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UNIVERSAL MATRIX TRANSFORMS OF HOLOMORPHIC FUNCTIONS

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Abstract. The phenomenon of overconvergence is related with the convergence of subsequences of the sequence of partial sums of Taylor series at points outside their disk of convergence. During the seventies Chui and Parnes and the third author provided a holomorphic function in the unit disk which is universal with respect to overconvergence. The generic nature of this kind of universality has been recently shown by Nestoridis. In this paper, we connect the overconvergence with the summability theory. We show that there are "many" holomorphic functions in the unit disk such that their sequences of A-transforms have the overconvergence property, A being an infinite matrix. This strengthens Nestoridis' result.

1. Introduction

A century ago Porter discovered that certain Taylor series with radius of convergence 1 enjoy the property that some subsequences $\{s_{n_k}(z)\}_{k=0}^{\infty}$ of their sequences of partial sums $\{s_n(z)\}_{n=0}^{\infty}$ converge at some points outside the closed unit disk $\{z: \ |z| \le 1\}$ of the complex plane $\mathbb{C}$. This phenomenon is called *overconvergence*. This idea was developed by the third author in 1970 [7] and by Chui and Parnes in 1971 [3]. They proved the existence of holomorphic functions $f(z) = \sum_{\nu=0}^{\infty} a_\nu z^\nu$ in the open unit disk $\mathbb{D}$ with the property that, given a compact set K having connected complement and satisfying $K \cap \{z: \ |z| \le 1\} = \emptyset$, and given $g \in A(K)$ –that is, g is continuous in K and holomorphic in its interior K^0– there exists a

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subsequence $\{s_{n_k}(f,z)\}_{k=0}^{\infty}$ of the sequence $\{s_n(f,z) = \sum_{\nu=0}^{n} a_\nu z^\nu\}_{n=0}^{\infty}$ of partial sums such that

$$s_{n_k}(f,z) \to g(z) \qquad \text{uniformly on } K \qquad (k \to \infty). \tag{1}$$

Let us denote, as usual, by $H(\mathbb{D})$ the space of holomorphic functions in the unit disk, endowed with the topology of uniform convergence on compact subsets. It is well known that $H(\mathbb{D})$ is a Fréchet space (= completely metrizable locally convex space), so it is a Baire space. In a Baire space X, a subset A is *residual* whenever its complement is of first category (= a countable union of sets whose closures have empty interior) or, equivalently, whenever A contains some dense G_δ subset. Hence, topologically speaking, a residual set is "very large" in X.

In 1996 Nestoridis [10] gave a new impulse to the idea of overconvergence. He was able to prove that this is in fact a generic phenomenon, in the sense that "most" holomorphic functions in $\mathbb{D}$ are universal with respect to overconvergence, even in a stronger way than before: There exists a residual set of functions $f \in H(\mathbb{D})$ satisfying that for each compact set K with $K \cap \mathbb{D} = \emptyset$ and connected complement, and for given $g \in A(K)$, the approximation property (1) holds for some $\{n_k\}$ [10, Theorem 2.6]. Observe that this time K is allowed to intersect the boundary $\partial\mathbb{D}$.

The results of Luh-Chui-Parnes-Nestoridis have been recently continued in many ways, for instance with properties of non-continuation, covering the plane, holomorphic monsters, and others (see [5, Section 4d] or [6, Section 4] for references). In this paper we want to provide a new way, connecting with summability methods given by infinite matrices. Specifically, we produce *generic* universality with respect to *overconvergence*, but this time the sequence of partial sums $s_n(f,z) = \sum_{\nu=0}^{n} a_\nu z^\nu$ of the Taylor series $f(z) = \sum_{\nu=0}^{\infty} a_\nu z^\nu$ is replaced to the sequence $\sigma_n(f,z) := \sum_{\nu=0}^{\infty} \alpha_{n\nu} s_\nu(f,z)$ of their *A-transforms*, $A = [\alpha_{n\nu}]_{n,\nu=0}^{\infty}$ being an adequate infinite matrix with complex entries, see Theorem 2.2 below.

