



Discrete breathers collisions: An overview and some recent results

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Outline

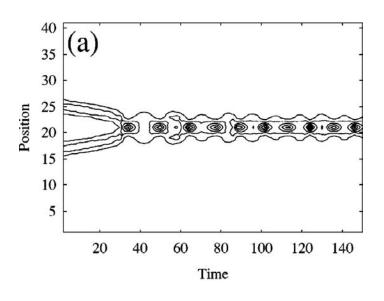
- Overview of previous studies:
 - Collisions in FPU lattices
 - Collisions in dissipative Klein–Gordon lattices
 - Collisions in DNLS lattices
- New results on DNLS lattices with saturable nonlinearity:
 - Properties of SDNLS lattices
 - Collisions in SDNLS lattices
- Future challenges

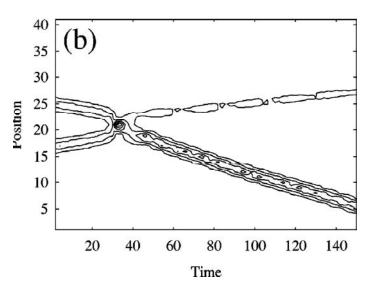
FPU lattices

- Y. Doi, PRE 68, 066608 (2003)
- β -FPU model is considered
- 6 Collisions of breathers with the same/different energies and the same/different phase
- Observed regimes:
 - Reflection
 - Fusion
 - Symmetry breaking (even if same energy and same phase)
- The energy exchange is throughoutly studied
- 6 Transferred energy may be very sensitive to phase difference

FPU lattices

Fusion and symmetry breaking regimes



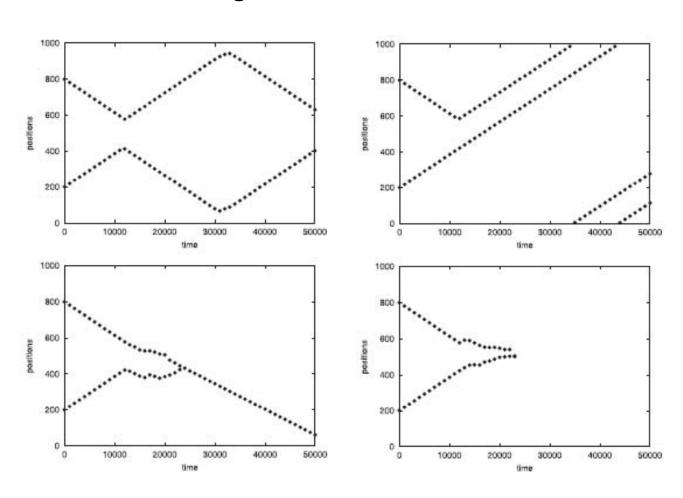


Dissipative Klein-Gordon lattices

- M. Meister & L.M. Floría, EPJB 37, 213 (2004)
- Frenkel–Kontorova model with dissipation and external force
- 6 Collisions of breathers with the same energy and phase
- Observed regimes:
 - Reflection
 - Destruction of one/both breathers
 - Bound state:
 - Pair of breathers subject to the force of the emitted phonons
 - The distance between breathers is related to the velocity
- The final states are attractors of the system

