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CHARACTERIZATION OF COMPLEX FILIFORM LIE ALGEBRAS OF DIMENSION 8 ACCORDING TO WHETHER THEY ARE OR NOT DERIVED FROM OTHERS

By

F.J. ECHARTE REULA[1], J.R. GOMEZ MARTIN[2] y J. NUÑEZ VALDES[1].

[1] Universidad de Sevilla. Facultad de Matemáticas. Dpto de Algebra, Computación, Geometría y Topología. C/ Tarfia s/n. 41012 Sevilla. España.

[2] Universidad de Sevilla. Facultad de Informática y Estadística. Dpto de Matemática Aplicada. C/ Tarfia s/n. 41012 Sevilla, España.

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Abstract.- In this paper, we characterize those Complex Filiform Lie Algebras of dimension 8 which are derived from other Solvable Lie Algebras of higher dimension. This result and the previous one given in ([8]) allow us to find a complete list of Characteristically Nilpotent Filiform Lie Algebras of dimension 8.

1.- Introduction and Notations.

There does not exist, at present, any classification of complex Nilpotent Lie Algebras (NLA) of dimension greater than 7. Goze and Ancochea, by the introduction of a new invariant which they call the "characteristic sequence", which corresponds to the maximal dimensions of Jordan blocks of a certain nilpotent matrix, obtained the classification of complex Filiform Lie Algebras (FLA) of dimension 8 ([1]). These FLA, as it is known, are a subset of NLA.

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However, the importance of these authors' work is not only to have obtained this classification, but, above all, to have devised techniques which can be applied in any dimension, which, apart from the fact of having permitted Gómez Martín to classify FLA of dimension 9 ([5]), totally solved the problem of this classification of FLA of greater dimension, although, of course, this requires hard and complicated calculations which are impossible without the use of a computer.

Therefore, although the classification of Lie Algebras (LA) of dimension greater than 7 has not been obtained yet, except for FLA, the main problem which is now considered is the search for new results, rather than the classification itself, such as the attainment of new invariants or the description of the irreducible components of varieties of LA. For this purpose, Valeiras proved recently that the variety of NLA of dimension 8 has 8 irreducible components and that only the two first of them meet the open set of FLA ([9]).

The main purpose of this paper is, therefore, starting from the classification of FLA of dimension 8 by Goze and Ancochea, to characterize two groups of these LA, according to whether or not they are derived from other Solvable Lie Algebras (SLA) of higher dimension. This characterization we propose could be interesting in the sense of knowing which of these FLA are also Characteristically Nilpotent Lie Algebras (CNLA) since, as we previously proved in an earlier paper ([8]), if a FLA is not derived from any LA then it is a CNLA. This theorem will allow us to write the list of Characteristically Nilpotent Filiform Lie Algebras (CNFLA) of dimension 8.

A complex FLA may be derived from an SLA of higher dimension. In ([8]) we proved the following:

(1.1) Theorem: A complex FLA of dimension $\underline{n}$ is either derived from a SLA of dimension $\underline{n+1}$ or not derived from any LA.

From now on, we write $(A,B,C) = 0$ to represent the Jacobi identity

$$[[A,B],C] + [[B,C],A] + [[C,A],B] = 0$$

and we denote by $\mathfrak{M}$ a FLA of dimension $\underline{n}$ ([8]), that is, a complex NLA admitting a basis

(1) $$\{X_1, X_2, \ldots, X_n\}$$

such that $X_1 \notin [\mathfrak{M}, \mathfrak{M}]$,

(2) $$[X_1, X_2] = 0$$

and

(3) $[X_1, X_j] = X_{j-1} \quad 3 \le j \le n$

Moreover, since $[X_8, X_j] = 0 \quad 1 < j < n$ and $[X_8, X_n] = a \; X_2$ ($a \in C$), if we consider the change of basis given by

$$X_j' = X_j \quad (1 \le j \le n-1) \quad ; \quad X_n' = X_n + a\; X_1$$

then it results that $[X_8', X_j'] = 0$ $(1 < j \le n)$. So, a basis (1) can be always chosen in a suitable way verifying

(4) $[X_8, X_j] = 0 \quad 1 < j \le n$

Moreover, $\mathfrak{L}$ will denote a complex SLA of dimension n+1 such that

(5) $\{X_1, .., X_n, U\}$

is a basis, where $\{X_1, X_2, ..., X_n\}$ is the basis (1) mentioned above, and U a derivation of $\mathfrak{M}$ such that