2. Preliminaries and statement of the main result

We will make use later of the following purely topological auxiliary assertion. Its special case $R_0 = +\infty$ can be found in [10, Lemma 2.1], which in turn is also a special instance of [9, Lemma 2.1]; see also [2, Lemma 2.9] for an earlier, similar property on general domains of the plane. The proof of Lemma 2.1 can be achieved by modifying suitably the proof of [10, Lemma 2.1]. As usual, $\mathbb{C}$

is the same as $\{|z| < +\infty\}$, $\mathbb{N}$ will stand for the set of positive integers, and $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$.

Lemma 2.1. *Let us fix R_0 with $1 < R_0 \le +\infty$. Then there exists a sequence $\{K_n : \ n \in \mathbb{N}\}$ of compact sets in $\{1 \le |z| < R_0\}$ which have connected complement, such that given a compact set $K \subset \{1 \le |z| < R_0\}$ with connected complement there exists an $m \in \mathbb{N}$ with $K \subset K_m$.*

The preceding lemma is useful in order to "enumerate" the adequate compact sets.

In other order of ideas, let $R_0 \in (1, +\infty]$ and let $A = [\alpha_{n\nu}]_{n,\nu=0}^{\infty}$ be an infinite matrix with complex entries, and consider the following five properties which may or may not be satisfied by A:

(a) For all $n \in \mathbb{N}_0$, $\limsup_{\nu\to\infty} |\alpha_{n\nu}|^{1/\nu} \le \dfrac{1}{R_0}$.

(b) For all $\nu \in \mathbb{N}_0$, $\lim_{n\to\infty} \alpha_{n\nu} = 0$.

(b') For all finite subsets $F \subset \mathbb{N}_0$, $\liminf_{n\to\infty}(\max_{\nu\in F} |\alpha_{n\nu}|) = 0$.

(c) For every $n \in \mathbb{N}_0$, the series $\sum_{\nu=0}^{\infty} \alpha_{n\nu}$ converges, and there exists an $\alpha \in \mathbb{C} \setminus \{0\}$ such that $\lim_{n\to\infty} \sum_{\nu=0}^{\infty} \alpha_{n\nu} = \alpha$.

(c') For every $n \in \mathbb{N}_0$, the series $\sum_{\nu=0}^{\infty} \alpha_{n\nu}$ converges, and some oscillation limit of the sequence $\left\{ \left| \sum_{\nu=0}^{\infty} \alpha_{n\nu} \right| \right\}_{n=0}^{\infty}$ is positive but finite.

Observe that (b) implies (b'), that (c) implies (c'), and that (a) implies the first part of (c)–(c'). Note also that the second part of (c') is equivalent to the existence of a strictly increasing sequence $\{n_j\}$ of natural numbers and some $\alpha \in \mathbb{C} \setminus \{0\}$ with $\lim_{j\to\infty} \sum_{\nu=0}^{\infty} \alpha_{n_j\nu} = \alpha$. Finally, we observe that, trivially, any row-finite matrix –and so any triangular matrix– satisfies (a) and the first part of (c)–(c'). We recall that A is said to be triangular if $\alpha_{n\nu} = 0$ for $\nu > n$, while A is row-finite whenever for each n there is $\nu(n)$ such that $\alpha_{n\nu} = 0$ for $\nu > \nu(n)$.

Our main statement, which can be labelled as a "matrix overconvergence generic phenomenon result", reads as follows.

Theorem 2.2. *Suppose that $1 < R_0 \le +\infty$ and that $A = [\alpha_{n\nu}]$ is an infinite matrix which satisfies at least one of the sets of properties $[(a), (b), (c')]$, $[(a), (b'), (c)]$.*

Let us denote by M the subset of all functions $f \in H(\mathbb{D})$ such that the sequence $\{\sigma_n(f,\cdot)\}_{n=1}^{\infty}$ of their A-transforms has the following property:
For every compact set $K \subset \{1 \leq |z| < R_0\}$ with connected complement and every function $g \in A(K)$ there exists a sequence $\{n_k\}$ with

$$\sigma_{n_k}(f,z) \to g(z) \qquad \textit{uniformly on } K \qquad (k \to \infty).$$

Then M is residual in $H(\mathbb{D})$.

