SENSITIVITY-BASED EXTERNAL EQUIVALENT FOR RELIABILITY ASSESSMENT OF SUBTRANSMISSION & DISTRIBUTION SYSTEMS

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Abstract - It is well known that topology changes in a power system strongly influences the results of reliability assessment studies. Besides, it is obvious that the topology of an external system also affect the reliability of the internal system through changes on the power flows within the rest of the system, and hence, through changes on the tie lines. In order to properly model the interconnection between the internal network and the rest of the system, an adaptive external equivalent model, obtained from the sensitivity matrix between tie-line power flows and injected powers, is proposed for reliability assessment studies.

The proposed technique has been applied to the IEEE-RTS, and test results show that the proposed methodology is suitable both for planning studies and for 24-hour-ahead reliability assessment studies.

Keywords - External equivalent, reliability assessment, subtransmission systems.

1 INTRODUCTION

SUBTRANSMISSION systems have been traditionally included in the reliability analysis of composite power systems [1] [2]. However, under the new regulatory context, subtransmission networks have the responsibility to manage power exchanges and load curtailment to fulfill the reliability commitments and requirements of customers [3]. The present work has been focussed on the subtransmission level, in order to present an accurate network model under changing system conditions for the security & reliability assessment problem faced by electrical utilities dealing with meshed subtransmission networks and the supply points of the distribution facilities.

In the security & reliability analysis of a system, Figure 1, the interconnection by means of tie lines (TL) implies that a contingency in the Internal System (IS) can disturb the operating conditions of the External System (ES), and this disturbance in turn can have a significant effect on the internal system (IS) in which the contingency occurred. In order to adequately model the influence of the external system, while reducing their size, several external equivalents have been proposed in the literature, being the Ward and REI the most used [4] [5].

Efforts on the external system modeling have been made mainly for power system load flow, optimal power flow, contingency analysis and state estimation [6] [7] [8]. However, few studies have addressed the problem of external equivalents from the point of view of subtransmission systems, typically meshed and with few manageable generation resources [3].

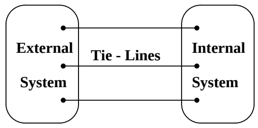


Figure 1: Power system including Internal and External networks.

A Sensitivity-based External Equivalent is presented in this paper. The proposed model is based on two sensitivity matrices, S_f and S_{TL}, the first one corresponding to sensitivities of the IS power flows with respect to IS power injections, and the second one relating changes in the TL power flows to changes in the IS power flows, taking into account the variations of the power flow in the ES branches. Working with this two matrices is similar to consider a simplified system such as the one shown in Figure 2. In this system, power injections in the boundary nodes represent the tie-line power flows, calculated by means of S_{TL}.

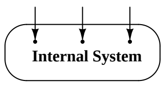


Figure 2: External System modelled as power injections.

The application of probabilistic techniques provides a quantitative prediction and more complete information of the system performance than the contingency analysis, and, more important, a way of consistently evaluating the respective reliability levels of alternative operational arrangements or network reinforcements [1] [9]. In this way, the methodology presented in this paper includes a statistical evaluation module and a reliability analysis supported on a DC-OPF. The State Enumeration technique has been applied in the statistical evaluation module, and the reliability indices LOLP (Loss Of Load Probability) and EPNS (Expected Power Not Served) have been used in the reliability assessment.

The reminder of the paper is organized as follows. Firs, the basic concepts of the Sensitivity-based External Equivalent and the probabilistic technique applied to the reliability assessment are presented. Then, the mathematical problem is also presented, followed by some numerical simulations and the subsequent analysis of the results. Finally, some conclusions derived from this work are summarized.

2 Sensitivity-based External Equivalent

To determine the proposed External Equivalent, the first step is to obtain the sensitivity matrix $\widehat{S}_f$ of the complete system, including both the IS and the ES, (Figure 1). $\widehat{S}_f$ expresses the sensitivity of the power flows in all branches of the bulk system to changes in the injected powers. Once $\widehat{S}_f$ is known, the rows corresponding to TL

branches are selected, and only the elements corresponding to nodes of the IS are retained and arranged into a new matrix of sensitivities, S_{TL}.

