sound production [2][3]. Among the different sources of latency found in sound systems [2], a noise shaped Digital-to-Analog (DAC) converter introduces a delay linked to the interpolation filter impulse response. In [4], it is even stated that “the dominant latency here is the FIR filter within the sigma-delta DAC.”

The research effort in interpolation filter has mainly been driven towards the reduction of the overall complexity. Two main approaches are 1) polyphase filters [5] and 2) using multiple interpolation stages [5][6][7][8]. In both cases, the idea is to apply filtering at lower working frequency. The polyphase filter method can represent any impulse response exactly. It can help reducing the latency by allowing a high level of parallelism. However, it does not allow reducing the group delay of the impulse response. The multiple stages method generally needs a longer equivalent single stage filter impulse response for a given filter requirements [2]. However, the overall delay in multistage filters can be reduced by allowing lower requirements for the later stages [9]. This is generally reasonable considering that a Delta-Sigma noise-shaping loop [10] and low-pass analog filtering following the interpolation filter.

The main idea presented in this paper is to apply to a single-stage filter a similar filter requirement reduction as in the multiple-stage case. To do so, the stop band constant ripple is replaced by a logarithmically increasing ripple. Hence, at the stop-band frequency, the full attenuation is still obtained. Although non-linear phase FIR and IIR filters are frequently used in both single and multiple stages interpolation filters [6], the discussion is limited to linear FIR filters.

The Chebyshev criterion is used for the FIR filter weights optimization. The Parks-McClellan algorithm [11] remains the gold-standard for finding the optimal FIR filters weights under Chebyshev criterion. The numerical results will use the Matlab® version of the algorithm. The Parks-McClellan algorithm is known to have convergence issues when filter length becomes high [12]. Hence, the numerical results will be shown for filter specifications that allows the Parks-McClellan algorithm to converge to the optimal solution.

It will be shown that for given pass-band ripple and attenuation at the stop-band frequency, a shorter symmetrical filter can be used with increasing stop-band ripple slope. It will be explained that fewer degrees of freedom (i.e. fewer ripples) will be necessary at high frequencies.

The most interesting finding is that the lower the delay of the filter, the more relative reduction of the delay is to be expected. The filters obtained can obtain all the usual benefits from polyphase structures. Also, it is expected that such results could be generalized to non-linear phase filters [13], IIR filters and complex filters, with Chebyshev or other optimality criterion.

By duality [10], the idea can be applied in Analog-to-Digital (ADC) if the band of interest is smaller than the Nyquist rate. If fact, the group delay reduction should be higher as the oversampling ratio of the ADC is high. It can also used when the ADC and DAC are used jointly such as in linearity testing [14][15] or background calibration [16][17].

The paper is organized as follows: Section 2 presents the methodology. Section 3 shows a design example and general results are presented and discussed in Section 4. Finally, conclusions are drawn in Section 5.

2. METHODOLOGY TO REDUCE THE LATENCY

This section presents the interpolation filters design methods that will be compared throughout this paper. The context of Delta-Sigma converter is explained. The performance comparison method is also developed.

2.1. Interpolation Filter Coefficients Optimization

Let suppose a sampled signal s_0. In a Delta-Sigma DAC, the first step is to upsample the signal by a ratio called oversampling ratio (OSR.) This upsampled signal v_0 is sent to an interpolation filter with an impulse response $h(n)$.

Two interpolation filters design methods for $h(n)$ are considered, a classical method and the proposed variation.

1) Parks-McClellan Algorithm

The Parks-McClellan algorithm [11] is a widely used method to optimize FIR filter coefficients. It does so by finding iteratively in the frequency domain points which are extremums for an interpolation polynomial. Although it is done in the frequency domain, it does not require full domain transformation. A direct relation between the polynomials and the time domain coefficients exists.

The Parks-McClellan algorithm seeks to minimize the maximum error on the filter template. This optimisation objective is generally called the Chebyshev criterion.

Most of programming software including signal processing package have the algorithm implemented, sometimes under the name of Remez-Exchange algorithm. The Matlab® version of the algorithm is found under the function name *firpm*. It is this function that is used here.

The Matlab's Parks-McClellan function allows to define multiple frequency bands with different targets and different weights to the errors. While the targets can be a function of the frequency, the weights can only be constant within a single frequency band.

