## Ph.D. Thesis

# Operating Theatre Planning 

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## Scheduling in Real-Life Settings:

## Problem Analysis, Models,

 and Solution ProceduresJosé Manuel Molina Pariente

Advisor: Dr. José Manuel Framiñán Torres<br>SCHOOL OF ENGINEERING<br>UNIVERSITY OF SEVILLE

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# Part I 

## Preliminaries

and

## Research Framework

## Chapter 1

## Introduction and Objectives of the Thesis

### 1.1 Context and problem statement

Nowadays health care organizations experience an increasing pressure in order to provide their services at the lowest possible costs as a response to the combination of restrictive budgets, increasing waiting lists, and the aging of the population. In general, hospital resources are expensive and scarce, being the operating theatre the most critical and expensive resource. In most hospitals, the operating theatre is a complex system composed of operating rooms (ORs) together with their specialized equipment, preoperative and postoperative facilities and, finally, a diversity of human resources, including surgeons, anesthetists, nurses, etc. To handle such complexity, decisions related to operating theatre management are usually decomposed into three hierarchical decision levels, i.e.: strategic, tactical and operational.

At the strategic level, hospital managers set the volume and the mix of surgeries that will be performed over a long-term horizon (typically, a year) to keep up acceptable size of waiting lists while achieving cost targets, thus making long-term decisions related to the dimensioning of surgical facilities (e.g. build new ORs, adding new recovery beds, etc.), the hiring of surgical staff (e.g. surgeons, nurses, etc.), the purchase of novel surgical devices, and the amount of operating theatre resources required by surgical specialties to perform their surgeries (OR time, number of beds, etc.).

Once decisions at strategic level have been made, the operating theatre resources are allocated over a medium-term planning horizon (ranging from few weeks to 6 months) in the tactical level. Since the OR is both a bottleneck and the most expensive facility for most hospitals, surgical specialties are first assigned to OR days (i.e. a pair of an OR and a day) over the planning horizon, until the OR time allocated to each surgical specialty in the strategic level is reached. Then, the above assignment defines aggregate resource requirements for specialties, such as the demand of nurses, drugs, diagnostic
procedures, laboratory tests, etc. Finally, the working shifts of human resources and their workload (e.g. the number of surgeries allocated to each surgeon) are defined over the medium-term planning horizon in order to achieve the volume of surgeries set by hospital managers.

Finally, the surgical schedule is determined over a short-term planning horizon (ranging from few days to few weeks) at the operational level. The operational level is usually solved into two steps. The first step involves the determination of the date and the OR for a set of surgeries in the waiting list; while in the second step, a sequence of surgeries for each OR within each day in the planning horizon is obtained. Note that only a set of surgeries will be performed during the planning horizon due to capacity constraints (both facilities and human resources). The decomposition of the operational level into the two aforementioned steps intends to reduce the complexity of the resulting problem, although the quality of the so-obtained surgery schedule may be reduced due to the high interdependence among these two steps, being the integrated approach a popular topic of research. At the operational level, a feature greatly influencing the performance is the uncertainty in the surgical activities, as frequently large discrepancies between the scheduled duration and the real duration of the surgeries appear, together with the availability of the resources reserved for emergency arrivals.

Despite the importance and the complexity of these hierarchical levels, decisions in practice are usually made according to the decision makers' experience without considering the underlying optimization problems. Furthermore, the lack of usage of decision models and solution procedures causes the decision makers to consume long times on performing management tasks (e.g. determine the surgical schedule, react to unforeseen events, carry out what-if analyses, etc.), instead of healthcare tasks.

The context discussed above stresses the need to provide healthcare decision makers with advanced operations research techniques (i.e. models and solution procedures) in order to improve the efficiency of the operating theatre resources and the quality of the healthcare services at the operational level. This Thesis is aimed at this goal.

### 1.2 Research Objectives and Outline of the thesis

This Thesis has been carried out in the framework of several research projects in the healthcare operations management area (see full list of projects in Section 8.2). The outcomes of these projects have been validated and implemented in the University Hospital "Virgen del Rocio" in Seville (Spain). This Hospital is one of the largest hospitals in Spain, with over 1,400 beds and 50 ORs, currently executing more than 60,000 surgeries per year. Several private companies (such as INGENIA, SIEMENS and EVERIS), two research groups (such as the Industrial Management Group (TEP134) of the School of Engineering of Seville -where the author of this Thesis has been integrated since 2007-- and the Technological Innovation Group of the University Hospital "Virgen del Rocio"), and a number of surgical specialties (such as Plastic Surgery and Major Burns, Urology and Pediatrics) are among the participants in some of these projects.

Due to the heavy implication of the University Hospital "Virgen del Rocio" in the aforementioned projects, the research issues tackled in this Thesis have been motivated by the analysis of the operational decision level in the surgical specialties of this specific hospital. In this sense, the Thesis is project- (or customer-) driven, although the problems addressed here are rather general and can be easily extrapolated to other hospitals.

As mentioned in the previous section, the goal of this Thesis is to provide healthcare professionals with operations research techniques in order to improve the efficiency of the operating theatre resources and the quality of the healthcare services at the operational level. In order to fulfill this general goal, the following research objectives were established:
i. To carry out a literature review on the operational level of the operating theatre management problem.
ii. To propose a testbed generator based on the literature review to analyze the operating theatre problems identified in i). This objective was set after detecting in the literature review the lack of a suitable experimental testbed for the problems to be addressed in iii and iv.
iii. To address the OR planning problem by proposing mathematical decision models and solution procedures under deterministic and stochastic surgery durations, emergency arrivals and resources capacity.
iv. To address a deterministic integrated OR planning and scheduling problem, taking into account the case where there is a surgical team composed by surgeons with different surgical experience. Although the stochastic version of this integrated problem is not addressed in this Thesis, a simulation approach has been carried out to analyze the robustness of the surgical schedules caused by stochastic surgery durations.
v. To demonstrate the validity of the decision models and the solution procedures developed in iii) and iv) for a real-life setting, by developing and deploying a decision support system (DSS) for OR planning and scheduling in the University Hospital "Virgen del Rocio".

This Thesis is organized in four parts:

- Part I is composed of three chapters. In Chapter 1 we have discussed the context, problem statement and the research objectives of the Thesis. Then, Chapter 2 first provides a background of the operating theatre management problem and, in the remaining of the chapter, a literature review presenting the research topics identified in the University Hospital "Virgen del Rocio" (see iii), iv) and v) is discussed. Finally, Chapter 3 presents a testbed procedure for experimentally generate scenarios to analyze the decision problems and the solution procedures to be proposed in iii), iv) and $v$ ).
- Part II covers the research objectives iii) and iv) of the Thesis. Chapter 4 analyzes the deterministic version of the OR planning problem, presenting a decision model that incorporates the main constraints identified in Chapter 2, and the objective function commonly used in all surgical specialties of the University Hospital "Virgen del Rocio". A set of solution procedures are proposed to solve the problem, including an exhaustive computational comparison with existing procedures identified in Chapter 2 by using the testbed procedure described in Chapter 3. Chapter 5 presents the stochastic OR planning problem in order to study the uncertainty in surgery durations, in the arrivals of emergency surgeries, and in the surgeons' capacity to
perform elective surgeries. A stochastic decision model and a Monte Carlo optimization method based on the Sample Average Approximation (SAA) method are presented. Finally, Chapter 6 analyzes the integrated OR planning and scheduling problem considering surgical teams composed by surgeons with different surgical experience, analyzing how the composition of a surgical team influences the length of the surgery duration. An iterative constructive method is presented to solve the problem, studying the robustness of the so-obtained surgical schedules by means of simulation.
- Part III presents the validation (both theoretical and practical) of the proposed solutions procedures to address the OR planning and scheduling problem in the University Hospital "Virgen del Rocio" (Chapter 7). Besides, the chapter includes the description of a DSS developed for the University Hospital "Virgen del Rocio", where the decision models and solution procedures presented in Part II are embedded.
- Part IV summarizes the main results and conclusions of the Thesis, and presents future research lines (Chapter 8).


## Chapter 2

## The Operating Room Planning and Scheduling Problem

### 2.1 Introduction

As described in Chapter 1, in this chapter we focus on the operational decision level of the operating theatre management problem. We introduce an overview of the general operating theatre management problem in Section 2.2, in which strategic, tactical and operational decision levels are described in detail. Section 2.3 and Section 2.4 present the OR planning problem under deterministic and stochastic considerations, and the integrated OR planning and scheduling problem considering surgical teams composed by surgeons with different surgical experience respectively. Finally, the conclusions gained from the literature review are discussed in Section 2.5.

### 2.2 An overview of the operating theatre management problem

The operating theatre consists of ORs as well as of preoperative and postoperative facilities such as the preoperative holding unit, the post anesthesia care unit (PACU) and, finally, the intensive care unit (ICU); as well as human resources (surgeons, anesthetists, nurses...). The operating theatre is among the most critical and expensive resource in the hospital (Guerriero and Guido, 2011), representing around $70 \%$ of revenues (Jackson, 2002) and $40 \%$ of costs (Macario et al., 1995), being the operating theatre management problem widely analyzed by the literature.

Decisions related to operating theatre management are usually decomposed into three hierarchical decision levels (Cardoen et al., 2010): strategic, tactical and operational. The main settings and assumptions for each decision level are described in several recent reviews on the topic (Cardoen et al., 2010; Guerriero and Guido, 2011; May et al., 2011). At the tactical level, decision makers determine the volume and the mix of
surgeries to keep up acceptable size of waiting lists achieving cost targets (see e.g. Adan and Vissers, 2002; Blake and Carter, 2002; Testi et al., 2007). Among other factors, the case mix depends on the disease processes effecting the population in the catchment area and the capacity of resources of the hospital (Blake and Carter, 2002). Once the case mix is set, operating theatre resources are allocated to surgical specialties of the hospital, determining how much amount of resource each specialty obtains (i.e. the OR time, the number of beds...). Once the strategic level has been decided, the operating theatre resources are allocated over a planning horizon of several weeks in the tactical level (Blake et al., 2002; Wachtel and Dexter, 2008). As the OR represents a bottleneck in most hospitals and, in addition, it is the most budget-consuming facility in the hospital (Jebali et al., 2006), most papers only consider the OR allocation problem at the tactical level (see e.g. Testi et al., 2007). The purpose is to define the so-called master surgical schedule that specifies which surgical specialties (at most two specialties due to large set-up times and costs, Beliën and Demeulemeester, 2007) are assigned to each OR during a day (in the following OR-day) over the planning horizon. The master surgical schedule also defines aggregate resource requirements, such as the demand of nurses, drugs, diagnostic procedures, laboratory tests, etc. (Blake et al., 2002). However, few approaches have considered beds (Beliën and Demeulemeester, 2007) and nurses (Beliën and Demeulemeester, 2008) in the construction of the master surgical schedule in order to reduce staffing costs.

Finally, at the operational level, the surgical schedule is obtained over a week or two week planning horizon (see e.g. Fei et al., 2009; Lamiri et al., 2009; Marques et al., 2012; Ozkarahan, 2000). At this level, the number, type and opening hours for each resource have been already set, as well as the relevant data from the patients in the waiting list (such as expected surgery duration, patient priority, deadline to be operated, etc.). Several decisions have to be into account by decision makers before the surgical schedule is determined. First, the assignment of the surgeon who is the responsible of a patient during his/her stay in the hospital, that is made at the first consultation in order to guarantee the continuity of care. This assignment is commonly made by the decision maker based on the surgeon's specialty (i.e. types of surgery which could be performed by the surgeon), his/her skills and workload. After surgeons are assigned to patients in the waiting list, the OR time assigned by the master surgical schedule to the specialty is allocated to individual surgeon or surgical groups. The assignment is made according to
surgeon preferences and/or the type of surgeries that they have to perform (i.e. they are assigned to well-equipped ORs where they can perform the assigned surgeries). One of the three management strategies proposed by Patterson (1996) can be used. The first one is the so-called block scheduling strategy, where each surgeon has been assigned to a number of OR time windows in which he/she will perform his/her surgeries. A surgeon cannot carry out one surgery outside his/her time windows. The second one is the socalled open scheduling in which the decision maker allocates ORs to surgeons according to their requests for planning their surgeries. According to Fei et al. (2009), the block scheduling policy is a special case of the open scheduling policy being the latter more flexible than the former (all solutions of the block scheduling policy are feasible for the open scheduling policy). Finally, the block scheduling strategy can be modified in order to increase its flexibility, yielding the so-called modified block scheduling. The flexibility is reached by two ways: some OR time windows are booked and others are left open, or unused windows are released at some time before surgery.

The operational decision level consists of the offline and the online levels (Hans et al., 2012). The offline operational level is traditionally solved into two steps (Magerlein and Martin, 1978): the first step (called advance scheduling), involves the determination of the OR-day (i.e. the date and the OR), while in the second step (called allocation scheduling), a sequence of surgeries for each OR within each day in the planning horizon is obtained. In the following, according to the definition proposed by Cardoen et al. (2010), the offline operational level is called the OR planning (advance scheduling) and scheduling (allocation scheduling) problem. Note that the decomposition of the operational level into the two aforementioned steps intends to reduce the complexity of the resulting problem (Riise and Burke, 2010). Nevertheless, the quality of the so-obtained surgery schedule is reduced due to the high interdependence among these two steps (Cardoen et al., 2009a), being the integrated approach a popular topic of research (see e.g. Marques et al., 2012; Riise and Burke, 2010; Van Huele and Vanhoucke, 2014; Vijayakumar et al., 2013). The online operational level involves control mechanisms that deal with monitoring the process and reacting to unforeseen or unanticipated events (Hans et al., 2012), such as the large discrepancies between the scheduled duration and the real duration of the surgeries (Min and Yih, 2010), and/or the availability of the resources reserved for uncertain arrivals (see e.g. Lamiri et al., 2009).

### 2.3 The operating room planning problem

The OR planning of surgeries (Cardoen et al., 2010) on the offline operational decision level (Hans et al., 2012) is a popular topic of research (see the literature reviews by Cardoen et al., 2010; Guerriero and Guido, 2011; May et al., 2011). In Table 2.1 we have categorized the literature contributions on OR planning, and have indicated for all these contributions what surgical resources are taken into account, the management strategy (open and block), as well as the modeling approach (deterministic and stochastic), decision types, objective functions, and solution approaches.

The OR planning problem is a heavily constrained problem, with constraints related to the following aspects:

- Capacity of the resources, since both surgical facilities and surgical personnel are not fully available during the planning horizon,
- Time periods, as each patient must be intervened within a release date and a deadline. The release date and the deadline represent the earliest and the latest date when a patient can be operated in the planning horizon,
- Limits on the number of ORs where surgeons can be assigned to perform surgeries on a given day, in order to reduce the surgeon idle time and to avoid the overlapping of consecutive surgeries performed by the same surgeon,
- OR eligibility, as for example to book OR-days for planning a certain type of surgery or to impose that some surgeries take place only in certain ORs,
- Patient priority, as patients are planned according to a certain priority indicator (for example, arrival date and urgency of the patient proposed by Ogulata and Erol, 2003), and
- Uncertainty, in order to consider the large discrepancies between the scheduled duration and the real duration in the use of resources (e.g. OR and ICU), and/or the availability of the resources reserved for uncertain arrivals.


Table 2.1. An overview of the OR planning problem

Regarding the objective function, usual goals considered in the literature include:

- Resource utilization, which includes the minimization of the OR under-utilization, the minimization of the OR over-utilization, and the ICU over-utilization,
- Costs, such as the minimization of the fixed costs of the patients, i.e. minimizing costs not related to the number of surgeries that have to be carried out,
- Leveling, in order to balance the distribution of total surgery time among surgeons, to evenly distribute planned surgeries across the days in the planning horizon, or to evenly distribute planned surgeries across the days in the planning horizon,
- Eligibility, in order to consider the preferences of the surgeons to perform their surgeries,
- Cancellations, such as the minimization of the risk of no realization of a surgery in its planned date, and
- Priority, as the minimization of the patient access time, i.e. the period time between the surgery is diagnosed and the execution date of the surgery.

Among exact methods proposed for solving OR planning problems in the literature, several Integer Linear Programming (ILP) models have been presented, but they are able to provide optimal solutions only for instances sizes substantially smaller than those found in practice. Besides, given the context of the decision problem, priority goes to finding good (although possibly not optimal) schedules in reasonable time rather than optimal schedules procured too late (Roland et al., 2010), due to the high number of unforeseen events, like emergencies (Roland et al., 2010) or the absence of the patient in the planned day (Weinbroum et al., 2003), that may lead to re-scheduling the planned interventions, and as a way for decision makers to quickly perform a what-if analysis over several possible scenarios. Therefore, several heuristics have been proposed for solving the OR planning problem in the literature, as are:

- Heuristics based on exact methods, as an extended version of the Hungarian method proposed by Guinet and Chaabane (2003), and the column-generation approaches (see e.g. Fei et al., 2009; Fei, Meskens and Chu, 2010; Lamiri et al., 2007; Lamiri, Xie and Zhang, 2008).
- Constructive heuristics, as bin packing methods (see e.g. Dexter, Macario and Traub, 1999; Hans et al., 2008), and a dynamic programming approach proposed by Liu et al. (2011).
- Improvement heuristics, as local search methods (see e.g. Hans et al., 2008; Lamiri et al., 2009), in which the solution improvement consists of swapping different patients between OR-days.
- Meta-heuristics, as a taboo search method (Lamiri et al., 2009), and a simulated annealing method (Lamiri et al., 2009).
- Stochastic heuristics, as a SAA method, which combines Monte Carlo simulation and mixed integer programming (see e.g. Lamiri, Xie, Dolgui et al., 2008; Min and Yih, 2010). Several authors propose approximate methods to try reduce the CPU time required by the integer linear programming, as a column generation approach (Lamiri, Xie and Zhang, 2008), improvement heuristics and meta-heuristics (Lamiri et al., 2009).


### 2.4 The integrated operating room planning and scheduling problem

In this section, we focus on the integrated OR planning and scheduling problem. The interest of an integrated approach is currently growing due to the interdependence among the OR planning and scheduling problems (Augusto et al., 2010; Ghazalbash et al., 2012; Hashemi Doulabi et al., 2014; M'Hallah and Al-Roomi, 2014; Marques et al., 2012, 2014; Meskens et al., 2013; Pham and Klinkert, 2008; Riise and Burke, 2010; Roland et al., 2010; Van Huele and Vanhoucke, 2014; Vijayakumar et al., 2013; Zhao and $\mathrm{Li}, 2014$ ). In Table 2.2 we have categorized the contributions on the integrated OR planning and scheduling problem, and have indicated for all these contributions what surgical resources are taken into account, the management strategy (open and block), as well as the decision types, constraints, and objective functions.

Most constraints considered in the integrated OR planning and scheduling problem have been previously described in the OR planning problem (see Section 2.3). In addition, material capacity constraints are considered as are the sterilization of medical trays or the availability of mobile equipment required for performing surgeries. Finally, new


Table 2.2. An overview of the integrated OR planning and scheduling problem
personnel constraints have been considered by Meskens et al. (2013), as are the surgical teams' preferences in the assignment of the OR time windows and their affinities to work together or not.

Regarding the objective function, OR utilization and priority goals have been previously described in the OR planning problem (see Section 2.3). Other goals considered in the literature for the integrated problem are:

- Time, such as the minimization of makespan (i.e. minimizing the completion time of the last scheduled surgery in the planning horizon) and the minimization of tardiness (i.e. minimizing the difference between the schedule date and the deadline of a surgery),
- Throughput, such as the maximization of the number of scheduled surgeries in the planning horizon,
- Resource utilization, which includes the minimization of the surgeon over-utilization,
- Costs, such as the minimization of the number of opened ORs, and finally
- Affinities, such as the maximization of collaborations according to staff preferences.

As shown in Table 2.2, most papers address the integrated OR planning and scheduling problem assuming a surgical team composed by a single surgeon, i.e. the responsible surgeon. However, studies related to general surgery procedures (Zheng et al., 2012) as well as to laparoscopic procedures (Cassera et al., 2009) show that around $90 \%$ of surgeries are performed by a surgical team composed by more than one surgeon, being the two-surgeons team (i.e. a responsible surgeon and an assistant surgeon) the most extended case (see e.g. Cassera et al., 2009; Chitwood Jr. et al., 2001; Giulianotti et al., 2003; Powers et al., 2008; Zheng et al., 2012). However, to the best of our knowledge, the integrated problem considering surgical teams composed by more than one surgeon has been studied only by Ghazalbash et al. (2012).

In surgical teams composed of several surgeons, the literature stresses that surgery duration depends on the experience of the assistant surgeon (see e.g. Cassera et al., 2009; Parker et al., 2012; Zheng et al., 2011). On the one hand, Bridges and Diamond (1999) study the financial impact of teaching surgical residents in the OR, showing that
the presence of a resident usually causes an increase in the surgery duration, although there are some situations (e.g. when a resident has a similar experience or skill than the teaching surgeon in some type of surgeries) in which he/she may cause a decrease of the surgery duration. On the other hand, a faculty surgeon acting as assistant decreases the surgery duration (around $30 \%$ time reduction in some urology procedures, see Ludwig et al., 2005). Hence, the literature attests that the assistant surgeon's experience clearly influences the surgery duration. However, to the best of our knowledge, this variability has not been previously addressed. For different decision problems in other research topics, resource dependent processing times are receiving growing attention (Akturk and Ilhan, 2011). In these problems, processing times are considered as a function both of the amount of resources assigned (see e.g. Demeulemeester et al., 2000; Tseng et al., 2009) and of the experience of the resources assigned to the task (see e.g. Dodin and Elimam, 1997; Drexl, 1991; Valls et al., 2009). As a particular case of experience, Heimerl and Kolisch (2010) consider a learning curve of the resources assigned. In this case, the processing time of a task decreases if the resource assigned has previously performed the same task, a phenomenon denoted as learning effect. The reference where the assumptions of processing times are most related to our problem is (Kara et al., 2011), as processing times depend on whether the task is performed with or without an assistant employee. However, these processing times do not depend of the specific assistant employee assigned to the task.

### 2.5 Conclusions

In this chapter, an overview of the operating theatre management problem is presented (see Section 2.2), focusing on the operational decision level. This decision level consists of the offline and the online levels. The offline operational level is traditionally solved into two steps (Magerlein and Martin, 1978): the first step (called advance scheduling), involves the determination of the OR-day (i.e. the date and the OR), while in the second step (called allocation scheduling), a sequence of surgeries for each OR within each day in the planning horizon is obtained. The online operational level involves control mechanisms that deal with monitoring the process and reacting to unforeseen or unanticipated events (Hans et al., 2012), such as the large discrepancies between the scheduled duration and the real duration of the surgeries (Min and Yih, 2010), and/or
the availability of the resources reserved for uncertain arrivals (see e.g. Lamiri et al., 2009).

The OR planning problem under deterministic and stochastic considerations, and the integrated OR planning and scheduling problem have been analyzed in Section 2.2 and 2.3 respectively. Summarizing the literature review, we conclude that:

- To the best of our knowledge, there are not benchmarks to analyze and evaluate the performance of solution approaches against the existing methods in the literature to solve a given decision problem, being a common practice in other research topics (see e.g. Taillard, 1993; Vallada et al., 2015). Therefore, in Chapter 3, we propose the procedure to create testbeds used in this Thesis, including necessary data to solve any OR planning and scheduling problem. The procedure integrates real-life data and parameters in the surgical specialties of the University Hospital "Virgen del Rocio", as well as data and parameters from the literature (both in real-life applications as in papers where problems are randomly generated).
- The deterministic OR planning problem has been extensively analyzed in the literature. The objective of the Thesis (see Chapter 4) is to propose a generic decision model to solve the deterministic version of the problem in surgical specialties of the Hospital, including the aspects identified by meetings with heads of surgical specialties and in the literature review. In addition, we propose several approximate methods to solve the problem, which have been compared against the adaptions of the existing methods in the literature, providing a benchmark.
- There are several interesting approaches to solve the stochastic OR planning problem. However, in our opinion, the following important aspects have been ignored:
- The block scheduling strategy is the only management strategy used in the stochastic OR planning problem for managing surgical resources (see Table 2.1). However, as described in Chapter 1, the block scheduling strategy is a special case of the open scheduling policy (Fei et al., 2009), where the latter is more flexible than the former (all solutions of the block scheduling strategy are feasible for the open scheduling strategy). Therefore, the open scheduling strategy should be analyzed for the stochastic OR planning problem.
- Responsible surgeons and their availabilities are not included in existing stochastic decision models. As described in Section 2.1, it is a common practice that the decision maker assigns a set of surgeries to be performed during the planning horizon by each surgeon based on surgeon's skills, surgeons' availabilities, etc. For this reason, responsible surgeons and their availabilities should be included in the problem under consideration, as in existing decision models for solving the deterministic version of the OR planning problem (see Fei, Meskens and Chu, 2010; Jebali et al., 2006).
- In these approaches, time period constraints are not considered for patients. However, in general, every patient in a waiting list must be operated before his/her maximum time before treatment (expressed in days). It depends on the patient's urgency-related group which is defined by National Healthcare Services based on a set of explicit clinical and social criteria (Valente et al., 2009).

Therefore, the objective of the Thesis for the stochastic OR planning problem is to propose a decision model that includes the above important aspects. In addition, we propose a stochastic mathematical model and a Monte Carlo optimization method based on the SAA method, which combines an iterative greedy local search method and Monte Carlo simulation. These aspects will be addressed in Chapter 5.

- The integrated OR planning and scheduling problem has been properly analyzed in the literature. However, the aforementioned approaches ignore the following important aspects of the problem:
- Only surgical teams composed by a single surgeon are considered in the integrated OR planning and scheduling approach. However, $90 \%$ of surgeries are performed by a surgical team composed by more than one surgeon, being the two-surgeon team the most extended case. Therefore, surgical teams composed by two surgeons should be analyzed for the integrated approach.
- The influence of the assistant surgeon's experience in the surgery duration is not considered in the existing literature. However, studies show how the duration of a surgery can increase or decrease depending on the experience
(see e.g. Bridges and Diamond, 1999). Hence, it should be included in the integrated approach.
- Most references assume surgery durations as a discrete variable, dividing it into time units: 10,15 or 30 minutes (see e.g. Augusto et al., 2010; Ghazalbash et al., 2012; Marques et al., 2012). This approach greatly increases the number of binary decision variables. In addition, surgery durations do not necessarily have to be multiple of these time units. In order to avoid these issues, we propose continuous time units (see e.g. Pham and Klinkert, 2008; Zhao and Li, 2014).

Therefore, the objective of the Thesis for the integrated OR planning and scheduling problem is to propose an ILP model to optimally solve the problem with surgical teams composed by one or two surgeons where surgery durations depend on their experience and skills. Given the high computation requirements of our decision model, we also propose an iterative constructive method. These aspects will be addressed in Chapter 6.

## Chapter 3

## Testbed Design

### 3.1 Introduction

In the literature review carried out in the previous chapter, it became clear the need of standard procedures to generate ample testbeds for the problems under consideration in this Thesis. In this chapter, we provide a testbed generator for building test instances to generate OR planning and scheduling problems in order to test the efficiency of the solution procedures for these problems. The generator integrates real-life data and parameters in the surgical specialties of the University Hospital "Virgen del Rocio", as well as data and parameters from the literature (both in real-life applications as in papers where problems are randomly generated) for generating the data involved on the decision problems. Section 3.2 discusses the data required to solve an OR planning and scheduling problem and how they are generated in the literature and in surgical specialties of the University Hospital "Virgen del Rocio". We distinguish between patients' data (Section 3.2.1) and resources data (Section 3.2.2). Section 3.4 describes the factors that define the size and the characteristics of an instance, and how they are determined. Finally, Section 3.5 provides a summarize of factors and parameters used to solve the problems considered in the Thesis.

### 3.2 Data generation

In this section, we carry out a literature review of the parameters required for solving the OR planning and scheduling problems proposed in the Thesis (see Table 3.1), and how they are generated. In addition, we also consider the parameters and the procedures identified in the surgical specialties of the University Hospital "Virgen del Rocio". We distinguish between patient data (Section 3.2.1) and resource data (Section 3.2.2).