The implication $(i) \Rightarrow (ii)$ of the following elementary lemma will be used in the proof of Theorem 2.2. But note that the lemma tells us that property (a) for a matrix A is *sharp* in order that the A-transforms are well defined in that theorem.

Lemma 2.3. *Let be provided R_0 with $1 < R_0 \leq +\infty$. Assume that $\{\alpha_\nu\}_{\nu=0}^{\infty}$ is a sequence of complex numbers. Then the following properties are equivalent:*

(i) $\limsup_{\nu\to\infty} |\alpha_\nu|^{1/\nu} \leq \dfrac{1}{R_0}$.

(ii) *The series $\displaystyle\sum_{\nu=0}^{\infty} \alpha_\nu s_\nu(f,z)$ converges uniformly on K for every $f(z) = \displaystyle\sum_{\nu=0}^{\infty} a_\nu z^\nu \in H(\mathbb{D})$ and every compact subset $K \subset \{|z| < R_0\}$.*

(iii) *The series $\sum_{\nu=0}^{\infty} \alpha_\nu s_\nu(f,z)$ converges for every $f \in H(\mathbb{D})$ and every point z with $1 < |z| < R_0$.*

Proof. It is trivial that (ii) implies (iii). Assume now that (i) holds and fix a compact subset $K \subset \{|z| < R_0\}$. Then there exists a constant $R \in (1, R_0)$ such that $|z| \leq R$ for all $z \in K$. Fix a function $f(z) = \sum_{\nu=0}^{\infty} a_\nu z^\nu \in H(\mathbb{D})$. Since $R/R_0 < 1$, we can select (and fix) an $\varepsilon > 0$ such that $\beta := (\dfrac{1}{R_0} + \varepsilon)(1+\varepsilon) < 1$. But $\limsup_{n\to\infty}(\nu|\alpha_\nu|)^{1/\nu} \leq 1/R_0$ and $\limsup_{\nu\to\infty}|a_\nu|^{1/\nu} \leq 1$, so $(\nu+1)|\alpha_\nu| < (\frac{1}{R_0} + \varepsilon)^\nu \cdot C'$ and $|a_\nu| < (1+\varepsilon)^\nu \cdot C''$ for adequate constants C', C'' and all $\nu \in \mathbb{N}_0$. Then

$$\begin{aligned}
|\alpha_\nu s_\nu(f,z)| &\leq |\alpha_\nu| \cdot \left|\sum_{\mu=0}^{\nu} a_\mu z^\mu\right| \leq |\alpha_\nu| \cdot \sum_{\mu=0}^{\nu} |a_\mu| R^\mu \\
&\leq |\alpha_\nu| \cdot R^\nu \cdot \sum_{\mu=0}^{\nu} (1+\varepsilon)^\mu C'' \\
&\leq (\nu+1)|\alpha_\nu| R^\nu (1+\varepsilon)^\nu C'' \\
&< C'C'' \cdot \left[\left(\frac{1}{R_0} + \varepsilon\right)(1+\varepsilon)R\right]^\nu = C\beta^\nu \qquad (z \in K,\ \nu \in \mathbb{N})
\end{aligned}$$

for some constant C. Then Weierstrass' M-test yields (ii).

Finally, suppose that (iii) is true and that, by way of contradiction,

$$\limsup_{\nu\to\infty} |\alpha_\nu|^{1/\nu} > 1/R_0.$$

Let us choose the point $z_0 = \gamma$, where $\max\left\{1, \frac{1}{\limsup_{\nu\to\infty} |\alpha_\nu|^{1/\nu}}\right\} < \gamma < R_0$. Then $1 < |z_0| < R$ and $|\alpha_\nu|\gamma^\nu > 1$ for infinitely many $\nu \in \mathbb{N}$. Consider the function $f(z) := \frac{1}{1-z} \in H(\mathbb{D})$. Then $|s_\nu(f, z_0)| = |1+\gamma+\gamma^2+\cdots+\gamma^\nu| \geq \gamma^\nu$ for all ν, therefore $|\alpha_\nu s_\nu(f, z_0)| > 1$ for infinitely many ν. Consequently, $\sum_{\nu=0}^{\infty} \alpha_\nu s_\nu(f, z_0)$ cannot converge, which is a contradiction. □

