DCOPF Contingency Analysis Including Phase Shifting Transformers

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Abstract—This work deals with a new formulation for the Direct Current Optimal Power Flow (DCOPF) including the corrective actions related to the phase shifting transformers. The formulation is based on the outage of generators and/or branches modelled as fictitious injections of active power. The inclusion of the sensitivities of the phase shifting transformer with respect to the injected powers is one of the novelties of this paper. By including the fictitious injections in the optimization problem, the injections are adjusted to the post-contingency state as a consequence of the corrective actions carried out by the DCOPF to bring the system back to its normal state. Consequently, when the analysis of contingencies is performed, the classical topological analysis and the subsequent analyses are avoided with this approach. The DCOPF includes as corrective control variables the rescheduling of active power generations, phase shifting transformers and, if required, permitted load shedding. The IEEE-RTS of 24 buses is used as benchmark network to assess the properties of the proposed approach.

I. INTRODUCTION

Load flow is the basic building block for the management of large, interconnected, multinational transmission systems [1] [2] [3] [4]. The possibility of controlling load flow in a power system by phase shifter transformers - PSTs - was recognized long ago [5], and the installation of PSTs is considered one way of increasing the utilization of bulk power system facilities [6].

In the analysis of a system, obtaining the optimum settings for PSTs and the generations schedule becomes a difficult task if ac power flow algorithm is adopted due to the nonlinearity. DC power flow method is much easier to use as the PST can be included in the modeling as a reactance and a phase shift [6] [7]. DC load flow models are inherently approximate, and it is well-known that their accuracies are very close to the system and the case analysed. When they are applied to Contingency Analysis, the full nonlinear power-flow solution is approximated to produce the long-term steady-state conditions after an outage. They are used primarily to verify that thermal line limits are not exceeded after the outage [8] [9].

This paper focuses on the analysis of the impact of potential outages in electric networks and on the use of PTSs as control variables in the DC problem for alleviating electric power transmission line overloads that occur during the loss of one or more transmission lines of a large scale interconnected power system. The work covers direct current modelling for its application in a corrective Optimal Power Flow (OPF), aiming to exploit the capabilities of the linear methods and improve the analysis techniques of energy systems. The main novelty of the formulation proposed is the fact that the contingency due to generator and/or branch outages (line or transformer) is managed as fictitious injections of active power. These fictitious injections are formulated by means of their corresponding elements in the matrix of sensitivities between the branch power flows and the powers injected in an electric power system.

An important aspect is that the sensitivity matrix is constant in the load scenario of each period of time to be analysed [9] [10] [11]. Therefore, there is no need to use the conventional Power Transfer Distribution Factors (PTDFs) and, consequently, the use of Topological Analysis and Direct Current Load Flow (DCLF) in the post-contingency state [12] [13] [14] [15], when a Contingency Analysis is performed.

The inclusion of the contingency (single or multiple) as fictitious nodal injections in the optimization problem implies that the fictitious injections are adapted to the post-contingency state by changing the optimization decision variables. By considering sensitivities of PSTs with respect to the injected powers, the injections are adjusted to the post-contingency state as a consequence of the corrective actions carried out by the DCOPF in the optimization problem.

The paper is arranged as follows. First, the basic concepts of the DC network analysis are set out. Second, the mathematical problem and the formulation proposed of the optimization algorithm are presented, followed by the numerical simulations and comparative studies applied to the 24 buses network of the well-known IEEE-RTS [16]. Finally, some conclusions derived from this work are summarized.

II. SENSITIVITY MATRIX, BRANCH POWER FLOWS AND NODAL POWERS INJECTED

The corrective DCOPF suggested in this paper is based upon the basic DC power flow model. This method focuses only on active power flows and is formulated on a few assumptions [9]. DC power flow method assumes a power system to have a flat voltage profile, negligible line resistances and small voltage angle differences thus making the power flow problem linear and easy to solve. These assumptions do cause some loss of accuracy, but it is stated in the literature that the errors due to DC method are within acceptable limits [11] [17] [18]. Hence the method is extensively used for contingency analysis and optimization studies of power systems.