### 3.2.1 Patient data

Table 3.2 gives an overview of patient data required to solve the proposed decision problems, and how they are generated in the literature. The following patient data are considered in the testbed:

| Category | Acronym | Description | Unit |
| :--- | :--- | :--- | :--- |
| Set and indices | $h \in H$ | Index of time period within the working planning horizon | - |
|  | $i \in I$ | Index of patient (surgery) in the waiting list | - |
|  | $j \in J$ | Index of ORs | - |
|  | $k \in K$ | Index of surgeons | - |
|  | $l \in L$ | Index of level experience | - |
| Patient | $t_{i l}$ | Surgery duration of surgery $i$ performed by an assistant | minute |
|  | $\mu$ | surgeon belonging to level of experience $l$ |  |
|  | $\sigma$ | Expected time of surgery duration | minute |
|  | $m p_{i}$ | Standard deviation of surgery duration | Minute |
|  | $M T B T_{i}$ | Maxical priority of surgery $i$ | - |
|  | $d w l_{i}$ | Days on waiting Before Treatment of surgery $i$ | day |
|  | $r d_{i}$ | Release date of surgery $i$ | day |
|  | $d_{i}$ | Deadline of surgery $i$ | day |
|  | $w_{i}$ | Clinical priority of surgery $i$ | day |
|  | $\tau_{i}$ | Surgeon in charge of surgery $i$ | - |
|  | $\gamma_{i l}$ | 1 if surgery $i$ can be performed by an assistant surgeon | - |
|  | $\delta_{i j h}$ | belonging to surgeon type $l ; 0$ otherwise | - |
|  | 1 if surgery $i$ can be performed in OR-day $(j, h) ; 0$ | - |  |
| Resources | $r_{j h}$ | otherwise | - |
|  | $o_{j h}$ | Regular capacity of OR $j$ on day $h$ | Overtime capacity of OR $j$ on day $h$ |
|  | $a_{k h}$ | Regular capacity of surgeon $k$ on day $h$ | minute/day |
|  | $m d s_{k}$ | Maximum number of available days to perform surgeries | day |
|  |  | in a weekly planning horizon | minute/day |
|  |  |  |  |

Table 3.1. Set and parameters of the operational level

- $t_{i}$, surgery duration of surgery $i$ (in minutes). We consider that $t_{i}$ follows a 2 parameter log-normal distribution (see e.g. Guinet and Chaabane, 2003; Lamiri et al., 2007; Min and Yih, 2010). The expected duration $\left(\mu_{i}\right)$ is randomly selected taking one of the values in the set $\{60,120,180,240\}$ as in Marcon et al. (2003) or setting to a constant value (e.g. 120 minutes). The standard deviation $\left(\sigma_{i}\right)$ is determined by using the coefficient of variation (CV) which is defined as the ratio of $\sigma$ to $\mu$. Note that $t_{i}$ includes not only the time needed to perform the surgery, but also the set-up time, the clean-up time, and the preparation time for the next surgery.
- $t_{i l}$, surgery duration of a surgery $i$ depends on the assistant surgeon level experience $l$ (in minutes). With loss of generality, the following levels of experience have been

| Reference | $t$ |  |  | $r d$ | $d$ | w | $\tau$ | $\delta$ | $a$ | $r$ | $o$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Dist. | $\mu$ | $\sigma$ |  |  |  |  |  |  |  |  |
| (Dexter, Macario and Traub, 1999) | LN | 124.2 | 55.1 | - | Real | - | Real | Real | Real | $\begin{aligned} & 59 \pm 74,76 \\ & \pm 96 \end{aligned}$ | - |
| (Fei et al., 2007) | PIII [ 30,150 ] | 80 | 10 | - | $\mathrm{U}[1,20]$ | - | - | - | - | - | - |
| (Fei et al., 2008) | U[15,480] | 248 | 134 | - | $\mathrm{U}[1,20]$ | - | - | - | - | U[0, 480] | U[0, 360] |
| (Fei et al., 2009) | PIII [40,150] | 90 | 15 | - | $\mathrm{U}[1,20]$ | - | $\mathrm{U}[1,\|S\|]$ | - | 180-720 | 240-480 | 0-180 |
| (Fei, Meskens and Chu, 2010) | - | Real | - | - | Real | - | Real | - |  |  |  |
| (Guinet and Chaabane, 2003) | LN | 120 | 60 | LN [2,1] | LN [4,1] | - | NC | NC | Relax. | 480 | 240 |
| (Hans et al., 2008) | Multinomial | Real | Real | - | - | - | - | - | - | 450 | - |
| (Jebali et al., 2006) | LN [30,420] | 180 | 60 | - | - | - | - | - | 480-720 | 480 | 240 |
| (Lamiri et al., 2007) | LN | U[60,180] | $\mathrm{R}[0.1 \mu \ldots 0.5 \mu]$ | $\mathrm{R}[-1 \ldots\|T\|]$ | - | - | - | Specialty | - | 480 | - |
| (Lamiri, Xie and Zhang, 2008) | U[30,180] | 120 | 43 | $\mathrm{R}[-1 \ldots \mid T]]$ | - | - | - | Specialty | - | 480 | 180 |
| (Lamiri, Xie, Dolgui et al., 2008) | U[30,180] | 120 | 43 | $\mathrm{R}[-1 \ldots\|T\|]$ | - | - | - | - | - | 480 | - |
| (Lamiri et al., 2009) | U[30,180] | 120 | 43 | $\mathrm{R}[-1 \ldots\|T\|]$ | - | - | - | - | - | 480 | - |
| (Liu et al., 2011) | PIII [40,150] | 90 | 15 | - | $\mathrm{U}[1,20]$ | - | $\mathrm{U}[1,\|S\|]$ | - | 180-720 | 240-480 | 0-180 |
| (Marcon et al., 2003) | N, LN | $\mathrm{R}[60,70,80, \ldots, 180]$ | $\mathrm{R}[0.1 \mu \ldots 0.5 \mu]$ | - | - | - | NS | - | 480 | 480 | - |
| (Min and Yih, 2010) | LN | Real | Real | - | - | - | - | Specialty | - | 480 | - |
| (Ogulata and Erol, 2003) | - | Real | - | - | - | $\mathrm{f}(d w l, m p)$ | - | - | - | 360 | - |
| (Ozkarahan, 2000) | - | Real | - | - | - | - | Real | Real | NS | 480 | - |
| (Pham and Klinkert, 2008) | - | Real | - | - | - | - | Real | Real | 480 | 480 | - |
| (Augusto et al., 2010) | U | 130, 210 | 52, 17 | - | - | - | - | - | - | NS | - |
| (Riise and Burke, 2010) | - | Real | Real | - | - | - | Real | - | 420 | 720 | - |
| (Roland et al., 2010) | - | 167 | 93 | Real | Real | - | Real | Real | Real | 720 | 180 |
| (Marques et al., 2012) | - | Real | Real | Real | Real | - | - | - | 480 | 690 | - |
| (Marques et al., 2014) | - | Real | Real | Real | Real | - | - | - | 480 | 690 | - |
| (Meskens et al., 2013) | - | Real | - | Real | Real | - | Real | - | Real | 480 | - |
| (Vijayakumar et al., 2013) | U[30,360] | 195 | 95 | - | - | Real | Real | - | Real | 480 | - |
| (Zhao and Li, 2014) | LN | 60,120,180 | 10,30,60 | - | - | - | - | Be[0.5] | - | 480 | 240 |
| (Hashemi Doulabi et al., 2014) | U[120,240] | 180 | 34 | - | - | - | - | - | - | 480 | - |
| (Van Huele and Vanhoucke, 2014) | LN | Real | Real | - | - | - | Real | Real | Real | 600 | - |

Table 3.2. Parameters considered in the design of the testbed
considered: 0 (for no consider assistant surgeon), 1 (for junior residents), 2 (for senior residents), and 3 (for faculty surgeons). For surgery $i$, the value $t_{i l}$ ( $l=1,2$, 3 ) is assumed to be related to $t_{i}$ (i.e. the length of the surgery when performed only by the responsible surgeon). Therefore, for each value of $l$, a variation interval affecting $t_{i}$ is defined as follows: (1) junior residents' surgeries are commonly trained surgeries, whereby the involvement of them always causes an increase of the surgery duration; (2) however, for senior residents, there are situations in which the resident has a similar level of experience that the faculty surgeon, causing a decrease of the surgery duration; (3) finally, the involvement of a faculty surgeon as assistant surgeon always produces a decrease of the surgery duration. According to Bridges and Diamond (1999) and Ludwig et al. (2005), the variation intervals for determining the values of $t_{i l}$ from $t_{i}$ for each surgery are: [20\%, 50\%], [-10\%, 20\%] and $[-30 \%,-10 \%]$ for level 1, level 2, and level 3, respectively. We assume that the coefficient of variation is randomly selected within these intervals. An example of the calculation of $t_{i l}$ is shown in Table 3.3.

- $\quad \gamma_{i}$, binary parameter yielding 1 if surgery of patient $i$ can be operated by an assistant surgeon with a level of experience $l, 0$ otherwise. We assume that $\gamma_{i l}$ follows a Bernoulli distribution. Note that each surgery must be assigned to at least one level of experience.
- $M T B T_{i}$, maximum time before treatment of patient $i$ (in days). $M T B T_{i}$ depends on the patient's Urgency-Related Group which are defined by National Healthcare Services based on a set of explicit clinical and social criteria (Valente et al., 2009). In this work, $M T B T_{i}$ is randomly generated from the set $\{45,180,360\}$ as in the University Hospital "Virgen del Rocio".
- $d w l_{i}$, number of days on the waiting list of surgery $i$ at the beginning of the planning horizon. $d w l_{i}$ is drawn from a uniform distribution $[-|H|, M T B T-1]$. Note that $-|H|$ is selected to consider admissions in the waiting list during the planning horizon.
- $\quad r d_{i}$, release date for performing the surgery of patient $i$ (in days). $r d_{i}$ represents the earliest date in which surgery $i$ can be planned in the planning horizon. Note that if $d w l \leq 0$, the release date of surgery $i\left(r d_{i}\right)$ takes the value $-d w l($ and $d w l=0)$, while if $d w l>0, r d$ is equal to 0 .

| Level of <br> experience, $\boldsymbol{l}$ | Variation Intervals <br> $($ Lower bound $(\boldsymbol{\%})$, Upper bound $(\%))$ | Variation coefficient <br> $(\boldsymbol{\%})$ | Surgery duration, $\boldsymbol{t}_{\boldsymbol{i l}}$ <br> $($ minutes $)$ |
| :--- | :--- | :--- | :--- |
| $\mathbf{0}$ | - | - | 120 |
| $\mathbf{1}$ | $(20,50)$ | 35 | $120 \cdot(1+0.35)=162$ |
| $\mathbf{2}$ | $(-10,20)$ | 10 | $120 \cdot(1+0.1)=132$ |
| $\mathbf{3}$ | $(-30,-10)$ | -20 | $120 \cdot(1-0.2)=96$ |

Table 3.3. An example of the calculation of $\boldsymbol{t}_{i l}$

- $\quad r d_{i}$, release date for performing the surgery of patient $i$ (in days). $r d_{i}$ represents the earliest date in which surgery $i$ can be planned in the planning horizon. Note that if $d w l \leq 0$, the release date of surgery $i\left(r d_{i}\right)$ takes the value $-d w l($ and $d w l=0)$, while if $d w l>0, r d$ is equal to 0 .
- $\quad d_{i}$, deadline for performing a surgery $i$ (in days). $d_{i}$ represents the latest date in which surgery $i$ can be planned in the planning horizon, being determined as the difference between $M T B T_{i}$ and $d w l_{i}$.
- $\tau_{i}$, surgeon in charge of performing the surgery of patient $i$. According to Bridges and Diamond (1999), $\tau_{i}$ must be a faculty surgeon. Therefore, we consider that $\tau_{i}$ is randomly selected from the available faculty surgeons (i.e. surgeons belong to level of experience 3). The procedure used to assign the responsible surgeon is the following: faculty surgeons are randomly sorted, assigning one surgery to each faculty surgeon at random. The procedure finishes when all surgeries in the waiting list have been assigned to any faculty surgeon.
- $\quad \delta_{i j h}$, binary parameter yielding 1 if surgery $i$ can be performed in OR day $(j, h), 0$ otherwise. $\delta_{i j h}$ is used to book OR-days for planning a certain type of surgery or to impose that some surgeries take place only in certain ORs. This parameter is taken into account by some authors (see e.g. Jebali et al., 2006; Pham and Klinkert, 2008; Roland et al., 2010), although the procedure employed in their works is not described. In this work, we use the data available from surgical specialties in University Hospital "Virgen del Rocio", in which there is a kind of surgeries (that make around $10 \%$ of the waiting list) that can be only performed in certain specialized ORs (that make $30 \%$ of the total ORs in the specialty). Therefore, $90 \%$ of the surgeries in the waiting list can be assigned to any OR (multifunctional or specialized), while $10 \%$ have to be performed in specialized ORs.
- $w_{i}$, clinical weight of surgery $i . w_{i}$ is calculated as a linear combination of the normalized values of the medical priority of the patient $\left(m p_{i}\right)$ and the number of days of the patient on the waiting list, i.e. $w_{i}=a \cdot m p^{*}{ }_{i}+(1-a) \cdot d w l^{*}{ }_{i} \cdot m p_{i}$ is generated from a discrete uniform distribution [1,5], being 5 the highest priority. In order to normalize both measures, $m p^{*}{ }_{i}=m p_{i} / 5$ and $d w l^{*}{ }_{i}=d w l_{i} / M T B T_{i}$.


### 3.2.2 Resource data

According to the literature review carried out in Chapter 2 and to the surgical specialties analyzed in the University Hospital "Virgen del Rocio", the main resources required to solve the OR planning and scheduling problem (since in most hospitals represent a bottleneck) are surgeons and ORs.

Regarding surgeons data, the following surgeon parameters are considered in the testbed:

- $m d s_{k}$, maximum number of available days of surgeon $k$ to perform surgeries in a weekly planning horizon. According to the literature, surgeons usually perform surgeries between 3 and 5 days per week (see e.g. Fei et al., 2009). In this work, $m d s_{k}$ can be drawn from a uniform distribution [3,5] or setting to a constant value (e.g. 3 or 4 days). If the planning horizon is lesser than a week, then surgeons are assumed to be fully available.
- $a_{k h}$, maximum available surgery time of surgeon $k$ to perform surgeries on day $h$. We assume that, for each surgeon, $a_{k h}$ can be randomly and uniformly taken from the set $\{240,360,480\}$ (see e.g. Fei et al., 2009; Marques et al., 2012; Pham and Klinkert, 2008) or setting to a constant value (e.g. 480 minutes, see Hans et al., 2008; Lamiri et al., 2009).

It is a common practice at the surgical specialties of the University Hospital "Virgen del Rocio" constructs a weekly schedule that specifies who surgeons are available for performing surgeries on each day. In addition, for each surgeon, the surgery time available for performing surgeries is specified. The reason of constructing a weekly schedule is that facilitates the integration with other tasks performed by surgeons in the specialty (as are doing consultations or looking after patients operated). The following three-step procedure is used to generate this schedule:

- In the first step, for each day, the number of surgeons equals to the number of ORs is randomly allocated, avoiding that an OR-day is idle.
- In the second step, a set of days are randomly assigned to each surgeon without exceeding $m d s_{k}$.
- Finally, in the third step, if the planning horizon is longer than a week, we consider the weekly schedule as a cycle schedule for each week in the planning horizon.

Regarding OR data, the following parameters are considered:

- $r_{j h}$, regular capacity of OR-day $(j, h)$ (in minutes). We consider a regular capacity of 8 hours for any OR-day (see e.g. Lamiri, Xie, Dolgui et al., 2008; Lamiri, Xie and Zhang, 2008).
- $o_{j h}$, overtime capacity of OR-day $(j, h)$. We consider an overcapacity of 4 hours for any OR-day (see e.g. Guinet and Chaabane, 2003).


### 3.3 Factors and levels

The main factors and the levels taken into account to build a testbed for solving the OR planning and scheduling problems are (see Table 3.4):

- $|H|$ : number of days in the planning horizon. Depending on the OR planning and scheduling problem, $|H|$ can vary from a few days to a few weeks (May et al., 2011). 1 and 2-days planning horizons are normally considered for the OR scheduling problem (see e.g. Cardoen et al., 2009a, 2009b; Jebali et al., 2006), while 1 and 2weeks planning horizons are considered for the OR planning problem (see e.g. Fei, Meskens and El-Darzi, 2010; Min and Yih, 2010; Ogulata and Erol, 2003). Finally, a working week planning horizon is also considered for the integrated OR planning and scheduling problem solved in an integrated way (see e.g. Roland et al., 2010).
- $|J|$ : number of ORs.
- $\beta$ : control factor to generate $|I|$. Some papers propose to generate surgeries one by one until the sum of expected surgery durations for the generated surgeries exceeds a proportion $\beta$ of the total OR time available in the whole planning horizon (see e.g.

| Reference | $\|\boldsymbol{H}\|$ | $\|\boldsymbol{J}\|$ | $\boldsymbol{\beta}$ | $\boldsymbol{\alpha}$ | $\boldsymbol{C V}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| (Dexter, Macario and Traub, 1999) | 1 | 6,22 | 1.00 | - | - |
| (Fei et al., 2007) | 5 | NS | - | - | 0.13 |
| (Fei et al., 2008) | 5 | 4 | - | - | 0.54 |
| (Fei et al., 2009) | 5 | 6 | - | $1.0,1.3$ | 0.17 |
| (Fei, Meskens and Chu, 2010) | 5 | 6 | - | - | - |
| (Guinet and Chaabane, 2003) | 5 | $1,2,3$ | - | - | - |
| (Hans et al., 2008) | 1,5 | 16 | - | $-1.1,1.7$ | 0.3 |
| (Jebali et al., 2006) | 1 | 3 | - | - | $0.1 \ldots 0.5$ |
| (Lamiri et al., 2007) | 5 | 3,6 | 0.75 | 0.4 |  |
| (Lammiri, Xie and Zhang, 2008) | 5 | $3,6,9,12$ | $0.75,1.00$ | - | 0.4 |
| (Lamiri, Xie, Dolgui et al., 2008) | 5 | 2 | 1.00 | - | 0.4 |
| (Lamiri et al., 2009) | 5 | $4,8,12$ | $0.85,1.00$ | - | 0.2 |
| (Liu et al., 2011) | 5 | 6 | - | $1,1.3$ | $0.1 \ldots 0.5$ |
| (Marcon et al., 2003) | 1 | 8 | - | - | $0.4,0.5,0.8$ |
| (Min and Yih, 2010) | 5 | 10 | - | - | - |
| (Ogulata and Erol, 2003) | 5 | 2 | - | - | - |
| (Ozkarahan, 2000) | 10 | 7 | - | - | $0.1,0.4$ |
| (Pham and Klinkert, 2008) | 2 | 2 | 0.71 | - |  |
| (Augusto et al., 2010) | 1,2 | $2,4,6$ | - | - | 0.6 |
| (Riise and Burke, 2010) | 1 | 5 | $3.5,4.3, \ldots, 6.4,7.2$ | - | $0.5 \ldots .8$ |
| (Roland et al., 2010) | 1,5 | 7 | $0.70,0.80$ | - | $0.5 \ldots 0.8$ |
| (Marques et al., 2012) | 5 | $1,5,6$ | $1.1,1.4, \ldots, 2.0,2.2$ | - | - |
| (Marques et al., 2014) | 4,5 | 6 | $0.9,1.1, \ldots, 7.2,7.4$ | - | 0.5 |
| (Meskens et al., 2013) | 1 | 4 | - | - | $0.1 \ldots 0.3$ |
| (Vijayakumar et al., 2013) | 2,5 | 2,5 | $0.5,0.9$ | 0.2 |  |
| (Zhao and Li, 2014) | 1 | $3,4,5$ | $0.5,0.6, \ldots, 1.3,1.5$ | - | - |
| (Hashemi Doulabi et al., 2014) | 5 | 6 | $0.5,1.0,1.5$ | - | - |
| (Van Huele and Vanhoucke, 2014) | 5 | 3 | 3.7 | - |  |

Table 3.4. Factors and levels for designing a testbed

Dexter, Macario and Traub, 1999; Lamiri et al., 2009). $\beta$ values, shown in Table 3.4, have been calculated using the expression $\beta=\frac{\sum_{i \in \epsilon} t_{i 0}}{\sum_{h \in H} \sum_{j \in J} r_{j h}}$ and data provided by authors.

- $\alpha$ : control factor to generate $|K|$. In this work, we determine the number of surgeons available for performing surgeries during a weekly planning horizon. A common procedure employed in the literature is used to determine the number of surgeons (Beliën and Demeulemeester, 2007). For each level of experience, the total surgeon time required to perform all assigned surgeries in the waiting list $\left(S_{l}\right)$ is first determined. More specifically, the surgeon time required to perform surgery $i$ when surgeon type $l$ is involved can be calculated as the quotient between the surgery duration required by level experience $l\left(t_{i l}\right)$ and the number of levels that can perform the surgery (in order to avoid surgeon overcapacity). Therefore, $S_{l}$ is calculated by the following expression:

$$
S_{l}=\alpha \sum_{i \in I}\left(\frac{\gamma_{i l} \cdot t_{i l}}{\sum_{l \in L} \gamma_{i l}}\right)
$$

Then, the number of surgeons is generated one by one until the total availability generated exceeds $S_{l}$. For each surgeon, $m d s_{k}$ and $a_{k h}$ is generated as described in Section 3.2.2. $\alpha$ values, shown in Table 3.4, have been calculated using the expression of $S_{l}$ and data provided by authors.

- $C V$ : the coefficient of variation of surgery duration. $C V$ is defined as the ratio of the standard deviation $(\sigma)$ to the mean $(\mu)$. By using $C V$, we are able to analyze the effects of homogeneous (low values) and heterogeneous (high values) waiting lists with respect to surgery duration on the OR planning problem. The coefficient of variation is randomly generated from an interval [0.1...0.5] (see e.g. Lamiri et al., 2007) or setting to a constant value (e.g. 0.5).


### 3.4 Conclusions

The contribution of this Chapter is to propose a testbed generator to create instances for analyzing OR planning and scheduling problems. The procedure is based on the literature and on surgical specialties of the University Hospital "Virgen del Rocio". We distinguish between parameters (i.e. the data involved in the decision models) and factors (that define the size and the characteristics of an instance). The testbed generator has been coded in the C programming language, and it has been used to generate the testbeds employed in the Thesis. Table 3.5 shows the levels of the factors used in every Chapter to generate the testbed. The size of the problem (in terms of the waiting list size) and the number of instances generated are presented. The testbed is available at http://taylor.us.es/componentes/jmmp.

| Factor/Parameter | Deterministic OR <br> planning <br> (Chapter 4) | Stochastic OR <br> planning <br> (Chapter 5) | Integrated OR <br> planning/scheduling <br> (Chapter 6) | Real Surgical <br> Specialty OR <br> planning <br> (Chapter 7) |
| :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{H} \mid$ | 5 | 5 | $1,2,5$ | $5,10,20,40,60$ |
| $\|\boldsymbol{J}\|$ | 3,9 | 4,8 | $3,6,9$ | 4,8 |
| $\boldsymbol{\beta}$ | $1.00,1.25$ | $1.00,1.25$ | $0.75,1.00,1.25$ | $1.00,1.25$ |
| $\boldsymbol{\alpha}$ | $1.5,2.0$ | $1.5,2.0$ | $1.5,2.0$ | $1.5,2.0$ |
| $\boldsymbol{C} \boldsymbol{V}$ | Ran $[0.1 \ldots 0.5]$ | $0.1,0.5$ | $\operatorname{Ran}[0.1 \ldots 0.5]$ | 0.1 |
| $\boldsymbol{u}$ | $1,\|J\|$ | $\|J\|$ | $\|J\|$ | $\|J\|$ |
| $\|\boldsymbol{I}\|$ | $50,61,146,182$ | $80,102,161,202$ | $10,20, \ldots, 222,294$ | $81,100, \ldots, 1925,2400$ |
| $\|\boldsymbol{K}\|$ | $6,8, \ldots, 23,30$ | $8,10,15,20$ | $10,20,30, \ldots, 62,75$ | 8,16 |
| $\boldsymbol{\mu}$ | $\operatorname{Ran}[60,120,180,240]$ | 120 | $\operatorname{Ran}[60,120,180,240]$ | 120 |
| $\boldsymbol{m d} \boldsymbol{\operatorname { s i n }}$ | 3,4 | $\operatorname{Ran}[3,4,5]$ | $\operatorname{Ran}[3,4,5]$ | $\operatorname{Ran}[3,4,5]$ |
| $\boldsymbol{a}$ | 480 | 480 | $\operatorname{Ran}[240,360,480]$ | 480 |
| Instances | 320,960 | 160 | 1080 | 120 |

Table 3.5. Factors and levels for the proposed OR planning and scheduling problems

## Part II

## Decision Problems

and

## Solution Procedures

## Chapter 4

## New Heuristics for the Operating Room Planning Problem

### 4.1 Introduction

In this chapter, we tackle a OR planning problem in which an intervention date and an OR are assigned to a set of surgeries on the waiting list, minimizing access time for patients with diverse clinical priority values. The clinical priority depends on the surgery priority and the number of days spent on the waiting list. Section 4.2 presents the problem formulation. We propose a set of 83 heuristics ( 81 constructive heuristics, a composite heuristic and a meta-heuristic) based on a new encoding of the solution (Section 4.3), and we compare these against existing heuristics from the literature for solving OR planning problems (Section 4.4). The heuristics are adapted to the problem under consideration (i.e. considering all constraints and the new objective function), being re-implemented using the information provided by the authors. In total, after a calibration procedure, we compare 17 heuristics. The computational experiments show that our proposed meta-heuristic is the best for the problem under consideration. Finally, conclusions are presented in Section 4.5.

### 4.2 Problem formulation

In the OR planning problem we consider a set $H$ of planning days ( $h=1 \ldots|H|$ ) where there is a set $J$ of parallel ORs $(j=1 \ldots|J|)$ available on each day $h$ in the planning horizon. The regular capacity of the $j$-th OR during day $h$ is denoted by $r_{j h}$. In the following, a pair $(j, h)$ is denoted as OR-day. Recovery facilities are also assumed to be always available during the planning horizon. There is a set $K$ of surgeons $(k=1 \ldots|K|)$ and, on each day $h$, each surgeon $k$ has a maximum available time $\left(s_{k h}\right)$ to perform surgeries. The remaining human and material resources are assumed to be available whenever needed. In this setting, a set $I$ of elective surgeries (i.e. patients) are in the waiting list $(i=1 \ldots|I|)$.

| Indices and Sets |  |
| :---: | :---: |
| $h \in H$ | Set of time periods within the planning horizon for perioperative resources |
| $i \in I$ | Set of patients (surgeries) on the waiting list |
| $j \in J$ | Set of ORs |
| $k \in K$ | Set of surgeons |
| Parameters |  |
| $r_{j h}$ | Regular capacity of OR $j$ on day $h$ (in minutes) |
| $a_{k h}$ | Regular capacity of surgeon $k$ on day $h$ (in minutes) |
| $u_{k}$ | Non-negative integer number of ORs in which surgeon $k$ can perform surgeries within the same day |
| $\tau_{i}$ | Surgeon in charge of patient $i$ |
| $r d_{i}$ | Release date for performing the surgery on patient $i$ |
| $d_{i}$ | Deadline for performing the surgery on patient $i$ |
| $\delta_{i j h}$ | Binary parameter yielding 1 if surgery of patient $i$ can be performed in OR $j$ on day $h$; 0 otherwise |
| $t_{i}$ | Expected time of surgery $i$ (in minutes) |
| $w_{i}$ | Clinical weight of surgery $i$ |
| Variables |  |
| $X_{i j h}$ | 1 if patient $i$ is to be operated in OR $j$ on day $h ; 0$ otherwise |
| $Z_{k j h}$ | 1 if surgeon $k$ is allocated to $\mathrm{OR} j$ on day $h ; 0$ otherwise |

Table 4.1. Sets, data and variables used in the ILP decision model

Each surgery $i$ should be performed before a given deadline $\left(d_{i}\right)$ according to the disease's characteristics and the waiting time in the waiting list. The binary parameter $\delta_{i j}$ yields 1 if surgery $i$ can be performed in OR $j, 0$ otherwise. Many realistic situations can be modeled with this parameter such as, for example, to book OR-days in order to plan a certain type of surgery or to forbid the assignment of a surgery to an OR that does not have the equipment required to perform this specific surgery.

Below, we present the ILP model to solve the OR planning problem of the Plastic Surgery and Major Burns Specialty. Table 4.1 summarizes sets, data and variables used in the decision model.

The objective function is

Maximize $\sum_{n \in H} \frac{1}{h}\left(\sum_{i \in I} \sum_{j \in J} w_{i} X_{i j n}\right)$

And the constraints are:

$$
\begin{align*}
& \sum_{j \in J} \sum_{h \in H} X_{i j h} \leq 1 \quad(\forall i \in I)  \tag{4.2}\\
& \sum_{j \in J} \sum_{h=1}^{r d_{i}-1} X_{i j h}=0 \quad(\forall i \in I)  \tag{4.3a}\\
& \sum_{j \in J} \sum_{n \in H} X_{i j h}=1 \quad\left(\forall i \in I\left|d_{i} \leq|H|\right)\right.  \tag{4.3b}\\
& \sum_{i \in I} t_{i} X_{i j h} \leq r_{j h} \quad(\forall j \in J, \forall h \in H)  \tag{4.4}\\
& \sum_{j \in \epsilon} \sum_{i \in I} t_{i} X_{i j h} \leq a_{k h} \quad(\forall k \in K, \forall h \in H)  \tag{4.5}\\
& \sum_{j \in J} Z_{k j h} \leq u_{k} \quad(\forall k \in K, \forall h \in H)  \tag{4.6}\\
& \sum_{i \in J} t_{i} X_{i j h} \leq r_{j h} Z_{k j h} \quad(\forall k \in K, \forall j \in J, \forall h \in H)  \tag{4.7a}\\
& \tau_{i}=k
\end{aligned}, \begin{aligned}
& \sum_{i \in I} t_{i} X_{i j h} \geq Z_{k j h} \quad(\forall k \in K, \forall j \in J, \forall h \in h)  \tag{4.7b}\\
& \tau_{i}=k  \tag{4.8}\\
& X_{i j h}=0 \quad\left(\forall i \in I, \forall j \in J, \forall h \in H \mid \delta_{i j}=0\right)  \tag{4.9}\\
& X_{i j h} \in\{0,1\} \quad(\forall i \in I, \forall j \in J, \forall h \in H)  \tag{4.10}\\
& Z_{k j h} \in\{0,1\} \quad(\forall k \in K, \forall j \in J, \forall h \in H)
\end{align*}
$$

The objective function (4.1) maximizes the service level of a surgical specialty prioritizing patients with higher values of w. The service level of a planned surgery is defined as the quotient between the clinical weight and the planned date. Note that, if a surgery is not planned within the planning horizon (therefore its planned date is equal to 0 ), then the value of the service level is unbounded. In order to avoid such unbounded solutions, we introduce the parameter h in the objective function to capture the planned date. $h$ represents the planned date for a scheduled surgery (at least one $X_{i j h}=1$ ), excluding unscheduled surgeries (all $X_{i j h}=0$ ). Constraints (4.2) enforce that each surgery is scheduled at most once during the planning horizon. The set of constraints (4.3a) and (4.3b) define the earliest and the latest date where a patient can be scheduled.

Constraints (4.3a) prohibit that the patient is scheduled before the release date, while constraints (4.3b) ensure that the surgery of a patient with a deadline within the planning horizon must take place before his/her latest date. Constraints (4.4) prohibit that the total amount of OR time assigned to surgeons in an OR-day is higher than its regular capacity. Constraints (4.5) prohibit that the total amount of time allocated to a surgeon is higher than his/her capacity in any day. Constraints (4.6) limit the number of OR-days that can be assigned to a surgeon in a day. The set of constraints (4.7a) and (4.7b) define whether a surgeon is allocated to an OR-day. Constraints (4.8) ensure that each surgery is carried out in a suitable OR-day. Finally, constraints (4.9)-(4.10) are binary constraints for decision variables.

### 4.3 Heuristics

In view of the NP-hard nature of the procedures employed for solving the ILP model, it is foreseeable that optimal solutions can only be obtained for relatively small problems (see the computational experience carried out in Section 4.4.2). Therefore, a novel encoding is proposed for solving the advance OR scheduling problem. A surgical schedule is encoded into a permutation vector $\pi$ and a bin packing (BP) operator, where $\pi$ represents a certain order of the surgeries in the waiting list, and it is determined considering the prioritization of surgeries with dude dates within the planning horizon. The BP operator is the algorithm used to allocate surgeries to OR-days, integrating the constraints (4.2)-(4.8) of the decision model. The following BP operators can be considered (see e.g. Dexter, Macario, Traub et al., 1999; Dexter, Macario and Traub, 1999):

- Next Fit (NF): the surgery is planned in the last OR-day occupied, if possible. Otherwise, the surgery is planned into the next available OR-day.
- First Fit (FF): the surgery is planned in the first OR-day where it fits.
- Best Fit (BF): the surgery is planned in the OR-day that has the least amount of available time and it fits.
- Level Fit (LF): the surgery is planned on the OR-day that has the most amount available time and it fits.