Corollary 2.4. *Under the assumption of Lemma 2.3 we assume that (i) of such lemma is satisfied. We consider the operator* $T : H(\mathbb{D}) \to H(\{|z| < R_0\})$ *given by* $(Tf)(z) = \sum_{\nu=0}^{\infty} \alpha_\nu s_\nu(f, z)$. *Then T is continuous, if $H(\mathbb{D})$ and $H(\{|z| < R_0\})$ are endowed with the topologies of uniform convergence in compacta.*

Proof. We observe that by (i) the series $\sum_{\nu=0}^{\infty} \alpha_\nu$ converges and $(Tf)(z) = \sum_{k=0}^{\infty}\left[\sum_{\nu=k}^{\infty} \alpha_\nu\right] \frac{f^{(k)(0)}}{k!} z^k$. The linearity of T_n together with the Closed Graph Theorem (see [13]) applied to the Fréchet spaces $H(\mathbb{D})$ and $H(\{|z| < R_0\})$ yield the continuity of T. □

3. Proof of the main result

We assume during the whole proof that properties (a), (b), (c') are satisfied by the matrix A. The proof under the set of conditions [(a), (b'), (c)] is similar and left to the interested reader.

Due to (a), Lemma 2.3 shows that for each $n \in \mathbb{N}$ and each $f \in H(\mathbb{D})$ the A-transform $\sigma_n(f, z)$ not only makes sense for $z \in K$, but also defines a function belonging to $A(K)$, whenever K is a compact subset of $\{|z| < R_0\}$.

Now, from Mergelyan's theorem (see [4] or [12]), each $g \in A(K)$ can be uniformly approximated by polynomials on K, where the compact $K \subset \{1 \leq |z| < R_0\}$ has connected complement. Let $\{K_n\}$ be the sequence given by Lemma 2.1. There is $m \in \mathbb{N}$ with $K \subset K_m$. Thus, it is not difficult to realize that if the sequence $\{p_j\}_{j=1}^{\infty}$ is an enumeration of all polynomials with rational real and imaginary parts then

$$M = \bigcap_{n\in\mathbb{N}} \bigcap_{j\in\mathbb{N}} \bigcap_{k\in\mathbb{N}} G(K_n, p_j, \frac{1}{k}). \tag{2}$$

We have denoted

$$G(K,p,\varepsilon) := \{f \in H(\mathbb{D}) : \text{ there exists } n \in \mathbb{N} \text{ such that } |\sigma_n(f,z) - p(z)| < \varepsilon \text{ for all } z \in K\},$$

where K is a compact subset of $\{1 \le |z| < R_0\}$ with connected complement, p is a polynomial and $\varepsilon > 0$.

Fix K, p, ε as before. For each $n \in \mathbb{N}$ consider the mapping

$$T_n : f \in H(\mathbb{D}) \mapsto \sigma_n(f,\cdot)_{|K} \in A(K).$$

We have already shown that T_n is well defined. But note also that every T_n is continuous by Corollary 2.4. On the other hand, we can write

$$G(K,p,\varepsilon) = \bigcup_{n\in\mathbb{N}} T_n^{-1}(B_K(p,\varepsilon))$$

where $B_K(p,\varepsilon) = \{g \in A(K) : \ |g(z) - f(z)| < \varepsilon \text{ for all } z \in K\}$, the open ball in $A(K)$ with center p and radius ε. Hence $G(K,p,\varepsilon)$ is an open subset of $H(\mathbb{D})$, so by (2) M is a G_δ subset. Since $H(\mathbb{D})$ is a Baire space, it is enough to show that each $G(K,p,\varepsilon)$ is dense in $H(\mathbb{D})$.