Considering the conventional DCLF (Direct Current Load Flow) [9] then a linear relationship between the active power flows P_f and the active power injections P can be obtained for the power system, as follows:

$$P_f = \left[X^{-1} A^T B^{-1} \right] P = S_f P \qquad (1)$$

where X expresses a diagonal matrix of branch reactances, A denotes the branch-to-node incidence matrix, reduced by removing the slack bus, B is a matrix (omitting the slack bus) defined as the matrix B' of the Fast Decoupled Load Flow, and S_f is the matrix of sensitivities between branch power flows and powers injected [9].

The Superposition Principle can be applied since we are dealing with a linear system, so power flows after a change on the powers injected can be computed as:

$$P_f = S_f \left[P^0 + \Delta P \right] = P_f^0 + \Delta P_f \Rightarrow \Delta P_f = S_f \Delta P \quad (2)$$

where ΔP_f is the vector of variations of branch power flows after a change ΔP on the powers injected, and P_f^0 is the vector of active power flows in the base case (base-point case).

In DC analysis, in case of contingencies due to a line or transformer outage, a usual procedure to perform Contingency Analysis is to use Power Transfer Distribution Factors (PTDFs) [9] [11] [17] [18] [19], but this needs a topological analysis and the subsequent current analyses.

In the approach proposed, the main novelty stems from the use in the optimization problem of the S_f matrix of sensitivities between branch power flows and powers injected, to model the fictitious injections. The S_f matrix remains constant during the Contingency Analysis performed for the generation-load scenario (base case) of each period of time to be analysed [9] [12] [15] [20] [21] [22] [23].

In next sections the power system showed in Figure 1 is considered as a reference and is assumed to be a lossless network in the following explanations.
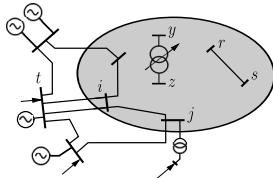

Fig. 1.   Electric Power System in Normal State.

III.   SENSITIVITY MATRIX AND PSTS

A Phase Shifting Transformer is an effective device that can be used for power flow control in tie lines within a system. Figure 2 shows a line connecting two busses between which the phase shifting transformer is connected, where X_{PST} is the leakage reactance of PST.
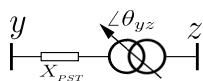

Fig. 2.   Phase Shifting Transformer.

As phase shifting transformer is an active power control device, the phase shift provided by the phase shifter needs to be introduced in the active power flow equations for carrying out the required optimization.

The power flow from node y to node z is increased by adding an angle θ_{yz} to the existing angle. This action, in branch rs (Figure 1), carries out a power flow

$$p_{rs} = S_{rs,y}\,\Delta p_{T,y} + S_{rs,z}\,\Delta p_{T,z} \quad (3)$$

where $S_{rs,y}$ and $S_{rs,z}$ express the sensitivity of branch rs to the changes of the injected power in nodes y and z, respectively, of the phase shift transformer T that has performed the action.

IV.   SENSITIVITY MATRIX AND FICTITIOUS INJECTIONS

The fictitious nodal injections that model the branch in a outaged state are obtained in the optimization algorithm from the S_f matrix, equation (1). In this way, the fictitious nodal injections modify their value in relation to the changes (corrective actions) performed by the optimization problem. Changes in the powers injected in the system nodes can be motivated by loss of generation, branch outages, load shedding or generation rescheduling [9] [24] [25] [26].

Figure 3 shows a multiple contingency due to the simultaneous failure of a generator and a branch in the power system (double contingency). The outaged generator is supposed to be placed at node t and the outaged branch is supposed to be placed from node i to node j (branch ij).
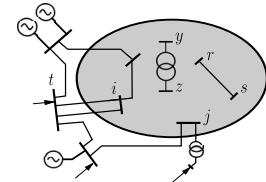

Fig. 3.   Generator at node t and branch ij simultaneous out of service.

For this double contingency, it is supposed that the generator placed at node t was generating $p_{g_t}^0$ (active power) and the active power flow in the branch ij was p_{ij}^0, both elements in the pre-contingency state. Also, it is supposed that the lost active power generation $\Delta p_{g_t} = -p_{g_t}^0$ is assumed by the generator of the reference bus (slack).