The following sections present a set of constructive heuristics, a composite heuristic and a meta-heuristic for solving the proposed OR planning problem.

### 4.3.1 Constructive heuristics

In the constructive heuristics, a permutation sequence $\pi$ composed of an order of the patients in the waiting list is constructed in two stages which simultaneously take into account the fulfillment of time period constraints (i.e. constrains 4.1 and 4.2) and the objective function.

- In stage I, a partial sequence is determined by only sorting patients whose deadline falls within the planning horizon in increasing order of deadlines, in order to fulfill time period constraints.
- In stage II, the remaining patients in the waiting list are added at the end of this partial vector according to $\left(S_{I}, S_{C}\right)$. $S_{I}$ is the sorting indicator, which is the parameter used to sort patients (surgery duration, clinical priority...), while $S_{C}$ is the sorting criterion which indicates how surgeries are sorted according to $S_{I}$ (descending, ascending...).

The following two types of constructive heuristics are considered: (1) Single-Tuple (ST) method in which one permutation sequence is considered applying a sorting tuple ( $S_{I}$, $S_{C}$ ), and (2) Multiple-Tuple (MT) method in which a set of sorting tuples ( $n^{S I}$ indicators and $n^{S C}$ criteria) are simultaneously considered, resulting $n^{S I} \cdot n^{S C}$ permutation sequences.

Regarding the sorting indicator $\left(S_{I}\right)$, the options usually considered are:

- $t$ : surgery duration (see e.g. Dexter, Macario and Traub, 1999; Hans et al., 2008).
- $d$ : deadline (see e.g. Fei et al., 2009).
- w: clinical weight (Ogulata and Erol, 2003; Ozkarahan, 2000).
- ran: random sorting. This is equivalent to not sorting the surgeries with deadlines outside the planning horizon.

| $\begin{aligned} & \text { P:1 } \\ & (240,95) \end{aligned}$ | $\begin{aligned} & \mathrm{P}: 2 \\ & (120,85) \end{aligned}$ | $\begin{aligned} & \text { P: } 3 \\ & (260,90) \end{aligned}$ | $\begin{aligned} & \text { P:4 } \\ & (180,80) \end{aligned}$ | $\begin{aligned} & \text { P:5 } \\ & (180,100) \end{aligned}$ | $\begin{aligned} & \text { P-6 } \\ & (320,85) \end{aligned}$ | $\begin{aligned} & \text { P:7 } \\ & (260,75) \end{aligned}$ | $\begin{aligned} & \text { P:8 } \\ & (220,70) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

Applying $\left(S_{I}, S_{C}\right)=(w, D E C)$

|  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $P: 5$ |  |  |  |  |  |
| $(180,100)$ | $P: 1$ |  |  |  |  |
| $(240,95)$ | $(260,90)$ | $P: 6$ <br> $(320,85)$ | $P: 4$ <br> $(120,85)$ | $(180,80)$ | $(260,75)$ |




OF Value $=363.6$


OF Value $=369.4$


OF Value $=351.8$

Figure 4.1. An example of a constructive heuristic

The sorting criteria $\left(S_{C}\right)$ that can be considered (see e.g. Framinan et al., 2003; Marcon and Dexter, 2006) are the following:

- INC: sorts the surgeries according to increasing values of indicator $S_{I}$.
- DEC: sorts the surgeries according to decreasing values of indicator $S_{I}$.
- HILL: sorts the surgeries as a "hill": i.e. high values of indicator $S_{I}$ in the middle of the waiting list and low figures in the beginning and in the end.
- VALLEY: sorts the surgeries as a "valley", i.e. low values of indicator $S_{I}$ in the middle of the waiting list and high figures in the beginning and in the end.
- LOHI: sorts the surgeries by choosing one surgery with a low value of indicator $S_{I}$ and one with a high value alternately.
- HILO: sorts the surgeries by choosing one surgery with a high value of indicator $S_{I}$ and one with a low value alternately.

Figure 4.1 shows an example of a ST method that applies the sorting tuple (w, DEC). Note that each box represents a surgery, specifying the surgery duration and the clinical weight $\left(t_{i}, w_{i}\right)$.

### 4.3.2 Composite heuristics

In the composite heuristic $\left(\mathrm{C}_{\mathrm{m}}\right), \pi$ is constructed based on the well-known heuristic proposed by Nawaz et al. (1983) and on the idea of re-inserting scheduled jobs (Rad et al., 2009) for the permutation flowshop scheduling problem. Starting from the best surgical schedule obtained using a constructive heuristic (see previous section), an initial permutation sequence ( $\pi_{\text {ini }}$ ) is determined by sorting patients in ascending order of planned date, being the unplanned patients added at the end of the vector. $\pi$ is constructed according to the following two steps (see Figure 4.2):

- Constructive step. For a surgery in position $l$ in $\pi_{i n i}$, this step consists of obtaining the best position $p$ to insert the surgery in the partial sequence composed by the previous $l-1$ surgeries, keeping the relative order of the last ones. Among these partial sequences, sequence $\pi_{l}$ yielding the best value of the objective function is selected.
- Bounded local search step. Considering one by one the surgeries placed in the $m$ positions around position $p$ in $\pi_{l}$ (i.e. the positions from $\max (1, p-m)$ to $\min (l$, $p+m)$ ), this step consists of inserting the surgery in all possible positions keeping the relative order of the $l-1$ surgeries, selecting the position yielding the best value of the objective function.


### 4.3.3 Random extraction-insertion meta-heuristic

The Random Extraction-Insertion algorithm (REI) is an iterated greedy local search based on the algorithm proposed by Ruiz and Stützle (2007) for the permutation flowshop scheduling problem. $\pi_{i n i}$ is constructed following the procedure used in $\mathrm{C}_{\mathrm{m}}$. The general procedure for determining $\pi$ from an incumbent permutation vector $\left(\pi_{i n c}\right)$ is composed by the following two steps (see Figure 4.3):

- Destruction step. It consists of randomly removing $n$ surgeries $\left(\pi_{d e s}\right)$ from $\pi_{i n c}$, obtained a permutation vector $\pi_{|| |-n}$ composed by $|I|-n$ surgeries.


Iteration $l=6$




Figure 4.2. An example of the composite heuristic $C_{2}$ starting form the best solution obtained by the constructive heuristic, $\mathrm{ST}(w, D E C, \mathrm{BF})$

| $\pi_{i n i}$ | $\begin{aligned} & \text { P:5 } \\ & \text { (180, 100) } \end{aligned}$ | $\begin{aligned} & \text { P:1 } \\ & \text { (240, 95) } \end{aligned}$ | $\begin{aligned} & \text { P:3 } \\ & (260,90) \end{aligned}$ | $\begin{aligned} & \text { P:4 } \\ & \text { (180, 80) } \end{aligned}$ | $\begin{aligned} & \text { P. } 6 \\ & (320,85) \end{aligned}$ | $\begin{aligned} & \text { P:2 } \\ & (120,85) \end{aligned}$ | $\begin{aligned} & \text { P:7 } \\ & (260,75) \end{aligned}$ | $\begin{aligned} & \text { P:8 } \\ & \text { (220. 70) } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |



Construction Step
$P_{8}$, best position 6

$\mathrm{P}_{2}$, best position 1

| $P: 2$ | $P: 5$ |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $(120,85)$ | $(180,100)$ | P:1 <br> $(240,95)$ | $P: 4$ <br> $(180,80)$ | P:6 <br> $(320,85)$ | $P: 7$ <br> $(260,75)$ | $P: 8$ <br> $(220,70)$ |

$P_{3}$, best position 5

| $P: 2$ |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $(120,85)$ | P:5 |  |  |  |  |  |
| $(180,100)$ | P:1 <br> $(240,95)$ | P:4 <br> $(180,80)$ | P:3 <br> $(260,90)$ | P:6 <br> $(320,85)$ | P:7 <br> $(260.75)$ | P:8 |

$\pi$


Figure 4.3. An example of the REI meta-heuristic starting form the best solution obtained by the constructive heuristic $\mathrm{ST}(w, D E C, \mathrm{BF})$

- Construction step. For a surgery in position $k$ in $\pi_{\text {des }}$, it consists of determining the best position to insert the surgery in $\pi_{|l|+k-1}$, keeping the relative order of the $|I|+k-1$ surgeries. The permutation vector $\pi_{|l|+k-1}$ that yields the best value of the objective function is selected.

The resulting sequence $\pi$ is considered as the new $\pi_{i n c}$, and therefore the best permutation vector, if the objective function value improves the best value obtained so far. A simulated annealing-like acceptance criterion with a constant temperature is implemented to avoid the stagnation in the search procedure. The constant temperature is set so that moves that deteriorate the solution more than a percentage $\theta$ of the maximal deterioration are accepted with a probability smaller than $\varphi$ (Lamiri et al., 2009). The termination criterion of REI is determined based on the size of the problem (the length of the planning horizon, the number of ORs and the number of surgeries on the waiting list).

### 4.4 Computational evaluation

An extensive computational experiments of the ILP decision model, the proposed heuristics and the adapted ones for solving the proposed advance OR scheduling problem is presented. Section 3.3.1 presents a Design of Experiments (DOE) approach carried for the calibration of ST, MT, $\mathrm{C}_{\mathrm{m}}$ and REI algorithms. Then, in Section 3.3.2, the effectiveness (in terms of the proportion of feasible solutions and the quality of the solution) of the proposed heuristics and of the adaptation of the existing heuristics is evaluated.

### 4.4.1 Calibration procedure

We have generated a 320 -instances calibration testbed using the procedure given in Chapter 3. We have considered 32 different combinations of factors (see Table 4.2) and, for each combination, we have generated 10 instances.

| Factor | Level |
| :--- | :--- |
| $\|\boldsymbol{H}\|$ | 5 |
| $\|\boldsymbol{J}\|$ | 3,9 |
| $\boldsymbol{\beta}$ | $1.00,1.25$ |
| $\boldsymbol{\alpha}$ | $1.5,2.0$ |
| $\boldsymbol{C} \boldsymbol{V}$ | Ran $[0.1 \ldots 0.5]$ |
| $\boldsymbol{\mu}$ | Ran $[60,120,180,240]$ |
| $\boldsymbol{m} \boldsymbol{d} \boldsymbol{s}$ | 3,4 |
| $\boldsymbol{a}$ | 480 |
| $\boldsymbol{u}$ | $1,\|J\|$ |

Table 4.2. Factors and levels considered in the OR planning problem

In order to select the best among each type of algorithm, we have considered two response variables: feasibility of the constructed solution, and Relative Percentage Deviation (RPD), according to the expression RPD $=\left(\right.$ Best $\left._{\text {sol }}-H e u_{\text {sol }}\right) /$ Best $_{\text {sol }}$. 100, where $H e u_{\text {sol }}$ is the solution given by any of the tested constructive heuristics and Best $_{\text {sol }}$ is the best solution found so far (either the optimum, or the best (highest) lower bound for a given generated instance). In our case, Best $_{\text {sol }}$ has been obtained for each instance by solving the related ILP model using the commercial software Gurobi version 4.5 .1 with a CPU time limit of 900 seconds. The experiment was analyzed by means of a multi-factor Analysis of Variance (ANOVA) technique with a $95 \%$ confidence level.

The procedure employed for the calibration of the different algorithms is the following:
I. We select the level(s) of the most significant factor yielding statistically significant differences with respect to feasibility, i.e. we select the sorting tuple(s) or BP algorithm(s) that obtain a higher number of feasible solutions over the instances in the testbed.
II. Among the instances for which the selected factors in Stage I yield feasible solutions, we select the remaining level/s by taking those that obtain the best (statistically significant) RPD.

Regarding constructive heuristics, ST algorithms are characterized by a permutation vector ordered by a tuple $\left(S_{I}, S_{C}\right)$ and a BP operator. In the following, we denote an ST algorithm as $\mathrm{ST}_{(S I, S C, B P)}$. Note that 19 sorting tuples are considered for each BP algorithm: (i) indicators $t, d$ and $w$ are combined for each sorting criterion (18 sorting tuples) and (ii) the random sorting. Therefore a total of 76 different ST algorithms are
tested. As a result from the experimental analysis, we can conclude that the $\mathrm{ST}_{(w, D E C, F F)}$ is the best. Regarding MT algorithms, the factor is the BP algorithm employed in the construction procedure, so the levels are: $\mathrm{MT}_{N F}, \mathrm{MT}_{F F}, \mathrm{MT}_{B F}$ and $\mathrm{MT}_{W F}$. Note that we have considered the 19 different combinations of $S_{I}$ and $S_{C}$ for each MT algorithm. Finally, we will also consider the $\mathrm{MT}_{A L L}$ algorithm in which all BP algorithms and all sorting tuples are applied to the instance, selecting the combination of BP and ( $S_{I}, S_{C}$ ) yielding the best results for the instance. The analysis shows that $\mathrm{MT}_{\mathrm{ALL}}$ is statistically the best algorithm.

Regarding local search method and meta-heuristic, we select $\mathrm{MT}_{\mathrm{ALL}}$ and FF as the constructive heuristic (employed to determine the initial waiting list) and the BP algorithm (employed to evaluate waiting lists) respectively based on the results obtained for constructive heuristics. As described above, the $\mathrm{C}_{\mathrm{m}}$ algorithm is characterized by the number of re-inserted surgeries $(m) . m$ is used with levels $0,6,12$ and 18 ; yielding $C_{6}$ the best results in terms of RPD and CPU time. Finally, as described above, REI is characterized by the number of extracted surgeries $(n)$, the percentage of the maximal deterioration $(\theta)$ and the probability of accepts a solution which deteriorates a solution $(\varphi)$. REI is tested with the following levels: $n$ is set to 1,3 and $5 ; \theta$ is set to $10 \%$ and $20 \%\}$ and; $\varphi$ is set to $1 \%, 5 \%$, and $10 \%$. The best setting was $n=3, \theta=10 \%$ and $\varphi=$ $1 \%$.

### 4.4.2 Computational experience

In this section we generate a testbed according to the procedure described in Chapter 3, considering the 32 different combinations of factors shown in Table 4.2. For each combination, we have generated 30 instances, resulting in a total of 960 instances. The size of the waiting list depends on the tuple $(|J|, \beta)$, being $50,61,146$ and 182 the average number of surgeries for $(3,1.00),(3,1.25),(9,1.00)$ and $(9,1.25)$ respectively. The experiments were carried out on a PC with 2.80 GHz Intel Core i7-930 processor and 16 GBytes of RAM memory. The 960 instances are solved by the best proposed heuristics obtained in the calibration $\left(\mathrm{ST}_{(w, D E C, F F)}, \mathrm{MT}_{\mathrm{ALL}}, \mathrm{C}_{6}\right.$ and REI) and the following adapted approximate heuristics existing in the literature:

## - Constructive heuristics

- An adaptation of the primal-dual method which is an extension of the Hungarian method proposed by Guinet and Chaabane (2003), referred to as HM. The order of surgeries on the waiting list has an important influence in the performance of HM. Therefore, we select the best sorting tuple ( $S_{I}, S_{C}$ ) using the sorting procedure employed in the proposed constructive heuristics, being a HM algorithm denoted by $\mathrm{HM}_{(S l, S C)}$. The analysis shows that $\mathrm{HM}_{(w, \mathrm{DEC})}$ is the best HM algorithm.
- An adaptation of the method based on dynamic programming proposed by Liu et al. (2011), referred to as DPH. Note that the column generation heuristics proposed by Fei et al. are not included in the comparison, since they are outperformed by DPH proposed by Liu et al. (2011).
- The off-line method of Dexter, Macario and Traub (1999), referred to as OFF. As ST algorithms, an OFF method is characterized by a sorting tuple ( $S_{I}, S_{C}$ ) and a BP algorithm, being denoted as $\operatorname{OFF}_{(S, S, B P)}$. We consider the 19 sorting tuples and BP algorithms proposed in this paper, being $\operatorname{OFF}_{(d, I N C, F F)}$ the best method.


## - Improvement heuristics

- An adaptation of the pair-wise swapping method of Lamiri et al. (2009), referred to as PS. Starting from the surgical schedule obtained by the best constructive heuristic (i.e. $\mathrm{MT}_{\mathrm{ALL}}$ ), the solution improvement consists of swapping two different patients between OR-days. For each iteration patients are considered one by one, determining and performing the exchange which yields the largest improvement and satisfies the constraints. The process stops when the solution cannot improve any more. We include the pair-wise swapping global method, referred to as PSG, based on the local optimization method proposed by Lamiri et al. (2009). For each iteration the largest improvement is selected among all patients' largest improvements, stopping when the solution cannot improve anymore.
- A triplet-wise swapping method, referred to as TS. The main difference between PS and TS is that the solution improvement consists of swapping a pair of patients
scheduled in the same OR-day with a patient scheduled in a different OR-day. As occurred with PS, we include the triplet-wise swapping global method (TSG).
- A hybrid swapping method, referred to as HS. For each iteration PS and TS methods are considered for determining the largest improvement for each patient or pair of patients respectively. A hybrid swapping global method (HSG) is also considered in the comparison.


## - Meta-heuristics

- An adaption of the taboo search proposed by Lamiri et al. (2009), referred to as TABOO. The procedure used to select moves is an iteration of the best improvement heuristic. A calibration procedure is carried out to determine the best swapping heuristic, being HS and HSG algorithms the best ones. We select the HS method due to its lower CPU time required. The taboo list size is set to $|H|$ and the stopping criterion is based on a computation time limit that depends on the size of the problem (the length of the planning horizon, the number of ORs and the number of surgeries on the waiting list).
- An adaptation of the Multi-start method proposed by Lamiri et al. (2009), referred to as MS. This method tries to avoid the problem of getting stuck in a local optimum (Lamiri et al., 2009). Let $S$ be the initial solution at a given iteration, and $R$ the solution provided by HS (the best improvement heuristic) starting from $S$. At the next iteration, a new initial solution $S^{\prime}$ is determined from $R$ by randomly modifying the planned time blocks of some patients. Each patient is selected with probability $1 \%$ according to Lamiri et al. (2009). If a patient is selected, a randomly feasible exchange with another patient is carried out (swapping ORdays). In order to compare among the other heuristics, the stopping criterion is modified by defining a computation time limit depending on the size of the instance.
- An adaptation of the simulated annealing method of Lamiri et al. (2009), referred to as SA. The procedure to build a new solution at any iteration is applied a random exchange to the solution obtained at the previous iteration. The patient (pair-wise) or the pair of patients (triplet-wise) is randomly chosen, selecting the exchange which yields the largest improvement in any case. The cooling factor is
set to 0.95 and the temperature is reduced after $|I|$ iterations. These values result after a calibration procedure using the values proposed by Lamiri et al. (2009). As in MS, the stopping criterion is based on a computation time limit depending on the size of the instance.
- A simulated annealing method in which the temperature is considered as a constant parameter, referred to as $\mathbf{S A}_{\mathbf{C}}$. The temperature is determined as in REI. $\theta$ and $\varphi$ are set to $\{10 \%, 20 \%\}$ and $\{1 \%, 5 \%, 10 \%\}$ respectively, yielding $10 \%$ and $1 \%$ the best results. As in SA, the stopping criterion is based on a computation time limit.

The results of the experiments for the advance scheduling ( $\left.u_{s}=|J|\right)$ and for the integrated approach ( $u_{s}=1$ ) are shown in Figure 4.4. The mean RPD and computation time values are obtained by averaging these results only for feasible solutions obtained by the heuristics. The computation time limit for meta-heuristics is fixed to $|I| \cdot|J| \cdot|H| \cdot$ $\eta$ seconds for meta-heuristics. $\eta$ is a time factor, which is set to 0.0125 and 0.025 . The value leading to the best results for every meta-heuristic is $\eta=0.025$, not being the difference big enough to consider a double computation time. Therefore, we only include the results for $\eta=0.0125$. Note that the solutions obtained by the heuristics are compared to the solution obtained for each instance by solving the related ILP model using the commercial software Gurobi version 4.5.1.

For each level of factor $|J|$ (the most influential on the performance of the methods), Table 4.3 shows the minimum, the maximum and the average GAP (i.e. (Best ${ }_{\text {bound }}-$ Solution)/Best ${ }_{\text {bound }} \cdot 100$ ) for the advance scheduling problem and the integrated approach. In addition, the percentage of optimal solutions and the average CPU time values are presented. The analysis shows that DPH and $\mathrm{MT}_{\mathrm{ALL}}$ are statistically the best constructive heuristics for the advance scheduling problem and the integrated approach respectively. However, we can conclude that the $\mathrm{MT}_{\mathrm{ALL}}$ heuristic as the best constructive heuristic, because of the reduction on the feasible solutions obtained by DPH (only $79 \%$ of feasible solutions) and the larger computation time required. Regarding improvement heuristics, $\mathrm{C}_{6}$ is statistically the best algorithm for both planning problems ( $3.9 \%$ and $8.7 \%$ respectively), with not much more computation time required ( 16.15 and 22.89 seconds respectively). It is important to point out that $\mathrm{C}_{6}$ outperforms the adaptations of the existing meta-heuristics for the integrated approach.


Means and 95.0 Percentage Tukey Intervals


Figure 4.4. Feasibility, CPU time and RPD results for algorithms

| Problem | $\|J\|$ | Min. <br> GAP $(\%)$ | Max. <br> GAP $(\%)$ | Average <br> GAP (\%) | Optimal <br> solutions (\%) | CPU time <br> (seconds) |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Advance | 3 | 0.00 | 1.18 | 0.17 | 57.0 | 473 |
| Scheduling | 9 | 0.23 | 0.72 | 0.72 | 0.0 | 900 |
|  | Average |  |  |  |  |  |
| Integrated | 3 | 0.00 | 1.52 | 0.45 | 28.5 | 686.2 |
| approach | 9 | 1.22 | 4.87 | 3.17 | 65.7 | 437 |
|  | Average |  |  |  |  |  |
|  |  |  | 1.64 | 0.0 | 900 |  |

Table 4.3. ILP approach performance for solving the off-line decision level

| Problem | $\|J\|$ | Heuristic | $\begin{aligned} & \text { Min. } \\ & \text { RPD (\%) } \end{aligned}$ | Max. RPD (\%) | Average RPD (\%) | Solutions RPD < $1 \%$ <br> (\%) | CPU time (seconds) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Advance Scheduling | 3 | $\mathrm{MT}_{\text {ALL }}$ | 2.59 | 17.54 | 9.75 | 0 | 0.02 |
|  |  | $\mathrm{C}_{6}$ | 0.20 | 12.00 | 3.60 | 1.7 | 0.51 |
|  |  | REI | 0.00 | 4.65 | 0.82 | 70 | 10.31 |
|  | 9 | $\mathrm{MT}_{\text {ALL }}$ | 5.34 | 16.67 | 10.35 | 0 | 0.30 |
|  |  | $\mathrm{C}_{6}$ | 1.19 | 9.97 | 4.22 | 0 | 31.8 |
|  |  | REI | 0.47 | 6.63 | 2.55 | 2.9 | 92.12 |
| Integrated approach | 3 | $\mathrm{MT}_{\text {ALL }}$ | 9.40 | 27.54 | 17.80 | 0 | 0.02 |
|  |  | $\mathrm{C}_{6}$ | 1.13 | 16.34 | 7.15 | 0 | 0.58 |
|  |  | REI | 0.00 | 7.28 | 2.64 | 14.6 | 10.27 |
|  | 9 | $\mathrm{MT}_{\text {ALL }}$ | 17.86 | 30.04 | 24.14 | 0 | 0.35 |
|  |  | $\mathrm{C}_{6}$ | 5.28 | 23.76 | 10.35 | 0 | 45.21 |
|  |  | REI | 4.93 | 24.96 | 13.00 | 0 | 92.21 |

Table 4.4. Heuristic performance for solving the off-line decision level

Regarding meta-heuristics, the results show that there are statistically significant differences between the REI algorithm and the remaining meta-heuristics for the advance scheduling problem and the integrated approach, yielding $1.7 \%$ and $7.8 \%$ of RPD values respectively. Finally, as described above, the number of ORs is the most influential factor on the performance of heuristics for off-line decision problems, especially for the integrated approach.

Table 4.4 shows the minimum, the maximum, and the average RPD for the best heuristics in the manuscript (constructive, improvement and meta-heuristic), along with the percentage of solutions with RPD values less than $1 \%$ and the average CPU times. Finally, Figure 4.5 shows the average RPD values of the proposed heuristics to solve the off-line decision problems for each level of $|J|$.


Figure 4.5. Influence of the number of ORs on the off-line decision problems

### 4.5 Conclusions

In this chapter, we have analyzed the advance OR scheduling problem on the off-line operational decision level. The problem consists of assigning an intervention date and OR to surgeries on the waiting list over a given planning horizon, taking into account the following constraints: resources availability (OR and surgeon), time period (a surgery must be performed between a release date and a deadline), eligibility (surgeries must be performed on a suitable OR) and resources assignment (surgeons has limited the number of ORs in where the can operated during a day). The objective function is related to minimizing access time for patients with higher clinical weight values (defining based on the priority of the patient --surgery's urgency-- and the number of days spent on the waiting list at the time of the planning).

A set of approximate methods have been proposed for solving the problem under consideration. To show the efficiency of our proposed heuristics, we have adapted
existing heuristics to the problem and compare them using a testbed we have developed based on the literature. In total, we have compared 17 efficient heuristics (i.e. the best parameters of any method have been selected by a calibration procedure). The computational experiments show that the proposed heuristics statistically outperform existing ones in the literature for every type of heuristic proposed (constructive, improvement and meta-heuristic).

## Chapter 5

## The Stochastic Operating Room Planning Problem

### 5.1 Introduction

In this chapter, we address a stochastic OR planning problem which consists of assigning an intervention date and OR to a set of surgeries on the waiting list, minimizing the unexploited OR time and overtime costs. Uncertainty in surgeries duration and in the arrivals of emergency surgeries and in surgeons' capacity is considered. To solve the problem we present a stochastic mathematical model (Section 5.2) and a Monte Carlo optimization method based on the SAA method (Section 5.3), which combines an iterative greedy local search (IGLS) method (Section 5.4) and Monte Carlo simulation. The performance of the IGLS method is evaluated against an exact method and two existing heuristics for solving the deterministic version of the problem, using a testbed generated based on the literature (Section 5.5). Finally, a computational experiment is presented to evaluate the performance of the Monte Carlo optimization method in a stochastic setting (Section 5.6). The results highlight that the objective function value obtained by our proposal converges to the optimal value of the problem and presents a high robustness in terms of the proportion of feasible simulations when the number of samples increases. Finally, Section 5.7 presents the conclusions.

### 5.2 Problem formulation

In this section, we formalize the stochastic advance OR scheduling problem with uncertainty in surgical activities, which has been analyzed in the literature review presented in Chapter 2 (see Section 2.3). Our problem is to determine the OR and the intervention date for surgeries on the waiting list, considering the open scheduling strategy, resources availabilities (OR and surgeons), and deadline constraints. The objective function is to minimize the total cost of the unexploited OR time and

| Indices and Sets |  |
| :---: | :---: |
| $\boldsymbol{h} \in \boldsymbol{H}$ | Set of days within the planning horizon |
| $i \in I$ | Set of patients (surgeries) on the waiting list |
| $\boldsymbol{j} \in \boldsymbol{J}$ | Set of ORs |
| $\boldsymbol{k} \in \boldsymbol{K}$ | Set of surgeons |
| $\zeta \in Z$ | Set of scenarios |
| Parameters |  |
| $r_{j h}^{\zeta}$ | Regular capacity (in minutes) of OR $j$ on day $h$ under scenario $\zeta$ |
| $\boldsymbol{o}_{\boldsymbol{j} \boldsymbol{h}}$ | Overtime capacity (in minutes) of OR $j$ on day $h$ |
| $a_{\text {kh }}^{\zeta}$ | Regular capacity (in minutes) of surgeon $k$ on day $h$ under scenario $\zeta$ |
| $\boldsymbol{\tau}_{\boldsymbol{i}}$ | Surgeon in charge of patient $i$ |
| $r d_{i}$ | Release date for performing the surgery on patient $i$ |
| $d_{i}$ | Deadline for performing the surgery on patient $i$ |
| $t_{i}^{\zeta}$ | Length of surgery $i$ (in minutes) under scenario $\zeta$ |
| $\varphi$ | Ratio of the cost of an hour of overtime to the cost of a regular working hour |
| Variables |  |
| $\boldsymbol{X}_{i j h}$ | 1 if patient $i$ is to be scheduled in $\mathrm{OR} j$ on day $h ; 0$ otherwise |

Table 5.1. Indices, sets, parameters and variables used in the decision model
overtime. The uncertainty in surgical activities is denoted by scenario $\zeta \in Z$ as in Min and Yih (2010). A scenario $\zeta$ is defined by three random variables: surgery durations, emergency surgeries' arrivals (by means of OR capacity that must be booked in advance for emergencies) and the surgeon time for performing emergency surgeries during a day (by reducing their regular capacity).

In the stochastic advance OR scheduling problem (see Section 2.3) we consider a set $H$ of planning days $(h=1 \ldots|H|)$ where there is a set $J$ of parallel ORs $(j=1 \ldots|J|)$ available on each day $h$ in the planning horizon. The regular capacity for performing elective surgeries of the $j$-th OR during day $h$ under scenario $\zeta(\zeta=1 \ldots|Z|)$ is denoted by $r_{j h}^{\zeta}$. In the following, a tuple $(j, h)$ is denoted as OR-day. On each OR-day $(j, h)$, the OR overtime is limited to $o_{j h}$. There is a set $K$ of surgeons ( $k=1 \ldots|K|$ ) and, on each day $h$ and each scenario $\zeta$, each surgeon $k$ has a maximum available time ( $a_{k h}^{\zeta}$ ) for performing elective surgeries. The remaining human and instrumental resources are assumed to be available whenever needed. Recovery facilities are also assumed to be always available during the planning horizon. A set $I$ of elective surgeries (i.e. patients) are on the waiting list $(i=1 \ldots|I|)$. For each surgery $i$ and each scenario $\zeta$, the surgery duration is represented by $t_{i}^{\zeta}$. Each surgery $i$ has a surgeon in charge $\left(\tau_{i}\right)$ and must be scheduled within the time period defined by its release date $\left(r d_{i}\right)$ and deadline $\left(d_{i}\right)$.