For this, fix a basic open subset of $H(\mathbb{D})$, of the shape

$$D(h,r,\delta) = \{f \in H(\mathbb{D}) : \ |f(z) - h(z)| < \delta \ \text{ for all } z \text{ with } |z| \le r\}$$

($h \in H(\mathbb{D})$, $0 < r < 1$, $\delta > 0$). Our goal is to prove that

$$G(K,p,\varepsilon) \cap D(h,r,\delta) \ne \emptyset. \tag{3}$$

Due to (c'), there are a sequence $n_1 < n_2 < \cdots$ of positive integers and a value $\alpha \in \mathbb{C} \setminus \{0\}$ satisfying

$$\lim_{j\to\infty} \sum_{\nu=0}^{\infty} \alpha_{n_j\nu} = \alpha. \tag{4}$$

Since that set $\{|z| \le r\} \cup K$ is a compact set with connected complement, Mergelyan's theorem guarantees the existence of a polynomial f such that

$$|f(z) - h(z)| < \delta \qquad \text{on } \{|z| \le r\} \tag{5}$$

and

$$\left| f(z) - \frac{p(z)}{\alpha} \right| < \frac{\varepsilon}{3|\alpha|} \qquad \text{for all } z \in K. \tag{6}$$

Choose $R > 0$ with $K \subset \{|z| \le R\}$. Set $d :=$ degree(f), in such a way that $f(z) = \sum_{\nu=0}^{d} a_\nu z^\nu$. Then $|f(z)| \le \beta := \max_{|t|\le R} |f(t)|$ on K and, by Cauchy's inequalities, $|a_\nu z^\nu| \le \beta$ for all $\nu \in \{0, 1, \dots, d\}$ and all $z \in K$. Hence,

$$|s_\nu(f, z) - f(z)| = \left| \sum_{\mu=\nu+1}^{d} a_\mu z^\mu \right| \le \sum_{\mu=0}^{d} |a_\mu z^\mu| \le (d+1) \cdot \beta \tag{7}$$

for every $z \in K$ and every $\nu \in \{0, 1, \dots, d-1\}$. On the other hand, $s_\nu(f, \cdot) = f$ for every $\nu \ge d$.

In order to get (3), we should verify that $f \in G(K, p, \varepsilon)$. By (b) and (4), there exists a positive integer $N \ge d$ satisfying the following properties:

$$\left| \alpha - \sum_{\nu=0}^{\infty} \alpha_{N\nu} \right| \le \frac{\varepsilon}{3(\beta+1)}, \tag{8}$$

$$\sum_{\nu=0}^{d-1} |\alpha_{N\nu}| \le \frac{\varepsilon}{3(d+1)(\beta+1)}. \tag{9}$$

Therefore, for all $z \in K$,

$$\begin{aligned} \sigma_N(f, z) - p(z) &= \sum_{\nu=0}^{\infty} \alpha_{N\nu} s_\nu(f, z) - p(z) \\ &= \sum_{\nu=0}^{d-1} \alpha_{N\nu} s_\nu(f, z) + \sum_{\nu=d}^{\infty} \alpha_{N\nu} f(z) - p(z) \\ &= \sum_{\nu=0}^{d-1} \alpha_{N\nu} (s_\nu(f, z) - f(z)) + \sum_{\nu=0}^{\infty} \alpha_{N\nu} f(z) - p(z) \\ &= \sum_{\nu=0}^{d-1} \alpha_{N\nu} (s_\nu(f, z) - f(z)) + \left(\sum_{\nu=0}^{\infty} \alpha_{N\nu} - \alpha \right) f(z) + \alpha f(z) - p(z). \end{aligned}$$

From (6), (7), (8), (9) and the triangle inequality we obtain

$$|\sigma_N(f, z) - p(z)| \le (d+1)\beta \frac{\varepsilon}{3(d+1)(\beta+1)} + \frac{\varepsilon}{3(\beta+1)} \cdot \beta + |\alpha| \cdot \frac{\varepsilon}{3|\alpha|} < \varepsilon \qquad (z \in K),$$

that is, $f \in G(K, p, \varepsilon)$. This and (5) give us (3). The proof is finished.

4. Final Remarks

(1) If we consider $A =$ the identity and $R_0 = +\infty$ in Theorem 2.2, then we obtain, as a particular case, the Nestoridis result about overconvergence of Taylor series.