If the IS is analyzed modelling the ES as power injections, Figure 2, a new sensitivity matrix S_f is obtained. In the proposed approach, S_f is used to calculate changes in power flows due to contingencies in IS, while using S_{TL} to update the injections in the boundary nodes to reflect the changes in tie-line flows. Both matrices, S_f and S_{TL}, are obtained from a DC linear model, and using this linear model the branch contingencies are modelled as two fictitious injections at both end nodes of the branch [10].

2.1 Modelling branch outages as power injections

The DC linear model facilitates the use of two fictitious injections to model the outage of a branch [10], avoiding the need to modify the topology of the network after an outage. As a consequence, sensitivity matrix S_f remains constant throughout the reliability assessment study.

In the case of failure of the ij branch (Figure 3) with a p_{ij}^0 pre-failure power flow, its outage can be modeled using two fictitious injections Δp_i and Δp_j at both ends of the branch. If the post-contingency power flow is represented by p_{ij}, it is evident that $p_{ij} = \Delta p_i = -\Delta p_j$.

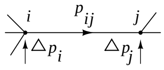


Figure 3: Simple outage of the ij branch and fictitious injections.

Then,

$$p_{ij} = p_{ij}^0 + S_{ij,i}\,\Delta p_i + S_{ij,j}\,\Delta p_j \qquad (1)$$

where $S_{ij,i}$ and $S_{ij,j}$ are the sensitivities between the ij power flow and injections in the nodes i and j, respectively. Note that the outaged branch ij is virtually "in service" in this approach. From (1) the power increase, Δp_i, in the node i is yielded as

$$\Delta p_i = p_{ij}^0\;[1-(S_{ij,i}-S_{ij,j})]^{-1} \qquad (2)$$

The Δp_{mn} flow increase in the mn branch, after the outage of the ij branch, is obtained as follows:

$$p_{mn} = p_{mn}^0 + S_{mn,i}\Delta p_i + S_{mn,j}\Delta p_j \;\;\Rightarrow \qquad (3)$$

$$\Delta p_{mn} = p_{ij}^0\,(S_{mn,i}-S_{mn,j})\,[1-(S_{ij,i}-S_{ij,j})]^{-1}$$

where $S_{mn,i}$ and $S_{mn,j}$ represent the sensitivities between the power flow in the branch mn and the changes of the injections in the nodes i and j, respectively.

Multiple outages (Figure 4) are analyzed in similar way. Thus, considering the interactions between the outages, the fictitious injections at both ends of the outaged branches are computed by solving a linear system of equations. In matrix form,

$$P_{f_c}^0 = T\cdot\Delta P_c \qquad (4)$$

where $P_{f_c}^0$ is the vector of precontingency power flows in outaged branches, ΔP_c is the vector of fictitious injections at both ends of the outaged branches, and T is the coefficient matrix of the system of equations, consisting of ones and the corresponding sensitivities.

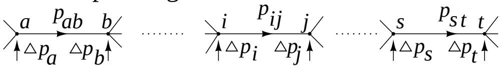


Figure 4: Simultaneous outages of branches and fictitious injections.

2.2 Sensitivity matrices

In the proposed approach the DC analysis, acceptable in subtransmission voltage levels [11] [12], has been used in order to reduce computational times.

Using the DC model of the power system, Figure 1, the active power flow $\hat{p}_{ij}$ in each branch ij is expressed as

$$\hat{p}_{ij} = \frac{\hat{\theta}_i - \hat{\theta}_j}{\hat{x}_{ij}}$$

If phase angles are removed, then a linear relationship between nodal active powers, $\widehat{P}$, and the active power flows, $\widehat{P}_f$, is yielded as follows

$$\widehat{P}_f = \left[\widehat{X}^{-1}\,\widehat{A}^T\,\widehat{B}^{-1}\right]\,\widehat{P} = \widehat{S}_f\,\widehat{P}$$

where $\widehat{S}_f$ is a sensitivity matrix between branch power flows and injected powers, $\widehat{X}$ expresses a diagonal matrix of branch reactances, $\widehat{A}$ denotes the branch-to-node incidence matrix, reduced by removing the slack row, and $\widehat{B}$ is a matrix defined as equal as the matrix B' of the "XB" version of the FDLF (Fast Decoupled Load Flow). The matrix $\widehat{S}_f$ can be expressed as a set of three sensitivity sub-matrices,

$$\widehat{S}_f = \left\{\widehat{S}_{ES},\,\widehat{S}_{TL},\,\widehat{S}_{IS}\right\}$$

corresponding to the rows of the branches of ES, TL and IS, respectively. A reduced matrix S_{TL} is obtained extracting the columns that correspond to the nodes of the IS from $\widehat{S}_{TL}$, and restricted by changes in ES power flows, S_{TL} provides the sensitivities of tie-line branch power flows to changes in the injected powers of the IS.