In this paper, the most basic use of Parks-McClellan function is used, namely the design of a symmetric (therefore linear phase) low-pass filter. It is used for comparison to the proposed variation presented next.

The parameters of the low-pass filter are the pass-band frequency f_p, the stop-band frequency f_s, the pass-band ripple δ_p, stop band ripple δ_s. The filter length N_1 is determined by these four parameters.

The group delay of such a filter is the half of the filter length N_1.

2) Proposed Variation

The proposed variation resides in the stop-band weighting function. Instead of using a constant weight, it is proposed to decrease the weight as frequency increases, hence reducing the overall effective constraint on the filter design.

Although the weighting function in the stopband could take lots of form, a simple linear one is considered. Hence, a new parameter is necessary. It is the slope of the relaxation β. This slope is found on the when the interpolation error is measured in decibels with respect to the logarithm of the frequency. The four slopes considered here will be 20, 40, 60 and 80 dB/decade.

Because the Matlab® does not allow continuous weighting function, logarithmically spaced multiple bands separated by an epsilon (approx. 1e-15 Hz) with decreasing weights are used. However, when to much bands are used, convergence problem arises. Hence, the number of bands is chosen to be a tenth of the filter length. This approach will lead to slightly worse performance of the proposed method, especially for shorter filter, mostly because the band at the stop-band frequency becomes larger.

The group delay of such the proposed filter is again the half of the filter length N_2.

2.2. Note on the Delta-Sigma Loop

In a digital-to-analog Delta-Sigma converter [10], the interpolation filter is followed by a Delta-Sigma loop. The Delta-Sigma loop is defined principally by its Noise-Transfer-Function (NTF). The NTF is principally defined by its order. In the frequency domain, the NTF has a slope similar to the inverse of weighting proposed. The slope is generally 20dB/decade times the order of the NTF. Hence, by choosing reasonable weighting slope for the interpolation filter with respect to the NTF order, it is possible to ensure that the added noise stays well below the residual noise shaped by the NTF. It is expected that the design of the interpolation filter will depend strongly on the NTF. This aspect is not treated in this paper.

The Delta-Sigma loop has also a Signal-Transfer-Function (STF). It is possible to have STF that are completely transparent, i.e. to have $STF = 1$. This case is supposed. The example shown will omit the noise caused by the Delta-Sigma loop and concentrate on the residual noise of the interpolation filter.

2.3. Analog Filtering

The digital-to-analog Delta-Sigma converter generally finishes by an analog low pass filter. In order to emulate the impact of such filter, a Butterworth filter is included. Although this filter adds to the overall delay, it is already present in the converter. Since it is considered that the

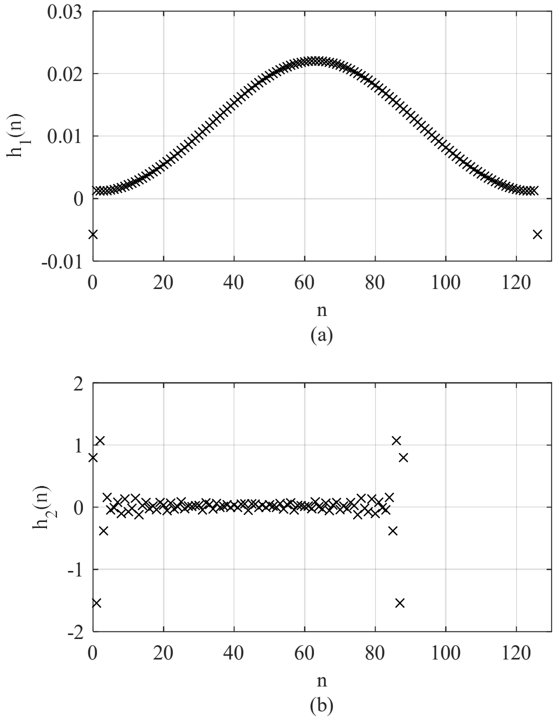


Fig. 1. Impulse responses of the classical filter h_1 (a) and proposed filter h_2 (b).

interpolation filter residual noise is much lower that the Delta-Sigma loop noise, it is reasonable to suppose that no modification of the analog filtering part is required.