We present below the stochastic integer programming model to solve the problem under consideration. Table 5.1 summarizes sets, data and variables used in the decision model.

$$
\begin{equation*}
\text { (P) Minimize } \sum_{\zeta \in Z} \sum_{h \in H} \sum_{j \in J} \max \left\{\left(r_{j h}^{\zeta}-\sum_{i \in I} t_{i}^{\zeta} \cdot x_{i j h}\right), \varphi\left(\sum_{i \in I} t_{i}^{\zeta} \cdot x_{i j h}-r_{j h}^{\zeta}\right)\right\} \tag{5.1}
\end{equation*}
$$

Subject to:

$$
\begin{align*}
& \sum_{j \in J} \sum_{h \in H} X_{i j h} \leq 1 \quad(\forall i \in I)  \tag{5.2}\\
& \sum_{j \in J} \sum_{\substack{h \in H \\
h \leq r_{i}-1}} X_{i j h}=0 \quad(\forall i \in I)  \tag{5.3a}\\
& \sum_{j \in J} \sum_{\substack{h \in H \\
h \leq d_{i}}} X_{i j h}=1 \quad\left(\forall i \in I\left|d_{i} \leq|H|\right)\right.  \tag{5.3b}\\
& \sum_{i \in I} t_{i}^{\zeta} \cdot X_{i j h} \leq r_{j h}^{\zeta}+o_{j h} \quad(\forall j \in J, \forall h \in H, \forall \zeta \in Z)  \tag{5.4}\\
& \sum_{j \in J} \sum_{\substack{i \in I \\
\tau_{i}=k}} t_{i}^{\zeta} \cdot X_{i j h} \leq a_{k h}^{\zeta} \quad(\forall k \in K, \forall h \in H, \forall \zeta \in Z)  \tag{5.5}\\
& X_{i j h} \in\{0,1\} \quad(\forall i \in I, \forall j \in J, \forall h \in H) \tag{5.6}
\end{align*}
$$

The objective function (5.1) minimizes the total cost of the unexploited OR time and overtime. Following Fei et al. (2009) we use a cost ratio $\varphi$ between a regular working hour and overtime to penalize overtime. Constraints (5.2) enforce that each surgery is scheduled at most once during the planning horizon. Constraints (5.3a) and (5.3b) define the earliest and the latest date on which a patient has to be scheduled. Constraints (5.3a) ensure that the patient is scheduled after his/her release date, while constraints (5.3b) ensure that the surgery of a patient with a deadline within the planning horizon must take place before his/her deadline. Constraints (5.4) ensure that the total amount of OR time assigned to surgeons in an OR-day under a scenario is lesser than its total capacity (i.e. regular plus overtime). Constraints (5.5) ensure that the total amount of time allocated to a surgeon is lesser than his/her capacity in any day and scenario. Finally, constraints (5.6) are binary constraints for decision variables.

### 5.3 Monte Carlo optimization method

In this section we present a Monte Carlo optimization method to solve the proposed stochastic advance OR scheduling problem. The Monte Carlo optimization method is based on the SAA method proposed by Min and Yih (2010) for solving a stochastic OR scheduling problem. The problem $(P)$ can be approximated by a SAA problem $\left(P_{N}\right)$ and formulated as the following decision model for a sample size $N$ :
( $P_{N}$ ) Minimize $z_{N}=\sum_{n=1}^{N} \frac{1}{N} \sum_{h \in H} \sum_{j \in J} \max \left\{\left(r_{j h}^{n}-\sum_{i \in I} t_{i}^{n} \cdot X_{i j h}\right), \varphi\left(\sum_{i \in I} t_{i}^{n} \cdot X_{i j h}-r_{j h}^{n}\right)\right\}$
Subject to:

$$
\begin{align*}
& \sum_{j \in J} \sum_{n \in H} X_{i j h} \leq 1 \quad(\forall i \in I)  \tag{5.8}\\
& \sum_{j \in J} \sum_{\substack{h \in H \\
h \leq r d_{i}-1}} X_{i j h}=0 \quad(\forall i \in I)  \tag{5.9.1}\\
& \sum_{j \in J} \sum_{n \in H} X_{i j h}=1 \quad\left(\forall i \in I\left|d_{i} \leq|H|\right)\right.  \tag{5.9.2}\\
& \sum_{i \in I} t_{i}^{n} \cdot X_{i j h} \leq r_{j h}^{n}+o_{j h} \quad(\forall j \in J, \forall h \in H, n=1 \ldots N)  \tag{5.10}\\
& \sum_{j \in J} \sum_{\substack{i \in I \\
\tau_{i}=k}} t_{i}^{n} \cdot X_{i j h} \leq a_{s h}^{n} \quad(\forall k \in K, \forall h \in H, n=1 \ldots N)  \tag{5.11}\\
& X_{i j h} \in\{0,1\} \quad(\forall i \in I, \forall j \in J, \forall h \in H) \tag{5.12}
\end{align*}
$$

The procedure of the Monte Carlo optimization method is shown in Figure 5.1. A number of replications $(M)$ are introduced in the procedure for reducing the effects of large variances in the calculation of the objective function value (Min and Yih, 2010). In step 2, the problem $P_{N}$ is heuristically solved using the IGLS method (see Section 5.4) because of the long computation times required by the integer programming for solving problems of realistic size (see e.g. Lamiri et al., 2009), providing an approximated objective function value $\left(z_{N}^{m}\right)$. In step 3, a Monte Carlo simulation is used to evaluate the objective function value. A good feasible solution is required to obtain a

```
form:= 1 to Mdo
% Step 1. Generate N samples
    %Resources capacities
```



```
    % Surgery durations
    Generate Nsamples of t }\mp@subsup{t}{i}{}(\mp@subsup{t}{i}{1}\ldots.t\mp@subsup{t}{i}{N}),i=1\ldots|
```

\% Step 2. Solve heuristically the $P_{N}$ problem
Solve the corresponding $P_{N}$ problem using an iterative greedy local search method and let $X_{N}^{m}$ be the resulting solution and $z_{N}^{m}$ be the resulting approximate objective function value.
\% Step 3. Evaluation of the true objective function using Monte Carlo simulation

## \% Step 3.1 Generate N' samples

$\%$ Resources capacities
Generate $N^{\prime}$ samples of $\gamma_{j h}\left(r_{j h}^{1} \ldots \gamma_{j h}^{N^{\prime}}\right)$ and $a_{s h}\left(a_{s h}^{1} \ldots a_{s h}^{N^{\prime}}\right), h=1 \ldots|H ; j=1 \ldots||; s=1 \ldots| S \mid$

## \% Surgery durations

Generate $N^{\prime}$ samples of $t_{i}\left(t_{i}^{1} \ldots t_{i}^{N^{\prime}}\right), i=1 \ldots \mid I$

## \% Step 3.2 Approximate the true value of the objective function

$$
z_{N^{\prime}}^{\prime m}=\sum_{n=1}^{N^{\prime}} \frac{1}{N^{\prime}} \sum_{n \in H} \sum_{i \in I} \max \left\{\left(r_{j n}^{n}-\sum_{i \in I} t_{i}^{n} \cdot X_{N i j n}^{m}\right), \gamma\left(\sum_{i \in I} t_{i}^{n} \cdot X_{N i j n}^{m}-r_{j n}^{n}\right)\right\}
$$

end for
$\%$ Determine $z_{N}^{M}$ and $z_{N^{\prime}}^{\prime M}$
$z_{N}^{M}=\frac{1}{M} \sum_{m=1}^{M} z_{N}^{m}, \quad z_{N^{\prime}}^{\prime M}=\frac{1}{M} \sum_{m=1}^{M} z_{N^{\prime}}^{\prime \prime}$

## \% Determine the Optimality Index

$O I=\frac{\left(z_{N^{\prime}}^{\prime N}-z_{N}^{N}\right)}{z_{N^{\prime}}^{\prime N}}$

Figure 5.1. Monte Carlo optimization method
good estimated objective function value $\left(z_{N^{\prime}}^{\prime m}\right)$. Even though several procedures to obtain a feasible solution exist, the solution of the problem $P_{N}\left(\right.$ i.e. $\left.X_{N}^{m}\right)$ is selected to evaluate the true value of the objective function of $P$, following what has been previously considered in the OR scheduling literature (see e.g. Lamiri et al., 2009) and because it yields the best results (Min and Yih, 2010). Finally, in the calculation of $z_{N^{\prime}}^{\prime}$, we only consider samples satisfying the stochastic constraints (i.e. constraints 5.4 and 5.5).

### 5.4 Iterative greedy local search method

In this section we propose an IGLS method for solving the problem $P_{N}$. The method is composed of two phases: the construction surgical schedule phase and the iterative greedy local search phase.

The construction surgical schedule phase determines a feasible surgical schedule as follows:

- Step 1: Waiting list sorting. In this step, a sorted waiting list $\left(W L_{\text {sort }}\right)$ is obtained by sorting the surgeries in the initial waiting list, following the procedure employed in the constructive heuristics proposed in Chapter 4 (see Section 4.3.1). Regarding sorting criteria, we consider the same criteria used in Section 4.3 .1 (i.e. INC, DEC, HILL, VALLEY, LOHI, HILO, and RAN), while we only consider the surgery duration $(t)$ as sorting indicator. As $t$ is a random variable, a sample of the $N$ samples is randomly selected ( $n_{r a n}$ ), considering the corresponding surgery duration values in the sorting procedure. Note that seven sorted waiting lists are obtained in Step 1.
- Step 2: Surgical schedule construction. In this step, a surgery schedule is obtained by applying a BP operator to each $W L_{\text {sort }}$ obtained in Step 1 taking into account constraints (5.8)-(5.12). We consider the following BP algorithms presented in Section 4.3: $F F, B F$ and $L F$. Due to the nature of the objective function, in this step, we assume that OR overtime is not allowed, considering only OR regular capacity. In order to select the suitable OR-day on BP algorithms (i.e. to determine the available OR time), the values of surgery durations and OR regular capacities considered are those corresponding to the $n_{\text {ran }}$ sample. Note that 21 surgical schedules are constructed in Step 2. We select the one that yields the best value of the objective function (5.7).
- Step 3: Iterative improvement. Starting from the best surgical schedule obtained in Step 2, the solution is improved by applying a swapping method. The method consists of swapping two surgeries between different OR-days (a pair-wise swap) and swapping a pair of surgeries scheduled in the same OR-day with another surgery scheduled in a different OR-day (a triplet-wise swap). Note that swaps between scheduled and unscheduled surgeries are also considered (i.e. a scheduled surgery is unscheduled, while an unscheduled surgery is assigned its OR-day). In order to
evaluate a given swap, the values of the random variables (i.e. surgeries duration, OR regular capacities and surgeons' capacities) are those corresponding to the $n_{\text {ran }}$ sample, and the total capacity of an OR-day is the regular capacity plus the overtime capacity (note that the overtime is not allowed in Step 2). In each iteration, for each OR-day, surgeries (pair-wise swap) and pairs of surgeries (triplet-wise swap) are considered one by one in the swapping method, determining and performing the swap that yields the largest improvement and satisfies constraints (5.8)-(5.12). The iterative improvement step finishes when the solution cannot improve any more.

The local search phase determines a new surgical schedule from the surgical schedule constructed in the previous phase by the following five steps:

- Step 4. Destruction step: randomly remove $q$ surgeries from an incumbent surgical schedule. This yields a surgical schedule composed of $|I|-q$ surgeries.
- Step 5. Construction step. It reinserts the $q$ surgeries (one by one) in the OR-day that yields the best value of the objective function, while satisfying constraints (5.8)(5.12). After the reinsertion of the $q$ surgeries, Step 3 is applied to the surgical schedule obtained. Note that a sample of the $N$ samples ( $n{ }^{\prime} r a n$ ) is randomly selected for evaluating swaps.
- Step 6. The resulting surgical schedule is considered as the incumbent surgical schedule if the value of the objective function improves the best value obtained so far. A simulated annealing-like acceptance criterion with a constant temperature is implemented to avoid the stagnation in the search procedure. The constant temperature is set such that moves that deteriorate the solution more than a percentage $\theta$ of the maximal deterioration are accepted with a probability smaller than $\varphi$ (Lamiri et al., 2009).
- Step 7. If the termination criterion is not satisfied, return to Step 4. The termination criterion of the iterative greedy local search phase is defined as a CPU time limit depending on the size of the problem (see Section 5.4).

| Combination | $\|\boldsymbol{J}\|$ | $\boldsymbol{\beta}$ | $\boldsymbol{\alpha}$ | $\boldsymbol{C} \boldsymbol{V}$ | $\overline{\boldsymbol{I}}$ | $\overline{\boldsymbol{S}}$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{1}$ | 4 | 1 | 1.5 | 0.1 | 81.1 | 8.1 |
| $\mathbf{2}$ | 4 | 1 | 1.5 | 0.5 | 82.3 | 8.3 |
| $\mathbf{3}$ | 4 | 1 | 2 | 0.1 | 80.7 | 10.1 |
| $\mathbf{4}$ | 4 | 1 | 2 | 0.5 | 81.2 | 11.1 |
| $\mathbf{5}$ | 4 | 1.25 | 1.5 | 0.1 | 101 | 8 |
| $\mathbf{6}$ | 4 | 1.25 | 1.5 | 0.5 | 102.8 | 7.9 |
| $\mathbf{7}$ | 4 | 1.25 | 2 | 0.1 | 100.8 | 10.7 |
| $\mathbf{8}$ | 4 | 1.25 | 2 | 0.5 | 105.7 | 11 |
| $\mathbf{9}$ | 8 | 1 | 1.5 | 0.1 | 161 | 15.7 |
| $\mathbf{1 0}$ | 8 | 1 | 1.5 | 0.5 | 163.2 | 15.3 |
| $\mathbf{1 1}$ | 8 | 1 | 2 | 0.1 | 161.1 | 20.3 |
| $\mathbf{1 2}$ | 8 | 1 | 2 | 0.5 | 162 | 20.7 |
| $\mathbf{1 3}$ | 8 | 1.25 | 1.5 | 0.1 | 200.4 | 15.8 |
| $\mathbf{1 4}$ | 8 | 1.25 | 1.5 | 0.5 | 204.9 | 15.5 |
| $\mathbf{1 5}$ | 8 | 1.25 | 2 | 0.1 | 201.9 | 20.8 |
| $\mathbf{1 6}$ | 8 | 1.25 | 2 | 0.5 | 207.4 | 21.2 |

Table 5.2. Size of problems considered in the proposed testbed

### 5.5 Analysis of deterministic solutions

In this section we present the results of the integer programming approach, the IGLS method and the existing heuristics for solving the advance OR scheduling problem in a deterministic way. In order to conduct the fairest computational experience, we carry out an experimental calibration of the parameters of the IGLS method. We have generated a 160 -instance calibration testbed using the procedure described in Chapter 3. We have considered 16 different combinations of $|J|, \beta, \alpha$ and $C V$ (see Table 5.2) and, for each combination, we have generated 10 instances. Table 5.2 details the average size of the problem for each combination. The values of $|H|, \mu$, and $a$ are 5, 120 and 480 respectively (see Section 3.5). Finally, $m d s$ is drawn from a uniform distribution [3, 5]. In order to select the best algorithm, we consider the following response variables:
(1) feasibility of the solution,
(2) Relative Deviation Index (RDI), according to the expression RDI $=\left(\mathrm{Heu}_{\text {sol }}-\right.$ Best $\left._{\text {sol }}\right) /\left(\right.$ Worst $_{\text {sol }}-$ Best $\left._{\text {sol }}\right)$ where Best $_{\text {sol }}$ and Worst ${ }_{\text {sol }}$ are the best and the worst solutions obtained among all the methods and $\mathrm{Heu}_{\text {sol }}$ is the solution obtained by a given algorithm configuration, and
(3) CPU time (in seconds) required for solving a given instance.

In the construction surgical schedule phase, the sorting criterion $\left(S_{C}\right)$ and the $B P$ algorithm have been calibrated. The results show that the 21 surgical schedules obtained
in the construction surgical schedule phase are feasible. Regarding the objective function value, the results show that $I N C$ is the worst sorting criterion, while there are no statistically significant differences among the remaining sorting criteria. Note that $D E C$ provides better results than the other sorting criteria. Finally, there are statistically significant differences between $L F$ and the remaining $B P$ algorithms at a $95 \%$ confidence interval, where $L F$ obtains the best average objective function value. Regarding CPU time, there are no statistically significant differences for the parameters at a $95 \%$ confidence interval. In the iterative greedy local search phase, the number of extracted surgeries $(q)$, the percentage of the maximal deterioration $(\theta)$ and the probability of accepting a solution which deteriorates a solution $(\varphi)$ have been calibrated. The IGLS method has been tested with the following levels: $q$ is set to 3,5 and $7 ; \theta$ is set to $10 \%$ and $20 \%$, and $\varphi$ is set to $1 \%, 5 \%$, and $10 \%$. According to RDI values, the best setting is $q=3, \theta=10 \%$ and $\varphi=10 \%$.

Regarding the existing heuristics for solving the problem under consideration, Fei et al. $(2009,2010)$ propose a column-generation-based heuristic (CGBH) procedure. Liu et al. (2011) propose a heuristic based on the dynamic programming idea (in the following dynamic programming heuristic, DPH ), where the objective is to partition the set of surgeries to be performed into subsets, and then assign an OR-day to each subset in order to optimize the objective function. In the computational experiments carried out by Liu et al. (2011), DPH outperforms CGBH for large size instances (120 and 160 surgeries on the waiting list) with respect to the feasibility of the surgical schedule and to CPU time requirements. Therefore, DPH can be considered the best-so-far heuristic method for the problem. In order to make a fair comparison regarding CPU time, we code the DPH algorithm.

The analysis of the effectiveness of the integer programming approach, the DPH method and the IGLS method is carried out using the testbed provided by Liu et al. (2011) and the 160 -instances testbed. The experiments have been executed on a PC with 2.40 GHz Intel Core i5-450 processor and 4 GBytes of RAM memory. The integer programming decision model is solved using the commercial software Gurobi version 5.6. The computation time limit is fixed to $|I| \cdot|J| \cdot|H| \cdot \eta$ seconds for the integer programming model and the IGLS method. $\eta$ is a time factor, which is set to 0.003125 , $0.00625,0.0125$ and 0.025 .


Figure 5.2. Feasibility, optimality and RDI results for methods using the testbed based on the literature

Figure 5.2 shows the results using the testbed generated in this chapter. Note that, for this testbed, RDI is calculated replacing Best $_{\text {sol }}$ by Best $_{\text {bound }}$, which is the best bound provided by the solver Gurobi. Regarding feasibility, it should be pointed out that the high percentage of unfeasible solutions ( $33 \%$ ) obtained by DPH algorithm is due to the non fulfilment of deadline constraints. The procedure used to sort the waiting list (see the waiting list sorting step) guarantees a feasible solution on every instance solved by the IGLS method. Furthermore, the high percentage of optimal solutions (52\%) obtained by the IGLS method should be noted. It finds an optimal solution whenever the factor $\beta$ is set to $125 \%$ (i.e. $50 \%$ of the instances). Regarding $R D I$, the results show that the IGLS method is statistically the best algorithm at a $95 \%$ confidence interval. The time factor does not affect the performance of the IGLS method, being 0.03 the average RDI value after an average CPU time limit of 20 seconds ( $\eta=0.003125$ ). The DPH algorithm yields a RDI value of 0.52 after an average CPU time of 38 seconds. Note that the DPH algorithm is a constructive heuristic and, therefore, no stopping criterion has to be considered. The results show that there are no statistically significant differences between the DPH algorithm and the integer programming method with $\eta=$ 0.00625 ( 40 seconds on average) and $\eta=0.0125$ ( 80 seconds on average).

Figure 5.3 shows the results using the testbed proposed by Liu et al. (2011). Note that the IGLS method is only analyzed considering the lowest time factor ( $\eta=0.003125$ ), and time factors are not taken into account for the integer programming method since optimal solutions are found in less CPU time. We observe that the DPH algorithm increases the percentage of feasible solutions obtained from $77 \%$ to $90 \%$, and we observe a significant increase of the optimal solutions obtained by the methods (especially the DPH method). In contrast with the performance in the testbed proposed in the paper, the algorithms obtain similar RDI average values in the testbed proposed by Liu et al. (2011) especially for 40,80 and 120 surgeries. This fact, together with the high unexploited OR time obtained (the mean objective function values are 2854.7, 1982.8 and 1150.50 for 40,80 , and 120 respectively), suggests that instances involve excessive resources (OR time) relative to the total surgery time in the waiting list and explains the significant increase of the proportion the optimal solutions.

In view of the results, we conclude that the IGLS method is the best method for solving the deterministic version of the advance OR scheduling problem, yielding a high percentage of optimal solutions for realistic size instances.



Figure 5.3. Feasibility, optimality and RDI results for methods using the testbed proposed by Liu et al. (2011)

### 5.6 Analysis of stochastic solutions

A computational experiment is presented to evaluate the performance of the Monte Carlo optimization method for solving the proposed stochastic advance OR scheduling problem. The Monte Carlo optimization method is used with a number of replications $(M)$ equals to 20 . The values of the number of samples $(N)$ for solving the SAA problem $\left(P_{N}\right)$ are $1,5,10,20,30,40,50,100,200$, and 300 . Finally, we consider 50,000 samples ( $N$ ') in the Monte Carlo simulation as in Min and Yih (2010). In this section, we have only considered the factors which statistically influence on the deterministic version of the problem, i.e. $|J|$ and $\beta$, considering the resulting 4 different combinations (see Table 5.2). Without loss of generality, factors $\alpha$ and $C V$ are set to 1.5 and 0.1 respectively, since these levels resulting in more difficult problems in terms of RDI. The computation time limit is fixed to $0.0003125 \cdot N \cdot|I| \cdot|J| \cdot|H|$ seconds for the IGLS method.

Regarding surgery durations, we assume that $t_{i}^{\zeta}$ follows a 2-parameter log-normal distribution. The expected duration is set to the deterministic surgery duration for surgery $i\left(t_{i}\right)$, while the standard deviation is calculated as $C V^{\prime} \cdot t_{i} . C V^{\prime}$ is randomly and uniformly drawn from the set $\{0.1,0.2,0.3\}$. Regarding emergency arrivals, the following statistical distributions are used to generate the total OR time of an OR-day required for emergency demands $\left(e_{j h}^{\zeta}\right)$ : an exponential distribution (Lamiri et al., 2009; Lamiri, Xie, Dolgui et al., 2008; Lamiri, Xie and Zhang, 2008), a log-normal distribution (Lamiri et al., 2007), a normal distribution (Lamiri et al., 2009), and an uniform distribution (Min and Yih, 2010). According to Lamiri et al. (2009), we assume an expected emergency capacity of 72 minutes and a coefficient of variation 0.5 . Finally, the surgeon capacity uncertainty due to emergency arrivals has not been previously addressed in the literature. In this paper, we propose the following procedure to generate the regular capacity of a surgeon considering emergency demands: In the first step, for each day, the total OR time for emergency surgeries is determined. In the second step, for each day, surgeons are randomly sorted (in order to randomize the allocation of emergency surgeries to surgeons) and, one by one, the regular capacity (i.e. 8 hours) is reduced by a $0 \%, 25 \%$ ( 2 hours) or $50 \%$ ( 4 hours) with an equal probability of $1 / 3$. The procedure stops when the total reduced time is equal to the total OR time for emergency surgeries determined in the first step.

For each $N$ sample size and each statistical distribution considered to generate emergency demands, the mean approximated objective function value $\left(z_{N}^{M}\right)$, the mean estimated objective function value $\left(z_{N^{\prime}}^{M}\right)$, the optimality index value, and the mean proportion of feasible simulations are shown in Figure 5.4. Note that only feasible simulations are considered for determining $z_{N^{\prime}}^{\prime m}$. The results highlight that, independently of the statistical distribution considered to generate emergency demands, the objective function value obtained by the IGLS method converges to the optimal value of the problem and presents a high robustness in terms of the proportion of feasible simulations when the number $N$ of samples increases (see red and orange curves in Figure 5.4). Depending on the statistical distribution considered to generate emergency demands, an optimality index value of around $1 \%$ is obtained with sample size 40 (exponential) and 100 (log-normal, normal, and uniform). However, the number $N$ of samples must be greater than the above sizes for yielding a reasonable proportion of feasible simulations (e.g. more than $85 \%$ of simulations are feasible with $N=200$ for any statistical distribution). As shown in the blue curve of Figure 5.4, a high robustness of the solution implies an important increase of the total cost of the unexploited OR time and overtime. Without loss of generality, we present the conclusions obtained for the exponential distribution since similar performances are observed for the statistical distributions.

To increase the percentage of feasible simulations obtained from $37.1 \%(N=40)$ to $88.3 \%(N=200)$, an increase of $95 \%$ of the objective function value is observed (from 2,186 to 4,263 ). In order to clarify the increase of the total cost of the unexploited OR time and overtime, nine performance indicators values are detailed in Table 5.3. First, the mean values of the number of scheduled surgeries, the total OR undertime, and the total OR overtime are calculated considering only feasible simulations. Second, in order to analyze the unfeasibility of simulations, Table 5.3 also shows the mean values of the number of cancelled surgeries, the mean OR time exceed (over the OR overtime allowed) and the surgeon overtime. The increase of $95 \%$ is because of both important increases of the mean OR undertime per OR-day (from 114.9 to 222.0 minutes) and the number of under-utilized OR-days (from 15.02 to 18.61 OR-days), as consequence of the decrease of the mean number of scheduled surgeries (from 58.7 to 34.7 surgeries) due to the high uncertainty considered (by increasing $N$ ) in emergency demands and surgeons' capacity for performing emergency surgeries.



|  |  | Feasible simulations |  |  |  | Unfeasible simulations |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{N}$ | \% feasible <br> simulations | Scheduled <br> surgeries | Unex. <br> OR-days | Undertime <br> (min/OR-day) | Overex. <br> OR-days | Overtime <br> (min/OR-day) | Cancelled <br> surgeries | Overtime <br> exceed <br> (min.) | Surgeon <br> overtime <br> (min.) |
| $\mathbf{1}$ | 0.6 | 72.30 | 9.99 | 81.34 | 9.93 | 67.99 | 8.34 | 39.76 | 324.49 |
| $\mathbf{5}$ | 4.2 | 70.84 | 10.90 | 70.32 | 9.01 | 66.93 | 6.78 | 28.91 | 176.60 |
| $\mathbf{1 0}$ | 10.1 | 68.47 | 11.72 | 78.72 | 8.19 | 67.41 | 5.42 | 26.06 | 117.49 |
| $\mathbf{2 0}$ | 20.3 | 64.86 | 13.44 | 86.31 | 6.49 | 66.03 | 4.85 | 20.64 | 74.48 |
| $\mathbf{3 0}$ | 30.7 | 61.10 | 14.46 | 102.79 | 5.47 | 66.27 | 4.41 | 18.84 | 50.22 |
| $\mathbf{4 0}$ | 37.1 | 58.65 | 15.02 | 114.88 | 4.92 | 66.31 | 3.90 | 18.04 | 42.43 |
| $\mathbf{5 0}$ | 43.2 | 56.30 | 15.94 | 120.99 | 4.01 | 64.60 | 3.74 | 14.19 | 39.45 |
| $\mathbf{1 0 0}$ | 66.2 | 46.30 | 17.42 | 169.34 | 2.54 | 63.62 | 2.87 | 11.95 | 30.44 |
| $\mathbf{2 0 0}$ | 88.3 | 34.65 | 18.61 | 221.96 | 1.37 | 61.59 | 2.22 | 13.43 | 23.85 |
| $\mathbf{3 0 0}$ | 94.6 | 27.63 | 19.04 | 256.14 | 0.95 | 59.42 | 1.94 | 18.57 | 21.41 |

Table 5.3. Mean values for problem $|J|=4$ and $\beta=1.25$ considering the exponential distribution to

## generate emergency demands

However, the latter decrease supposes that the proportion of feasible simulations increases by reducing the surgeon overtime (from 42.43 to 23.85 minutes) and the OR time exceed (from 18.04 to 13.43 minutes). Note that the latter values imply a reduction of the number of cancelled surgeries (from 3.90 to 2.22 cancelled surgeries). In view of the results, the setting of the number of samples will depend on a tradeoff between costs and robustness.

Finally, Table 5.4 shows the 95.0 confidence intervals of $z_{N}^{M}$ for each statistical distribution and each $N$ sample size. The results highlight that the IGLS method is robust for solving the problem $P_{N}$, since reasonable confidence intervals of $z_{N}^{M}$ are obtained considering $M=20$ (number of replications in the Monte Carlo optimization method). Given that the low error margins obtained for solving the problem $P_{N}(8.93 \%$ for the exponential distribution and $N=300$ in the worst case), the CPU time (see Table 5.4) required for solving the problem can be reduced by decreasing the value of $M$.

### 5.7 Conclusions

In this chapter, we have addressed a stochastic advance OR scheduling problem under the open scheduling strategy, taking into account resources availability (OR and surgeons) and time period constraints (release and deadlines) in order to minimize the unexploited OR time and overtime costs. A stochastic decision model is proposed for

\left.|  |  | Statistical Distribution |  |  | Normal |
| :--- | :--- | :--- | :--- | :--- | :--- |$\right)$| CPU Time |
| :--- |
| (sec./replication) |

Table 5.4. 95.0\% Confidence intervals of $z_{N}^{M}$ and CPU time values
solving the problem, taking into account the uncertainty in the surgery duration, in the total emergency surgery time in the planning horizon and in the surgeons' regular capacity. A Monte Carlo optimization method, based on the SAA method proposed by Min and Yih (2010), is proposed for solving the problem. The method combines an iterative local search method and Monte Carlo simulation. The performance of the iterative local search method is analyzed against a column-generation-based heuristic procedure proposed by Fei et al. (2009) and a heuristic based on the dynamic programming idea proposed by Liu et al. (2011) for solving the deterministic version of the problem. These methods constitute the up-to-now state of the art heuristics for the (deterministic) problem. The analysis is carried out using the testbed proposed by Liu et al. (2011) and a testbed generated based on the literature. The results show that the iterative local search method is the best method for solving the deterministic version of the advance OR scheduling problem, yielding a high percentage of optimal solutions for realistic size instances. We also carry out a computational experiment to evaluate the performance of the Monte Carlo optimization method for solving the proposed stochastic advance OR scheduling problem. The results highlight that, regardless the statistical distribution employed to generate the arrivals of emergency surgeries, the objective function value obtained by the IGLS converges to the optimal value of the problem and presents a high robustness in terms of the proportion of feasible simulations when the number of samples increases.

## Chapter 6

## The Integrated Operating Room Planning and Scheduling Problem

### 6.1 Introduction

As described in Chapter 2, the offline operational level is traditionally solved into two steps (the OR planning and scheduling problems), intending to reduce the complexity of the integrated problem. In Chapters 4 and 5, we have addressed the OR planning problem considering deterministic and stochastic surgery durations, emergency arrivals and resources capacity. However, due to the high interdependence among these problems, an integrated approach would improve the quality of the surgery schedule. Therefore, in this chapter, we address the integrated OR planning and scheduling problem.