From the conditions mentioned above, the active power flow p_{ij} in the branch ij (virtually in service) is:

$$p_{ij} = p_{ij}^0 + S_{ij,i}\,\Delta p_i + S_{ij,j}\,\Delta p_j + S_{ij,t}\,\Delta p_{g_t} \Rightarrow \quad (4)$$

$$\Delta p_i = \left[p_{ij}^0 + S_{ij,t}\,\Delta p_{g_t} \right] \left[1 - (S_{ij,i} - S_{ij,j}) \right]^{-1}$$

where $\quad \Delta p_i = -\Delta p_j = p_{ij} \quad$ and $\quad \Delta p_{g_t} = -p_{g_t}^0$

In the post-contingency state (Figure 3), the Δp_{rs} variation of the active power flow in the rs branch is:

$$\Delta p_{rs} = p_{rs} - p_{rs}^0 = S_{rs,t}\,\Delta p_{g_t} + (S_{rs,i} - S_{rs,j})$$
$$\cdot \left[p_{ij}^0 + S_{ij,t}\,\Delta p_{g_t} \right] \left[1 - (S_{ij,i} - S_{ij,j}) \right]^{-1} \quad (5)$$

At this point, it is important to note that considering the interactions between the simultaneous outages of branches (Figure 4), in the post-contingency state, the active power flow in any rs branch in service is:

$$\Delta p_{rs} = p_{rs}^0 + (S_{rs,a} - S_{rs,b})\,p_{ab} + \cdots$$
$$+ (S_{rs,i} - S_{rs,j})\,p_{ij} + \cdots\cdots \quad (6)$$
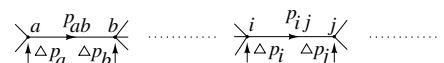
Fig. 4.   Multiple contingency modelled as fictitious nodal injections.

The injections at both ends of the outaged branches are computed by solving the next linear system of equations.

$$
\begin{aligned}
p_{ab} &= p_{ab}^0 + (S_{ab,a} - S_{ab,b})\, p_{ab} + \cdots \\
&\quad + (S_{ab,i} - S_{ab,j})\, p_{ij} + \cdots\cdots \\
\cdots &= \cdots\cdots + \cdots\cdots + \cdots\cdots \\
p_{ij} &= p_{ij}^0 + (S_{ij,a} - S_{ij,b})\, p_{ab} + \cdots \\
&\quad + S_{ij,i} - S_{ij,j})\, p_{ij} + \cdots\cdots \\
\cdots &= \cdots\cdots + \cdots\cdots + \cdots\cdots
\end{aligned} \tag{7}
$$

For both cases of multiple contingency have been considered that there is not the possibility to carry out any action with the transformers of the power system (Figure 3).

V. DC Corrective OPF (DC-COPF): Problem Formulation

The applied optimization problem is based on linear programming [27] [9] [28] [29] [10] [22] and is carried out to obtain remedial actions (generation re-dispatch and/or load shedding) to bring the system back to its normal state when a contingency occurs [30] [24] [9] [28] [14] [22]. The cost assigned to generation rescheduling is significantly lower than the cost of load shedding with the aim of minimizing the use of these measures.

By considering the sensitivity matrix S_f, the OPF problem of active power dispatch based on fictitious active power injections, including phase shifting transformers and with limited number of corrective actions to face line and/or generator outages (single or multiple contingency), can be formulated as follows:

$$
\textbf{min} \quad \sum_{g \in \mathcal{G}} c_{g,u}\, \Delta p_{g,u} + \sum_{g \in \mathcal{G}} c_{g,d}\, \Delta p_{g,d} + \sum_{l \in \mathcal{L}} c_{l,s}\, \Delta p_{l,s}
$$

subject to

$$
\sum_{g \in \mathcal{G}} (\Delta p_{g,u} - \Delta p_{g,d}) + \sum_{l \in \mathcal{L}} \Delta p_{l,s} + \sum_{gf \in \mathcal{G}^k} \Delta p_{gf}^k = 0
$$

$$
\Delta p_i^k = -\Delta p_j^k, \ \Delta p_{gf}^k = -p_{gf}^0
$$

$$
\Delta p_{T,y}^k = -\Delta p_{T,z}^k, \ |\Delta p_{T,y}^k| \le 0.1 \cdot \Delta p_{y,z}^{max}
$$

$$
\Delta p_{g,u} \le (p_{g,u}^{max} - p_{g,u}^0), \ \Delta p_{g,d} \le (p_{g,d}^0 - p_{g,d}^{min})
$$

$$
\Delta p_{l,s} \le p_{l,s}^{max}, \ \Delta p_{g,u} \ge 0, \ \Delta p_{g,d} \ge 0, \ \Delta p_{l,s} \ge 0
$$

$$
\forall ij \in \mathcal{B}^k, \ \forall i,j \in \mathcal{N}, \ k \in \mathcal{C}
$$