2.4. Performance Comparison Method

To quantify the reduction of group delay, a design of filter with the classical is done with certain specification, yielding a filter h_1, of length N_1. This filter might not exactly match the specification, generally because the filter length must be an integer number. A second design is done with the proposed variation. The filter length is chosen such as to have slightly better performances in terms of weighted error than with the classical method. The filter obtained is named h_2 of length N_2. The relative group delay reduction is simply:

$$GDRR = \frac{N_1 - N_2}{N_1}, \quad (1)$$

Table 1 Simulation Conditions

Input Sinus Frequency		8820 Hz
OSR		32
Input sampling frequency		44.1 kHz
Pass-band frequency	f_p	12 kHz
Stop-band frequency	f_s	22.05 kHz
Pass-band ripple	δ_p	10 dB
Stop-band ripple	δ_s	40 dB
Slope of the relaxation	β	40 dB/decade
Analog filter order		2
Analog filter Stop-Band Frequency		18 kHz

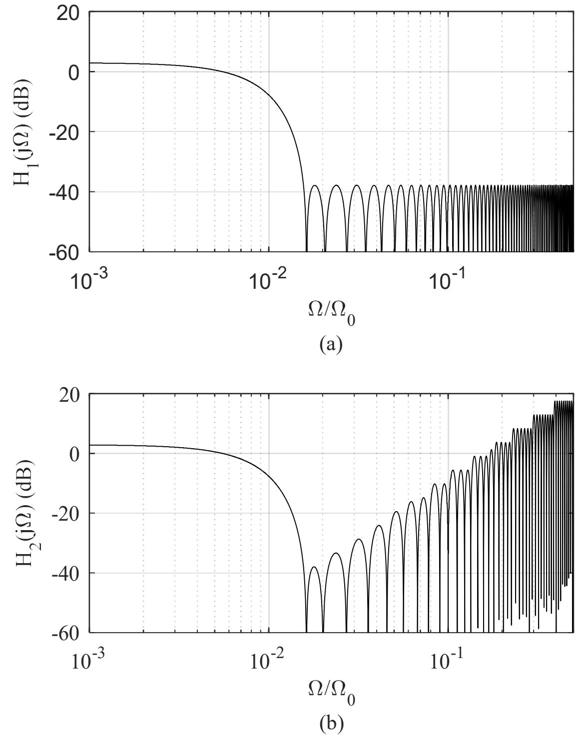


Fig. 2. Transfer functions of the classical filter H_1 (a) and proposed filter H_2 (b).

Where $GDRR$ stands for Group Delay Reduction Ratio.

It is well known that for large filters, the Parks-McClellan algorithm is prone to convergence problem [12]. While, approaching the thousand coefficients suboptimal results may appear, larger filter may simply not converge. Hence, the comparison will be limited to specifications that yields reasonable convergence. Also, because too short filters reduce the number of frequency bands that can be used, as stated before, the analysis is also limited to filters that have more than 50 coefficients.

3. TYPICAL DESIGN EXAMPLE

This section presents an illustrative example in sound production context of designs using the classical method and the proposed variation. The simulation conditions are given and the graphs of the results explained.

3.1. Design Example Simulation Conditions

The simulation condition for the Design example are shown in Table 1.

These conditions chosen are an example of relatively short filters in comparison to what is simulated later. The frequency of the input sinus was chosen to be exactly periodic relative to the input sampling frequency, thus avoiding any windowing problem in frequency domain analysis. Also, the plots are scale to an amplitude of approximately 1 and with a time offset to remove the initial transient phase.