On the other hand, if IS is analyzed as a separate zone of power system, then the S_f sensitivity matrix is obtained. S_f verifies the relation:

$$P_f = S_f\,P$$

where P_f denotes the vector of the active power flows in the IS branches, and P is the vector of injected powers in the nodes of the IS.

S_{TL} relates the IS to its ES supply area by means of the TL power flows that are included as injections, P_{TL}, in the vector P. Hence, the vector P_{TL} of injected powers from ES to IS can be obtained as [10]:

$$P_{TL} = P_{TL}^0 + (S_{TL}\,\Delta P_{IS}) \qquad (5)$$

where P_{TL}^0 are the injected powers from ES to IS in the base case, and ΔP_{IS} is the vector of injected power changes in IS nodes. In consequence, the vector ΔP_f is yielded as follows

$$\Delta P_f \;=\; [\,S_f\,\Delta P_{IS}\,] + [\,S_f\,S_{TL}\,\Delta P_{IS}] \qquad (6)$$

Changes on the injected powers in IS nodes can be motivated by loss of generation, branch outages, load shedding or corrective/preventive actions.

Basically, equation (5) is a linear model of External Equivalent based on adaptive injections to changes in IS.

2.3 Adaptive External Equivalent in the DC-OPF

Taking advantage of the decoupling between active and reactive power equations, the study of overloads have been made trough a DC-OPF [10] [14]. The applied DC-OPF is based on linear programming and is carried out to obtain remedial actions and to bring the system back to a normal state, as quickly as possible, whether a contingency occurs. A lower cost to generation rescheduling than to possible load shedding has been assigned with the objective of minimizing load curtailment.

In the case of a DC-OPF (linear system) the sensitivity matrix S_A can be considered in the OPF formulation to obtaining a more compact network model, yielding

Min

$$c_u^T \, \Delta P_g^+ + c_d^T \, \Delta P_g^- + c_{ls}^T \, \Delta P_d \tag{7}$$

s.t.

$$\begin{aligned}
& S_A \left(\Delta P_g^+ - \Delta P_g^- + \Delta P_d + \Delta \tilde{P}_c \right) = \Delta P_f \\
& P_{f_c}^0 + \tilde{S}_A \left(\Delta P_g^+ - \Delta P_g^- + \Delta P_d \right) = T \cdot \Delta P_c \\
& -P_f^{max} \leq P_f^0 + \Delta P_f \leq P_f^{max} \\
& \Delta P_g^+ \, \leq \, P_g^{max} - P_g \\
& \Delta P_g^- \, \leq \, P_g - P_g^{max} \\
& \Delta P_d \, \leq \, P_d^{max} \\
& \Delta P_g^+ \geq 0 \;\; \Delta P_g^- \geq 0 \;\; \Delta P_d \geq 0
\end{aligned}$$

where ΔP_g^+, ΔP_g^- and ΔP_d are vectors that correspond to generation control actions (up and down) and to load shedding, respectively, with c_u, c_d and c_{ls} being penalty vectors. S_A is the sensitivity matrix of the Internal System, but including the sensitivity elements of the Tie-Line. In compact form: $S_A = \{S_f, \, S_{TL}\}$ The sub-matrix S_{TL} adapts the injected powers in the boundary nodes depending on the changes of the injected powers in the nodes of the Internal System, but restricted by the changes in the power flows in the branches of the External System.

Note that outages are modeled using fictitious injections ΔP_c, being $\Delta \tilde{P}_c$ a vector with the corresponding injections in the correct buses. Furthermore, note that the fictitious injections modeling outages must be also computed by the DC-OPF, as the precontingency flows are affected by the control actions, i.e.,

$$P_{f_c}^0 + \tilde{S}_A \left(\Delta P_g^+ - \Delta P_g^- + \Delta P_d \right) = T \cdot \Delta P_c \tag{8}$$

with $\tilde{S}_A$ being the sensitivity matrix relating flows in outaged lines with power injections.