Branches in service

$$
\begin{aligned}
-p_{rs}^{max} &\le p_{rs}^0 + \sum_{g \to n} S_{rs,n} (\Delta p_{g,u} - \Delta p_{g,d}) \\
&+ \sum_{l \to n} S_{rs,n}\, \Delta p_{l,s} + \sum_{gf \to n} S_{rs,n}\, \Delta p_{gf}^k \\
&+ \sum_{ij \in \mathcal{B}^k} \left(S_{rs,i}\, \Delta p_i^k + S_{rs,j}\, \Delta p_j^k \right) \\
&+ \sum_{yz \in \mathcal{T}} \left(S_{rs,y}\, \Delta p_{T,y}^k + S_{rs,z}\, \Delta p_{T,z}^k \right) \le p_{rs}^{max}
\end{aligned}
$$

$$
\forall rs \in \mathcal{B}, \ \forall rs \ne ij, \ \forall i,j,n,r,s \in \mathcal{N}, \ k \in \mathcal{C}
$$

Branches virtually in service (outaged branches)

$$
\begin{aligned}
p_{ab}^0 &= [1 - (S_{ab,a} - S_{ab,b})]\, \Delta p_a^k \\
&- \sum_{gf \to n} S_{ab,n}\, \Delta p_{gf}^k - \sum_{l \to n} S_{ab,n}\, \Delta p_{l,s} \\
&- \sum_{g \to n} S_{ab,n} (\Delta p_{g,u} - \Delta p_{g,d}) \\
&- \sum_{ij \in \mathcal{B}^k} \left(S_{ab,i}\, \Delta p_i^k + S_{ab,j}\, \Delta p_j^k \right) \\
&- \sum_{yz \in \mathcal{T}} \left(S_{ab,y}\, \Delta p_{T,y}^k + S_{ab,z}\, \Delta p_{T,z}^k \right) \le p_{rs}^{max}
\end{aligned}
$$

$$
\forall ab \in \mathcal{B}^k, \ \forall ab \ne ij, \ \forall i,j,n,a,b \in \mathcal{N}, \ k \in \mathcal{C}
$$

where,

- Superscript 0 (resp. k) refers to the base case (resp. contingency k state).

- $\mathcal{C}$ is the set of contingencies postulated.

- $\mathcal{N}$ and $\mathcal{L}$ are the set of nodes and the set of loads, respectively.

- $\mathcal{G}$ is the set of generators in service in the post-contingency state.

- $\mathcal{G}^k$ is the set of outaged generators, but virtually in service in the post-contingency state k.

- $\mathcal{B}$ is the set of branches in service in the post-contingency state.

- $\mathcal{B}^k$ is the set of outaged branches, but virtually in service in the post-contingency state k.

- $\mathcal{T}$ is the set of active Phase Shifting Transformers (PST).

- $g \to n$ is the set of generators in service connected to node n.

- $gf \to n$ is the set of outaged generators connected to node n, but virtually in service in the post-contingency state.

- $l \to n$ is the set of loads (demands) connected to node n and affected by a load-shedding.

- Δp_{gf}^k correspond to the vector of fictitious active power injections that model the outaged generators at contingency k.

- Δp_i^k y Δp_i^k expresses the fictitious active power injections used for modelling the outage of the ij branch.

- $\Delta p_{T,y}^k$ y $\Delta p_{T,z}^k$ expresses the fictitious active power injections used for modelling the PST action in contingency k.

- $\Delta p_{g,u}$, $\Delta p_{g,d}$, $\Delta p_{l,s}$ correspond to generation control actions (up and down) and to load shedding, respectively, with $c_{g,u}$, $c_{g,d}$ y $c_{l,s}$ being penalty indices, respectively, for active power cost of generator g, and load shedding l.

Note that fictitious injections, Δp_i^k and Δp_j^k, modelling outaged branches are also computed by the optimization

formulation, as the power flows are affected by the control actions. Similarly, the power flows are affected in the branches in service, whether fictitious injections are modified as a consequence of corrective actions. Also, the elements of the S_f matrix of sensitivity depend only on the network topology at the pre-contingency state, and, consequently, they have been computed off-line, in a time before the post-contingency.