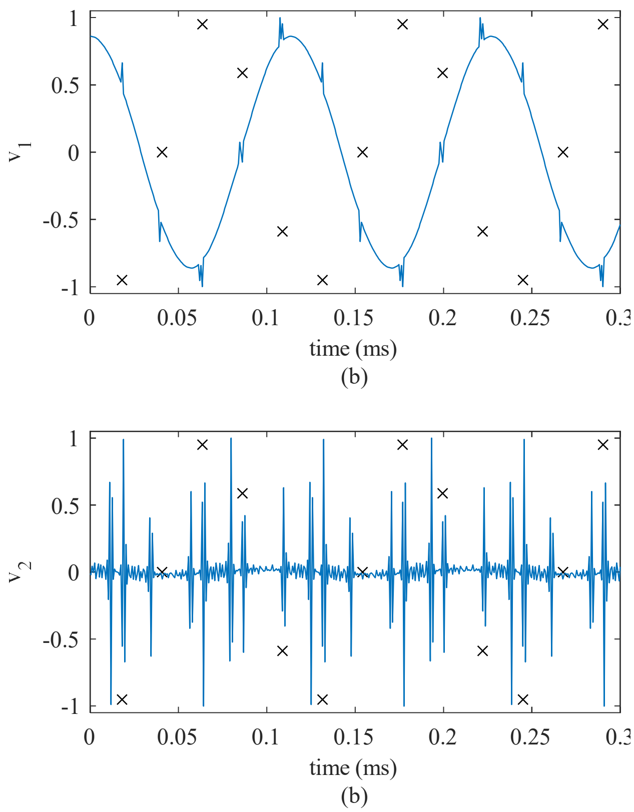


Fig. 3. Filtered upsampled signals for the classical filter (a) and proposed filter (b). The input is an 8820Hz sinusoidal signal showed with “x” points.

3.2. Design Example Results

The filter length obtained were $N_1 = 126$ for the classical metho and $N_2 = 88$ for the proposed variation. This yields a GDRR of 30%. In terms of time delay, the proposed filter has a group delay of 31µs compare to 45µs for the classic filter a reduction 14µs. It worth noting that as the group delay reduces, so the complexity and pre-ringing artefact, both by the same ratio as the GDRR.

The Fig. 1 presents the obtained time domain interpolation filter impulse response for classical method (h_1) and the proposed variation (h_2). It is clear that the proposed filter is shorter.

The Fig. 2 shows the power spectrum of the two filters. The staircase weighting function can be easily deducted from the proposed filter power spectrum. It is interesting to note that at very high frequency, the residual error power is above the desired signal power. This noise will however be easily removed by the analog filter. Also, the weights of this region could be increased with only minor effect on the group delay to reduce the signal dynamics if necessary.

The Fig. 3 shows the interpolated signal of a sinusoidal input by the two method along the input signal given as reference. The residual noise is clearly much stronger for the proposed method, but again, it is mostly at high frequency which can be easily removed.

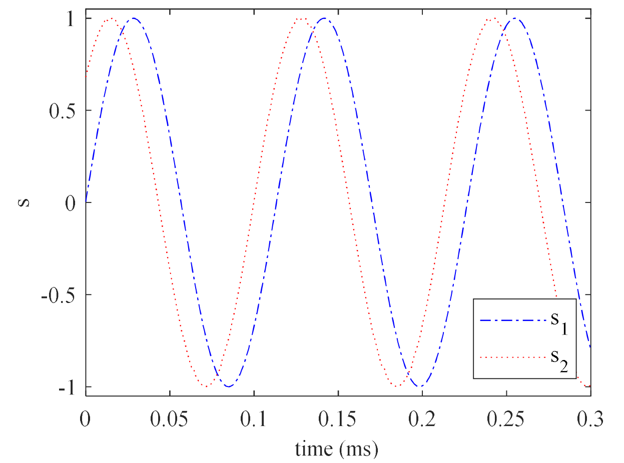


Fig. 4. Analog filter output signals for the classical filter (a) and proposed filter (b). The input is an 8820Hz sinusoidal signal.

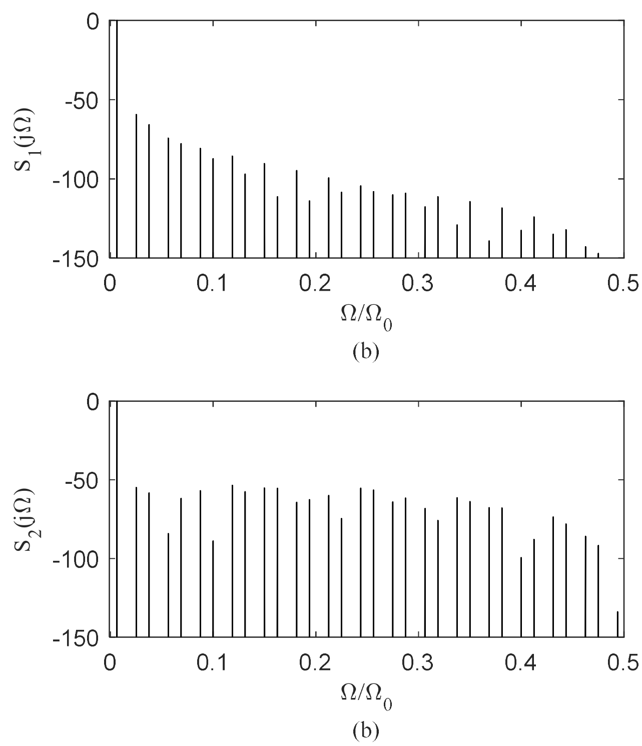


Fig. 5. Power spectrum of analog filter output signals for the classical filter (a) and proposed filter (b). The input is an 8820 Hz sinusoidal signal.