There is evidence in the literature that most surgeries in hospitals are performed by a team composed of two surgeons, and that their experience largely influences the surgery duration. However, to the best our knowledge, only one contribution has addressed the OR planning and scheduling problem with surgical teams, but in such case surgery durations did not depend on the experience of surgeons. In this chapter we address an integrated OR planning and scheduling problem with surgical teams composed by one or two surgeons where surgery durations depend on their experience and skills (Section 6.2). We propose an ILP model to optimally solve this problem (Section 6.3). Given the high computation requirements of our ILP model, Section 6.4 proposes an iterative constructive method. The computational experience presented in Section 6.5 shows that the proposed algorithm is able to find feasible solution for all problems requiring shorter CPU time and average relative percentage deviation than the ILP model. In addition, the robustness of the so-obtained surgical schedules is analyzed using simulation. Finally, conclusions and further research are presented in Section 6.6.

### 6.2 Problem formulation

In the integrated OR planning and scheduling problem we consider a set $H$ of planning days $(h=1, \ldots,|H|)$ where there is a set $J$ of parallel ORs $(j=1, \ldots,|J|)$ available on each day $h$ in the planning horizon. The regular capacity of the $j$-th OR during day $h$ is denoted by $r_{j h}$. In the following, a pair $(j, h)$ is denoted as OR-day. Recovery facilities are also assumed to be always available during the planning horizon. There is a set $K$ of surgeons ( $k=1, \ldots,|K|$ ) and, on each day $h$, each surgeon $k$ has a maximum available time $\left(s_{k h}\right)$ to perform surgeries without limits in the number of surgeries performed and in the number of different ORs that he/she may visit in an OR-day. Additionally, we consider a set $L$ of levels of experience of the assistant surgeons $(l=1, \ldots,|L|)$. We denote the set of surgeons belonging to level $l$ as $K_{l}\left(k^{\prime}=1, \ldots,\left|K_{l}\right|\right)$, and $\sum_{l}\left|K_{l}\right|=|K|$. Note that each surgeon may belong only to one level of experience. The remaining human and material resources are assumed to be available whenever needed. In this setting, a set $I$ of elective surgeries (i.e. patients) are in the waiting list ( $i=1, \ldots,|I|$ ). Each surgery $i$ should be performed before a given deadline $\left(d_{i}\right)$ according to the disease's characteristics and the waiting time in the waiting list. The binary parameter $\delta_{i j}$ yields 1 if surgery $i$ can be performed in OR $j, 0$ otherwise. Many realistic situations can be modeled with this parameter such as, for example, to book OR-days in order to plan a certain type of surgery or to forbid the assignment of a surgery to an OR that does not have the equipment required to perform this specific surgery. Finally, the following additional assumptions will help in formulating the problem.

- Surgical team composition.

In this chapter, we consider that a surgery can be performed by a surgical team composed by either one surgeon (the most extended assumption in the literature), or two surgeons (a realistic setting in many cases, see the previous section). In the first case, the surgery is performed only by the responsible surgeon $\left(\tau_{i}\right)$, which is assigned to each surgery in the waiting list before solving the integrated OR planning and scheduling problem. Such decision is usually made by the head of the surgical specialty according to surgeon's specialty, availability, workload, etc. In the second case, the responsible surgeon is accompanied by an assistant surgeon, due to the complexity of a surgery, the need of training residents, etc. As a surgical specialty is usually composed by faculty and resident surgeons, we assume that any of them can perform a surgery as assistant
surgeon. In order to simplify the exposition of the ILP model, a dummy assistant surgeon $(k=0)$ is introduced for considering a surgical team of two surgeons when a surgery is carried out only by the responsible surgeon.

- Assistant surgeon's level of experience.

Due to the medical characteristics of each surgery, only assistant surgeons with the required level of experience are able to perform the surgery. We define parameter $\gamma_{i l}$ to indicate whether surgery $i$ can be performed by an assistant surgeon with a level of experience $l\left(\gamma_{i l}=1\right)$, or not $\left(\gamma_{i l}=0\right)$. In this chapter, the following levels of experience have been considered: 0 (for the dummy surgeon), 1 (for junior residents), 2 (for senior residents), and 3 (for faculty surgeons).

- Impact of the level of experience on the surgery duration.

Depending on the experience of the responsible surgeon assigned to a surgery, the surgery duration (the duration required by one surgeon surgical team) is established by the head of the surgical specialty. As discussed in Chapter 2, the assistant surgeon's experience also influences on this duration. The effect might be positive (reducing the duration, see e.g. Ludwig et al., 2005) or negative (increasing the duration, see e.g. Bridges and Diamond, 1999). In our notation, the parameter $t_{i l}$ represents the expected duration (in minutes) of surgery $i$ when it is performed with an assistant surgeon with a level of experience $l$. For surgery $i$, the value $t_{i l}(l=1,2,3)$ is assumed to be related to $t_{i 0}$ (i.e. the length of the surgery when performed only by the responsible surgeon). Therefore, for each value of $l$, a variation interval affecting $t_{i 0}$ is defined as follows: (1) junior residents' surgeries are commonly trained surgeries, whereby the involvement of them always causes an increase of the surgery duration; (2) however, for senior residents, there are situations in which the resident has a similar level of experience that the faculty surgeon, causing a decrease of the surgery duration; (3) finally, the involvement of a faculty surgeon as assistant surgeon always produces a decrease of the surgery duration.

Hence, the problem can be considered as an integrated OR planning and scheduling problem under an open scheduling strategy where the assistant surgeons may influence surgery's duration (see Section 2.4). Additionally, the interventions may require a certain level of experience of the assigned assistant surgeon.

```
Indices and Sets
i\inI Index of surgery in the waiting list
j\inJ Index of OR
k}\in\boldsymbol{K}\quad\mathrm{ Index of surgeon
l\in\boldsymbol{L}}\quad\mathrm{ Index of level experience
k},\in\mp@subsup{\boldsymbol{K}}{l}{}\quad\mathrm{ Index of surgeon in level of experience l
h}\in\boldsymbol{H}\quad\mathrm{ Index of day within the planning horizon
Parameters
\mp@subsup{\boldsymbol{t}}{\boldsymbol{i}}{}\quad\mathrm{ Expected time of surgery i performed by an assistant surgeon belonging to level of experience l}
d
\mp@subsup{\tau}{i}{}}\quad\mathrm{ Responsible surgeon of surgery i
\delta}\mp@subsup{\boldsymbol{\delta}}{ij}{}\quad1\mathrm{ if surgery i can be performed in OR j;0 otherwise
\gammail }\quad1\mathrm{ if surgery i can be performed by an assistant surgeon belonging to surgeon type l;0 otherwise
skh}\quad\mathrm{ Maximum available time for surgeon }k\mathrm{ to conduct surgeries in day }
r rin Regular capacity of OR j in day }
B Maximum value of OR regular capacity (max jeJ,h\inH}\mp@subsup{r}{jh}{}
```



```
\mp@subsup{\boldsymbol{w}}{\boldsymbol{P}}{}}\quad\mathrm{ Weighted factor for the maximization of surgeries scheduled
\mp@subsup{\boldsymbol{w}}{\boldsymbol{T}}{}\quad\mathrm{ Weighted factor for the minimization of tardiness}
w
Variables
\mp@subsup{X}{ikjh}{}}\quad1\mathrm{ if surgery i is performed by assistant surgeon k in OR-day (j,h);0 otherwise
\mp@subsup{Y}{ii}{\prime}
C}\mp@subsup{\boldsymbol{C}}{\boldsymbol{h}}{}\quad\mathrm{ Completion time of surgery }i\mathrm{ in day }
C}\mp@subsup{\boldsymbol{C}}{\boldsymbol{max}}{\mp@subsup{k}{h}{}}\quad\mathrm{ Maximum surgery completion time of surgeon }k\mathrm{ in day }
Imin}\mp@subsup{}{k\boldsymbol{h}}{m}\quad\mathrm{ Minimum surgery starting time of surgeon }k\mathrm{ in day }
Z}\mp@subsup{\boldsymbol{k}}{\boldsymbol{h}}{}\quad\mathrm{ Idle time between surgeries of surgeon }k\mathrm{ in day }
```

Table 6.1. Sets, data and variables used in the ILP decision model

The objective of the problem is to maximize a weighted objective function which includes the number of surgeries scheduled, the tardiness of each surgery, and the idle time of each surgeon between consecutive surgeries.

### 6.3 ILP model formulation

In this section, we present an ILP model to solve the integrated OR planning and scheduling problem. Table 6.1 summarizes sets, data and variables used in the decision model.

The objective function is:
$\operatorname{Max} \frac{w_{P}}{|I|} \sum_{i \in 1} \sum_{k \in K} \sum_{j \in J} \sum_{h \in H} X_{i k j h}$
$-\frac{w_{T}}{|H| \cdot|I|}\left(\sum_{\substack{i \in \in \\ d_{i} \leq|H|}} \sum_{k \in K} \sum_{j \in \in} \sum_{\substack{n \in H \\ h>d_{i}}}\left(h-d_{i}\right) x_{i k j h}+\sum_{\substack{i \in\left| \\d_{i} \leq|H|\right.}}\left(|H|-d_{i}+1\right)\left(1-\sum_{k \in K} \sum_{j \in \in} \sum_{h \in H} x_{i k j h}\right)\right)$
$-\frac{w_{S}}{A} \sum_{k \in K} \sum_{h \in H} z_{k h}$

And the constraints are:
$\sum_{k^{\prime} \in K_{l}} \sum_{j \in J} \sum_{h \in H} X_{i k^{\prime} j h} \leq \gamma_{i l}, \quad \forall i \in I, \forall l \in L$
$C_{i^{\prime} h}+B\left(2-\sum_{k \in K} X_{i k j h}-\sum_{k^{\prime} \in K} X_{i^{\prime} k^{\prime} j h}\right) \geq C_{i h}+\sum_{l \in L} \sum_{k^{\prime} \in K_{l}} t_{i^{\prime} l} X_{i^{\prime} k^{\prime} j h}-B\left(1-Y_{i i^{\prime}}\right)$,
$\forall i \in I, \forall i^{\prime} \in I, \forall j \in J, \forall h \in H \mid i<i^{\prime}, \delta_{i j}=\delta_{i^{\prime} j}=1$
$C_{i h}+B\left(2-\sum_{k \in K} X_{i k j h}-\sum_{k^{\prime} \in K} X_{i^{\prime} k^{\prime} j h}\right) \geq C_{i^{\prime} h}+\sum_{l \in L} \sum_{k \in K_{l}} t_{i l} X_{i k j h}-B Y_{i i^{\prime}}$,
$\forall i \in I, \forall i^{\prime} \in I, \forall j \in J, \forall h \in H \mid i<i^{\prime}, \delta_{i j}=\delta_{i^{\prime} j}=1$
$C_{i^{\prime} h}+B\left(2-\sum_{j \in J} X_{i k j h}-\sum_{j^{\prime} \in J} X_{i^{\prime} k^{\prime} j^{\prime} h}\right) \geq C_{i h}+\sum_{j^{\prime} \in J} t_{i^{\prime} l} X_{i^{\prime} k^{\prime} j^{\prime} h}-B\left(1-Y_{i i^{\prime}}\right)$,
$\forall i \in I, \forall i^{\prime} \in I, \forall l \in L, \forall k^{\prime} \in K_{l}, \forall k \in K, \forall h \in H \mid i<i^{\prime},\left(\tau_{i^{\prime}}=\tau_{i}\right) \vee\left(k=k^{\prime} \wedge k \neq 0\right) \vee\left(\tau_{i^{\prime}}=k\right) \vee\left(\tau_{i}=k^{\prime}\right)$
$C_{i h}+B\left(2-\sum_{j \in J} X_{i k j h}-\sum_{j^{\prime} \in J} X_{i^{\prime} k^{\prime} j^{\prime} h}\right) \geq C_{i^{\prime} h}+\sum_{j \in J} t_{i l} X_{i k j h}-B Y_{i i^{\prime}}$,
$\forall i \in I, \forall i^{\prime} \in I, \forall l \in L, \forall k \in K_{l}, \forall k^{\prime} \in K, \forall h \in H \mid i<i^{\prime},\left(\tau_{i^{\prime}}=\tau_{i}\right) \vee\left(k=k^{\prime} \wedge k \neq 0\right) \vee\left(\tau_{i^{\prime}}=k\right) \vee\left(\tau_{i}=k^{\prime}\right)$
$I_{k h}^{\min } \leq C_{i h}-\sum_{l \in L} \sum_{k^{\prime} \in K_{l}} \sum_{j \in J} t_{i l} X_{i k^{\prime} j h}, \quad \forall k \in K, \forall i \in I, \forall h \in H \mid \tau_{i}=k, k \neq 0$
$I_{k h}^{\min }-B\left(1-\sum_{j \in J} X_{i k j h}\right) \leq C_{i h}-\sum_{l \in L} \sum_{k^{\prime} \in K_{l}} \sum_{j \in J} t_{i l} X_{i k^{\prime} j h}, \quad \forall k \in K, \forall i \in I, \forall h \in H \mid \tau_{i} \neq k, k \neq 0$
$C_{k h}^{\max } \geq C_{i h}, \quad \forall k \in K, \forall i \in I, \forall h \in H \mid \tau_{i}=k, k \neq 0$
$C_{k h}^{\max }+B\left(1-\sum_{j \in J} X_{i k j h}\right) \geq C_{i h}, \quad \forall k \in K, \forall i \in I, \forall h \in H \mid \tau_{i} \neq k, k \neq 0$
$Z_{k h} \geq C_{k h}^{m a x}-I_{k h}^{m i n}-\sum_{\substack{i \in I \\ \tau_{i}=k}} \sum_{j \in J} \sum_{\substack{l^{\prime} \in L}} \sum_{\substack{k^{\prime} \in K_{l^{\prime}} \\ k^{\prime} \neq k}} t_{i l^{\prime}} X_{i k^{\prime} j h}-\sum_{i \in I} \sum_{j \in J} t_{i l} X_{i k j h}$,
$\forall l \in L, \forall k \in K_{l}, \forall h \in H \mid k \neq 0$
$C_{i n} \geq \sum_{j \in J} \sum_{l \in L} \sum_{k \in K_{l}} t_{i l} X_{i k j h}, \quad \forall i \in I, \forall h \in H$
$C_{i h} \leq \sum_{k \in K} \sum_{j \in J} r_{j h} X_{i k j h}, \quad \forall i \in I, \forall h \in H$
$\sum_{\substack{i \in I \\ \tau_{i}=k}} \sum_{j \in J} \sum_{\substack{l^{\prime} \in L}} \sum_{\substack{k^{\prime} \in K_{l^{\prime}} \\ k^{\prime} \neq k}} t_{i l^{\prime}} X_{i k^{\prime} j h}+\sum_{i \in I} \sum_{j \in J} t_{i l} X_{i k j h} \leq s_{k h}, \quad \forall l \in L, \forall k \in K_{l}, \forall h \in H \mid k \neq 0$
$X_{i k j h}=0, \quad \forall i \in I, \forall k \in K, \forall j \in J, \forall h \in H \mid \tau_{i}=k$
$X_{i k j h}=0, \quad \forall i \in I, \forall k \in K, \forall j \in J, \forall h \in H \mid \delta_{i j}=0$
$X_{i k j h} \in\{0,1\}, \quad \forall i \in I, \forall k \in K, \forall j \in J, \forall h \in H$
$Y_{i i^{\prime}} \in\{0,1\}, \quad \forall i \in I, \forall i^{\prime} \in I \mid i^{\prime}>i$
$C_{i h} \geq 0, \quad \forall i \in I, \forall h \in H$
$C_{k h}^{\max }, I_{k h}^{\min } \geq 0, \quad \forall k \in K, \forall h \in H$
$Z_{k h} \geq 0, \quad \forall k \in K, \forall h \in H$
Equation (6.1) represents the objective function. As describe above, the first term is related to the maximization of the number of patients scheduled. The second term considers the minimization of the tardiness (i.e. the difference between the scheduled date and the deadline when the scheduled date is higher than the deadline). Surgeries which are scheduled after their deadlines as well as non-scheduled surgeries with deadline within the planning horizon are considered. Finally, the third term is related to the minimization of surgeons' waiting time between their surgeries. Note that each objective is normalized since these are measured in different units.

Constraints (6.2) enforce that each surgery is performed by only one assistant surgeon in only one suitable OR-day at most once. The set of constraints (6.3) ensures that a surgery is performed by an assistant surgeon with the level of experience required. Constraints (6.4) define the precedence relationship in each OR-day avoiding the overlap of surgeries. For each pair of surgeries ( $i, i^{\prime}$ ), the constraints do not apply if any of these surgeries are not scheduled in the same OR-day $(j, h)$. Therefore, $B$ is defined as the maximum value of OR regular capacity available in the planning horizon, being the completion times of these surgeries non positive numbers. The variable $Y_{i i}$, is introduced to consider whether surgery $i$ is finished before surgery $i^{\prime}$ (6.4a) or vice versa (6.4b) when both surgeries are scheduled in the same OR-day $(j, h)$. The set of constraints (6.5) defines the precedence relationship for each surgeon $k$ avoiding the overlap of surgeries ( $i, i^{\prime}$ ) in the same day. As in constraints (6.4), $B$ makes redundant the constraint if surgeries are not performed in the same day and $Y_{i i}$, is introduced to obtain the precedence between surgeries if both are scheduled in the same day. In this manner, the following situations can be considered for two consecutive surgeries ( $i, i$ ') in day $h: \eta_{i}=\eta_{i}$, assumes that a given surgeon is the responsible surgeon in both surgeries; $k=k^{\prime}$ supposes that a given surgeon is the assistant surgeon in both surgeries ; $\eta_{i}=k^{\prime}$ assumes that surgeon $k^{\prime}$ operates surgeries $i$ and $i^{\prime}$ as responsible and assistant surgeon, respectively; and finally, $\eta_{i^{\prime}}=k$ assumes that surgeon $k$ operates surgeries $i^{\prime}$ and $i$ as responsible and assistant surgeon respectively. Constraints (6.6a)-(6.6b)-(6.7a)(6.7b) define the earliest starting time $\left(I^{\min }{ }_{k h}\right)$ and the latest completion time ( $C^{\max }{ }_{k h}$ ) for each surgeon during a day respectively, working as a responsible surgeon (6.6a)-(6.7a) or assistant surgeon (6.6b)-(6.7b). The waiting time of a surgeon during a day $\left(Z_{k h}\right)$ is determined by constraints (6.8). Constraints (6.9) and (6.10) ensure that the completion time of a surgery in an OR-day must be higher or equal than the surgery duration -- set (6.9)--, and lesser or equal than OR-day regular capacity --set (6.10)--, respectively. Constraints (6.11) prohibit that the total surgery time allocated to a surgeon during a day is higher than his/her availability in this day. Note that the first and the second term take into account surgeries performed by the responsible surgeons and assistant surgeons, respectively. Constraints (6.12) prohibit that a surgeon perform a surgery as responsible and assistant, while constraints (6.13) ensure that each surgery is performed in a suitable OR. Finally, constraints (6.14)-(6.15) and constraints (6.16)-(6.17)-(6.18) are binary and non-negative continuous constraints for decision variables, respectively.

```
Procedure approximate algorithm
    W L : = ~ w a i t i n g ~ l i s t ~ i n c l u d i n g ~ p a t i e n t s ~ d a t a ; ~ \% ~ s u r g e r y ~ d u r a t i o n s , ~ d u e ~ d a t e . . .
    SL:= surgeon list including surgeons characteristics; % availability, level of experience...
    ORL:= OR list including regular capacity;
    AS:= 0;% assistant surgeon assigned to each surgery (initially zero)
    W L _ { S } : = 0 ; \% \text { sorted waiting list employed to build the surgery schedule (initially zero)}
    V:=0;% the value of the objective function
% Phase I
    (V,AS,WLS) := Phase I (WL,SL,ORL);
% Phase II
    (V,AS,WLS) := Phase II (WL,SL, ORL, V,AS,WLS);
```

Figure 6.1. Iterated Constructive algorithm

### 6.4 Iterated constructive method

In this section we propose an Iterated Constructive (IC) method for the problem. The algorithm is composed of two phases (see Figure 6.1): Phase I and Phase II. Phase I is aimed to obtain a fast feasible surgery schedule for a given problem. Then, the surgery schedule is improved using Phase II by changing the assistant surgeons and the order in the sequence of several surgeries.

The pseudocode of Phase I is shown in Figure 6.2, and consists of three steps:

- Step 1: Surgeon Assignment. In this step, an assistant surgeon is assigned to each surgery in the waiting list according to the following procedure: If possible ( $\gamma_{i 0}=1$ ), the dummy surgeon is assigned to the surgery (i.e. the surgery is only performed by the responsible surgeon). If not, a suitable assistant surgeon is randomly selected from the surgeon list. The result of this step is a vector ( $A S$ ) containing the assistant surgeon for each surgery.
- Step 2: Waiting list sorting. In this step, surgeries in the waiting list are grouped into two lists: surgeries whose deadline falls within the planning period $\left(W L_{A}\right)$, and the rest of patients $\left(W L_{B}\right)$. A sorted waiting list $\left(W L_{S}\right)$ is obtained by sorting the surgeries in $W L_{A}$ in ascending order of deadline (ties are broken by selecting the surgery with the lowest surgery duration), and then adding at the end the remaining surgeries (i.e. surgeries in $W L_{B}$ ) sorted in ascending order of surgery duration (note that surgery

```
Procedure Phase I (WL, SL, ORL)
\% Step 1: Surgeon Assignment
    AS := 0; \% assistant surgeon assigned to each surgery (initially zero)
    for \(i:=1\) to \(|I|\) do
            if the dummy surgeon is available for surgery \(i\) then
                \(A S[i]:=\) the dummy surgeon assigned to surgery \(i\) as assistant surgeon
            else
                \(A S[i]:=\) a random suitable assistant surgeon is selected from \(S L ;\)
            end if
    end for
\% Step 2: Waiting list sorting
    \(W L_{A}:=\) waiting list obtained by sorting surgeries with due date within the planning horizon in
                ascending order of due date (ties are broken with surgery duration);
    \(W L_{B}:=\) waiting list obtained by sorting sugeries with due date out of the planning horizon in ascending
                order of surgery duration;
    \(W L_{S}:=W L_{A} \cup W L_{B} ;\)
\% Step 3: Surgery schedule construction
    \(V:=\) construction_surgery_schedule \((A S, W L s) ; \%\) the objective function value
    return \(V, A S, W L_{S}\);
end
```

Figure 6.2. Phase I procedure
durations are already set as the assistant surgeons have been assigned to each surgery in the previous step).

- Step 3: Surgery schedule construction. A surgery schedule (date, OR and time indication for each surgery scheduled) is obtained here once $A S$ and $W L_{S}$ have been determined in the previous steps. Then, ORs in day $h$ are ordered in descending order of the amount of time that has been previously assigned to the responsible or assistant surgeon of the surgery. This order is denoted by $R O$. Each surgery is assigned to the earliest feasible day according to the order $R O$ of ORs. Using this procedure, the completion times of the surgeries are determined, as well as their OR and day where they take place. When trying to assign surgery $i$ to OR $R O[j]$ in day $h$, it is tried to be placed as soon as possible i.e. with a completion time equal to its surgery duration. Then, the feasibility with the rest of surgeries in the waiting list is checked. Note that infeasibilities due to other surgeries or due to the surgeons may appear. In case of infeasibility with surgery $j$ ', the completion time of surgery $j$ is replaced by the completion time of $j$ ' plus the surgery duration of $j$ and again, this completion time is checked against each other surgery. If it is not possible to further assign the surgery in $R O[j], R O[j+1]$ is tried.

```
Procedure construction_surgery_schedule (AS,WLS)
    for }i:=1\mathrm{ to }|I|\mathrm{ do
        for }h:=1\mathrm{ to }|H|\mathrm{ do
            % Determine the maximum OR time for each OR-day
            MOT := 0;% maximum OR time assigned to the responsible and/or the assistant surgeon in each OR-day (initially zero)
            for }j:=1\mathrm{ to }||\mathrm{ do
                    if the responsible surgeon and/or the assistant surgeon have been previously allocated to OR-day (j,h) then
                    MOT[j,h]:= maximum OR time assigned to one of them;
                    end if
            end for
            % Sort OR-days
            R O : = ~ l i s t ~ o b t a i n e d ~ b y ~ s o r t i n g ~ M O T ~ i n ~ d e s c e n d i n g ~ o r d e r ~ f o r ~ d a y ~ h ; \% ~ T i e s ~ a r e ~ b r o k e n ~ b y ~ s e l e c t i n g ~ t h e ~ n e x t ~ O R - d a y ~ t o ~
                    the OR-day where the last surgery was allocated
            % Schedule surgery i
            for j:=1 to }||\mathrm{ do
                if surgery }W\mp@subsup{L}{S}{}[i] can be scheduled in RO[j] satisfying all constraints then
                    Surgery }W\mp@subsup{L}{S}{}[i]\mathrm{ is scheduled in RO[j];
                    j:= |J| +1;% exit the loop
                end if
            end for
        end for
    end for
    Determine the objective function value (V);
    return }V\mathrm{ ;
end
```

Figure 6.3. Construction surgery schedule procedure

After assigning the surgeries, the new weighted objective function is calculated. The detailed procedure employed for obtaining such schedule is shown in Figure 6.3.

In order to improve the solution obtained in Phase I, successive calls of the construction surgery schedule step are made in Phase II. First, $N$ surgeries are randomly selected from $W L_{S}$. Then, new assistant surgeons $\left(A S_{n e w}\right)$ and new positions in the waiting list ( $W L_{\text {Snew }}$ ) are randomly chosen for the $N$ surgeries and the constructive surgery schedule is invoked. Then, the new weighted objective function is calculated. The procedure is iteratively called while the stopping criterion is not reached. The stopping criterion is defined as a CPU time limit depending on the size of the problem (see section 6.5.2). The pseudo-code of Phase II is shown in Figure 6.4.

### 6.5 Computational evaluation

In this section, we formulate the integrated OR planning and scheduling problem by slightly modifying the multi-mode blocking job shop model proposed by Pham and Klinkert (2008). Then, we compare the performance of the proposed model with that of the model proposed by Pham and Klinkert (2008). Finally, we carry out an extensive computational analysis to compare the quality of the solution obtained by solving the proposed model in an exact way and by using the proposed approximate method.

```
Procedure Phase II ( \(W L, S L, O R L, V, A S, W L S\) )
    \(V_{\text {best }}:=V\);
    \(A S_{\text {best }}:=A S\);
    \(W L_{\text {Sbest }}:=W L_{S}\);
    while the stopping criterion is not reached do
        \(N\) surgeries are randomly selected \((R L)\);
        \(A S_{\text {new }}:=A S_{\text {best }}\);
        \(W L_{\text {Snew }}:=\) waiting list obtained from \(W L_{\text {Sbest }}\) by excluding the N selected surgeries ;
        for \(i:=1\) to \(N\) do
            \(A S_{\text {new }}[R L[i]]:=\) assign a new random assistant surgeon whose level of experience can operate surgery \(R L[i] ;\)
            Introduce surgery \(R L[i]\) in a random position in \(W L_{\text {Snew }}\);
        end for
        \(V_{\text {new }}:=\) construction_surgery_schedule \(\left(A S_{\text {new }}, W L_{\text {Snew }}\right)\);
        if \(V_{\text {new }}>V_{\text {best }}\) and the new solution is feasible then
            \(V_{\text {best }}:=V_{\text {new }}\);
            \(A S_{\text {best }}:=A S_{\text {new }}\);
            \(W L_{\text {Sbest }}:=W L_{\text {Snew }}\);
        end if
    end while
    return \(V_{\text {best, }} A S_{\text {best }}, W L_{\text {best }}\),
end
```

Figure 6.4. Phase II procedure

We generate a testbed according to the procedure described in Chapter 3, considering the 54 different combinations of $|H|,|J|, \beta$ and $\alpha$ shown in shown in Table 6.2. Additionally, four different scenarios were defined by means of the following values of the objective weights:

- Scenario I: $w_{P}=0.33, w_{T}=0.33$ and $w_{S}=0.33$.
- Scenario II: $w_{P}=0.6, w_{T}=0.2$ and $w_{S}=0.2$.
- Scenario III: $w_{P}=0.2, w_{T}=0.6$ and $w_{S}=0.2$.
- Scenario IV: $w_{P}=0.2, w_{T}=0.2$ and $w_{S}=0.6$.

The different scenarios have been chosen in order to determine the influence of each objective. Thereby, in Scenario I, all objectives are equally weighted, while in scenarios II, III and IV, an objective (the number of surgeries scheduled, the tardiness for each surgery scheduled and the total idle time of each surgeon during a day) is prioritized above the others. For each combination of the parameters and scenarios, 10 instances are generated, resulting in a total of 1,080 instances. The experiments were carried out on a PC with 2.80 GHz Intel Core i7-930 processor and 16 GBytes of RAM memory.

| Factor | Level |
| :--- | :--- |
| $\boldsymbol{\| H \|}$ | $1,2,5$ |
| $\|\boldsymbol{J}\|$ | $3,6,9$ |
| $\boldsymbol{\beta}$ | $0.75,1.00,1.25$ |
| $\boldsymbol{\alpha}$ | $1.5,2.0$ |
| $\boldsymbol{C} \boldsymbol{V}$ | Ran $[0.1 \ldots 0.5]$ |
| $\boldsymbol{\mu}$ | Ran $[60,120,180,240]$ |
| $\boldsymbol{m} \boldsymbol{d} \boldsymbol{s}$ | Ran $[3 \ldots 5]$ |
| $\boldsymbol{a}$ | Ran $[240,360,480]$ |

Table 6.2. Factors and levels in the integrated OR planning and scheduling problem

### 6.5.1 Comparison to the multi-mode blocking job shop model proposed by Pham and Klinkert (2008)

Pham and Klinkert (2008) propose a multi-mode blocking job shop model to solve the elective surgical case scheduling problem, considering the makespan minimization objective (i.e. the maximum completion time). A mode is defined as a set of resources required to perform a surgery. Preoperative (a nurse), perioperative (a suitable OR, the responsible surgeon, a nurse and an anesthetist) and postoperative (a post-anesthesia care unit bed or a recovery bed) resources modes are considered. They assume constraints related to the availability of the resources as well as OR eligibility constraints. In order to adapt the multi-mode blocking job shop model proposed by Pham and Klinkert (2008) to our assumptions, the following modifications are made:

- Preoperative and postoperative stages are not taken into account. Blocking constraints are not allowed since the postoperative stage is not considered.
- Nurses and anesthetists are excluded from the model. Therefore, each surgical mode is composed by an OR, a responsible surgeon and an assistant surgeon.
- Since both deadlines constraints and surgeons idle time are not considered in the multi-mode proposed by Pham and Klinkert (2008), we only consider the maximization of the number of surgeries scheduled as objective function for the comparison, i.e. the following objective weights are taken into account: $w_{P}=1, w_{T}=$ 0 and $w_{S}=0$.

| Problem |  |  |  |  | Our model |  |  |  |  |  | Pham and Klinkert model adaptation |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \|H| | \|J| | B | $\|I\|$ | $\|K\|$ | Variables | Constraints | OS | FS | NS | Time (sec.) | Variables | Constraints | OS | FS | NS | Time (sec.) |
| 1 | 3 | 0.75 | 10 | 11 | 446 | 3,253 | 10 | 0 | 0 | 0 | 243 | 10,337 | 9 | 1 | 0 | 67 |
|  | 6 | 2.00 | 40 | 42 | 11,820 | 242,466 | 7 | 3 | 0 | 202 | 6,731 | 6,211,743 | 0 | 1 | 9 | 600 |
|  | 9 | 1.50 | 45 | 47 | 21,339 | 350,753 | 6 | 4 | 0 | 367 | 11,463 | 16,039,818 | 0 | 0 | 10 | 600 |
| 2 | 3 | 2.00 | 40 | 22 | 6,900 | 257,125 | 8 | 2 | 0 | 158 | 4,139 | 2,977,093 | 0 | 9 | 1 | 600 |
|  | 6 | 1.50 | 59 | 32 | 26,077 | 874,369 | 0 | 10 | 0 | 600 | 14,383 | 32,245,172 | 0 | 0 | 10 | 600 |
|  | 9 | 0.75 | 47 | 25 | 22,799 | 429,593 | 10 | 0 | 0 | 69 | 12,327 | 26,646,042 | 0 | 0 | 10 | 600 |
| 5 | 3 | 1.50 | 74 | 21 | 29,073 | 2,332,420 | 0 | 10 | 0 | 600 | 16,211 | 54,082,673 | 0 | 0 | 10 | 600 |
|  | 6 | 0.75 | 72 | 21 | 51,668 | 2,295,601 | 0 | 10 | 0 | 600 | 26,827 | $1.55 \cdot 10^{8}$ | 0 | 0 | 10 | 600 |
|  | 9 | 2.00 | 294 | 75 | 609,159 | 57,793,120 | 0 | 0 | 10 | 600 | 323,058 | $6.33 \cdot 10^{8}$ | 0 | 0 | 10 | 600 |

Table 6.3.Comparison of decision models.