(6) $[X_j, U] = \sum_{h=1}^{n} a_{jh} \; X_h$

with $(X_j, X_h, U) = 0, \quad 1 \le j,h \le n.$

The FLA $\mathfrak{M}$ is said to be derived from the SLA $\mathfrak{L}$ if $[\mathfrak{L},\mathfrak{L}] \equiv \mathfrak{M}$, where $[\mathfrak{L},\mathfrak{L}]$ represents any linear combination of all brackets among fields of the basis (1).

Characteristically Nilpotent Lie Algebras are those LA in which all their derivations are nilpotent. The first examples of CNLA given in the literature were of LA that are not derived from any LA, until Luks gave an example of a CNLA which is derived from a LA of dimension 18 ([7]). However, it is very easy to give examples of Nilpotent Lie Algebras (NLA) which are derived and not CNLA.

As we mentioned above, in ([8]) the following two theorems are proved:

(1.2) Theorem A necessary and sufficient condition for a FLA $\mathfrak{M}$ to be derived from the SLA $\mathfrak{L}$ is that $a_{11} \neq 0$.

(1.3) Theorem A complex FLA $\mathfrak{M}$ is a CNLA if and only if $\mathfrak{M}$ is not derived from any LA.

So, to separate FLA of dimension 8 into two groups according to whether or not they are derived from another SLA (Theorem (1.1)) we will use a method previously indicated by us for the case of NLA of dimension 7 ([4]). This proceedure, which we called the "method of determination of the vanishing of the coefficient a_{11}", can be applied to FLA. It consists of a set of iterative calculations based on all possible Jacobi identities of the kind $(X_i, X_j, U) = 0$ with $1 \leq i,j \leq 8$, from which we can determine whether the coefficient a_{11} vanishes or not. This in turn determines, by theorem (1.2), whether the FLA $\mathfrak{M}$ is derived from the SLA $\mathfrak{L}$ with $\{X_1,..,X_8,U\}$ as a basis.

These iterative steps are the following:

1.- Using the Jacobi identity $(X_1, X_{h_1}, U) = 0$ with $h_1 > 1$, deriving from such identity the set of equations obtained by setting to zero the coefficient of each field X_j.

2.- Using each equation obtained in this way to express in terms of the others the coefficient a_{ij} whose pair of subindices is the greater (in lexicographic order). (For instance, from the equation $a_{22} - a_{11} - a_{44} - a_{17} = 0$ we will have $a_{44} = a_{22} - a_{11} - a_{17}$).

3.- Replacing in (6) the coefficients a_{ij} with their corresponding values obtained previously in step 2.

4.- Repeating the former three steps with U, X_2 and X_{h_2} with $h_2 > 2$ in the first place. Secondly, with U, X_3 and X_{h_3} with $h_3 > 3$ and so on, until terminating this proceeding with U, X_{n-1} and X_n.

5.- Finally, observing if in the new expressions (6) obtained in this way the coefficient a_{11} appears explicity or not; this will indicate, according to theorem (1.2) whether the FLA $\mathfrak{M}$ which we are studying is derived from another SLA $\mathfrak{L}$ of dimension one more than the dimension of $\mathfrak{M}$.

In the case that the FLA $\mathfrak{M}$ is derived from another SLA of dimension one greater than the dimension of $\mathfrak{M}$, we can obtain the simplest SLA $\mathfrak{L}$ by taking $a_{11} = 1$ and the rest of the coefficients $a_{ij} = 0$ $(i,j \neq 1)$. Moreover, we can check that in FLA of dimension ≤ 9, either directly or by suitable changes of base, it is always possible to get

(7) $$[X_h, U] = a_h X_h \text{ with } h = 1, 2, \ldots, n$$

(although this is not proved in the general case of dimension $\mathfrak{M} = n$).

2.- <u>Division of complex FLA of dimension 8 into two groups depending on whether they are or not derived from other LA.</u>

We separate FLA of dimension 8 into two groups depending on whether they are derived from another SLA, starting from the classification of these FLA by Goze and Ancochea ([1]). This classification (which is indicated in an Appendix at the end of this paper) contains a list of 13 isolated FLA and 7 one-parameter families of FLA.