The Fig. 4 presents the classical filter and the proposed filter results after analog filtering in the same graph. To remove confusion, the input signal was not shown because the second order filter used to emulate the analog part introduces a delay of 27 µs. This delay would be present anyway in a complete Delta-Sigma converter.

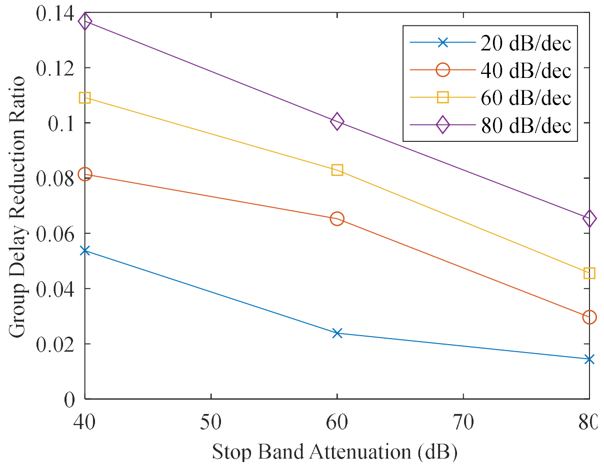
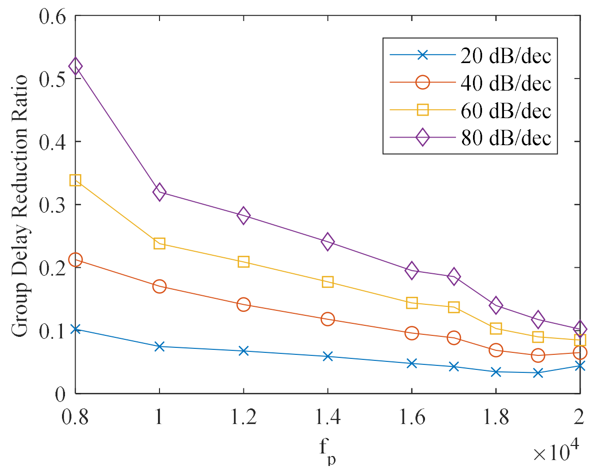
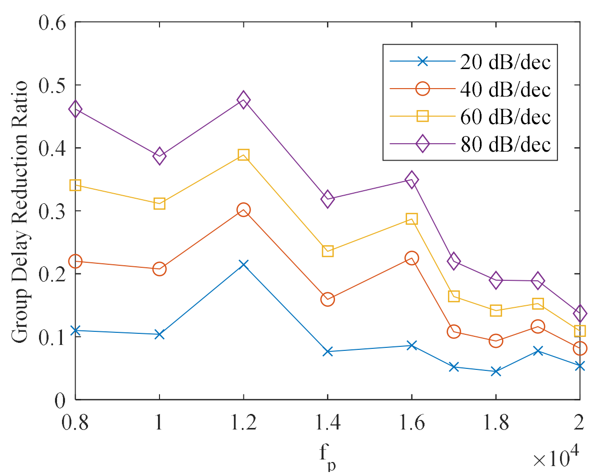


Fig. 6. GDRR for different stop band attenuation at relaxation slope β ranging from 20 to 80 dB/decade. The pass-band ripple is limited at 10 dB, the pass-band frequency is 20 kHz and the stop-band frequency is 22.05 kHz.



(a)



(b)

Fig. 7. GDRR vs stop band frequency at relaxation slope β ranging from 20 to 80 dB/decade. The stop-band attenuation is 40 dB and the stop-band frequency is 22.05 kHz. In (a), the pass-band ripple is at 20 dB while it is at 10 dB in (b).