The reliability indices, considering the load-shedding from the DC-OPF and the probability of the particular state, are updated in a Statistical Evaluation module based on a non-sequential State Enumeration process [16].

3 Simulations and results

The IEEE-RTS (Figure 5) and its common Ward external equivalent (Figure 6) have been used in the comparative analysis presented in relation to the results obtained when the proposed Sensitivity-based External Equivalent is used for the reliability assessment of the 138 kV subtransmission area (IS) of the IEEE-RTS.

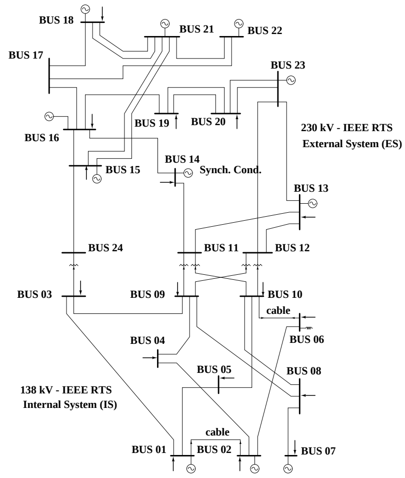


Figure 5: IEEE Reliability Test System, 24 nodes

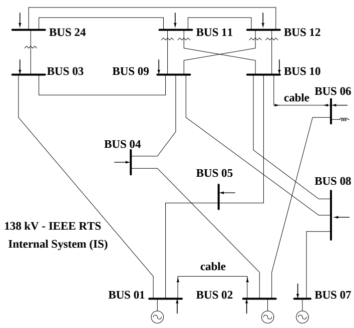


Figure 6: IEEE Ward External Equivalent

In the IEEE-RTS, the 230 kV transmission system (External System) has been assumed perfectly reliable, unavailability equal to zero, and only contingencies of generators, transformers and lines of the 138 kV subtransmission level have been considered. The load peak in the 138 kV sub-transmission area, and the rest of data of the IEEE-RTS, necessary for the reliability assessment, have been obtained from reference [17]. Respect to the Ward model, the external equivalent is a meshed network which has been obviously supposed as ideal (perfectly reliable), and as equal as in the case of the complete IEEE-RTS, only contingencies of generators, transformers and lines of the 138 kV level (IS) have been considered in the analyzed test cases.

The simulations have been performed for the usual contingency levels N-1 and N-2, and for each one, first the comparative analysis of results are exposed for the case of overloads studies, and next the results are shown for the case of reliability assessment. A thermal limit of 166 MVA for the 138 kV lines and a power base of 100 MVA have been applied in the simulations.

3.1 The N-1 test case

As an example of the analysis of overloads in the branches of the 138 kV sub-transmission system (IS), the outage of the branch from BUS03 to BUS09 of the 138 kV level (Figure 5) is supposed in the simulations.

When the complete model of the IEEE-RTS system (Figure 5) is used in the analysis of this case, and it is applied a AC (AC LF) and a DC (DC LF) load flow post outage, the branch power flows are shown in Table 1 for the 138 kV level. The fourth column of this table shows the percentage error of using DC versus AC load flow and relative to the branch thermal limit (1.66 pu). Maximum, minimum and average errors using the DC LF instead of the AC LF are, respectively, $8.10\,\%$, $0.22\,\%$ and $3.16\,\%$. The DC LF is the reference for the comparison of the simulations with the Ward and proposed external model in the third column of Table. 1.

Line From To	AC LF Post Outage (pu)	DC LF Post Outage (pu)	DC vs AC Thermal Error (%)
01-02	0.2021	0.2856	5.03
01-03	-0.4426	-0.5771	8.10
01-05	0.4958	0.5467	3.07
02-04	0.2478	0.3018	3.25
02-06	0.2777	0.3075	1.79
04-09	-1.1607	-1.1040	3.42
05-10	-0.8990	-0.8404	3.53
06-10	-1.5189	-1.4838	2.12
08-09	-0.4970	-0.4791	1.08
08-10	-0.2100	-0.2063	0.22

Table 1: N-1 test case. Load flow using IEEE-RTS system

Table 2 shows the results when the Ward model (Figure 6) is applied and the outage of the branch from BUS03 to BUS09 is considered. The fourth column, Table 2, shows the percentage error of using the DC load flow in the Ward model (second column in Table 2) respect to use DC load flow in the complete IEEE-RTS (third column in Table 1) and referred to the branch thermal limit. Note that maximum, minimum and average errors are, respectively, $8.00\,\%$, $1.07\,\%$ and $3.08\,\%$.