By the method mentioned above, we find that 6 of the 13 FLA and 2 of the 7 families are <u>derived</u> from SLA of dimension 8 whereas the other 7 FLA and the 5 families <u>are not</u>.

So we obtain the following

<u>Theorem 1</u>.- The complex FLA of dimension 8 with laws μ_8^1, μ_8^2, μ_8^3, μ_8^4, $\mu_8^{6,\alpha}$, μ_8^8, $\mu_8^{9,\alpha}$, $\mu_8^{10,\alpha}$, μ_8^{11}, $\mu_8^{13,\alpha}$, $\mu_8^{15,\alpha}$ and μ_8^{17} <u>are not derived</u> from any other LA.

Proof:

By appliying the method mentioned above to each of them we obtain the following results:

(2.1) The FLA of dimension 8 with law μ_8^1 is not derived from any LA, since:

$$
\begin{aligned}
[X_1,U] &= a_{12} X_2 + a_{13} X_3 + a_{14} X_4 + a_{15} X_5 + a_{16} X_6 + a_{17} X_7 \\
[X_2,U] &= 0 \\
[X_3,U] &= a_{32} X_2 \\
[X_4,U] &= (a_{16} + a_{17}) X_2 + (a_{32} + a_{17}) X_3 \\
[X_5,U] &= a_{52} X_2 + 3/5\, a_{17} X_3 + (a_{32} + a_{17}) X_4 \\
[X_6,U] &= (3/5\, a_{52} + a_{14} + 3/5\, a_{15} + 6/25\, a_{16}) X_2 + \\
&\quad + (a_{52} + a_{15}) X_3 + 1/5\, a_{17} X_4 + (a_{32} + a_{17}) X_5 \\
[X_7,U] &= a_{72} X_2 + (3/5\, a_{52} + a_{15} + 6/25\, a_{16}) X_3 + \\
&\quad + (a_{52} + a_{15} + 2/5\, a_{16}) X_4 + 1/5\, a_{17} X_5 + (a_{32} + a_{17}) X_6 \\
[X_8,U] &= a_{82} X_2 + (a_{72} - a_{14}) X_3 + (3/5\, a_{52} - a_{14} + 2/5\, a_{15} + 6/25\, a_{16}) X_4 + \\
&\quad + (a_{52} + 1/5\, a_{16}) X_5 - a_{16} X_6 + a_{32} X_7
\end{aligned}
$$

(2.2) The FLA of dimension 8 with law μ_8^2 is not derived from any LA, since:

$[X_1,U] = a_{12} X_2 + a_{13} X_3 + a_{14} X_4 + a_{15} X_5 + a_{16} X_6 + a_{17} X_7$
$[X_2,U] = 0$
$[X_3,U] = a_{32} X_2$
$[X_4,U] = a_{16} X_2 + (a_{32} + a_{17}) X_3$
$[X_5,U] = a_{52} X_2 + (a_{32} + a_{17}) X_4$
$[X_6,U] = (a_{17} + a_{16} + a_{14}) X_2 + (a_{52} + a_{17} + a_{15}) X_3 + (a_{32} + a_{17}) X_5$
$[X_7,U] = a_{72} X_2 + a_{17} X_3 + (a_{52} + a_{15} + a_{17}) X_4 + (a_{32} + a_{17}) X_6$
$[X_8,U] = a_{32} X_2 + (a_{72} - a_{16}) X_3 - (a_{16} - a_{14}) X_4 + a_{52} X_5 - a_{16} X_6 + a_{32} X_7$

(2.3) The FLA of dimension 8 with law μ_8^3 is not derived from any LA, since:

$[X_1,U] = a_{12} X_2 + a_{13} X_3 + a_{14} X_4 + a_{15} X_5 + a_{16} X_6 + a_{17} X_7$
$[X_2,U] = 0$
$[X_3,U] = a_{32} X_2$
$[X_4,U] = a_{16} X_2 + (a_{32} + a_{17}) X_3$
$[X_5,U] = a_{52} X_2 + (a_{32} + a_{17}) X_4$
$[X_6,U] = (a_{17} + a_{14}) X_2 + (a_{52} + a_{15}) X_3 + (a_{32} + a_{17}) X_5$
$[X_7,U] = a_{72} X_2 + a_{17} X_3 + (a_{52} + a_{15}) X_4 + (a_{32} + a_{17}) X_6$
$[X_8,U] = a_{32} X_2 + (a_{72} - a_{16}) X_3 - a_{14} X_4 + a_{52} X_5 - a_{16} X_6 + a_{32} X_7$