The following indicators are considered in the comparison:

- Size of both models (average number of variables and constraints),
- Effectiveness of both models, according to the type of solution found after a given CPU time limit: Number of optimal solutions (OS), number of feasible (not optimal) solutions (FS), and number of problems for which no feasible solution is found (NS).
- Average CPU time required for both models.

Regarding the solver employed to analyze both models, Gurobi 5.6 and CPLEX 12.4 were initially tested. The best results were obtained by Gurobi, so it was selected for solving both models. The results are shown in Table 6.3 (for several sizes of the testbed). Note that the mean CPU time is obtained by averaging these results only for optimal and feasible solutions among the 10 instances of each size. It can be seen that our ILP model is more effective than the adaptation of the multi-mode decision model due to the much lesser number of constraints. Together with the fact that our ILP model always find better solutions than the adapted model, the proposed ILP model provides $88.9 \%$ feasible solutions ( $45.5 \%$ optimal solutions), while the adaptation of the multimode decision model yields $22.2 \%$ feasible solutions ( $10 \%$ optimal solutions).

Despite its efficiency, the proposed ILP model requires very long CPU times to obtain good feasible solutions (most instances reach the CPU time limit) and it is not able to obtain feasible solutions for planning horizons employed in practice (most commonly, the planning horizon length is 5 days). Therefore, in the next section, we evaluate the performance of the proposed approximate algorithm in terms of the CPU time required
and the relative percentage deviation of feasible solutions for large planning horizons such as those appearing in real cases.

### 6.5.2 Performance of the iterated constructive method

In order to compare the quality of the solution obtained by solving the proposed model in an exact way or by using the proposed approximate method, we consider the following response variables:

- CPU time required for solving a given instance, and
- Relative Percentage Deviation ( $R P D$ ) and Relative Percentage Deviation' ( $R P D^{\prime}$ ), according to expressions $R P D=100 \cdot\left(M_{b}-M_{\text {sol }}\right) / M_{b} \quad$ and $R P D^{\prime}=100 \cdot\left(M_{\text {bound }}-M_{\text {sol }}\right) / M_{\text {bound }}$, where $M_{\text {sol }}$ is the value of the objective function obtained by a given method for a given instance, $M_{b}$ is the value of the objective function corresponding to the best solution found and $M_{b o u n d}$ is the upper bound obtained by solving the instance by means of Gurobi (version 5.6) with a CPU time limit of 600 seconds. Note that the upper bound is determined by taking the maximum of the optimal objective values of all of the leaf nodes in the branch-andbound procedure used by Gurobi.

In order to determine the best parameter setting for the approximate method, different values of the parameter $N(3,5,7,9)$ are tested, obtaining $N=5$ the best results. Regarding the CPU time limit, it is calculated as $(|I| \cdot|J| \cdot|H| / 2) \cdot v$ milliseconds, being $v$ an integer parameter (25 and 100).

For each level of parameter $|H|$, the results are classified with respect to $|J|, \beta$ and the number of each scenario. The average number of patients ( $|\bar{I}|)$ and the average number of surgeons $(|\bar{K}|)$ are presented for each set of instances. Regarding the ILP model, Table 6.4 shows the number of optimal solutions ( $O S$ ), the number of feasible (nonoptimal) solutions ( $F S$ ), and the number of instances for which no feasible solution is found ( $N S$ ). Note that no statistically significant differences at a $99 \%$ confidence interval between the levels of $\alpha$ were found, setting to 1.5 without loss of generality. The results highlight the difficulty for the ILP model to find optimal solutions, or even feasible solutions as the problem size increases. Thereby, 178, 41 and 0 optimal
solutions are found for 1, 2 and 5 days respectively. Regarding feasible (non-optimal) solutions, the ILP model is able to find solutions in 182, 282 and 167 instances for 1,2 and 5 days respectively. No solution has been found for the 230 remaining instances. To sum up, the ILP model is able to obtain 219 optimal solutions and 631 feasible (nonoptimal) solutions while there are 230 unsolved instances. The CPU time (seconds) values for the ILP model and the IC (with $v=25$ and $v=100$ ) are also detailed in Table 6.4. The CPU time required by the ILP model increases with the problem size. The time limit of 600 seconds is reached in 861 times by the ILP model with an average runtime of 488.6 seconds for the total testbed. This represents a huge amount of time as compared to the average CPU time required by the IC with $v=25$ and $v=100$ for the total testbed ( 24.5 and 98.1 seconds respectively). The Average RPD (ARPD) and the Average RPD' (ARPD') values for the ILP model and the IC (with $v=25$ and $v=100$ ) are detailed in Table 6.5. Note that ARPD and ARPD' values are obtained by averaging these results only for optimal and feasible solutions. The approximate methods clearly outperform the ILP model. Thereby, the global ARPD-ARPD' values (for the whole testbed) are $6.90 \%-8.02 \%, 0.38 \%-1.69 \%$ and $0.11 \%-1.41 \%$ for the ILP model, the IC with $v=25$ and $v=100$ respectively.

For one-day planning horizons, ARPD and ARPD' values obtained by the ILP model $(0.4 \%-1.2 \%)$ are closer to those obtained by the heuristics. However, these values significantly increase with the size of the problem, being $9 \%-10.6 \%$ and $12.1 \%-15.8 \%$ for 2 and 5 days, respectively. This fact, together with the CPU time requirements, justifies the implementation of approximate methods to find acceptable solutions in short period of times.

| Problem |  |  | Type of solutions obtained by MILP |  |  |  |  |  |  |  |  |  |  |  |  |  |  | CPU time (sec.) |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\|H\|=1$ |  |  |  |  | $\|H\|=2$ |  |  |  |  | $\|H\|=5$ |  |  |  |  | $\|H\|=1$ |  |  | \|H| = 2 |  |  | \|H| = 5 |  |  |
| \|J| | $\beta$ (\%) | Scenario | $\|\bar{I}\|$ | $\|\overline{\boldsymbol{K}}\|$ | OS | FS | NS | $\|\bar{I}\|$ | $\|\bar{K}\|$ | OS | FS | NS | $\|\bar{I}\|$ | $\|\bar{K}\|$ | OS | FS | NS | MILP | $\begin{aligned} & \text { IC } \\ & (\nu=25) \end{aligned}$ | $\begin{aligned} & \text { IC } \\ & (v=100) \end{aligned}$ | MILP | $\begin{aligned} & \text { IC } \\ & (v=25) \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { IC } \\ & (v=100) \end{aligned}$ | MILP | $\begin{aligned} & \text { IC } \\ & (v=25) \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { IC } \\ & (v=100) \end{aligned}$ |
| 3 | 0.75 |  | 10.0 | 10.8 | 10 | 0 | 0 | 16.3 | 9.9 | 9 | 1 | 0 | 39.6 | 11.8 | 0 | 10 | 0 | 0.5 | 0.4 | 1.5 | 87.0 | 1.2 | 4.9 | 600.0 | 7.4 | 29.7 |
| 3 | 1.50 |  | 16.2 | 18.4 | 8 | 2 | 0 | 29.4 | 17.7 | 0 | 10 | 0 | 73.7 | 20.9 | 0 | 10 | 0 | 142.4 | 0.6 | 2.4 | 600.0 | 2.2 | 8.8 | 600.0 | 13.8 | 55.3 |
| 3 | 2.00 |  | 21.1 | 23.8 | 5 | 5 | 0 | 39.6 | 21.9 | 0 | 10 | 0 | 97.7 | 27.1 | 0 | 10 | 0 | 324.5 | 0.8 | 3.2 | 600.0 | 3.0 | 11.9 | 600.0 | 18.3 | 73.3 |
| 6 | 0.75 |  | 15.6 | 18.5 | 10 | 0 | 0 | 31.1 | 17.9 | 1 | 9 | 0 | 71.7 | 21.4 | 0 | 10 | 0 | 2.1 | 1.2 | 4.7 | 551.1 | 4.7 | 18.7 | 600.0 | 26.9 | 107.6 |
| 6 | 1.50 | I | 31.8 | 34.2 | 0 | 10 | 0 | 59.1 | 31.5 | 0 | 10 | 0 | 144.5 | 39.6 | 0 | 0 | 10 | 600.0 | 2.4 | 9.5 | 600.0 | 8.9 | 35.5 | 600.0 | 54.2 | 216.8 |
| 6 | 2.00 |  | 40.3 | 41.7 | 0 | 10 | 0 | 80.5 | 41.7 | 0 | 10 | 0 | 198.2 | 51.9 | 0 | 0 | 10 | 600.0 | 3.0 | 12.1 | 600.0 | 12.1 | 48.3 | 600.0 | 74.3 | 297.3 |
| 9 | 0.75 |  | 25.9 | 26.2 | 10 | 0 | 0 | 46.5 | 24.5 | 0 | 10 | 0 | 107.2 | 29.9 | 0 | 2 | 8 | 22.0 | 2.9 | 11.7 | 600.0 | 10.5 | 41.9 | 600.0 | 60.3 | 241.2 |
| 9 | 1.50 |  | 45.0 | 47.3 | 0 | 10 | 0 | 89.5 | 47.7 | 0 | 10 | 0 | 221.4 | 57.1 | 0 | 0 | 10 | 600.0 | 5.1 | 20.3 | 600.0 | 20.1 | 80.6 | 600.0 | 124.5 | 498.2 |
| 9 | 2.00 |  | 59.6 | 62.8 | 0 | 10 | 0 | 117.8 | 61.9 | 0 | 1 | 9 | 293.5 | 75.1 | 0 | 0 | 10 | 600.0 | 6.7 | 26.8 | 600.0 | 26.5 | 106.0 | 600.0 | 165.1 | 660.4 |
| 3 | 0.75 |  | 10.0 | 10.8 | 10 | 0 | 0 | 16.3 | 9.9 | 9 | 1 | 0 | 39.6 | 11.8 | 0 | 10 | 0 | 0.5 | 0.4 | 1.5 | 73.8 | 1.2 | 4.9 | 600.0 | 7.4 | 29.7 |
| 3 | 1.50 |  | 16.2 | 18.4 | 8 | 2 | 0 | 29.4 | 17.7 | 0 | 10 | 0 | 73.7 | 20.9 | 0 | 10 | 0 | 169.3 | 0.6 | 2.4 | 600.0 | 2.2 | 8.8 | 600.0 | 13.8 | 55.3 |
| 3 | 2.00 |  | 21.1 | 23.8 | 7 | 3 | 0 | 39.6 | 21.9 | 0 | 10 | 0 | 97.7 | 27.1 | 0 | 10 | 0 | 239.8 | 0.8 | 3.2 | 600.0 | 3.0 | 11.9 | 600.0 | 18.3 | 73.3 |
| 6 | 0.75 |  | 15.6 | 18.5 | 10 | 0 | 0 | 31.1 | 17.9 | 0 | 10 | 0 | 71.7 | 21.4 | 0 | 10 | 0 | 2.4 | 1.2 | 4.7 | 600.0 | 4.7 | 18.7 | 600.0 | 26.9 | 107.6 |
| 6 | 1.50 | II | 31.8 | 34.2 | 0 | 10 | 0 | 59.1 | 31.5 | 0 | 10 | 0 | 144.5 | 39.6 | 0 | 0 | 10 | 600.0 | 2.4 | 9.5 | 600.0 | 8.9 | 35.5 | 600.0 | 54.2 | 216.8 |
| 6 | 2.00 |  | 40.3 | 41.7 | 0 | 10 | 0 | 80.5 | 41.7 | 0 | 10 | 0 | 198.2 | 51.9 | 0 | 0 | 10 | 600.0 | 3.0 | 12.1 | 600.0 | 12.1 | 48.3 | 600.0 | 74.3 | 297.3 |
| 9 | 0.75 |  | 25.9 | 26.2 | 10 | 0 | 0 | 46.5 | 24.5 | 0 | 10 | 0 | 107.2 | 29.9 | 0 | 1 | 9 | 27.8 | 2.9 | 11.7 | 600.0 | 10.5 | 41.9 | 600.0 | 60.3 | 241.2 |
| 9 | 1.50 |  | 45.0 | 47.3 | 0 | 10 | 0 | 89.5 | 47.7 | 0 | 10 | 0 | 221.4 | 57.1 | 0 | 0 | 10 | 600.0 | 5.1 | 20.3 | 600.0 | 20.1 | 80.6 | 600.0 | 124.6 | 498.2 |
| 9 | 2.00 |  | 59.6 | 62.8 | 0 | 10 | 0 | 117.8 | 61.9 | 0 | 1 | 9 | 293.5 | 75.1 | 0 | 0 | 10 | 600.0 | 6.7 | 26.8 | 600.0 | 26.5 | 106.0 | 600.0 | 165.1 | 660.4 |
| 3 | 0.75 |  | 10.0 | 10.8 | 10 | 0 | 0 | 16.3 | 9.9 | 8 | 2 | 0 | 39.6 | 11.8 | 0 | 10 | 0 | 0.5 | 0.4 | 1.5 | 144.5 | 1.2 | 4.9 | 600.0 | 7.4 | 29.7 |
| 3 | 1.50 |  | 16.2 | 18.4 | 9 | 1 | 0 | 29.4 | 17.7 | 0 | 10 | 0 | 73.7 | 20.9 | 0 | 10 | 0 | 89.8 | 0.6 | 2.4 | 600.0 | 2.2 | 8.8 | 600.0 | 13.8 | 55.3 |
| 3 | 2.00 |  | 21.1 | 23.8 | 8 | 2 | 0 | 39.6 | 21.9 | 0 | 10 | 0 | 97.7 | 27.1 | 0 | 10 | 0 | 173.4 | 0.8 | 3.2 | 600.0 | 3.0 | 11.9 | 600.0 | 18.3 | 73.3 |
| 6 | 0.75 |  | 15.6 | 18.5 | 10 | 0 | 0 | 31.1 | 17.9 | 2 | 8 | 0 | 71.7 | 21.4 | 0 | 10 | 0 | 2.0 | 1.2 | 4.7 | 531.9 | 4.7 | 18.7 | 600.0 | 26.9 | 107.6 |
| 6 | 1.50 | III | 31.8 | 34.2 | 0 | 10 | 0 | 59.1 | 31.5 | 0 | 10 | 0 | 144.5 | 39.6 | 0 | 0 | 10 | 600.0 | 2.4 | 9.5 | 600.0 | 8.9 | 35.5 | 600.0 | 54.2 | 216.8 |
| 6 | 2.00 |  | 40.3 | 41.7 | 0 | 10 | 0 | 80.5 | 41.7 | 0 | 10 | 0 | 198.2 | 51.9 | 0 | 0 | 10 | 600.0 | 3.0 | 12.1 | 600.0 | 12.1 | 48.3 | 600.0 | 74.3 | 297.3 |
| 9 | 0.75 |  | 25.9 | 26.2 | 10 | 0 | 0 | 46.5 | 24.5 | 0 | 10 | 0 | 107.2 | 29.9 | 0 | 3 | 7 | 17.5 | 2.9 | 11.7 | 600.0 | 10.5 | 41.9 | 600.0 | 60.3 | 241.2 |
| 9 | 1.50 |  | 45.0 | 47.3 | 0 | 10 | 0 | 89.5 | 47.7 | 0 | 10 | 0 | 221.4 | 57.1 | 0 | 0 | 10 | 600.0 | 5.1 | 20.3 | 600.0 | 20.1 | 80.6 | 600.0 | 124.6 | 498.2 |
| 9 | 2.00 |  | 59.6 | 62.8 | 0 | 10 | 0 | 117.8 | 61.9 | 0 | 1 | 9 | 293.5 | 75.1 | 0 | 0 | 10 | 600.0 | 6.7 | 26.8 | 600.0 | 26.5 | 106.0 | 600.0 | 165.1 | 660.4 |
| 3 | 0.75 |  | 10.0 | 10.8 | 10 | 0 | 0 | 16.3 | 9.9 | 9 | 1 | 0 | 39.6 | 11.8 | 0 | 10 | 0 | 0.8 | 0.4 | 1.5 | 77.0 | 1.2 | 4.9 | 600.0 | 7.4 | 29.7 |
| 3 | 1.50 |  | 16.2 | 18.4 | 7 | 3 | 0 | 29.4 | 17.7 | 0 | 10 | 0 | 73.7 | 20.9 | 0 | 10 | 0 | 192.3 | 0.6 | 2.4 | 600.0 | 2.2 | 8.8 | 600.0 | 13.8 | 55.3 |
| 3 | 2.00 |  | 21.1 | 23.8 | 6 | 4 | 0 | 39.6 | 21.9 | 0 | 10 | 0 | 97.7 | 27.1 | 0 | 10 | 0 | 310.3 | 0.8 | 3.2 | 600.0 | 3.0 | 11.9 | 600.0 | 18.3 | 73.3 |
| 6 | 0.75 |  | 15.6 | 18.5 | 10 | 0 | 0 | 31.1 | 17.9 | 3 | 7 | 0 | 71.7 | 21.4 | 0 | 10 | 0 | 5.0 | 1.2 | 4.7 | 516.9 | 4.7 | 18.7 | 600.0 | 26.9 | 107.6 |
| 6 | 1.50 | IV | 31.8 | 34.2 | 0 | 10 | 0 | 59.1 | 31.5 | 0 | 10 | 0 | 144.5 | 39.6 | 0 | 0 | 10 | 600.0 | 2.4 | 9.5 | 600.0 | 8.9 | 35.5 | 600.0 | 54.2 | 216.8 |
| 6 | 2.00 |  | 40.3 | 41.7 | 0 | 10 | 0 | 80.5 | 41.7 | 0 | 10 | 0 | 198.2 | 51.9 | 0 | 0 | 10 | 600.0 | 3.0 | 12.1 | 600.0 | 12.1 | 48.3 | 600.0 | 74.3 | 297.3 |
| 9 | 0.75 |  | 25.9 | 26.2 | 10 | 0 | 0 | 46.5 | 24.5 | 0 | 10 | 0 | 107.2 | 29.9 | 0 | 1 | 9 | 20.1 | 2.9 | 11.7 | 600.0 | 10.5 | 41.9 | 600.0 | 60.3 | 241.2 |
| 9 | 1.50 |  | 45.0 | 47.3 | 0 | 10 | 0 | 89.5 | 47.7 | 0 | 10 | 0 | 221.4 | 57.1 | 0 | 0 | 10 | 600.0 | 5.1 | 20.3 | 600.0 | 20.1 | 80.6 | 600.0 | 124.5 | 498.2 |
| 9 | 2.00 |  | 59.6 | 62.8 | 0 | 10 | 0 | 117.8 | 61.9 | 0 | 0 | 10 | 293.5 | 75.1 | 0 | 0 | 10 | 600.0 | 6.7 | 26.8 | 600.0 | 26.5 | 106.0 | 600.0 | 165.1 | 660.4 |
|  | erage |  |  |  | 178 | 182 | 0 |  |  | 41 | 282 | 37 |  |  | 0 | 167 | 193 | 315.1 | 2.6 | 10.2 | 538.4 | 9.9 | 39.6 | 600.0 | 60.6 | 242.2 |

Table 6.4. Number of optimal solutions (OS), feasible solutions (FS), not feasible solution found (NS) and CPU time values

|  | Problem |  | ARPD (\%) |  |  |  |  |  |  |  |  | ARPD'(\%) |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\|H\|=1$ |  |  | $\|H\|=2$ |  |  | $\|H\|=5$ |  |  | $\|H\|=1$ |  |  | $\|H\|=2$ |  |  | $\|H\|=5$ |  |  |
| $\|J\|$ | $\beta$ (\%) | Scenario | MILP |  | IC | MILP |  | IC | MILP |  | IC | MILP |  | IC | MILP |  | IC | MILP |  | IC |
|  |  |  |  | $(v=25)$ | ) $(\nu=100)$ |  | $(v=25)$ | $(\nu=100)$ |  | $(v=25)$ | $(\nu=100)$ |  | $(v=25)$ | $(\nu=100)$ |  | $(\nu=25)$ | $(v=100)$ |  | $(\nu=25)$ | $(v=100)$ |
| 3 | 0.75 |  | 0.0 | 0.1 | 0.1 | 0.0 | 0.1 | 0.0 | 0.7 | 0.4 | 0.2 | 0.0 | 0.1 | 0.1 | 0.0 | 0.1 | 0.0 | 1.0 | 0.7 | 0.5 |
| 3 | 1.50 |  | 0.0 | 0.4 | 0.3 | 1.8 | 0.4 | 0.1 | 9.6 | 0.5 | 0.0 | 0.2 | 0.6 | 0.5 | 3.4 | 1.9 | 1.6 | 12.0 | 3.2 | 2.7 |
| 3 | 2.00 |  | 0.0 | 0.8 | 0.5 | 2.9 | 0.3 | 0.1 | 21.5 | 0.4 | 0.1 | 0.0 | 1.1 | 0.8 | 4.6 | 2.0 | 1.8 | 23.5 | 2.9 | 2.6 |
| 6 | 0.75 |  | 0.0 | 0.1 | 0.0 | 0.1 | 0.5 | 0.2 | 13.3 | 0.5 | 0.0 | 0.0 | 0.1 | 0.0 | 0.3 | 0.6 | 0.4 | 14.4 | 1.8 | 1.3 |
| 6 | 1.50 | I | 0.1 | 0.5 | 0.3 | 7.8 | 0.4 | 0.1 | -- | 0.3 | 0.0 | 1.6 | 1.9 | 1.8 | 10.5 | 3.3 | 3.1 | -- | -- | -- |
| 6 | 2.00 |  | 0.7 | 0.7 | 0.0 | 16.9 | 0.4 | 0.1 | -- | 0.3 | 0.0 | 2.3 | 2.3 | 1.6 | 19.0 | 2.8 | 2.6 | -- | -- | -- |
| 9 | 0.75 |  | 0.0 | 0.6 | 0.1 | 6.8 | 0.7 | 0.0 | 26.7 | 0.4 | 0.0 | 0.0 | 0.6 | 0.1 | 7.9 | 1.9 | 1.2 | 28.3 | 2.8 | 2.0 |
| 9 | 1.50 |  | 0.6 | 0.9 | 0.1 | 24.7 | 0.2 | 0.1 | -- | 0.2 | 0.0 | 2.8 | 3.2 | 2.4 | 27.1 | 3.4 | 3.3 | -- | -- | -- |
| 9 | 2.00 |  | 1.1 | 0.8 | 0.0 | 21.6 | 0.3 | 0.0 | -- | 0.2 | 0.0 | 3.2 | 2.9 | 2.1 | 23.8 | 2.9 | 2.6 | -- | -- | -- |
| 3 | 0.75 |  | 0.0 | 0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4 | 0.3 | 0.1 | 0.0 | 0.1 | 0.0 | 0.0 | 0.1 | 0.0 | 0.5 | 0.4 | 0.2 |
| 3 | 1.50 |  | 0.0 | 0.5 | 0.5 | 1.1 | 0.8 | 0.1 | 12.4 | 0.5 | 0.0 | 0.0 | 0.6 | 0.6 | 2.4 | 2.2 | 1.5 | 15.1 | 3.5 | 3.1 |
| 3 | 2.00 |  | 0.0 | 0.9 | 0.8 | 1.7 | 0.1 | 0.1 | 25.2 | 0.4 | 0.1 | 0.2 | 1.0 | 1.0 | 3.4 | 1.9 | 1.8 | 27.5 | 3.5 | 3.2 |
| 6 | 0.75 |  | 0.0 | 0.2 | 0.0 | 0.0 | 0.4 | 0.1 | 30.8 | 0.5 | 0.0 | 0.0 | 0.2 | 0.0 | 0.0 | 0.5 | 0.2 | 31.7 | 1.8 | 1.3 |
| 6 | 1.50 | II | 0.1 | 0.7 | 0.6 | 5.5 | 0.2 | 0.0 | -- | 0.5 | 0.0 | 1.6 | 2.1 | 2.1 | 8.6 | 3.5 | 3.4 | -- | -- | -- |
| 6 | 2.00 |  | 0.3 | 0.7 | 0.1 | 25.6 | 0.3 | 0.2 | -- | 0.3 | 0.1 | 2.2 | 2.6 | 2.0 | 27.6 | 3.1 | 3.0 | -- | -- | -- |
| 9 | 0.75 |  | 0.0 | 0.4 | 0.2 | 16.0 | 0.5 | 0.0 | 37.8 | 0.4 | 0.0 | 0.0 | 0.4 | 0.2 | 16.7 | 1.5 | 1.0 | 39.0 | 2.5 | 2.0 |
| 9 | 1.50 |  | 0.3 | 0.9 | 0.6 | 34.3 | 0.5 | 0.1 | -- | 0.3 | 0.0 | 2.7 | 3.3 | 3.0 | 36.6 | 4.1 | 3.7 | -- | -- | -- |
| 9 | 2.00 |  | 3.5 | 0.5 | 0.2 | 30.5 | 0.2 | 0.1 | -- | 0.4 | 0.0 | 5.9 | 3.0 | 2.7 | 32.3 | 3.7 | 3.0 | -- | -- | -- |
| 3 | 0.75 |  | 0.0 | 0.0 | 0.1 | 0.0 | 0.2 | 0.1 | 0.4 | 0.4 | 0.1 | 0.0 | 0.0 | 0.1 | 0.0 | 0.2 | 0.1 | 0.5 | 0.5 | 0.2 |
| 3 | 1.50 |  | 0.0 | 0.5 | 0.1 | 0.9 | 0.1 | 0.0 | 4.0 | 0.2 | 0.0 | 0.1 | 0.6 | 0.2 | 2.0 | 1.2 | 1.1 | 5.9 | 2.1 | 2.0 |
| 3 | 2.00 |  | 0.0 | 0.6 | 0.3 | 1.6 | 0.2 | 0.0 | 17.5 | 0.1 | 0.1 | 0.0 | 0.7 | 0.4 | 2.8 | 1.4 | 1.2 | 19.0 | 2.0 | 1.9 |
| 6 | 0.75 |  | 0.0 | 0.1 | 0.1 | 0.2 | 0.3 | 0.1 | 16.0 | 0.3 | 0.0 | 0.0 | 0.1 | 0.1 | 0.4 | 0.4 | 0.2 | 16.7 | 1.2 | 0.9 |
| 6 | 1.50 | III | 0.3 | 0.6 | 0.1 | 4.5 | 0.4 | 0.0 | -- | 0.2 | 0.1 | 1.1 | 1.3 | 0.9 | 6.3 | 2.2 | 1.9 | -- | -- | -- |
| 6 | 2.00 |  | 0.3 | 0.6 | 0.0 | 17.1 | 0.3 | 0.1 | -- | 0.2 | 0.0 | 1.5 | 1.7 | 1.2 | 18.5 | 2.0 | 1.8 | -- | -- | -- |
| 9 | 0.75 |  | 0.0 | 0.3 | 0.2 | 12.5 | 0.1 | 0.2 | 22.9 | 0.4 | 0.0 | 0.0 | 0.3 | 0.2 | 13.2 | 1.0 | 1.0 | 23.2 | 1.6 | 1.4 |
| 9 | 1.50 |  | 0.4 | 0.4 | 0.1 | 22.8 | 0.1 | 0.1 | -- | 0.3 | 0.0 | 2.0 | 1.9 | 1.6 | 24.5 | 2.3 | 2.3 | -- | -- | -- |
| 9 | 2.00 |  | 0.8 | 0.5 | 0.0 | 20.1 | 0.3 | 0.0 | -- 0. | 0.2 | 0.0 | 2.2 | 1.9 | 1.5 | 22.6 | 1.8 | 2.0 | -- | -- | -- |
| 3 | 0.75 |  | 0.0 | 0.3 | 0.1 | 0.0 | 0.4 | 0.1 | 1.4 | 0.4 | 0.0 | 0.0 | 0.3 | 0.1 | 0.0 | 0.4 | 0.1 | 2.0 | 0.9 | 0.6 |
| 3 | 1.50 |  | 0.0 | 0.5 | 0.3 | 1.6 | 0.3 | 0.0 | 7.0 | 0.4 | 0.0 | 0.3 | 0.8 | 0.6 | 2.8 | 1.5 | 1.2 | 9.3 | 2.8 | 2.4 |
| 3 | 2.00 |  | 0.1 | 0.7 | 0.5 | 4.0 | 0.3 | 0.0 | 12.7 | 0.4 | 0.1 | 0.2 | 0.8 | 0.6 | 5.3 | 1.7 | 1.4 | 14.5 | 2.5 | 2.2 |
| 6 | 0.75 |  | 0.0 | 0.2 | 0.1 | 0.5 | 0.5 | 0.1 | 12.4 | 0.4 | 0.1 | 0.0 | 0.2 | 0.1 | 0.8 | 0.8 | 0.4 | 13.6 | 1.8 | 1.5 |
| 6 | 1.50 | IV | 0.6 | 0.5 | 0.1 | 6.7 | 0.5 | 0.1 | -- 0.3 | 0.3 | 0.1 | 1.9 | 1.8 | 1.4 | 9.0 | 2.9 | 2.5 | -- | -- | -- |
| 6 | 2.00 |  | 1.1 | 0.5 | 0.1 | 12.3 | 0.3 | 0.1 | -- 0. | 0.2 | 0.1 | 2.6 | 1.9 | 1.5 | 14.4 | 2.6 | 2.4 | -- | -- | -- |
| 9 | 0.75 |  | 0.0 | 0.7 | 0.5 | 5.3 | 0.3 | 0.1 | 17.2 | 0.3 | 0.1 | 0.0 | 0.7 | 0.5 | 6.9 | 2.0 | 1.8 | 18.9 | 2.6 | 2.3 |
| 9 | 1.50 |  | 1.3 | 0.4 | 0.0 | 15.6 | 0.5 | 0.0 | -- 0.4 | 0.4 | 0.0 | 3.3 | 2.4 | 2.1 | 18.0 | 3.3 | 2.8 | -- | -- | -- |
| 9 | 2.00 |  | 3.2 | 0.5 | 0.0 | -- | 0.1 | 0.1 | -- | 0.2 | 0.0 | 5.0 | 2.3 | 1.8 | -- | -- | -- | -- | -- | -- |
|  |  |  | 0.4 | 0.5 | 0.2 | 9.0 | 0.3 | 0.1 | 12.1 | 0.3 | 0.0 | 1.2 | 1.3 | 1.0 | 10.6 | 1.9 | 1.7 | 15.8 | 2.1 | 1.7 |

Table 6.5. ARPD and ARPD' values

| Objective | Scenario |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | I | II | III | IV |
| No. scheduled surgeries | 0.778 | 0.801 | 0.782 | 0.765 |
| Tardiness | 0.016 | 0.023 | 0.010 | 0.019 |
| Surgeon idle time | 0.022 | 0.035 | 0.023 | 0.011 |

Table 6.6. Objective functions normalized values

Finally, Table 6.6 shows the normalized average value for each objective under the different scenarios, providing to the OR manager an overview of the implications of choosing any of the proposed scenarios. Obviously, scenario II, III and IV maximize the number of surgeries, minimize the tardiness of the surgeries and minimize the surgeons' waiting time respectively. Thereby, for example under the scenario III, the tardiness sharply decreases from 0.016 to 0.010 with respect to the equally weighted scenario (i.e. scenario I) as well as the number of scheduled surgeries increases from 0.778 to 0.782 , while the surgeon idle time stay very similar (from 0.022 to 0.023 ).