Line From To	Ward DC Post Outage (pu)	Ward DC vs IEEE-RTS DC Error (pu)	Ward DC vs IEEE-RTS DC Thermal Error (%)
01-02	0.2448	0.0408	2.46
01-03	-0.5070	0.0701	4.22
01-05	0.5174	0.0293	1.77
02-04	0.2788	0.0230	1.38
02-06	0.2897	0.0178	1.07
04-09	-1.1270	0.0230	1.39
05-10	-0.8697	0.0293	1.77
06-10	-1.5016	0.0178	1.07
08-09	-0.6119	0.1328	8.00
08-10	-0.3342	0.1279	7.71

Table 2: N-1 test case. Load flow using the Ward model

When the Sensitivity-based External Equivalent is applied, the branch power flows of the 138 kV level are shown in Table 3. The failure of the line from BUS03 to BUS09, using two fictitious injections at both ends of this branch, has been also included in this simulation. The injected powers in the 138 kV boundary nodes are given by S_{TL}, but note that these injected powers are the representation of the power flows in the tie-lines, and it has restrictions linked to the variations of the power flows in the 230 kV branches (ES), such as it was exposed in Section 2.2. In Table 3 the maximum, minimum and average errors are, respectively, $3.07\,\%$, $0.53\,\%$ and $1.27\,\%$.

It is important to take into account that, on the one hand, the results are more accurate with the proposed model than with the Ward model and, on the other hand, the proposed model needs a low computational effort because it is also avoided the topological analysis.

Line From To	Sensitivity DC Post Outage (pu)	Sensitivity DC vs IEEE-RTS DC Error (pu)	Sensitivity DC vs IEEE-RTS DC Thermal Error (%)
01-02	0.3210	0.0354	2.13
01-03	-0.6281	0.0510	3.07
01-05	0.5624	0.0156	0.94
02-04	0.3284	0.0266	1.60
02-06	0.3162	0.0087	0.53
04-09	-1.0773	0.0266	1.60
05-10	-0.8247	0.0156	0.94
06-10	-1.4750	0.0087	0.53
08-09	-0.4671	0.0120	0.72
08-10	-0.2182	0.0120	0.72

Table 3: N-1 test case. Load flow using the Sensitivity-based model

Next, the results of the state enumeration process till a N-1 contingency level are presented and discussed. This process considers, one by one, the outage of the elements of the 138 kV level and determines, for each N-1 outage, the PNS (Power Not Supply) and the reliability indices EPNS (Expected Power Not Served) and LOLP (Loss Of Load Probability).

First, the reliability indices of the 138 kV system has been calculated using the complete model of the IEEE-RTS (Figure 5), and these have been used as the reference. Subsequently, considering the Ward model and the proposed Sensitivity-based model their respective reliability indices have been also obtained. Finally, errors associated to the obtained reliability indices of these models have been determined as a percentage of the results produced using the complete IEEE-RTS. In the N-1 process, the total number of outages included in the simulation are 39 and combined with the ten nodes of the 138 kV area of the IEEE-RTS give 390 cases to analyze. It is important to note that the simple failure of a generator of the 138 kV level involves no loss of load in any of the used models.

For the Ward and proposed external model, the percentage error of the PNS results have been classified in intervals such it is shown in Table 4. In the table, the rows named W and S correspond to the Ward and the Sensitivity-based model, respectively. For simplicity, the comparative is only exposed for PNS and EPNS indices, but the conclusions are also applicable to others indices.