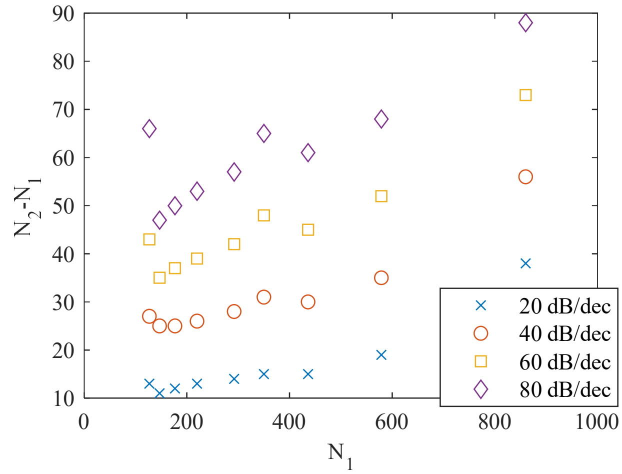


Fig. 8. Number of filter coefficient reduction vs the classical filter length at relaxation slope β ranging from 20 to 80 dB/decade. The stop-band attenuation is 40 dB, the stop-band frequency is 22.05 kHz and the pass-band ripple is at 20 dB. The data points are taken from the same simulation as in fig. 7 (a).

The Fig. 5 finally shows the frequency domain representation of the signals after analog filtering. For the proposed signal, the noise level at high frequency is strongly reduced. Moreover, in the case of sound production context, this residual noise being of high frequency is not audible. However, it is useful to keep it under control to avoid saturation problem of the speakers. With reasonably good design, it is expected that the high frequency residual noise of the interpolation filter can be much lower than the Delta-Sigma loop noise-shaped residual noise after filtering from the Delta-Sigma loop (Signal Transfer Function [10]) analog filtering by possibly orders of magnitude.

4. RESULTS AND DISCUSSION

This section presents the results of the Group Delay Reduction Ratio obtained with respect to some parameters and discusses their implications.

It is shown that with everything else being equal, the GDRR is higher for lesser quality filters, i.e. with lower stop-band attenuation filter in Fig. 6 and with larger distance between the pass-band frequency and stop-band frequency in Fig. 7.

Comparing the Fig. 7 (a) and (b) with pass-band ripple of 20 dB and 10 dB respectively, when the pass-band frequency is high, there is clearly better GDRR for the filter of lesser quality in (b). On the other hand, when the pass-band frequency is low, the opposite is found. It is expected that the reason is that the stairs in in the staircase weighting function are much larger for the smaller filter, hence yielding a more constrained specification.

The Fig. 8 reuse the data presented in Fig. 7 (a). It shows the number of coefficient reduction with respect to the number of filter coefficients for the classical method. It shows clearly that the number of coefficients saved becomes greater as the classical filter is longer. The delay is more reduced for longer filter. There is therefore still a gain to use the proposed method even when the filter length is long.

With reasonably good design, it is expected that the high frequency residual noise of the interpolation filter can be much lower than the Delta-Sigma loop noise-shaped residual noise after filtering from the Delta-Sigma loop (Signal Transfer Function [10]) analog filtering by possibly orders of magnitude. However, if the method is pushed to the limit, obviously the residual noise will become significative with respect to the Delta-Sigma loop noise. In that case, spurious effects may appear. While a full study of the interaction between the residual interpolation error and the Delta-Sigma Loop is out of the scope of this paper, the two main questions arising from it should be mentioned:

1) Does the added high frequency noise can trigger instabilities in the Delta-Sigma loop, especially since the residual noise, although somewhat random, is not identically distributed. From a Surrogate Analysis point of view [18], the residual noise has likely non-random phase.

2) Does the added noise may cause saturation, resulting in the need of reducing the output amplitude.

5. CONCLUSION

The most crucial aspect of the paper was that the relaxation of the interpolation filer requirements at high frequency should be considered to reduce the group delay when a Delta-Sigma converter is used. The analysis showed that the relative reduction of the group delay was stronger for already filters of shorter length. The results could be further improved by using a modified version of the Parks-McClellan algorithm that would allow a continuous weight function. Similar results should be obtainable in Analog-to-Digital converters (ACD) as there is duality between the DACs and ADCs [10]. Further research should be done to explore the interaction between the interpolation stop-band weighting variation, the Delta-Sigma loop and the analog filtering.