(2.4) The FLA of dimension 8 with law μ_8^4 is not derived from any LA, since:

$[X_1,U] = a_{12} X_2 + a_{13} X_3 + a_{14} X_4 + a_{15} X_5 + a_{16} X_6 + a_{17} X_7$
$[X_2,U] = 0$
$[X_3,U] = a_{32} X_2$
$[X_4,U] = a_{16} X_2 + (a_{32} + a_{17}) X_3$
$[X_5,U] = a_{52} X_2 + (a_{32} + a_{17}) X_4$
$[X_6,U] = (a_{16} + a_{14}) X_2 + (a_{52} + a_{15} + a_{17}) X_3 + (a_{32} + a_{17}) X_5$
$[X_7,U] = a_{72} X_2 + (a_{52} + a_{15} + a_{17}) X_4 + (a_{32} + a_{17}) X_6$
$[X_8,U] = a_{32} X_2 + a_{72} X_3 - (a_{14} + a_{16}) X_4 + a_{52} X_5 - a_{16} X_6 + a_{32} X_7$

(2.5) The FLA of dimension 8 with law $\mu_8^{6,\alpha}$ is not derived from any LA, since:

$[X_1,U] = a_{12} X_2 + a_{13} X_3 + a_{14} X_4 + a_{15} X_5 + a_{16} X_6 + a_{17} X_7$
$[X_2,U] = 0$
$[X_3,U] = a_{32} X_2$
$[X_4,U] = \alpha\, a_{17} X_2 + a_{32} X_3$
$[X_5,U] = (a_{16} + a_{17}) X_2 + (1 + \alpha)\, a_{17} X_3 + a_{32} X_4$
$[X_6,U] = a_{62} X_2 + (a_{16} + a_{17}) X_3 + (2 + \alpha)\, a_{17} X_4 + a_{32} X_5$

$$[X_7,U] = a_{72} X_2 + (a_{62} - a_{15}) X_3 + a_{17} X_4 + (2+\alpha)\, a_{17} X_5 + a_{32} X_6$$

$$[X_8,U] = a_{82} X_2 + (a_{72} - \alpha\, a_{14} - a_{15}) X_3 + (a_{62} - (2+\alpha)\, a_{15} - a_{16}) X_4 -$$
$$- (2+\alpha)\, a_{16} X_5 + a_{32} X_7$$

(2.6) The FLA of dimension 8 with law μ_8^3 is not derived from any LA, since:

$$[X_1,U] = a_{12} X_2 + a_{13} X_3 + a_{14} X_4 + a_{15} X_5 + a_{16} X_6 + a_{17} X_7$$
$$[X_2,U] = 0$$
$$[X_3,U] = a_{32} X_2$$
$$[X_4,U] = - a_{17} X_2 + a_{32} X_3$$
$$[X_5,U] = a_{16} X_2 + a_{32} X_4$$
$$[X_6,U] = a_{62} X_2 + (a_{16} + a_{17}) X_3 + a_{17} X_4 + a_{32} X_5$$
$$[X_7,U] = a_{72} X_2 + (a_{62} - a_{15} - a_{16}) X_3 + a_{17} X_4 + a_{17} X_5 + a_{32} X_6$$
$$[X_8,U] = a_{82} X_2 + (a_{72} + a_{14}) X_3 + (a_{62} - a_{15} - 2\, a_{16}) X_4 - a_{16} X_5 + a_{32} X_7$$