Fictitious injections in both ends of a PSTs are also used to model the changes performed by these transformers. The maximum variation of power flow that a PST can perform has been restricted to the 10% of the maximum power flow in the branch. Obviously, the phase shifting transformers are also included in the outage list. When the outaged branch is the PST between the nodes y and z, then: $\Delta p_{T,y}^k = \Delta p_{T,z}^k = 0$

Finally, it must be considered that the fictitious injections are linked to variations of power flows in the branches of the system and to the power-injection variations carried out by the corrective actions. Consequently, fictitious injections are adaptive injections to changes of power injections in the system.

VI. NUMERICAL SIMULATIONS AND RESULTS

It is important to note that the methodology proposed - by using fictitious injections of active power - carries out an accurate result without a previous topological analysis and the rest of traditional consequent analyses. The formulation proposed is handled by using the GAMS platform and the same software has been used to handle the formulation corresponding to the traditional DC network analysis applied in the simulations. The minimization problem has been solved using the CPLEX solver running under GAMS. CPLEX implements a dual simplex algorithm for solving the linear programming problem [31] [32]. All tests have been performed on a 2.01-GHz, 2-GB RAM, PC AMD-Athlon.

A. Description of the test system of 24 buses

Representative numerical results obtained by applying the approach proposed in the IEEE-RTS of 24 buses [16] are presented in this section. A summary of the characteristics of this test system is given in Table I, where: $\mathcal{N}$, $\mathcal{G}$, $\mathcal{D}$, $\mathcal{B}$, $\mathcal{L}$, $\mathcal{T}$, and $\mathcal{S}$ denote the number of buses, generators, loads (demands), branches, lines, all transformers, and shunt elements, respectively.

TABLE I.   TEST SYSTEM SUMMARY

System	$\mathcal{N}$	$\mathcal{G}$	$\mathcal{D}$	$\mathcal{B}$	$\mathcal{L}$	$\mathcal{T}$	$\mathcal{S}$
IEEE-RTS	24	33	17	34	29	5	1

The load-generation scenario for the active problem and the rest of data of the IEEE-RTS have been obtained from reference [16]. The selected scenario (Base Case) corresponds to the $18:00$ h of the Summer season and has been used as the pre-contingency state (N-0 Contingency Level). This scenario - for 100 MVA power base - is shown per unit (pu) in Table II including both 138 kV and 230 kV levels of the IEEE-RTS.

The key issue of this section is to present the approach proposed as an acceptable tool for the contingency analysis of transmission and sub-transmission systems. So test simulations

TABLE II.   GENERATION-LOAD SCENARIO: BASE CASE.
N-0 CONTINGENCY LEVEL

* 138 kV level

BUS	01	02	03	04	05
Generation (pu)	1.87	1.87	—	—	—
Load (pu)	1.62	1.55	2.06	1.41	1.39
BUS	06	07	08	09	10
Generation (pu)	—	2.86	—	—	—
Load (pu)	1.79	1.72	1.82	2.03	2.16

* 230 kV level

BUS	13	14	15	16	18
Generation (pu)	5.18	—	2.15	1.55	3.90
Load (pu)	2.65	1.94	3.17	1.00	3.33
BUS	19	20	21	22	23
Generation (pu)	—	—	3.90	3.00	6.45
Load (pu)	1.81	1.28	—	—	—

have been performed for contingencies considering both transmission (230 kV level) and sub-transmission (138 kV level) systems of the IEEE-RTS.

Note that the DC network analysis is also acceptable in sub-transmission voltage levels, as far as the requirements for using this model are completed by subtransmission networks [17] [18] [9] [15]. These requirements are satisfied by the 138 kV level of the IEEE-RTS (sub-transmission area).

B. First N-2 test case

In this test case no actions on PSTs are considered, so the DC Corrective OPF proposed only shows how contingencies are modelled as fictitious injections thanks to sensitivity matrix and how these injections are modified when corrective actions are taken into account in the optimisation problem.

The case associated with a contingency (N-2) which implies an overloaded state is set out in this section. This contingency considers the simultaneous outage of the lines from from BUS 12 to BUS 13 and from from BUS 12 to BUS 23, both lines of the transmission area (230 kV) of the IEEE-RTS. The pre-contingency state is shown in Figure 5(a).