ACKNOWLEDGEMENTS

This work has been funded by the Natural Sciences and Engineering Research Council of Canada grant, the CMC Microsystems, Simaudio Ltd, and the Research Chair in Signals and Intelligence of High Performance Systems.

REFERENCES

[1] M. Palattella et al., "Internet of Things in the 5G Era: Enablers, Architecture, and Business Models", IEEE Journal on Selected Areas in Communications, vol. 34, no. 3, pp. 510-527, 2016.

[2] Y. Wang, (2018). "Low Latency Audio Processing," (Doctoral dissertation, Queen Mary University of London), 2018.

[3] A. McPherson, V. Zappi, "An environment for submillisecond-latency audio and sensor processing on BeagleBone Black," In Audio Engineering Society Convention 138. Audio Engineering Society, 2015.

[4] A. McPherson, R. Jack, G. Moro, "Action-sound latency: Are our tools fast enough?" in Proceedings of the Conference on New Interfaces for Musical Expression, 2016.

[5] R. Crochiere and L. Rabiner, "Interpolation and decimation of digital signals—A tutorial review", Proceedings of the IEEE, vol. 69, no. 3, pp. 300-331, 1981.

[6] S. Chu and C. Burrus, "Optimum FIR and IIR multistage multirate filter design", Circuits, Systems, and Signal Processing, vol. 2, no. 3, pp. 361-386, 1983.

[7] Shuni Chu and C. Burrus, "Multirate filter designs using comb filters", IEEE Transactions on Circuits and Systems, vol. 31, no. 11, pp. 913-924, 1984.

[8] S. Park, "Multi-stage decimation filter design technique for high-resolution sigma-delta A/D converters," in IEEE Transactions on Instrumentation and Measurement, vol. 41, no. 6, pp. 868-873, Dec. 1992.

[9] X. Li and A. Lee, "An FPGA implemented 24-bit audio DAC with 1-bit sigma-delta modulator," 2010 IEEE Asia Pacific Conference on Circuits and Systems, Kuala Lumpur, pp. 768-771, 2010.

[10] R. Schreier and G. C. Temes, "Understanding Delta-Sigma Data Converters," Piscataway, NJ: IEEE Press/Wiley, 2005.

[11] T. Parks and J. McClellan, "Chebyshev Approximation for Nonrecursive Digital Filters with Linear Phase", IEEE Transactions on Circuit Theory, vol. 19, no. 2, pp. 189-194, 1972.

[12] P. Lesso, A. J. Magrath, "An Ultra High Performance DAC with Controlled Time-Domain Response," In Audio Engineering Society Convention 119. Audio Engineering Society, 2005.

[13] L. Rabiner, J. McClellan and T. Parks, "FIR digital filter design techniques using weighted Chebyshev approximation", Proceedings of the IEEE, vol. 63, no. 4, pp. 595-610, 1975.

[14] A. Di Nisio, A. M. L. Lanzolla and M. Savino, "Simultaneous A/D and D/A converters linearity testing with deterministic dithering," 2011 IEEE International Instrumentation and Measurement Technology Conference, Binjiang, 2011, pp. 1-5.

[15] Y. Zhuang and D. Chen, "Cost-effective accurate DAC-ADC co-testing and DAC linearization," 2018 IEEE International Instrumentation and Measurement Technology Conference (I2MTC), Houston, TX, 2018, pp. 1-6.

[16] J. McNeill, C. David, M. C. W. Coln, K. Y. Chan and C. Brenneman, "Split ADC background self-calibration of a 16-b successive approximation ADC in 180nm CMOS," 2013 IEEE International Instrumentation and Measurement Technology Conference (I2MTC), Minneapolis, MN, 2013, pp. 310-313.

[17] S. Tan, Y. Miao, M. Palm, J. Rodrigues and P. Andreani, "A continuous-time delta-sigma ADC with integrated digital background calibration", Analog Integrated Circuits and Signal Processing, vol. 89, no. 2, pp. 273-282, 2016.

[18] M. Caza-Szoka and D. Massicotte, "Detection of Non Random Phase Signal in Additive Noise with Surrogate Analysis," ICASSP 2019 - 2019 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), Brighton, United Kingdom, 2019, pp. 1159-1163.