(2) Recently, the authors have proved –by using a *constructive* way– the existence of *one* function f whose sequence of A-transforms is universal in the sense of Theorem 2.2, but on the weaker framework of any compact set $K \subset \partial\mathbb{D}$, $K \neq \partial\mathbb{D}$ (see [1, Theorem 4]). The matrix A is this time triangular and satisfies (b)–(c). From this f, they also provided a trigonometric

series $\sum_{\nu=0}^{\infty} a_\nu(\cos \nu t + i \sin \nu t)$ whose sequence of A-transforms $\{\sigma_n(t)\}$ is universal in the following sense:

For any two real-valued measurable functions φ and ψ on $[0, 2\pi]$, there exists a sequence $\{n_k\}$ such that

$$\begin{array}{rcl} \mathrm{Re}\{\sigma_{n_k}(t)\} & \to & \varphi(t) \\ \mathrm{Im}\{\sigma_{n_k}(t)\} & \to & \psi(t) \end{array} \qquad \text{almost everywhere on } [0, 2\pi].$$

Analogously, we can obtain from our theorem the existence of "many" trigonometric series with the above universal behaviour.

(3) Under the hypotheses of Theorem 2.2, "most" functions in $H(\mathbb{D})$ satisfy its statement and, simultaneously, have power series expansions with radius of convergence 1. Indeed, the set N of functions in $H(\mathbb{D})$ such that $\partial\mathbb{D}$ is a natural boundary is residual (see [6, Section 3]), hence $M \cap N$ is also residual.

(4) By using Baire categories, Melas and Nestoridis have recently shown [9, Theorem 3.4] a strong overconvergence-universality result which also covers that in [10]. They even consider a simply connected domain Ω instead of $\mathbb{D}$ and their matrices A (which they called "admissible") have entries $\alpha_{n\nu}(z)$ that are holomorphic functions on a certain connected open neighbourhood of $\mathbb{C} \setminus \Omega$. Nevertheless, these matrices were row-finite and, even in the case that their entries were just numbers, they were allowed to satisfy conditions stronger than (b') and (c'). We also point out that the third author had constructed in [7] a function $f \in H(\mathbb{D})$ whose A-transforms exhibited universality on every bounded simply connected domain G with $G \cap \{|z| \leq 1\} = \emptyset$, where this time the matrix $A = [\alpha_{n\nu}]_{n,\nu=0}^{\infty}$ was triangular and satisfied (b) and (c) with $\alpha = 1$.

(5) By following the proof of Theorem 2.2 it is not difficult to realize that if conditions (a), (b), (c) are imposed on A, then one would in fact obtain that for any subsequence $\{m_j\}$ of $\mathbb{N}$ there is a residual set of functions M such that, for every compact set $K \subset \{1 \leq |z| < R_0\}$ with connected complement and every $f \in M$, the set $\{\sigma_{m_j}(f, \cdot) : \ j \in \mathbb{N}\}$ is dense in $A(K)$. In the terminology of [5], the sequence $\{T_n\}$ of operators considered in the proof would be densely hereditarily hypercyclic in this case.

(6) Finally, we recall that Toeplitz–Silverman's theorem (see [11]) asserts that an infinite matrix $A = [\alpha_{n\nu}]_{n,\nu=0}^{\infty}$ is regular –that is, it preserves convergence and limits of sequences– if and only if

(A) $\lim_{n\to\infty} \alpha_{n\nu} = 0$ for all $\nu \in \mathbb{N}_0$,

(B) $\lim_{n\to\infty} \sum_{\nu=0}^{\infty} \alpha_{n\nu} = 1$, and

(C) $\sup_n \sum_{\nu=0}^{\infty} |\alpha_{n\nu}| < +\infty$.

As for a weaker property, the third author proved in [8] that A is P-regular –that is, regular for power series– if and only if (A), (B) and (C') hold, where (C') is the condition $\left[\sup_n \sum_{\nu=0}^{\infty} |\alpha_{n\nu}|\rho^{\nu} < +\infty \text{ for all } \rho \in (0,1)\right]$. We observe that (B) is (c) with $\alpha = 1$. Thus, our matrix in Theorem 2.2 may be far from being regular, even far from being P-regular. In the opposite direction, regular matrices generate dense hereditary hypercyclicity in the sense of the preceding remark.

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