(2.7) The FLA of dimension 8 with law $\mu_8^{9,\alpha}$ is not derived from any LA, since:

$$[X_1,U] = a_{12} X_2 + a_{13} X_3 + a_{14} X_4 + a_{15} X_5 + a_{16} X_6 + a_{17} X_7$$
$$[X_2,U] = 0$$
$$[X_3,U] = a_{32} X_2$$
$$[X_4,U] = a_{17} X_2 + a_{32} X_3$$
$$[X_5,U] = a_{52} X_2 + a_{17} X_3 + a_{32} X_4$$
$$[X_6,U] = a_{62} X_2 + (a_{52} + a_{17}) X_3 + a_{17} X_4 + a_{32} X_5$$
$$[X_7,U] = a_{72} X_2 + (a_{62} - a_{16}) X_3 + (a_{52} + a_{17}) X_4 + a_{17} X_5 + a_{32} X_6$$
$$[X_8,U] = a_{82} X_2 + (a_{72} - a_{14} - \alpha\, a_{16}) X_3 + (a_{62} - a_{15} - 2\, a_{16} - \alpha\, a_{17}) X_4 +$$
$$+ (a_{52} - a_{16}) X_5 + a_{32} X_7$$

(2.8) The FLA of dimension 8 with law $\mu_8^{10,\alpha}$ is not derived from any LA, since:

$$[X_1,U] = a_{12} X_2 + a_{13} X_3 + a_{14} X_4 + a_{15} X_5 + a_{16} X_6 + a_{17} X_7$$
$$[X_2,U] = 0$$
$$[X_3,U] = a_{32} X_2$$
$$[X_4,U] = a_{42} X_2 + a_{32} X_3$$
$$[X_5,U] = a_{52} X_2 + a_{42} X_3 + a_{32} X_4$$
$$[X_6,U] = a_{62} X_2 + a_{52} X_3 + a_{42} X_4 + a_{32} X_5$$
$$[X_7,U] = a_{72} X_2 + a_{62} X_3 + a_{52} X_4 + a_{42} X_5 + a_{32} X_6$$
$$[X_8,U] = a_{82} X_2 + (a_{72} - \alpha\, a_{17} - a_{16} - a_{14}) X_3 + (a_{62} - a_{17} - a_{15}) X_4 +$$
$$+ (a_{52} - a_{16}) X_5 + (a_{42} - a_{17}) X_6 + a_{32} X_7$$

(2.9) The FLA of dimension 8 with law μ_8^{11} is not derived from any LA, since:

$$[X_1,U] = a_{12} X_2 + a_{13} X_3 + a_{14} X_4 + a_{15} X_5 + a_{16} X_6 + a_{17} X_7$$
$$[X_2,U] = 0$$
$$[X_3,U] = a_{32} X_2$$
$$[X_4,U] = a_{42} X_2 + a_{32} X_3$$
$$[X_5,U] = a_{52} X_2 + a_{42} X_3 + a_{32} X_4$$
$$[X_6,U] = a_{62} X_2 + a_{52} X_3 + a_{42} X_4 + a_{32} X_5$$
$$[X_7,U] = a_{72} X_2 + a_{62} X_3 + a_{52} X_4 + a_{42} X_5 + a_{32} X_6$$
$$[X_8,U] = a_{82} X_2 + (a_{72} - a_{17} - a_{14}) X_3 + (a_{62} - a_{15}) X_4 + (a_{52} - a_{16}) X_5 + + (a_{42} - a_{17}) X_6 + a_{32} X_7$$

(2.10) The FLA of dimension 8 with law $\mu_8^{13,\alpha}$ is not derived from any LA, since:

$$[X_1,U] = a_{12} X_2 + a_{13} X_3 + a_{14} X_4 + a_{15} X_5 + a_{16} X_6 + a_{17} X_7 + a_{18} X_8$$
$$[X_2,U] = 0$$
$$[X_3,U] = a_{32} X_2$$
$$[X_4,U] = a_{42} X_2 + a_{32} X_3$$
$$[X_5,U] = a_{52} X_2 + (a_{42} + \alpha a_{18}) X_3 + a_{32} X_4$$
$$[X_6,U] = a_{62} X_2 + (a_{52} + a_{17} + a_{18}) X_3 + (a_{42} + (1 + 2\alpha) a_{18}) X_4 + a_{32} X_5$$
$$[X_7,U] = a_{72} X_2 + (a_{62} - a_{16}) X_3 + (a_{52} + a_{17} + 2 a_{18}) X_4 + + (a_{42} + (2+3\alpha) a_{18}) X_5 + a_{32} X_6$$
$$[X_8,U] = a_{82} X_2 + (a_{72} - \alpha a_{15} - a_{16}) X_3 + (a_{62} - (2+\alpha) a_{16} - a_{17}) X_4 + + (a_{52} + 2 a_{18} - \alpha a_{17}) X_5 + (a_{42} + (2+ 3\alpha) a_{18}) X_6 + a_{32} X_7$$