### 6.5.3 Analysis of surgery duration uncertainty: a simulation approach

In Section 6.5.2, the computational experience of this section has been carried out assuming deterministic surgery durations. However, the surgical schedule is usually influenced by the stochastic nature of the surgery duration (see e.g. Cardoen et al., 2010). For these reasons, a number of simulations have been carried out to analyze the robustness of the so-obtained surgical schedules. More specifically, for each instance of the testbed, the surgical schedule obtained in a deterministic way was simulated 100 times by modifying the surgery duration of surgeries scheduled. Surgery durations were varied according to a log-normal distribution where the expected duration is the deterministic surgery duration and the standard deviation is $5 \%, 10 \%, 15 \%$ or $20 \%$ of the expected duration (i.e. $C V=0.05,0.10,0.15$ and 0.20 ). Note that 432,000 simulations were performed, being the feasibility of each surgical schedule analyzed for each value of $C V$. The robustness is measured by: the average OR-day utilization, the percentage of surgical resources (OR and surgeons) with overtime and the average overtime. These results and the average OR-day utilization are shown in Table 6.7.

| $\boldsymbol{C V}$ | Av. OR-day <br> Utilization (min.) | \% OR-days <br> with overtime | Av. OR-day <br> Overtime (min.) | \% Surgeons <br> with overtime | Av. Surgeon <br> Overtime (min.) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{0 . 0 5}$ | 376.46 | $5.58 \%$ | 9.24 | $1.99 \%$ | 8.46 |
| $\mathbf{0 . 1 0}$ | 376.46 | $11.91 \%$ | 18.99 | $4.21 \%$ | 17.12 |
| $\mathbf{0 . 1 5}$ | 376.51 | $16.88 \%$ | 29.49 | $6.27 \%$ | 26.10 |
| $\mathbf{0 . 2 0}$ | 376.50 | $20.66 \%$ | 40.63 | $8.14 \%$ | 35.41 |

Table 6.7. Simulation results for analyzing the stochasticity of surgery durations

It can be seen that the average OR utilization is not influenced by the stochasticity of the surgery duration. Thereby, the average utilizations are $376.46,376.46,376.51$ and 376.50 minutes by using $C V=0.05,0.1,0.15$ and 0.20 respectively. Due to the large number of simulations performed, $95 \%$ confidence intervals lengths are very narrow for each case, being, for example, 0.85 minutes for $C V=0.20$. Regarding OR overtime, only $5.58 \%$ of OR-days have overtime for $C V=0.05$, being the average overtime 9.24 minutes. Even for a high stochasticity of surgery durations represented by $C V=0.20$, $20.66 \%$ of the OR-days have overtime, being the average overtime 40.63 minutes (which represents $8.47 \%$ of the capacity of the ORs). Finally, surgeons' overtime are still more favorable since only the $1.99 \%, 4.21 \%, 6.27 \%$ and $8.14 \%$ of the surgeons have overtime for $C V=0.05,0.10,0.15$ and 0.20 respectively. In case of overtime, it is 8.46, 17.12, 26.10 and 35.41 minutes on average for $C V=0.05,0.10,0.15$ and 0.20 respectively. Summarizing the results of the simulation, we can conclude that the surgical schedules proposed by the approximate method are robust in terms of: (i) ORs, since the worst overtime value (i.e. 40.53 minutes, a $8.47 \%$ of the OR-day regular capacity for $C V=0.20$ ) is acceptable in real/literature settings, in which the overtime allowed varies from $25 \%$ to $50 \%$ of the regular capacity (see e.g. Roland et al., 2010); (ii) Surgeons, since the average surgeon overtime is 35.41 minutes, which represents a $9.84 \%$ of the average available time of surgeons. In the case overtime is not allowed, the simulation results can be used to determine planned slacks for reducing/avoiding the overtime (see e.g. Hans et al., 2008).

### 6.6 Conclusions

In this chapter, we have addressed the integrated OR planning and scheduling problem which consists on assigning the date, the OR and the time indication for each surgery in the waiting list over a given planning horizon. In practice, surgeries are usually performed by two-surgeon surgical teams (the responsible surgeon and the assistant surgeon), and the surgery duration depends on the type of the assistant surgeon assigned to the surgery. To the best of our knowledge, this decision problem has not been addressed in the literature. The novelty of our contribution is that surgery durations depend on the surgical team, which may be composed by one or two surgeons with different level of experience.

We have proposed a ILP model and an adaptation of the multi-mode blocking job shop model (Pham and Klinkert, 2008) to solve the problem. The performance of both models is compared by generating a set of instances based on the literature. The results show that the proposed model is more effective than the adapted multi-mode model. Nevertheless, both approaches are not able to find feasible solutions for real-life instance sizes in an acceptable CPU time. Therefore, we propose an approximate algorithm for obtaining good feasible solutions in short CPU times. The computational experience shows that the proposed algorithm is able to find feasible solutions for all problems in the testbed, requiring shorter CPU times than the ILP model. Additionally, the algorithm provides better average relative percentage deviations than the ILP model for each planning horizon of the testbed, resulting in an ARPD of $0.11 \%$ for IC ( $\mathrm{v}=$ 100), which is a $7.52 \%$ lower than that of Gurobi. Finally, the robustness of surgical schedules calculated in such deterministic way has been analyzed via simulation, resulting that, in the worst case, $20.66 \%$ of OR-days and $8.14 \%$ of surgeons would have overtime. Nevertheless, the average overtime for both surgeons and ORs is $8.47 \%$ of the OR-day regular capacity and $9.84 \%$ of the average available surgeon time. These results are acceptable in real settings and hence, a deterministic approach is suitable for solving the proposed problem.

# Part III 

Real

## Application

## Chapter 7

## Validation of Solution Procedures: A Real Application

### 7.1 Introduction

In this chapter we present the results of the implementation of the decision models and solution approaches described in previous chapters in the University Hospital "Virgen del Rocio", focusing on the Plastic Surgery and Major Burns Specialty, the pilot surgical specialty. In order to give a clear idea of this surgical specialty, first, the specific OR planning and scheduling problem of the Plastic Surgery and Major Burns Specialty is described in Section 7.2. Then, the decision models and solution approaches presented in Chapter 4 are validated for this Specialty. The validation is carried out both with experimental (i.e. generating specific problem instances based on the sizes and patterns of past interventions in this department, together with the specific constraints and policies employed in this specialty), and historical data (i.e. by using pasts waiting lists to compare the solution obtained by the procedures with the schedules applied in practice) in Section 7.3. By conducting the experimental validation we aim to ensure the quality of the solution procedures (already tested in testbeds extracted from the literature) when applied to this specific. The so-called historical validation provides us with a quantification of the advantages of using the proposed models, which serves us to increase the acceptance of the DSS by the responsible of the surgical and to set goals for and after its implementation. Furthermore, the capabilities of such DSS are explored by conducting a what-if analysis on several allocation policies, and on different objectives. In Section 7.4., the DSS implemented and currently in use in the Specialty is outlined. Finally, in Section 7.5, the conclusions of the chapter are presented.

### 7.2 The OR planning and scheduling problem of the Plastic Surgery and Major Burns Specialty

In this Section, we describe the specific OR planning and scheduling problem in the Plastic Surgery and Major Burns Specialty of the University Hospital "Virgen del Rocio". Note that the problem under consideration is modeled by the decision model presented in Chapter 4.

The Plastic Surgery and Major Burns Specialty performs around 3,000 surgeries per year, including emergency, deferred urgency, elective and ambulatory surgeries. More specifically, the specialty has 14 surgeons and 4 multifunctional ORs for performing deferred urgency, elective and ambulatory surgeries. Emergency surgeries are not considered as a part of our problem, since these surgeries are performed using additional resources (called urgent surgical resources). Currently, on each day, 3 ORs are available for performing deferred urgency and elective surgeries from 8.30 a.m. to 3 p.m., and 1 OR is reserved for performing ambulatory surgeries from 3 p.m. to 8 p.m. Regarding surgeons availability, a weekly schedule is defined by the responsible of the surgical unit, specifying who surgeons are available for performing surgeries (the maximum surgery time is 6.5 hours per day), and for doing other tasks (consultations, look after patients operated, etc.). The number of ORs where a surgeon could be allocated is limited in order to reduce surgeon idle time and overlapping of consecutive surgeries by the same surgeon. Finally, the remaining human and instrumental perioperative resources and recovery facilities are assumed to be available whenever needed, thus not representing bottlenecks.

The modified block scheduling strategy is used by the Decision Maker to manage ORs. Burn surgeries (i.e. deferred urgency surgeries) have two reserved OR-days (i.e. a tuple of an OR and a day) every week because of their unpredictable arrivals and their high priority (they have to be operated as soon as possible), and because they can only be operated by only few surgeons. Most plastic surgeries can be performed in any available OR by any available surgeon, with the exception of microsurgeries which have two reserved OR-days every week because of their complexity, the special surgical equipment required, and the high estimated length of the surgery (around 10 hours).

At the consultation stage, each patient on the waiting list is assigned to a surgeon who is the responsible for performing the surgery. This assignment is made by the responsible of the surgical specialty (Decision Maker) based on surgeon's specialty (i.e. types of surgery which could be performed by the surgeon), his/her skills and workload. The expected surgery duration is forecasted by the Decision Maker based on the historical data and patient's characteristics. Each surgery must be scheduled within a time period defined by its release and deadlines. The release date is the earliest date in which the patient could be operated (i.e. once all medical tests are completed). The deadline (i.e. the latest date for performing the surgery) depends on the maximum time before treatment (in days) established by the patient's urgency-related group, which is defined by National Healthcare Services based on a set of explicit clinical and social criteria. The maximum times considered in the Specialty are 45, 180 and 365 days.

The objective function is derived from the performance indicators employed by the Regional Healthcare System in Andalusia (Spain), and it is related to minimizing access time for patients with higher clinical weight values. The clinical weight depends on a linear combination of the priority of the surgery (so a higher urgency of the surgery leads to a greater weight) and the number of days per patient spent on the waiting list at the time (patients with longer stays on the waiting list have higher weights and thus it aims to reduce access time). It is to note that this weighting function yields a higher priority to the single patient with the highest weight as long as a set of patients whose sum of weights is highest than that of the single patient and they all together can be planned in the available slot. For this reason, it is necessary to give greater weight to a single patient's clinical weight as compared to the sum of patients' clinical weight. In Section 7.4, we take into account this issue in the objective function by means of a parameter $g$ which is the exponent of the patient's clinical weight, so that if $g=1$ we consider the first scenario, while $g>1$ indicates the second one.

Finally, the OR planning and scheduling problem is performed on a two weekly base and is finalized on Friday for the following two weeks. In addition, other decisions are made over medium-long planning horizons (four-week, eight-week or twelve-week planning horizons) in order to inform patients several weeks or even months in advance of their surgeries (reducing the number of cancellations, and improving the quality of service) or to negotiate surgical resources for future planning periods (managerial whatif analyses).

### 7.3 Validation and analysis

In this section we carry out computational experiments in order to validate the proposed solution procedures for solving the OR planning and scheduling problem in the Plastic Surgery and Major Burns Specialty of the University Hospital "Virgen del Rocio". Note that, the problem is modeled using the decision model proposed in Chapter 4, and solved using the proposed constructive heuristics and the Random Extraction-Insertion (REI) algorithm. Although the REI metaheuristic is the best method for solving the OR planning problem (see Chapter 4), the constructive heuristics are also taken into account in this analysis due to the lower CPU times required to solve the problem, which would be an important advantage to make decisions over medium-long planning horizons. First, an experimental validation is carried out to ensure the quality of the solution approaches for solving the problem (see Section 7.3.1). We focus on the performance for solving the problem over medium-long planning horizons, since the efficiency for solving the problem over a short planning horizon (week) has been showed in Chapter 4. Once the efficiency of the proposed solution approaches is showed for solving the problem, we present a historical validation to quantify the advantages of using the decision model and the REI method (see Section 7.4.2). Finally, Section 7.4.3 presents what-if analyses for solving the problem to compare the impact on several patient allocation policies, several objective functions, several resource management strategies and several planning horizons.

### 7.3.1 Experimental validation

As described above, the Decision Maker makes decisions related to inform patients several weeks or even months in advance of their surgeries or negotiate surgical resources for future planning periods. In this context, the complexity of the problem increases due to the huge number of decision variables (i.e. the number of surgeries in the waiting list, and the number of OR-days) and constraints. With this consideration in mind, we propose a solution approach to solve the problem over medium-long planning horizons. The solution approach is as follows:

- First, the planning horizon is divided into weekly planning horizons, since a week is typically used for solving the OR planning problem (see Chapter 4).
- Second, a partial waiting list is determined for each weekly planning horizon. The partial waiting list is determined using the following procedure: (i) patients that could not be scheduled in previous planning horizons are sorted according to their clinical weight (i.e. the best sorting indicator as shown in Chapter 4); (ii) patients are selected one by one until the total sum of their expected surgery duration exceeds a percentage $\gamma$ of the OR regular capacity available in the weekly planning horizon. For each partial waiting list, the planning problem is solved by the constructive heuristics and the REI metaheuristic proposed in this Thesis.

A calibration procedure is carried out to determine the best parameter setting for the REI metaheuristic. We generate a test bed according to the procedure described in Chapter 3, considering the 20 different combinations of $\beta,|H|$ and $|J|$ (see Table 7.1). For each combination of the factors, 10 instances are generated, resulting in a total of 120 instances. The experiments were carried out on a PC with 2.40 GHz Intel Core i5 processor and 4 GBytes of RAM memory. The RPD is considered as the response variable, being define as $R P D=100 \cdot\left(M_{b}-M_{\text {sol }}\right) / M_{b} \cdot M_{\text {sol }}$ is the value of the objective function obtained by a given method for a given instance, and $M_{b}$ is the value of the objective function corresponding to the best solution found.

Regarding the percentage used to determine the waiting list considered at each weekly planning horizon, we test $100 \%, 125 \%$ and $150 \%$ as levels of $\gamma$, being $\gamma=125 \%$ the best level. As describe in Chapter 4, REI is characterized by the constructive heuristic used to generate the initial solution, the number of extracted surgeries ( $n$ ), the percentage of the maximal deterioration $(\theta)$ and the probability of accepts a solution which deteriorates a solution $(\varphi)$. Regarding the constructive heuristic used to generate the initial solution, we consider ST and MT heuristics proposed in Chapter 4. The $\mathrm{MT}_{A L L}$ heuristic yields the best results to determine the surgical schedule from which the initial waiting list is constructed. In order to reduce CPU time values, we only consider the sorting tuples ( $t, H I L L$ ) and ( $w, D E C$ ), which are involved in the best ST heuristics (see the best ST heuristics for each level of $|H|$ in Table 7.2). REI is tested with the following levels: $n$ is set to 1,3 and $5 ; \theta$ is set to $10 \%$ and $20 \%\}$ and; $\varphi$ is set to $1 \%, 5 \%$, and $10 \%$. The best setting was $n=1, \theta=10 \%$ and $\varphi=5 \%$.

| Factor | Level |
| :--- | :--- |
| $\boldsymbol{\| \boldsymbol { \| } \|}$ | $5,10,20,40,60$ |
| $\|\boldsymbol{J}\|$ | 4,8 |
| $\boldsymbol{\beta}$ | $1.00,1.25$ |
| $\boldsymbol{\|} \boldsymbol{K} \mid$ | 8,16 |
| $\boldsymbol{C} \boldsymbol{V}$ | 0.1 |
| $\boldsymbol{\mu}$ | 120 |
| $\boldsymbol{m} \boldsymbol{d s}$ | Ran $[3 \ldots 5]$ |
| $\boldsymbol{a}$ | 480 |
| $\boldsymbol{u}$ | $\|J\|$ |

Table 7.1. Factors and levels considered in the OR planning and scheduling problem in the

> University Hospital "Virgen del Rocio"

| $\|H\|$ | $\overline{\mathbf{B P}}$ <br> algorithm | I | C | $\begin{aligned} & \hline \text { RPD } \\ & (\%) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| 5 | FF | $\boldsymbol{w}$ | DEC | 0.10 |
|  | BF |  |  | 1.97 |
|  | LF |  |  | 10.64 |
| 10 | FF | $\boldsymbol{w}$ | DEC | 0.16 |
|  | BF |  |  | 0.89 |
|  | LF |  |  | 5.43 |
| 20 | FF | $\boldsymbol{w}$ | DEC | 0.50 |
|  | BF |  |  | 0.52 |
|  | LF |  |  | 2.52 |
| 40 | FF | $\boldsymbol{w}$ | DEC | 1.40 |
|  | BF |  |  | 1.11 |
|  | LF |  |  | 1.23 |
| 60 | FF | w | DEC | 2.54 |
|  | BF |  |  | 1.76 |
|  | LF | $t$ | HILL | 1.08 |

Table 7.2. TSBP heuristics calibration results

The results are classified with respect to $|H|,|J|$ and $\beta$, being the average number of patients in the waiting list $(|\bar{I}|)$ and the average number of surgeons $(|\bar{K}|)$ presented for each set of instances. Table 7.3 shows the Average RPD (ARPD), the number of instances in which a feasible solution is found and the CPU time (in seconds) required for each approach (i.e. ILP, best ST heuristics, $\mathrm{MT}_{\mathrm{ALL}}$ and REI). Note that ARPD values are obtained by averaging these results only for feasible solutions. The ILP approach is solved by using the commercial software Gurobi version 5.6 with a stopping criterion.

|  | $\|J\|$ | $\beta(\|\bar{I}\|)$ | $\|\overline{\boldsymbol{K}}\|$ | ARPD (\%)/No. solutions found |  |  |  | CPU time (sec.) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \|H| |  |  |  | ILP | ST | $\mathrm{MT}_{\text {ALL }}$ | REI | ILP | ST | $\mathrm{MT}_{\text {ALL }}$ | REI |
| 5 | 4 | 1.00 (81.1) | 8.1 | 0.26 / 10 | $4.31 / 10$ | $4.31 / 10$ | 0.12 / 10 | 20.0 | 0.015 | 0.016 | 20.0 |
|  |  | 1.25 (100.4) | 8.2 | 0.55 / 10 | 4.71 / 10 | 4.52 / 10 | $0.00 / 10$ | 25.0 | 0.012 | 0.016 | 25.0 |
|  | 8 | 1.00 (161.5) | 15.7 | $0.27 / 10$ | $3.37 / 10$ | $3.33 / 10$ | 0.06/10 | 40.0 | 0.020 | 0.053 | 40.0 |
|  |  | 1.25 (201.6) | 15.7 | 0.29 / 10 | 3.47 / 10 | $3.30 / 10$ | $0.35 / 10$ | 50.0 | 0.019 | 0.070 | 50.0 |
| 10 | 4 | 1.00 (160.9) | 8.2 | 0.90 /10 | 4.79 / 10 | 4.70 / 10 | $0.01 / 10$ | 40.0 | 0.020 | 0.030 | 40.0 |
|  |  | 1.25 (201.8) | 8.1 | 1.51/10 | $5.33 / 10$ | 4.87 / 10 | $0.00 / 10$ | 50.0 | 0.020 | 0.031 | 50.0 |
|  | 8 | 1.00 (321.2) | 15.4 | $0.24 / 10$ | 3.87 / 10 | $3.85 / 10$ | $0.07 / 10$ | 80.0 | 0.040 | 0.118 | 80.0 |
|  |  | 1.25 (402.5) | 15.7 | 0.72 / 10 | $4.26 / 10$ | $4.08 / 10$ | $0.14 / 10$ | 100.0 | 0.039 | 0.130 | 100.0 |
| 20 | 4 | 1.00 (322.2) | 8.1 | 1.39 / 10 | 5.70 / 10 | $5.33 / 10$ | $0.00 / 10$ | 80.0 | 0.041 | 0.066 | 80.0 |
|  |  | 1.25 (401.7) | 8.6 | 1.45 / 10 | $7.00 / 10$ | 6.26 / 10 | $0.00 / 10$ | 100.0 | 0.047 | 0.066 | 100.0 |
|  | 8 | $1.00 \text { (643.7) }$ | 15.4 | 1.33 / 10 | 3.99 / 10 | 3.74 / 10 | $0.00 / 10$ | 160.0 | 0.091 | 0.268 | 160.0 |
|  |  | 1.25 (804.7) | 15.7 | $0.91 / 10$ | $5.65 / 10$ | $5.53 / 10$ | $0.11 / 10$ | 200.0 | 0.102 | 0.276 | 200.0 |
| 40 | 4 | 1.00 (643.0) | 7.9 | 1.89 / 10 | 6.63 / 10 | $5.50 / 10$ | $0.09 / 10$ | 160.0 | 0.104 | 0.150 | 160.0 |
|  |  | 1.25 (803.2) | 8.1 | 1.53 / 10 | $7.05 / 10$ | 5.62 / 10 | $0.00 / 10$ | 200.0 | 0.122 | 0.166 | 200.0 |
|  | 8 | $1.00 \text { (1284.5) }$ | 15.3 | 1.58 / 10 | 5.93 / 10 | $5.05 / 10$ | $0.00 / 10$ | 320.0 | 0.265 | 0.648 | 320.0 |
|  |  | 1.25 (1606.2) | 14.9 | 0.76/10 | 6.30 / 10 | $5.51 / 10$ | $0.01 / 10$ | 400.0 | 0.337 | 0.696 | 400.0 |
| 60 | 4 | 1.00 (963.8) | 8.1 | $2.33 / 10$ | 6.81 / 10 | 5.44 / 10 | $0.00 / 10$ | 240.0 | 0.206 | 0.284 | 240.0 |
|  |  | 1.25 (1206.0) | 7.9 | 1.34 / 8 | $6.51 / 10$ | $5.28 / 10$ | $0.05 / 10$ | 300.0 | 0.264 | 0.333 | 300.0 |
|  | 8 | 1.00 (1925.9) | 15.8 | -- / 0 | $5.26 / 10$ | 4.16 / 10 | $0.00 / 10$ | 480.0 | 0.610 | 1.149 | 480.0 |
|  |  | 1.25 (2408.2) | 15.9 | -- / 0 | $4.71 / 10$ | 4.77 / 10 | $0.00 / 10$ | 600.0 | 0.828 | 1.367 | 600.0 |
|  |  | Average |  | 1.07 / 8.4 | $5.28 / 10$ | 4.76 / 10 | $0.01 / 10$ | 182.3 | 0.160 | 0.297 | 182.3 |

Table 7.3. ARPD, No. solutions found and CPU time values

The stopping criterion is defined as a CPU time limit for the ILP approach and using the REI heuristic. The CPU time limit depends on the size of the problem, being calculated as $|H| \cdot|J| \cdot \beta$. Results highlight that REI heuristic is better than the ILP approach, being $0.01 \%$ and $1.07 \%$ the ARPD values respectively. It is also important to remark that constructive heuristics ( ST and $\mathrm{MT}_{\mathrm{ALL}}$ ) yield good quality of solutions requiring short CPU times ( $5.28 \%$ and $4.76 \%$ in 0.160 and 0.297 seconds respectively). Regarding the number of solutions found, the proposed heuristics always find feasible solutions, while the ILP approach presents difficulties to find feasible solutions when the size of the problem increases (it only finds feasible solutions for $45 \%$ of the instances for a twelveweek planning horizon).

### 7.3.2 Validation with historical data

In this Section we present results of a historical validation of the decision model and the REI metaheuristic for solving the OR planning and scheduling problem in the Plastic Surgery and Major Burns Specialty of the University Hospital "Virgen del Rocio".

First, we present computational experiments to evaluate and compare the results obtained by using the decision model against the real results obtained by the Decision Maker from February 2009 to July 2009. As an example, we show the results for February. The waiting list was composed by 365 patients, getting the data for several
information systems of the Hospital. In order to consider the prioritization criteria proposed in Section 7.2, we introduce the parameter $g$ in the objective function in the following manner:

$$
\begin{equation*}
\text { Maximize } \sum_{h \in H} \frac{1}{h}\left(\sum_{i \in I} \sum_{j \in J} w_{i}^{g} X_{i j h}\right) \tag{7.1}
\end{equation*}
$$

We propose the following scenarios to compare the results obtained by the decision model against the real results obtained by the Specialty:

- Scenario I, the parameter $g$ is set to 1 , i.e. the objective of maximizing the sum of patients' weights scheduled in the planning horizon is considered.
- Scenario II, the parameter $g$ is set to 4 , i.e. the objective of maximizing the service level by planning patients with greater clinical weight as soon as possible in the planning horizon.
- Scenario III, same scenario as in I, increasing a $10 \%$ the length of each surgery.
- Scenario IV, same scenario as in II, increasing a $10 \%$ the length of each surgery.

Table 7.4 shows the value of the service level, and the number of scheduled patients in the considered planning horizon. The real schedule column represents the value of the service level and the number of patients scheduled in the Specialty. In order to compare the scenarios, the service level showed on Table 7.4 is determined as the sum of the quotients between the clinical weight and the date of the intervention of scheduled patients with $g=1$. The values of the service level and the numbers of patients scheduled in the tested scenarios are greater than the ones in the real schedule.

In view of these results, the proposed decision model were used to solve the OR planning problem in the Plastic Surgery and Major Burns Specialty from October 2009 to May 2010. The values of the service level and the number of scheduled patients were better than the real results obtained in last years, turning out surgical schedules with a high adhesiveness ( $80 \%$ of surgeries were performed in the OR-day proposed by the solution approaches). In addition, it also has to be noted that the time that the Decision Maker devotes to planning surgeries is greatly reduced by the use of the decision.

| Scenario | Decision model |  | Real schedule |  |
| :--- | :--- | :--- | :--- | :--- |
|  | Service level | No. of scheduled patients | Service level | No. of scheduled patients |
| I | $7617(116 \%)$ | $120(35 \%)$ |  |  |
| II | $4723(34 \%)$ | $111(25 \%)$ | 3525 | 89 |
| III | $6821(93 \%)$ | $112(26 \%)$ |  |  |
| IV | $4001(13 \%)$ | $105(18 \%)$ |  |  |

Table 7.4. Decision model result vs. real results

Then, we generate an extensive testbed to validate the performance of the REI metaheuristic for solving the problem. The testbed (100 instances) is generated based on meetings with the Decision Maker, data provided by the annual management report and historical data from the Specialty following the procedure proposed in Chapter 3. Each instance contains an initial waiting list along with the patient arrivals for each week of the year (i.e. 52 weeks). Surgery parameters are generated by empirical statistical distributions. Note that the results are compared against the real results obtained by the Specialty during 2012. The number of surgeries performed by the Specialty during 2012 was 2,823 . Using the same surgical resources (ORs and surgeons) and the same initial waiting list, the REI metaheuristic is able to schedule 2,962 surgeries, which means an average increase of 2.67 surgeries per week.

### 7.3.3 What-if analysis

In this section we present results of different managerial what-if analyses for solving the OR planning and scheduling problem in the Plastic Surgery and Major Burns Specialty. Several managerial decisions were identified by meetings with the Decision Maker during two years, as were the selection of the patient allocation strategy (how patients are allocated to surgeons), the objective function (which is the impact on the size of the waiting list and on the use of resources), the planning horizon (what is the best planning horizon to solve the problem), and, finally, the resource management strategy (how the operating theatre resources are managed). The results presented in this section have been obtained from solving testbeds used in Section 7.3.1 using the decision model and the REI metaheuristic proposed in Chapter 4.