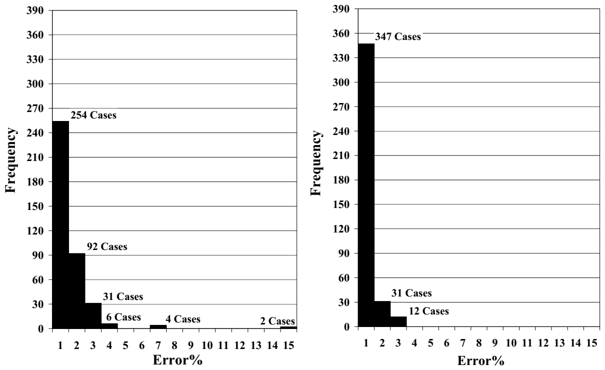
%	≤ 5	≤ 15	≤ 25	≤ 35	≤ 45	≤ 55	≤ 65	≤ 75	≤ 85	≤ 95	≤ 100
W	283	6	0	0	0	0	0	0	0	0	1
S	390	0	0	0	0	0	0	0	0	0	0

Table 4: Frequency of PNS Error% for the N-1 test reliability analysis

As an example of the results obtained for the Ward external model (row W of Table 4), there are 6 cases in the $5\% <$Error$\% \leq 15\%$ interval which means a PNS different at most in a 15% respect to the obtained PNS using the complete IEEE-RTS model in the N-1 reliability analysis. Finally, notice that the case belonging to the $95\% <$Error$\% \leq 100\%$ interval is very important because this means that the Ward model assigns a load curtailment that is not necessary.

On the other hand, if the proposed Sensitivity-based model, row named S in Table 4, is considered, then the errors are all in the $0\% <$ Error % $\leq 5\%$, and, as it can be noticed, there are no cases in the rest of intervals.

For the intervals from 0 % to 15 % the PNS-Error% Frequency (number of cases) is also shown as bar plots in Figure 7(a) and Figure 7(b) for the Ward and proposed external models, respectively.



(a) Ward (b) Sensitivity-based model

Figure 7: PNS-Error% Frequency for the N-1 reliability analysis

It is noticeable that in the N-1 enumeration process, the Ward External Equivalent carries out a case in the $95\% <$ Error % $\leq 100\%$ interval, and this is equal to a no-necessary load shedding because this result is not given when the complete IEEE-RTS network is used in the simulation. Besides, when the Sensitivity-based model is used in the N-1 reliability analysis, the results are accurate and near to the obtained with the complete IEEE-RTS.

When the Ward model is used, the no-necessary loss of load is associated to a system state post-outage of the branch from BUS09 to BUS11, and the LOLP and the PNS are, respectively, $9.4246 \cdot 10^{-6}$ pu and 0.0515 pu for this system state. But, in the case of the application of the Sensitivity-based External Equivalent, the PNS for this outage is $zero$, as equal as the result obtained using the complete model of the IEEE-RTS. Finally, and regardless of the PNS value, the important issue is that there is a wrong result, due to a no-necessary load shedding when the Ward model is used in the N-1 assessment.

3.2 The N-2 test case

In this section, and in similar form to the N-1 test case, only the outages of components of the 138 kV level (Internal System) have been studied, and the outages of components of the 230 kV level (External System) have been neglected. Also, a power-base of 100 MVA and a thermal limit of 166 MVA for the 138 kV lines have been applied in the simulations.

Respect to the N-2 analysis focussed to look for overloads, the branches from BUS03 to BUS09 and from BUS02 to BUS06, both in the 138 kV sub-transmission system (Figure 5), are considered in simultaneous outage. Firstly, the N-2 analysis of the 138 kV system has been carried out using the complete model of the IEEE-RTS, and for the post-outage state, an AC and a DC load flow have been applied to this system. The obtained results for these AC and DC load flows are those that are used as reference. And, secondly, the N-2 analysis of the 138 kV system is presented in two ways, and in both cases in a comparative study in relation to the obtained results when the DC load flow is used: the first corresponds to the analysis applying the Ward External Equivalent, and the second to the analysis applying the proposed Sensitivity-based model.

When the complete IEEE-RTS model (Figure 5) is used in the N-2 analysis, the branch power flows of the 138 kV level are shown in Table 5. The fourth column of the table shows the percentage error of using DC versus AC load flow and relative to the thermal limit of the branches (1.66 pu). The errors maximum, minimum and medium using the DC instead of the AC load flow are, respectively, 7.08 %, 0.01 % and 3.01 %. The DC load flow, third column in Table 5, is the reference for the comparative of the simulations performed with the Ward and the proposed external model.