NOTE: In this algebra it is verified that

$$a_{42} = 3/2 (\alpha - 1) (\alpha - 2/3)$$

(2.11) The FLA of dimension 8 with law $\mu_8^{15,\alpha}$ is not derived from any LA, since:

$$[X_1,U] = a_{12} X_2 + a_{13} X_3 + a_{14} X_4 + a_{15} X_5 + a_{16} X_6 + a_{17} X_7$$
$$[X_2,U] = 0$$
$$[X_3,U] = a_{32} X_2$$
$$[X_4,U] = a_{42} X_2 + a_{32} X_3$$
$$[X_5,U] = a_{52} X_2 + a_{42} X_3 + a_{32} X_4$$
$$[X_6,U] = a_{62} X_2 + a_{52} X_3 + a_{42} X_4 + a_{32} X_5$$
$$[X_7,U] = a_{72} X_2 + a_{62} X_3 + a_{52} X_4 + a_{42} X_5 + a_{32} X_6$$
$$[X_8,U] = a_{82} X_2 + (a_{72} - \alpha a_{17} - a_{16} - a_{15}) X_3 + (a_{62} - a_{17} - a_{16}) X_4 + + a_{52} X_5 + a_{42} X_6 + a_{32} X_7$$

(2.12) The FLA of dimension 8 with law μ_8^{17} is not derived from any LA, since:

$[X_1,U] = a_{12}X_2 + a_{13}X_3 + a_{14}X_4 + a_{15}X_5 + a_{16}X_6 + a_{17}X_7 + a_{18}X_8$
$[X_2,U] = 0$
$[X_3,U] = a_{32}X_2$
$[X_4,U] = a_{42}X_2 + a_{32}X_3$
$[X_5,U] = a_{52}X_2 + a_{42}X_3 + a_{32}X_4$
$[X_6,U] = a_{62}X_2 + (a_{52} + a_{13})X_3 + a_{42}X_4 + a_{32}X_5$
$[X_7,U] = a_{72}X_2 + (a_{62} + a_{13})X_3 + (a_{52} + 2a_{13})X_4 + a_{42}X_5 + a_{32}X_6$
$[X_8,U] = a_{82}X_2 + (a_{72} - a_{17} - a_{16})X_3 + (a_{62} + a_{13} - a_{17})X_4 +$
$(a_{52} + 2a_{13})X_5 + a_{42}X_6 + a_{32}X_7$ ■

<u>Corollary</u> <u>1</u>.- There exist exactly 12 CNFLA of dimension 8. These are the following: μ_8^1, μ_8^2, μ_8^3, μ_8^4, $\mu_8^{6,\alpha}$, μ_8^8, $\mu_8^{9,\alpha}$, $\mu_8^{10,\alpha}$, μ_8^{11}, $\mu_8^{13,\alpha}$, $\mu_8^{15,\alpha}$ and μ_8^{17}.

Proof:

It is an immediate consequence of Theorems 1 and (1.3).

■

<u>Theorem</u> <u>2</u>.- The complex FLA of dimension 8 with laws μ_8^5, $\mu_8^{7,\alpha}$, μ_8^{12}, $\mu_8^{14,\alpha}$, μ_8^{16}, μ_8^{18}, μ_8^{19} and μ_8^{20} <u>are derived</u> from SLA of dimension 9.

Proof:

By applying the method mentioned above to each of them we find that each of them is respectively <u>derived</u> from a SLA of dimension 9, with basis $\{X_1, ..., X_8, U\}$, verifying $[X_i, U] = a_i X_i$ $(1 \le i \le 8)$ where the coefficients a_i for each of them are the following:

<u>FLA</u>	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	obs.
μ_8^5	1	7	6	5	4	3	2	1	
$\mu_8^{7,\alpha}$	1	8	7	6	5	4	3	2	
μ_8^{12}	1	8	7	6	5	4	3	2	
$\mu_8^{14,\alpha}$	1	9	8	7	6	5	4	3	
μ_8^{16}	3	27	24	21	18	15	12	9	(*)
μ_8^{18}	1	10	9	8	7	6	5	4	
μ_8^{19}	1	11	10	9	8	7	6	5	
μ_8^{20}	1	8	7	6	5	4	3	2	