When this contingency was simulated, Figure 5(b), two branches were detected in overloaded state. The first one, the transformer from BUS 10 to BUS 11 and, the second one, the line from BUS 11 to BUS 13. It is important to note that two transformers of 4.00 pu each one in BUS 12 are out of service as a consequence of this multiple contingency, Figure 5(b). These transformers interconnect the 230 kV area with the 138 kV area, and their out of service reduce to $3/5$ pu the available transfer capability (ATC) between these two areas.

The meaning of the terms in Figure 5(b) corresponds to fictitious-active power flow:

- $p_{(12-13)} =$ in line from BUS 12 to BUS 13.

- $p_{(12-23)} =$ in line from BUS 12 to BUS 23.

and to fictitious-active power injection:
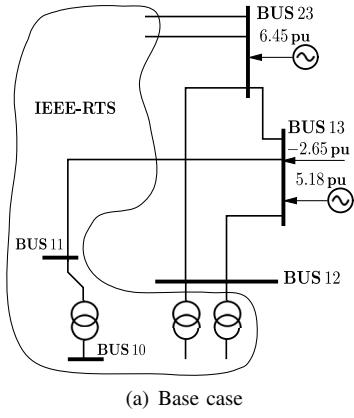
(a) Base case
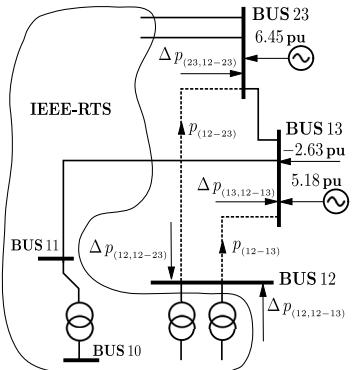
(b) Post-contingency state after corrective actions

Fig. 5. First N-2 test case modelled as fictitious injections.

- $\Delta p_{(12,12-13)} =$ in BUS 12 associated with the outaged line from BUS 12 to BUS 13.

- $\Delta p_{(13,12-13)} =$ in BUS 13 associated with the outaged line from BUS 12 to BUS 13.

- $\Delta p_{(12,12-23)} =$ in BUS 12 associated with the outaged line from BUS 12 to BUS 23.

- $\Delta p_{(23,12-23)} =$ in BUS 23 associated with the outaged line from BUS 12 to BUS 23.

By applying the formulation proposed (without actions on PSTs), the obtained fictitious injections were the following:

$$\Delta p_{(12,12-13)} = -\Delta p_{(13,12-13)} = p_{(12-13)} = -4.92\,\text{pu}$$

$$\Delta p_{(12,12-23)} = -\Delta p_{(23,12-23)} = p_{(12-23)} = -5.12\,\text{pu}$$

Table III shows the power flow in the two overloaded branches in pre-contingency (Base case) and post-contingency states. In the case of the post-contingency state (Overloaded) the results are previous to corrective actions.

TABLE III. BRANCHES POWER FLOW IN THE BASE CASE AND POST CONTINGENCY STATES

Branch	Base Case Power flow (pu)	Thermal Limit (pu)	Post-Contingency Overloaded (pu)	Post DC-OPF (pu)
10 - 11	- 2.20	4.00	- 4.01	- 3.99
11 - 13	- 2.43	5.00	- 5.01	- 4.99

To eliminate overloads, corrective actions focused on re-scheduling generation were applied, but load-shedding and actions on PSTs weren't necessary. These corrective actions are shown in Table IV.