Line From To	AC LF Post Outage (pu)	DC LF Post Outage (pu)	DC vs AC Thermal Error (%)
01-02	0.0139	0.0739	3.61
01-03	-0.4060	-0.5235	7.08
01-05	0.6473	0.7049	3.47
02-04	0.3372	0.3975	3.63
04-09	-1.0729	-1.0082	3.90
05-10	-0.7506	-0.6822	4.12
06-10	-1.7910	-1.7912	0.01
08-09	-0.5231	-0.5039	1.15
08-10	-0.1840	-0.1814	0.16

Table 5: N-2 test case. Load flow using IEEE-RTS system

Table 6 shows the results when the Ward model (Figure 6) is used in the N-2 simulation. The percentage error of using the DC load flow in the Ward model respect to use the DC load flow in the complete IEEE-RTS is showed in the fourth column of Table 6 and referred to the branch thermal limit. The errors maximum, minimum and medium are, respectively, 7.91 %, 0.02 % and 3.31 % in this case.

Line From To	Sensitivity DC Post Outage (pu)	Sensitivity DC vs IEEE-RTS DC Error (pu)	Sensitivity DC vs IEEE-RTS DC Thermal Error (%)
01-02	0.0450	0.0289	1.74
01-03	-0.4553	0.0682	4.11
01-05	0.6655	0.0394	2.37
02-04	0.3683	0.0292	1.76
04-09	-1.0374	0.0292	1.76
05-10	-0.7216	0.0394	2.37
06-10	-1.7909	0.0003	0.02
08-09	-0.6353	0.1314	7.91
08-10	-0.3108	0.1294	7.79

Table 6: N-2 test case. Load flow using the Ward External Equivalent

When the Sensitivity-based External Equivalent is applied, the power flows in the 138 kV branches (IS) are shown in Table 7. Fictitious injections at both ends of the branches from BUS03 to BUS09 and from BUS02 to BUS06, that are in outage state, have been also included in this simulation. Errors maximum, minimum and medium are, respectively, 4.37 %, 0.92 % and 2.42 %, Table 7.

As equal as in the N-1 test case, note that, the results presented in the N-2 analysis are more accurate with the proposed model than with the well-known Ward model.

Line From To	Sensitivity DC Post Outage (pu)	Sensitivity DC vs IEEE-RTS DC Error (pu)	Sensitivity DC vs IEEE-RTS DC Thermal Error (%)
01-02	0.0999	0.0260	1.56
01-03	-0.5960	0.0725	4.37
01-05	0.7514	0.0465	2.80
02-04	0.4526	0.0551	3.32
04-09	-0.9532	0.0551	3.32
05-10	-0.6357	0.0465	2.80
06-10	-1.8203	0.0291	1.75
08-09	-0.4886	0.0153	0.92
08-10	-0.1967	0.0153	0.92

Table 7: N-2 test case. Load flow using the Sensitivity-based model

The results of the state enumeration process till a N-2 contingency level are presented and discussed in the next. This process considers the possibility of simultaneous outage of two elements of the 138 kV level and determines, for each N-2 state, the PNS, LOLP and EPNS reliability indices. Similarly to the exposed in the previous section, the reliability indices of the 138 kV sub-transmission system have been calculated using the complete model of the IEEE-RTS, and these indices have been used as reference. The Ward and proposed Sensitivity-based models are subsequently analyzed and their respective reliability indices are also obtained. Finally, errors associated to the calculated reliability indices have been determined as a percentage of the results produced using the complete model of the IEEE-RTS.

In the N-2 process, the total outages considered are 741 and combined with the ten nodes of the 138 kV area of the IEEE-RTS, the result is: 7410 cases to analyze. For simplicity, the N-2 comparative analysis only involves PNS and EPNS indices, but similar conclusions are also applicable to others indices. The percentage error of PNS results have been classified in intervals (Table 8). The rows named W and S in Table 8 correspond to the Ward and the Sensitivity-based model, respectively. Fixing as reference the results obtained (column 3 Table 5) when the complete model of IEEE-RTS is used in the analysis, then the PNS-Error% have been obtained in percentage format respect to this reference.