(*) In fact, if we apply the method mentioned above to this FLA we do not obtain these coefficients directly, but instead obtain the following expressions:

$$[X_1,U] = 3X_1 + 2X_3 \qquad [X_2,U] = 27X_2$$
$$[X_3,U] = 24X_3 \qquad [X_4,U] = 21X_4$$
$$[X_5,U] = 2X_3 + 18X_5 \qquad [X_6,U] = 4X_4 + 15X_6$$
$$[X_7,U] = 2X_3 + 6X_5 + 12X_7 \qquad [X_8,U] = 2X_4 + 6X_6 + 9X_8$$

Starting from these and by a suitable procedure we can obtain new fields Y_i, deduced from fields X_i, such that $[Y_i,U] = K_i Y_i$ (with $K_i \in C$); in fact, if we call respectively

$$Y_1 = 6X_1 - 2X_3 + X_6 \quad ; \quad Y_5 = X_3 - 3X_5$$
$$Y_6 = 2X_4 - 3X_6 \quad ; \quad Y_7 = X_5 - X_7$$
$$Y_8 = X_4 - 6X_6 + 6X_8$$

we obtain:

$$[Y_1,U] = 3Y_1 \qquad [Y_5,U] = 18Y_5 \qquad [Y_6,U] = 15Y_6$$
$$[Y_7,U] = 12Y_7 \qquad [Y_8,U] = 9Y_8$$

■

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APPENDIX

Algebras de Lie Filiformes Complejas de dimensión 8

μ_8^1 : $[X_1,X_i] = X_{i-1}$ $3 \le i \le 8$
$[X_4,X_7] = X_2$
$[X_4,X_8] = X_2 + X_3$
$[X_5,X_6] = - X_2$
$[X_5,X_7] = - (2/5)\ X_2$
$[X_5,X_8] = X_4 + (3/5)\ X_3$
$[X_6,X_7] = - (2/5)\ X_3$
$[X_6,X_8] = X_5 + (1/5)\ X_4$
$[X_7,X_8] = X_6 + (1/5)\ X_5$

μ_8^2 : $[X_1,X_i] = X_{i-1}$ $3 \le i \le 8$
$[X_4,X_7] = X_2$
$[X_4,X_8] = X_3$
$[X_5,X_6] = - X_2$
$[X_5,X_8] = X_4$
$[X_6,X_7] = X_2$
$[X_6,X_8] = X_2 + X_3 + X_5$
$[X_7,X_8] = X_3 + X_4 + X_6$

μ_8^3 : $[X_1,X_i] = X_{i-1}$ $3 \le i \le 8$
$[X_4,X_7] = X_2$
$[X_4,X_8] = X_3$
$[X_5,X_6] = - X_2$
$[X_5,X_8] = X_4$
$[X_6,X_8] = X_2 + X_5$
$[X_7,X_8] = X_3 + X_6$

$$\mu_8^4 : \quad [X_1,X_i] = X_{i-1} \qquad 3 \le i \le 8$$
$$[X_4,X_7] = X_2$$
$$[X_4,X_8] = X_3$$
$$[X_5,X_6] = - X_2$$
$$[X_5,X_8] = X_4$$
$$[X_6,X_7] = X_2$$
$$[X_6,X_8] = X_3 + X_5$$
$$[X_7,X_8] = X_4 + X_6$$

$$\mu_8^5 : \quad [X_1,X_i] = X_{i-1} \qquad 3 \le i \le 8$$
$$[X_4,X_7] = X_2$$
$$[X_5,X_6] = - X_2$$
$$[X_i,X_8] = X_{i-1} \qquad 4 \le i \le 7$$

$$\mu_8^{6,\alpha} : \quad [X_1,X_i] = X_{i-1} \qquad 3 \le i \le 8$$
$$[X_4,X_8] = \alpha X_2$$
$$[X_5,X_7] = X_2 \qquad \alpha \in C-[-1]$$
$$[X_5,X_8] = (1+\alpha) X_3 + X_2$$
$$[X_6,X_7] = X_3$$
$$[X_6,X_8] = (2+\alpha) X_4 + X_3$$
$$[X_7,X_8] = (2+\alpha) X_5 + X_4$$