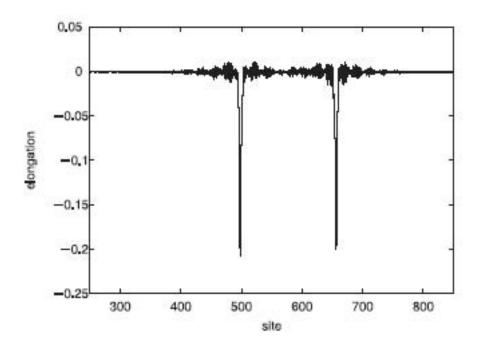
Dissipative Klein-Gordon lattices

6 Different regimes



Dissipative Klein-Gordon lattices

6 A bound state

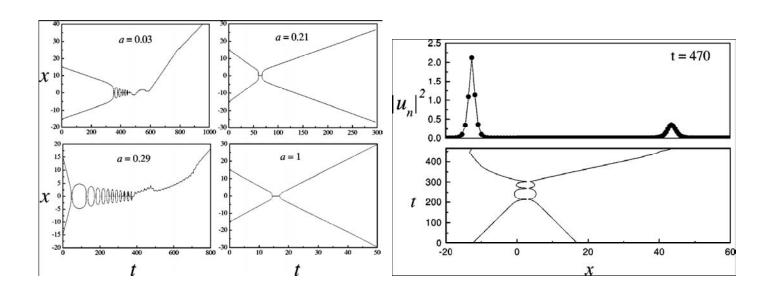


Nonintegrable DNLS lattices

- 6 I.E. Papacharalampous et al, PRE 68, 046604 (2003)
- 6 Collisions of breathers with the same energy and the same/different phase
- 6 Observed regimes (in-phase breathers):
 - Reflection
 - Bound state
 - Spontaneous symmetry breaking (due to numerical errors):
 - Bound state moves with a well-defined value of the velocity
 - Mutual bounce after multiple collisions
- 6 High Peierls-Nabarro barrier → Strong differences between on-site and inter-site collisions.

Nonintegrable DNLS lattices

6 Several regimes



DNLS lattices with saturable nonlinearity

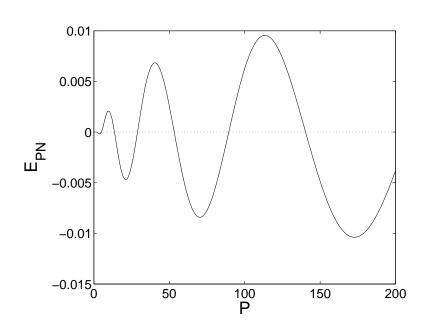
- M. Stepic et al, PRE 69, 066618 (2004)
- 6 L. Hadzievski et al, PRL 93, 033901 (2004)
- Oiscrete version of the Vinetskii-Kukhtarev equation (SDNLS equation):

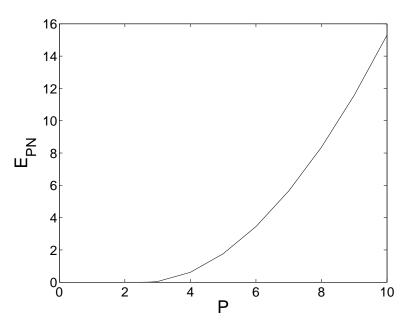
$$i\dot{u}_n - \beta \frac{u_n}{1 + |u_n|^2} + (u_{n+1} - 2u_n + u_{n-1}) = 0.$$

- Models 1D waveguide arrays of photorefractive materials \rightarrow SBN61 (Sr_{0.61}Ba_{0.39}Nb₂O₆)
- Main feature: Bounded Peierls-Nabarro barrier → High power moving breathers can be found.