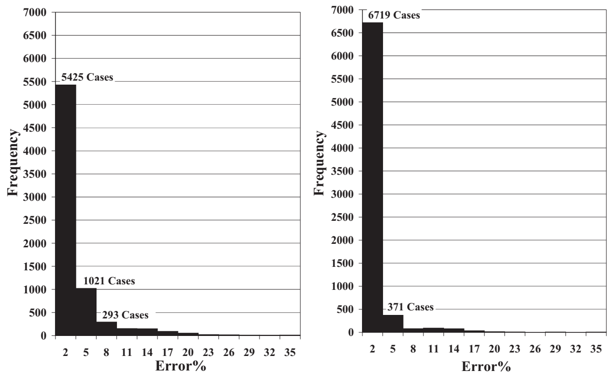
%	≤ 5	≤ 15	≤ 25	≤ 35	≤ 45	≤ 55	≤ 65	≤ 75	≤ 85	≤ 95	≤ 100
W	6446	634	129	41	12	1	1	1	1	2	142
S	7090	281	22	17	0	0	0	0	0	0	0

Table 8: Frequency of PNS Error% for the N-2 test reliability analysis

Respect to the results obtained for the Sensitivity-based external model, row S of Table 8, in the case of the interval $0\% <$ Error % $\leq 5\%$, there are cases 7090 cases of PNS-Error%. And, in others word, if the proposed external model is used in a N-2 reliability analysis of the internal system (138 kV level) of the IEEE-RTS, then there are 7090 cases over 7410 cases in which the PNS is different at most a 5% in relation to the value of the PNS index calculated when the complete model of IEEE-RTS is used in the reliability analysis.

If the results obtained for the Ward external model are analyzed, in the $95\% <$ Error % $\leq 100\%$ interval there are 142 cases (row W Table 8). This means that 142 times over 7410 are assigned wrong load curtailments. As a show of these 142 cases, one is fixed to the simultaneous failure of the branches from BUS03 to BUS09 and from BUS04 to BUS09. This no-necessary load-shedding implies a PNS of 0.2408 pu, a LOLP of $0.4208 \cdot 10^{-6}$ pu, and a EPNS of $0.1013 \cdot 10^{-6}$ pu for this system state. But, actually, the important issue is that there are 142 cases of PNS-Error% whose carry out a load shedding in the Ward External Equivalent that is not carry out in the reference system (complete IEEE-RTS).

In bar plots, the PNS-Error% Frequency (number of cases) is also show in Figure 8(a) and Figure 8(b) for the Ward and proposed external models, respectively. These plots only show the intervals from 0 % to 35 %, because errors, using the proposed external model, are within the mentioned intervals.



(a) Ward (b) Sensib-Based

Figure 8: PNS percentage Error for N-2

As it has been exposed, on the one hand, it is clear that the Ward External model produces several load curtailments that are no necessary in relation to a N-2 reliability assessment with the complete model of the IEEE-RTS. And, on the other hand, the proposed Sensitivity External model produces accurate and near results to the obtained with the model used as reference.

4 CONCLUSIONS

A study focused on a new Sensitivity-based External Equivalent and a analysis on its influence on the reliability assessment of sub-transmission networks is presented in this paper. The selection of the external equivalent model is based on the matrix of sensitivities between branch power flows and injected powers in a power system, and it is proposed in order to properly model the interconnection between the internal networks and the rest of the system.

The IEEE-RTS has been applied in the simulations and test results show that the proposed methodology is amenable to track the changes of operating-point data through boundary matching, since network data changes rather infrequently as compared to operating-point data. A comparative study in relation to the well-known Ward external model is also presented, and this comparative gives that the proposed external model carried out results more accurate and near to the real results than the obtained results when the Ward model is used. Besides, in the case of the Ward model, it must be consider that the operator center of the internal system is only making the management of the internal measurements and unknown the external system operating-point, and this is a disadvantages for the Ward model because in this situation is an unobservable system, by difference the Sensitivity-based external model only needs the management measurements of the injected powers in the internal system.

Finally, the work presented in this paper also leads to the conclusion that the proposed external equivalent is flexible and easy to implement in regard to provide detailed information appropriate to both planning and 24 hour-ahead analysis, and to perform detailed adequate reliability assessment of a selected area of interest.

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