$$\mu_8^{7,\alpha} : \quad [X_1,X_i] = X_{i-1} \qquad 3 \le i \le 8$$
$$[X_4,X_8] = \alpha X_2$$
$$[X_5,X_7] = X_2$$
$$[X_5,X_8] = (1+\alpha) X_3$$
$$[X_6,X_7] = X_3 \qquad \alpha \in C$$
$$[X_6,X_8] = (2+\alpha) X_4$$
$$[X_7,X_8] = (2+\alpha) X_5$$

$$\mu_8^8 : \quad [X_1,X_i] = X_{i-1} \qquad 3 \le i \le 8$$
$$[X_4,X_8] = - X_2$$
$$[X_5,X_7] = X_2$$
$$[X_6,X_7] = X_2 + X_3$$
$$[X_6,X_8] = X_3 + X_4$$
$$[X_7,X_8] = X_4 + X_5$$

$$\mu_8^{9,\alpha} : \quad [X_1,X_i] = X_{i-1} \qquad 3 \le i \le 8$$
$$[X_4,X_8] = X_2$$
$$[X_5,X_8] = X_3$$
$$[X_6,X_7] = X_2 \qquad \alpha \in C$$
$$[X_6,X_8] = \alpha X_2 + X_3 + X_4$$
$$[X_7,X_8] = \alpha X_3 + X_4 + X_5$$

$$\mu_8^{10,\alpha} : [X_1,X_i] = X_{i-1} \quad 3 \le i \le 8$$
$$[X_4,X_8] = X_2$$
$$[X_5,X_8] = X_3 \qquad \alpha \in C$$
$$[X_6,X_8] = X_2 + X_4$$
$$[X_7,X_8] = \alpha X_2 + X_3 + X_5$$

$$\mu_8^{11} : [X_1,X_i] = X_{i-1} \quad 3 \le i \le 8$$
$$[X_i,X_8] = X_{i-2} \quad 4 \le i \le 6$$
$$[X_7,X_8] = X_2 + X_5$$

$$\mu_8^{12} : [X_1,X_i] = X_{i-1} \quad 3 \le i \le 8$$
$$[X_i,X_8] = X_{i-1} \quad 4 \le i \le 7$$

$$\mu_8^{13,\alpha} : [X_1,X_i] = X_{i-1} \quad 3 \le i \le 8$$
$$[X_5,X_8] = \alpha X_2$$
$$[X_6,X_7] = X_2 \qquad \alpha \in C$$
$$[X_6,X_8] = (1+\alpha) X_3$$
$$[X_7,X_8] = (1+\alpha) X_4 + X_3$$

$$\mu_8^{14,\alpha} : [X_1,X_i] = X_{i-1} \quad 3 \le i \le 8$$
$$[X_5,X_8] = \alpha X_2$$
$$[X_6,X_7] = X_2 \qquad \alpha \in C$$
$$[X_6,X_8] = (1+\alpha) X_3$$
$$[X_7,X_8] = (1+\alpha) X_4$$

$$\mu_8^{15,\alpha} : [X_1,X_i] = X_{i-1} \quad 3 \le i \le 8$$
$$[X_5,X_8] = X_2$$
$$[X_6,X_8] = X_2 + X_3 \qquad \alpha \in C$$
$$[X_7,X_8] = \alpha X_2 + X_3 + X_4$$

$$\mu_8^{16} : [X_1,X_i] = X_{i-1} \quad 3 \le i \le 8$$
$$[X_5,X_8] = X_2$$
$$[X_6,X_8] = X_3$$
$$[X_7,X_8] = X_2 + X_4$$

$$\mu_8^{17} : [X_1,X_i] = X_{i-1} \quad 3 \le i \le 8$$
$$[X_6,X_8] = X_2$$
$$[X_7,X_8] = X_2 + X_3$$

$$\mu_8^{18} : [X_1,X_i] = X_{i-1} \quad 3 \le i \le 8$$
$$[X_6,X_8] = X_2$$
$$[X_7,X_8] = X_3$$

$$\mu_8^{19} : [X_1,X_i] = X_{i-1} \quad 3 \le i \le 8$$
$$[X_7,X_8] = X_2$$

$$\mu_8^{20} : [X_1,X_i] = X_{i-1} \quad 3 \le i \le 8$$