TABLE IV. CORRECTIVE ACTIONS: FIRST N-2 TEST CASE

BUS	07	13
Generation-up (pu)	0.02	—
Generation-down (pu)	—	0.02

C. Second N-2 test case

This test case is related to the simultaneous outage of the transformers from BUS 09 to BUS 12 and from BUS 10 to BUS 12. Both of them of the interconnection of the transmission area (230 kV) with the the sub-transmission area (138 kV) of the IEEE-RTS. Note that nodes BUS 12 and BUS 10 break connection between them in the post-contingency state. The pre and post-contingency state are presented in Figure 6(a) and in Figure 6(b), respectively, as is modelled in the DC COPF proposed.
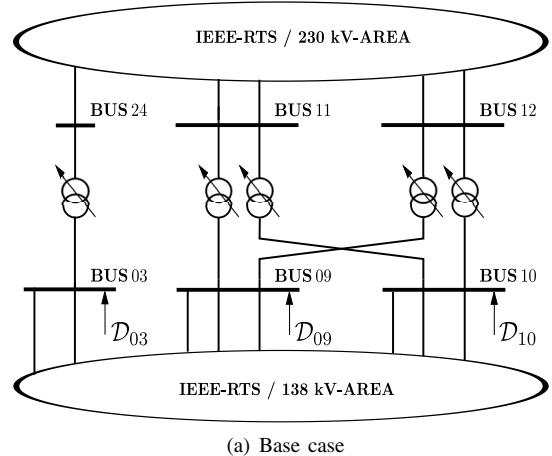
(a) Base case
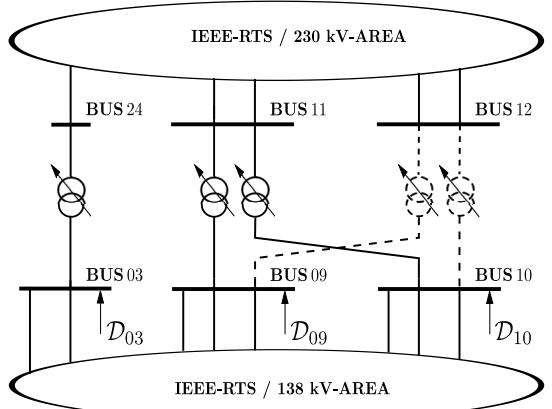
(b) Post-contingency state after corrective actions

Fig. 6. Second N-2 test case modelled as fictitious injections.

The post-contingency state, Figure 6(b), can be applied as much to the case of no actions on PSTs as to the case of actions on PSTs. The meaning of the terms in Figure 6(b) corresponds to fictitious-active power flow:

- $p_{(09-12)} =$ in transformer from BUS 09 to BUS 12.

- $p_{(10-12)} =$ in transformer from BUS 10 to BUS 12 .

to fictitious-active power injection:

- $\Delta p_{(09,09-12)} =$ in BUS 09 associated with the outaged transformer from BUS 09 to BUS 12 .

- $\Delta p_{(10,10-12)} =$ in BUS 10 associated with the outaged transformer from BUS 10 to BUS 12 .

- $\Delta p_{(12,09-12)} =$ in BUS 12 associated with the outaged transformer from BUS 09 to BUS 12 .

- $\Delta p_{(12,10-12)} =$ in BUS 12 associated with the outaged transformer from BUS 10 to BUS 12 .

and $\mathcal{D}_{10LS}$ expresses a load-shedding action in BUS 10 .

Next are presented the results of the methodology proposed, firstly, when in the corrective DC OPF no actions on PSTs are considered and, secondly, when all remedial actions (generation re-dispatch, load shedding and PSTs) are taken into account. By applying the DC COPF proposed to this N-2 contingency, load-shedding was necessary. But as it is shown below, when the PSTs actions are used the necessary load shedding is lower than in the case where only active power rescheduling is employed.

1) No actions on PSTs: The power flow in the transformers (interconnection) between the transmission area (230 kV) and the sub-transmission area (138 kV), are showed in Table V.

TABLE V. TRANSFORMERS POWER FLOW IN THE BASE CASE AND POST CONTINGENCY STATE. NO ACTIONS ON PSTS

Branch	Base Case Power flow (pu)	Thermal Limit (pu)	Post-Contingency state	Post DC-OPF (pu)
09 - 12	1.90	4.00	Outaged	-6.47
10 - 12	2.50	4.00	Outaged	-7.48
03 - 24	-2.75	4.00	In Service	-3.54
09 - 11	-1.59	4.00	In Service	-2.98
10 - 11	-2.19	4.00	In Service	-4.00

In the case of transformers from BUS 09 to BUS 12 and from BUS 10 to BUS 12, the fourth column of Table V corresponds to the fictitious power flow obtained in the DC OPF proposed. Accordingly, the fictitious injections are:

$$\Delta p_{(09,09-12)} = -\Delta p_{(12,09-12)} = p_{(09-12)} = -6.47 \, \text{pu}$$

$$\Delta p_{(10,10-12)} = -\Delta p_{(12,10-12)} = p_{(10-12)} = -7.48 \, \text{pu}$$

Obviously, when the contingency is analysed, corrective actions are taking into account. These corrective actions are shown in Table VI.