DNLS lattices with saturable nonlinearity

Peierls-Nabarro barrier in 1D chains



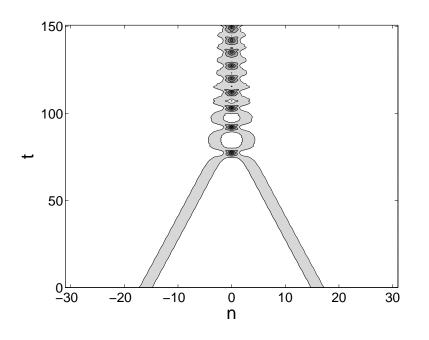


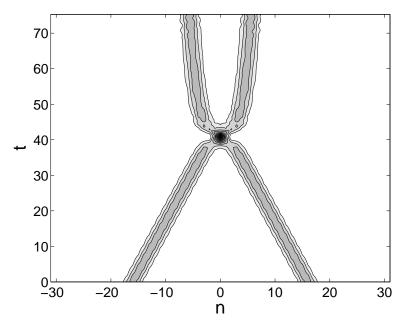
Saturable Nonlinearity

Kerr Nonlinearity

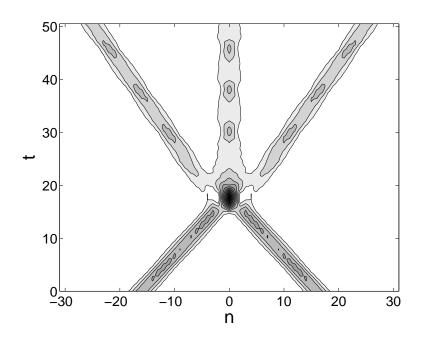
- J. Cuevas and J.C. Eilbeck, Arxiv:nlin.PS/0501050
- 6 Collisions of breathers with the same energy and phase
- Observed regimes (in-phase breathers):
 - Reflection
 - Bound state
 - Bound state + Reflection (for high powers)
 - Bound state + spontaneous symmetry breaking
- Symmetry breaking due to numerical errors

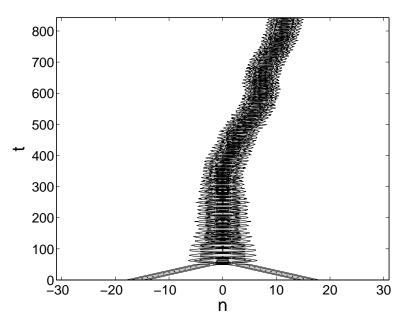
6 Reflection and bound state regimes



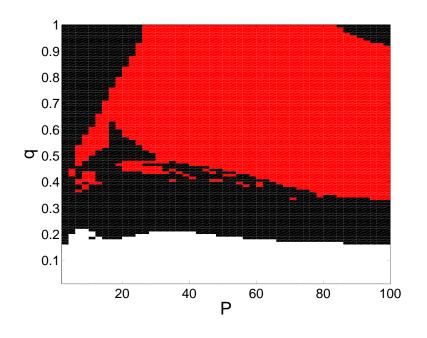


6 Reflection + bound state and symmetry breaking regimes

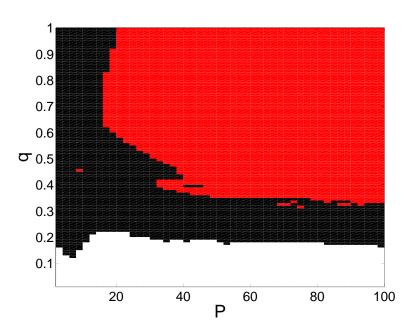




Oifferent regimes

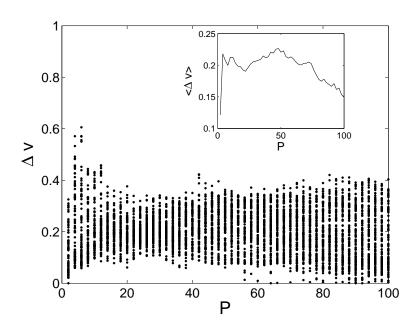


Inter-site collisions

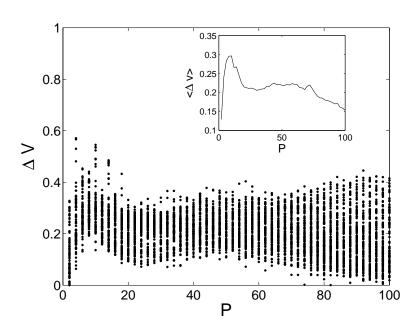


On-site collisions

Velocity difference (Incoming-Outgoing)



Inter-site collisions



On-site collisions

Conclusions

- In all cases, reflection and bound state formation is observed
- Symmetry breaking is also observed, with different origins:
 - DNLS: Numerical errors
 - Dissipative KG: Attractor of the system
 - FPU: Unexplained
- Saturable nonlinearity allows high-power breathers
- Future challenge: Study the case of Hamiltonian Klein-Gordon lattices.