2) With actions on PSTs: As equal as in the previous section, power flow in the transformers (interconnection) are shown Table VII. Accordingly, the fictitious injections are:

$$\Delta p_{(09,09-12)} = -\Delta p_{(12,09-12)} = p_{(09-12)} = -6.64 \, \text{pu}$$

$$\Delta p_{(10,10-12)} = -\Delta p_{(12,10-12)} = p_{(10-12)} = -7.49 \, \text{pu}$$

The corrective actions in this case are shown in Table VIII.

TABLE VI. GENERATION-LOAD SCENARIO: POST DC OPF PROPOSED. N-2 CONTINGENCY LEVEL.

* 138 kV level

BUS	01	02	07	10
Generation-up (pu)	0.05	0.05	0.14	—-
Generation-down (pu)	—-	—-	—-	—-
Load-shedding (pu)	—-	—-	—-	0.18

* 230 kV level

BUS	13	18	21	23
Generation-up (pu)	—-	0.10	0.10	—-
Generation-down (pu)	0.47	—-	—-	0.14
Load-shedding (pu)	—-	—-	—-	—-

TABLE VII. TRANSFORMERS POWER FLOW IN THE BASE CASE AND POST CONTINGENCY STATE. WITH ACTIONS ON PSTS

Branch	Base Case Power flow (pu)	Thermal Limit (pu)	Post-Contingency state	Post DC-OPF (pu)
09 - 12	1.90	4.00	Outaged	-6.64
10 - 12	2.50	4.00	Outaged	-7.48
03 - 24	-2.75	4.00	In Service	-3.32
09 - 11	-1.59	4.00	In Service	-3.16
10 - 11	-2.19	4.00	In Service	-4.00

TABLE VIII. GENERATION-LOAD SCENARIO: POST DC OPF PROPOSED. N-2 CONTINGENCY LEVEL.

* 138 kV level

BUS	01	02	07	10
Generation-up (pu)	0.05	0.05	0.14	—-
Generation-down (pu)	—-	—-	—-	—-
Load-shedding (pu)	—-	—-	—-	0.07

* 230 kV level

BUS	13	18	21	23
Generation-up (pu)	—-	0.10	0.10	—-
Generation-down (pu)	0.47	—-	—-	0.04
Load-shedding (pu)	—-	—-	—-	—-

Apart from other considerations, when actions on PTSs are used joint to remedial actions such as generation re-dispatch and load-shedding, the important issues are the fact that load shedding in BUS 10 has been reduced from 0.18 pu (Table VI) to 0.07 pu (Table VIII), and re-dispatch in generator of BUS 23 (generation-down) has been reduced from 0.14 pu (Table VI) to 0.04 pu (Table VIII).

VII. CONCLUSIONS

In this paper, a novel formulation (DC COPF) is proposed in the field of the DC network analysis. Basically, this novel formulation is a corrective power-system rescheduling and load-shedding problem that exploits problem structure significantly better than previous DC analysis supported by power flow.

The novelty of this work stems from the inclusion in the optimization problem of actions on PTSs and of fictitious injections modelling contingencies. So, the DC COPF proposed directly adjusts these fictitious injections to the post-contingency state as a consequence of the corrective actions carried out to bring the system back to its normal state.

The method efficiently handles the outaged branches because they are treated as branches virtually in service.

The linear model proposed (DC COPF) is applied to direct the solution to the "normal" operating region. Obviously, once the normal state is placed, then an AC analysis should be used to verify the results and modify limits accordingly. But, this last AC analysis is a matter not focused on in this work.

Simulations show that even if the initial operating point is far from the solution, large generation shifts are allowed and the linearization of the line flows is fairly good. Finally, simulations also show that the approach proposed is suitable to deal with both transmission and sub-transmission systems, as was shown by applying the approach proposed in the $230\,\mathrm{kV}$ and the $138\,\mathrm{kV}$ areas of the IEEE-RTS. Anyway, despite these excellent results, cases where not all binding contingencies are identified are to be expected, e.g. due to the reactive power flows which also contribute to branches current are neglected, lossless grid assumption of the DC model.

VIII. Acknowledgements

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