### 7.3.3.1 Patient allocation strategies

The following patient allocation policies are analyzed to solve the OR planning problem of the Plastic Surgery and Major Burns Specialty:

- In the P-S-OR policy, it is assumed that patients have been previously assigned to a surgeon. Therefore the set "patient-surgeon" is allocated to an OR-day where the surgery can be performed. This policy guarantees the continuity of care, i.e. each patient is operated by the surgeon who examined him/her from his/her arrival to the hospital (see e.g. Guinet and Chaabane, 2003; Jebali et al., 2006).
- In the P-OR-S policy, patients are first assigned to an OR-day, and then surgeons are allocated to the set "patients-OR-day" (see e.g. Hans et al., 2008). This policy is more flexible than P-S-OR, as the patient does not depend on the capacity of a particular surgeon, but it may present problems both from social and professional point of view. On one hand, it is possible that a patient does not want to be operated by a surgeon who did not examine him/her before. On the other hand, it may happen that a surgeon does not want to operate a patient who has been initially examined by another surgeon.
- In order to reduce the drawbacks of the so-called P-OR-S policy, we propose a hybrid policy in which there are patients who are scheduled based on the so-called P -S-OR policy and others are scheduled based on the P-OR-S. Patients, who are scheduled according to the P-OR-S policy, must be a level of medical priority established by the Decision Maker. These patients are assigned to a "knapsack surgeon" available in each OR-day in the planning horizon. In fact, surgeries assigned to the fictitious surgeon in a day will be performed by surgeons in the Specialty who are not assigned to any OR-day.

Note that the P-S-OR policy is the strategy used in the Specialty, and it is modeled by using the decision model proposed in Chapter 4. However, minor modifications are needed to model the P-OR-S and hybrid policies:

- To model the P-OR-S policy, we replace constraints (4.5)-(4.7), and adding the following ones:

$$
\begin{equation*}
\sum_{j \in J} Z_{k j h} \leq 1 \quad\left(\forall k \in K, \forall h \in H \mid a_{k h}>0\right) \tag{7.1}
\end{equation*}
$$

$$
\begin{equation*}
\sum_{k \in K} Z_{k j h}=1 \quad(\forall j \in J, \forall h \in H) \tag{7.2}
\end{equation*}
$$

| Scenario | Policy |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | P-S-OR | Hybrid | Peal schedule |  |
|  | $7617(116 \%)$ | $7705(119 \%)$ | $8075(129 \%)$ |  |
| II | $4723(34 \%)$ | $4820(37 \%)$ | $5027(43 \%)$ | 3525 |
| III | $6821(93 \%)$ | $7030(99 \%)$ | $7332(108 \%)$ | 3525 |
| IV | $4001(13 \%)$ | $4062(15 \%)$ | $4205(19 \%)$ |  |

Table 7.5. Service level of the surgical schedule

| Scenario | Policy |  |  | Real schedule |
| :--- | :--- | :--- | :--- | :--- |
|  | P-S-OR | Hybrid | P-OR-S |  |
| I | $120(35 \%)$ | $120(35 \%)$ | $126(42 \%)$ |  |
| II | $111(25 \%)$ | $112(26 \%)$ | $116(30 \%)$ | 89 |
| III | $112(26 \%)$ | $112(26 \%)$ | $117(31 \%)$ |  |
| IV | $105(18 \%)$ | $105(18 \%)$ | $108(21 \%)$ |  |

Table 7.6. Number of scheduled patients

Constraints (7.1) specify that a surgeon can be assigned at most to one OR-day during a day if he/she is available to perform surgeries, while constraints (7.2) ensure that each OR-day must be assigned to a surgeon.

- To model the hybrid policy, we include the knapsack surgeon in the model by extending the set $K$ (i.e. $k=1 \ldots|K|+1$ ). We assume that the regular capacity of the knapsack surgeon $\left(a_{|K|+l h}\right)$ is equal to the total OR capacity during the day in order to consider the extreme scenario in that all surgeries scheduled during a day belong to the knapsack surgeon. Finally, we replace constraints (4.6) for the knapsack surgeon due to surgeons belonged to the Specialty will perform the surgeries allocated to the knapsack surgeon.

The value of the service level and the number of scheduled patients in February 2009 are shown in Table 7.5 and 7.6 respectively. In the P-OR-S policy, the value of the service level and the number of scheduled patients is the largest for each scenario. A patient is not assigned to a specific surgeon, and therefore can be scheduled earlier in the planning horizon. Regarding the hybrid policy, the number of patients assigned to the "knapsack surgeon" influences the quality of service of a surgical specialty. According to the results, if the percentage of patients assigned to the "knapsack surgeon" in the waiting list is high, then the value of the service level is close to the value obtained in the P-OR-S policy. In our case, only 20 patients are assigned to the "knapsack surgeon" so the value is closer to the results of the P-S-OR policy. On the
other hand we can choose to schedule as many surgeries as possible in the planning period by setting $g=1$ or to prioritize the scheduling of patients with greater clinical weight $g=4$. As a consequence, the value of the service level and the number of scheduled patients in scenarios with $g=1$ is bigger than scenarios with $g=4$.

### 7.3.3.2 Selection of planning horizons and objective functions

In this section, with the help of the REI metaheuristic, we evaluate different objectives functions using several planning horizons under several guidelines (patient prioritization, waiting list reduction, etc.). Besides analyzing the service level (O1) under the modified block scheduling strategy that is used in the Specialty, the following objectives have been considered: number of scheduled surgeries maximization (O2), ORs utilization maximization (O3), and a weighted objective that maximizes the service level during the first 6 months and the number of scheduled surgeries during the second 6 months (O4) using different planning horizons of a week, a two-week, and four-week.

On Table 7.7, the average annual values for each objective and planning horizon are shown, taking into account the average values of service level, the number of scheduled surgeries (increase in number of patients with respect to the effectively intervened is shown in brackets), ORs utilization, number of patients on the waiting list at the end of the year (within brackets, the difference with respect to the size of the waiting list at the beginning of the year), and CPU time required to solve the instance. Note that the termination criterion of REI for each planning horizon is determined by the length (1, 2 and 4 seconds for each weekly, two-weekly and four-weekly planning horizons respectively). The results show that the selection of the planning horizon greatly depends on the indicator selected by the Decision Maker. More specifically, the fourweek horizon seems the best one regarding the service level, as there are a larger number of high-priority surgeries that can be schedule as compared to shorter horizons. With respect to the number of scheduled surgeries, the best horizon is a week, as in this case there are lesser surgeries on the waiting list whose deadline is within the planning horizon and therefore there is more flexibility to build the surgical schedule. Finally, the planning horizon does not seem to be a significant factor when the objective involves OR utilization. With respect to the time required to generate the surgical schedules, it has to be noted that the maximum average time for the evaluation of a scenario is 467.5 seconds.

| Objective <br> Function | Planning <br> horizon | Service <br> Level | Scheduled <br> surgeries | OR <br> utilization | Waiting list | CPU time <br> (sec.) |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{O}_{\boldsymbol{1}}$ | Weekly | 1322.0 | $2970(2.83)$ | $89 \%$ | $666(29.1 \%)$ | 159.1 |
|  | Two-weekly | 1641.0 | $962(2.67)$ | $89 \%$ | $674(30.7 \%)$ | 243.5 |
|  | Four-weekly | 1816.3 | $2943(2.31)$ | $89 \%$ | $693(34.2 \%)$ | 399.0 |
| $\boldsymbol{O}_{\boldsymbol{2}}$ | Weekly | 1298.3 | $3151(6.30)$ | $88 \%$ | $485(-6.0 \%)$ | 122.2 |
|  | Two-weekly | 1586.9 | $3138(6.06)$ | $88 \%$ | $498(-3.5 \%)$ | 184.5 |
|  | Four-weekly | 1960.6 | $3097(5.27)$ | $87 \%$ | $539(4.4 \%)$ | 313.5 |
| $\boldsymbol{O}_{3}$ | Weekly | 1220.0 | $2857(0.65)$ | $91 \%$ | $780(51.1 \%)$ | 188.7 |
|  | Two-weekly | 1464.7 | $2821(-0.04)$ | $91 \%$ | $816(58.0 \%)$ | 296.6 |
|  | Four-weekly | 1720.9 | $2836(0.25)$ | $91 \%$ | $800(55.1 \%)$ | 467.5 |
| $\boldsymbol{O}_{\boldsymbol{4}}$ | Weekly | 1250.6 | $3143(6.15)$ | $88 \%$ | $493(-4.5 \%)$ | 134.9 |
|  | Two-weekly | 1522.9 | $3134(5.98)$ | $88 \%$ | $502(-2.7 \%)$ | 207.7 |
|  | Four-weekly | 1725.4 | $3100(5.33)$ | $88 \%$ | $536(3.8 \%)$ | 348.7 |

Table 7.7. Analysis of the objectives and horizons under the modified block scheduling strategy

### 7.3.3.3 Resource management strategies

In this section we use the REI metaheuristic to assess the impact of the different strategies to manage the resources. Table 7.8 shows the results assuming an open scheduling strategy, releasing ORs reserved to burn surgeries. In general, there are substantial improvements for all objectives and planning horizons. Regarding the service level, a maximum improvement of $7.71 \%$ is achieved (over the value obtained assuming the modified block scheduling strategy). Again, a four-week planning horizon seems to offer the best results. The number of scheduled surgeries increases from 6.30 to 9.65 patients per week, which translates into a $39.8 \%$ reduction of the waiting list at the end of the year. Finally, ORs utilization increases a $6 \%$ average for all objectives and planning horizons under consideration.

Finally, the heuristics help the Decision Maker to negotiate with the hospital manager with respect to the services (blood tests, anesthesia tests...) and the resources required (ORs, surgeons...) in order to reduce surgery cancellations (e.g. expired tests). The graph in Figure 7.1 shows the evolution of the number of scheduled surgeries depending on the objectives sought. As shown in this figure, the number of scheduled surgeries greatly depends on surgeries' deadlines and on the objective. For O1, the deadline has no influence on the scheduled surgeries, because the surgeries with a deadline within the planning horizon are those with highest clinical weight, and are therefore gradually scheduled. For O 2 , the effect of the deadline can be clearly seen, reaching the maximum values for months 2 and 9 .

| Objective <br> Function | Planning <br> horizon | Service <br> level | Scheduled <br> surgeries | OR <br> utilization | Waiting list | CPU time <br> (sec.) |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{O}_{\boldsymbol{1}}$ | Weekly | 1348.1 | $3151(6.30)$ | $95 \%$ | $485(-6.1 \%)$ | 124.4 |
|  | Two-weekly | 1681.6 | $3143(6.15)$ | $95 \%$ | $493(-4.5 \%)$ | 191.5 |
|  | Four-weekly | 1956.0 | $3129(5.88)$ | $95 \%$ | $507(-1.7 \%)$ | 311.2 |
| $\boldsymbol{O}_{\boldsymbol{2}}$ | Weekly | 1297.8 | $3325(9.65)$ | $93 \%$ | $311(-39.8 \%)$ | 108.1 |
|  | Two-weekly | 1584.3 | $3317(9.5)$ | $93 \%$ | $319(-38.1 \%)$ | 139.9 |
|  | Four-weekly | 1963.7 | $3256(8.33)$ | $92 \%$ | $380(-26.4 \%)$ | 243.1 |
| $\boldsymbol{O}_{\mathbf{3}}$ | Weekly | 1260.8 | $3050(4.36)$ | $98 \%$ | $587(13.7 \%)$ | 147.8 |
|  | Two-weekly | 1454.2 | $3132(5.94)$ | $98 \%$ | $504(-2.3 \%)$ | 198.8 |
|  | Four-weekly | 1691.8 | $3084(5.02)$ | $97 \%$ | $552(6.9 \%)$ | 358.8 |
| $\boldsymbol{O}_{\boldsymbol{4}}$ | Weekly | 1252.2 | $3319(9.54)$ | $94 \%$ | $317(-38.5 \%)$ | 107.2 |
|  | Two-weekly | 1515.8 | $3310(9.36)$ | $94 \%$ | $326(-36.9 \%)$ | 162.5 |
|  | Four-weekly | 1770.58 | $3263(8.46)$ | $94 \%$ | $373(-27.8 \%)$ | 273.5 |

Table 7.8. Analysis of the objectives and horizons under the open scheduling strategy


Figure 7.1. Plot of the evolution of the scheduled surgeries based on objectives $O_{1}$ and $O_{2}$

In both months there are patients close to their deadline and with long duration of the intervention (otherwise they would have been previously scheduled according to O2).

### 7.4 The Decision Support System

In view of the results presented in Section 7.3, the decision model and the solution approaches (ST, $\mathrm{MT}_{\mathrm{ALL}}$ and REI) presented in Chapter 4 were embedded in a DSS for solving the problem in the Plastic Surgery and Major Burns Specialty. The DSS is currently in use in the Hospital. In this section we briefly discuss the main design and implementation issues of the DSS. We first outline the main requirements, secondly we
briefly explain the framework architecture and then we present the main use cases of the DSS.

### 7.4.1 Requirement and Design

The main features of the DSS are:

- It must accomplish with DPA (Data Protection Act), i.e. the system must be secured by checking user identity and that the host is licensed before executing the DSS tool.
- Since the Decision Maker usually decides the surgical schedule using his/her own laptop (sometimes out of working hours), the required tool is conceived to be a standalone system. As a consequence, the DSS is not integrated with the Hospital Information System, but imports from it the relevant data of patients in the waiting list and the corresponding surgery data, such as surgery duration, surgeon (or group of surgeons) in charge, OR (or group of ORs) where the patient can be intervened, clinical weight, etc.
- The optimization engine should provide a surgical schedule that can be manually modified by the Decision Maker, so he/she can incorporate 'soft' constraints that cannot be easily integrated in the model, such as the preference of using the first hours of a shift for certain types of surgeries (not only depending on the type of surgery, but on the specific patient), or some days in the beginning/end of the week due to the specific needs of post-surgery recovery. Therefore, easy manual finetuning of the solution is required.
- The DSS tool should provide detailed analysis tools and drill-down capabilities so the Decision Maker can analyze the so-called scenario (i.e. a surgery schedule arisen from a waiting list and staffed ORs for a specific planning horizon) with great detail. Consequently, the system should be capable of handling different possible scenarios, that is: several solutions of the decision problem with the same/different data and using same/different parameter settings must be maintained so that the Decision Maker may explore their feasibility, introduce manual changes, etc. and ultimately choose one as an 'executable' schedule.
- The DSS is required to be flexible and extensible, so that it satisfies the currently identified business rules while makes it easy to add new ones. Consequently, the tool should be modular to allow incorporating new decision problems (decision models/ solution approaches) to the system.
- Since, in most surgical specialties, surgeons can be organized in groups (i.e. patients may be assigned to a group of surgeons instead of to a single surgeon), the DSS should allow for setting groups of surgeons and defining surgeons' capacities within each group. In order to include groups of surgeons in the decision model presented in Chapter 4, we introduce a fictitious surgeon for each group of surgeons existing in the Specialty, defining the maximum time for performing surgeries in a given day from the regular capacity of the surgeons belonged to the group.

The most appropriate architecture for the required system is composed of three modules: Database Management, Model Management and Dialogue Management. As in other DSSs designs (see e.g. Moormann and Lochte-Holtgreven, 1993; Power and Sharda, 2007), splitting a software system into these three modules allows a greater degree of flexibility to independently renew the database technology, the decision model embedded or the user interface.

The Database Management module is based on a relational database including the relevant input data, as data about patients (name, age, address), surgeries (duration, clinical weight, medical priority), human resources (surgeons and their capacity), and material resources (number of ORs), etc. This component includes different mechanisms for storing, handling, updating and retrieving these data, which are used for efficient scenario management. More specifically, the module is in charge of reading input data from database, gathering the computed solution (i.e. obtained by the resolution of the optimization model), and then creating an scenario by saving both together, so they can be used when conducting "what-if" analysis.

The Model Management module brings together data from the database, models/solution approaches from an optimization repository, and user preferences (parameter sets by the Decision Maker) from the Dialogue Management module. More specifically, it is responsible for generating a problem instance and for controlling the launching of optimization calls. Note that the Model Management module includes the


Figure 7.2. Overview of the DSS
decision model proposed in Chapter 4, along with the decision models required for solving the problem under the P-OR-S and hybrid strategies. Regarding the solution procedures, the module includes the constructive heuristics ( ST and $\mathrm{MT}_{\mathrm{ALL}}$ ) and the REI metaheuristic presented in Chapter 4.

Finally, the Dialogue Management module is responsible to handle the communication between the DSS and the Decision Maker.

### 7.4.2 Implementation and Main Use Cases

Taking into account the above requirements and the design proposed, the DSS was implemented using Microsoft's C\# and Visual Studio as Integrated Development Environment (IDE), and MySQL as database management system. An overview of the system functionalities is provided in Figure 7.2. The main use cases of the DSS (shown in Figure 7.3) are:

- Medium term estimation, with the objective of generating a tentative surgical schedule for a period of up to six months by assuming a weekly pattern (i.e. same ORs and surgeons capacity in all weeks). The purpose is twofold: Check whether the available surgical resources pattern (ORs, surgeons, and working shifts) is sufficient to accomplish the surgeries in the waiting list in a proper manner, and to notify the patients with an estimated week for their schedule dates. To develop this surgical schedule, ST, $\mathrm{MT}_{\text {ALL }}$ and REI (considering short termination criteria) proposed in Chapter 4 are employed.
- Short term scheduling. The objective of this use case is to obtain a detailed surgical schedule for a short planning period (typically the next two weeks) over a rollinghorizon basis. More specifically, at the end of each week, the Decision Maker imports the waiting list from the Hospital Information System, refines the availability pattern of resources along the next two weeks by incorporating specific events (closure of certain OR, punctual non-availability of a surgeon, etc.) and generates a detailed surgical schedule for the next two weeks using the REI metaheuristic presented in Chapter 4. The choice of approximate methods is left to the Decision Maker in view of the size of the problem. It is also possible to specify the maximum running time allowed to generate the surgical schedule so the DSS may choose the best method.
- Manual fine-tuning. As stated before, a requirement for the DSS was that the Decision Maker will be able to move any of the scheduled surgeries within the short term surgical schedule, whether to postpone it (e.g. a patient has flu or some health complication impeding the intervention), or to put them into a specific OR-day. Moreover, not scheduled surgeries could also be manually allocated into a specific OR-day.


Figure 7.3. Main use cases


Figure 7.4. Generation of surgeons' groups and availabilities assignment


Figure 7.5. Availabilities refinement within the planning horizon: the ORs example


Figure 7.6. The user-friendly graphical interface: Example of short term scheduling within a three

## ORs

### 7.4.3 Friendly operational decision level

As mentioned before, the DSS allows for setting groups of surgeons and defining surgeons' capacities inside each group. Similarly, ORs sharing certain properties (e.g. equipped for certain specific procedures) can be also grouped to define group of ORs where a certain type of surgeries can be performed (see parameter $\delta_{i j}$ in Chapter 3). Starting from this initial assignment, there is an easy procedure for refining availabilities within the planning horizon, to obtain the so-called 'refined availability'. The DSS guides the Decision Maker through a road map to specify the day-to-day availability of staffed ORs, which comprises both facilities' and surgeons' capacities (see the sequence in the upper part of Figure 7.5).

As mentioned in the requirements, data from patients and their surgeries are imported from the Hospital information systems. The last step in the sequence shown in Figure 7.5 allows specifying patients' unavailability in a very intuitive manner. Detailed tools for analysis and drill-down capabilities have been also built in the DSS so the Decision Maker can study their scenarios in greater detail. All use cases invoke the heuristics for either scheduling or rescheduling. For manual fine-tuning, the Decision Maker can "freeze" a number of formerly staffed and scheduled OR-days so the surgeries who have already been notified remain unmodified. Once the optimization engine produces a solution (either exact or approximate), the resulting surgical schedule is displayed in a user-friendly graphical interface so the Decision Maker can visualize the available information of every surgery, the surgical timetable for every surgeon, and the graphic representation of the surgical schedules (sketched as a time-space matrix drawing, see Figure 7.6).

The above mentioned functionalities help the Decision Maker to conduct "what-if" analyses. Figure 7.7 shows an example in which the Decision Maker may use the DSS to assess the impact of using additional ORs and surgeons in order to discuss with the Hospital Managers future budget/OR-time allocation for his/her surgical specialty.

### 7.5 Conclusions

The purpose of the objective is to validate the decision model and the solution procedures presented in the Thesis for solving the specific OR planning and scheduling


Figure 7.7. "What-if" analysis
problem in the Plastic Surgery and Major Burns Specialty of the University Hospital "Virgen del Rocio".

First we introduce the problem in the Specialty, which is modeled and solved by the decision model and the solution procedures proposed in Chapter 4. Then, the decision model and solution procedures have been validated both experimental and historical manners:

- By the experimental manner, we ensure the quality of solution procedures to solve the problem over medium-long planning horizons in order to make decisions as inform patients several weeks or even months in advance of their surgeries or to negotiate surgical resources for future planning periods. With these considerations in mind, a solution approach for handling medium-long planning horizons is incorporated in the solution procedures presented in Chapter 4, since the complexity of the problem increases due to the huge number of decision variables. The results of the computational experiments show that the REI metaheuristic clearly outperforms the ILP approach (both in the quality of the solution and in the number of feasible solutions found), and the good performance of the constructive heuristics (ST and $\mathrm{MT}_{\mathrm{ALL}}$ ) with short CPU times.
- By the historical manner, we have quantified the advantages obtained by the responsible of the Specialty using the decision model and the solution procedures.

Once decision models and solutions procedures are validated, several managerial decisions identified by meetings with the Decision Maker during two years are analyzed. The main findings are: (1) the selection of a flexible patient allocation strategy yields a considerable reduction of the waiting list; (2) the selection of the planning horizon (a week, a two-weeks and a four-week) has a great impact on the problem, depending on the objective function optimized; (3) the evolution of the number of scheduled surgeries over a year depends on the selected objective function, being an important issue for the Decision Maker to negotiate the availability of shared services and resources (blood tests, anesthesia test...) with the hospital management; and (4) an important improvement is observed by changing from a modified block scheduling to an open scheduling strategy (the ORs reserved are released).

Finally, we present the DSS for solving the OR planning problem in the Plastic Surgery and Major Burns Specialty of the University Hospital "Virgen del Rocio", which is currently in use in the Hospital.

# Part IV 

## Conclusions

## and

## Further Research

## Chapter 8

## Conclusions and Future Research Lines

### 8.1 Conclusions

This Thesis focuses on operating theatre planning and scheduling. This decision problem is commonly decomposed into three hierarchical decision levels: strategic, tactical, and operational (see Chapter 1). Despite the importance and the complexity of decisions related to these hierarchical levels, it is a common practice that decision makers make such decisions based on their experience without considering the underlying optimization problems, providing solutions that are far from being optimal, and consuming long times on performing management tasks instead of healthcare tasks. In this context, the goal of this thesis is to develop models and solution procedures from operations research techniques that can help healthcare professionals to improve the efficiency of the operating theatre resources and the quality of the healthcare services at the operational level.

In order to fulfill the general goal of the Thesis, a number of research objectives were established in Chapter 1. Next we present a review of these objectives and how they have been addressed in this document:
i. To carry out a literature review on the operational level of the operating theatre management problem.

This objective has been extensively addressed in Chapter 2. Usually, this decision level is decomposed into two separate steps: the OR planning problem and the OR scheduling problem. This decomposition reduces the complexity of the whole problem, although the quality of the decisions is reduced due to the high interdependence among these steps. Therefore, we study both the OR planning problem (which is the most extended operational problem in the University Hospital "Virgen del Rocio"), and the integrated OR planning and scheduling problem. For each decision problem, their main features are presented (see Section 2.3 and Section
2.4), and the literature is extensively reviewed and classified. The main conclusions are:

- There are not experimental benchmarks to analyze and evaluate the performance of the different solution approaches. Comparisons are carried out mostly in ad-hoc data sets, which makes difficult to extract conclusions on the general validity of existing methods, and to compare new ones.
- The deterministic OR planning problem has been extensively analyzed in the literature, with efficient decision models and solution procedures. However, given the computation times required for the exact methods (i.e. procedures yielding the optimal solution), there is room for investigating more efficient approximate methods (i.e. yielding better quasi-optimal solutions in less CPU time), particularly in view of the need of a) an interactive approach to re-plan the interventions, and b) a long-term planning that allows the Decision Maker to have a higher visibility of the plan in order to check for the availability of additional resources.
- While the stochastic OR planning problem assuming a block scheduling strategy is considered in the literature, there is no such analysis assuming an open scheduling management strategy taking into account the responsible surgeons and their availabilities, together with time period constraints (specially, the deadline constraints established by the National Healthcare Services).
- The integrated OR planning and scheduling problem has been analyzed by considering that surgeries are performed by only one surgeon. In addition, the influence of the assistant surgeon's experience on the length of the surgery has not been considered in the literature. Since, according to the literature, $90 \%$ surgeries are performed by surgical teams composed by more than one surgeon --being twosurgeons team the most extended-- addressing this problem remains a research opportunity.
ii. To propose a testbed generator to analyze the operating theatre problems identified in $i$ ).

Chapter 3 presents a testbed generator for solving the OR planning and scheduling problems identified in the Thesis, providing the literature with a set of benchmarks. The proposed testbeds allow to researches to solve any OR planning and scheduling that involve constraints and objectives related to patients, OR and surgeons.
iii.To address the OR planning problem by proposing decision models and solution procedures under deterministic and stochastic surgery durations, emergency arrivals and resources capacity.

The deterministic OR planning problem is analyzed in Chapter 4. We propose a mathematical decision model to solve the problem of assigning the intervention date and the OR where a set of surgeries will be performed, minimizing access time for patients with diverse clinical priority values. A set of approximate methods are proposed for solving the problem under consideration. To show the efficiency of the heuristics proposed, existing heuristics for the problem are adapted and compared in a testbed based on the procedure presented in Chapter 3. The main conclusion is:

- The proposed heuristics statistically outperform existing ones in the literature for every type of heuristic proposed (constructive, improvement and meta-heuristic), providing the literature with a benchmark for the deterministic version of the problem.

The stochastic OR planning problem is addressed in Chapter 5. We propose a mathematical decision model considering resources availability (OR and surgeons) and time period constraints in order to minimize the unexploited OR time and overtime costs. Uncertainties in surgeries duration, in the arrivals of emergency surgeries and in surgeons' capacity are considered. A Monte Carlo optimization method, based on the SAA method, is proposed for solving the problem. The method combines an iterative local search method and Monte Carlo simulation. The main conclusions are:

- The performance of the iterative local search method is analyzed against the up-to-now state of the art heuristics for solving the deterministic version of the problem, yielding the best results.
- The results of the computational experiments highlight that, regardless the statistical distribution considered to generate the arrivals of emergency surgeries, the solution obtained by the Monte Carlo optimization method converges to the optimal solution of the problem and presents a high robustness in terms of the proportion of feasible simulations when the number of samples increases.
iv. To address a deterministic integrated OR planning and scheduling problem, taking into account the case where there is a surgical team composed by surgeons with different surgical experience.

Chapter 6 analyzes an integrated OR planning and scheduling problem which consists on assigning the date, the OR and the time indication for each surgery in the waiting list over a given planning horizon, maximizing a weighted objective function. The objective function includes the number of surgeries scheduled, the tardiness of each surgery, and the idle time of each surgeon between consecutive surgeries. We assume that surgery durations depend on the surgical team, which may be composed by one or two surgeons with different level of experience. We propose an ILP decision model to optimally solve the problem. Given the high computation requirements of our MILP model, we also propose an iterative constructive method. The main conclusions are:

- The computational experience shows that the proposed algorithm is able to find feasible solution for all problems requiring shorter CPU time and average relative percentage deviation than the ILP-based approach.
- A simulation analysis shows that the deterministic approach is suitable for solving the proposed problem considering random surgery durations, yielding acceptable values of OR and surgeon overtime.
v. To demonstrate the validity of decision models and solution procedures developed in the Thesis for solving the OR planning and scheduling problem in the University Hospital "Virgen del Rocio".

Chapter 7 presents the OR planning and scheduling problem in the Plastic Surgery and Major Burns Specialty, which is modeled and solved using the decision model and solution procedures proposed in Chapter 4. A solution approach integrated in the
solution methods is proposed to help the Decision Maker to make decisions over medium-long planning horizons. Computational experiments are carried out to validate the decision model and the solution procedures with experimental and historical data. The main conclusions are:

- The REI metaheuristic clearly outperforms the ILP approach (both in the quality of the solution and in the number of feasible solutions found), and the good performance of the constructive heuristics ( ST and $\mathrm{MT}_{\mathrm{ALL}}$ ) with short CPU times.
- The usage of the decision model and the solution procedures clearly improves the operating theatre management, providing the Decision Maker with a tool to analyze managerial decisions under different scenarios.

Finally, we present a DSS developed for the University Hospital "Virgen del Rocio", where the decision models and solution procedures presented in Chapter 4 are embedded.

### 8.2 Contributions

This section summarizes the research output of the Thesis. Section 8.2.1 describes the framework (i.e. research projects and grants) in which the Thesis has been carried out. Section 8.2.2 and Section 8.2.3 present the research outcomes published on international journals and conferences respectively.

### 8.2.1 Research projects

The Thesis has been carried out in the framework of several healthcare research projects carried out by the Industrial Management Research Group, where the author of the Thesis has been member since 2007. These projects are:

- "Operations Research \& Operating Room (OR2)" funded by the Spanish Ministry of Science and Innovation (reference ACC-300100-07-5),
- ASSYST funded by the Progress and Healthcare Foundation of the Andalusian Government (reference PI-0661/2010),
- PLAGES-IDQ funded by INGENIA company (reference PI-0502/2010), and
- SUPPORT funded by the Andalusian Government (reference PI-0502/2010).

The agent for validation and implementation of these projects has been the University Hospital "Virgen del Rocio" in Seville (Spain).

### 8.2.2 Journals

The following journal publications have derived from the contributions in this Thesis:

- Molina-Pariente J.M., Fernandez-Viagas, V., Framinan, J.M., (2015). Integrated operating room planning and scheduling problem with assistant surgeon dependent surgery durations. Computers and Industrial Engineering, 88, 8-20 (2014 Impact Factor: 1.783).
- Dios, M., Molina-Pariente J.M., Fernandez-Viagas, V., Andrade-Pineda J.L., Framinan, J.M., (2015). A decision support system for operating room scheduling. Computers and Industrial Engineering, 88, 430-443 (2014 Impact Factor: 1.783).
- Molina-Pariente, J.M., Hans, E.W., Framinan, J.M., Gomez-Cia, T. New heuristics for planning operating rooms. Computers and Industrial Engineering (2014 Impact Factor: 1.783). Accepted.
- Molina-Pariente, J.M., Hans, E.W., Framinan, J.M. A stochastic approach for solving the operating room scheduling problem. Under review


### 8.2.3 Conferences

- Dios, M., Molina-Pariente, J.M., Hans, E.W., Framinan, J.M. A decision support system for solving the stochastic operating theater tactical problem. Proceedings of the $40^{\text {th }}$ International Conference on Operational Research Applied to Health Services (ORAHS), Lisbon, July 20-25, 2014.
- Molina-Pariente, J.M., Framinan, J.M., Perez-Gonzalez, P. Employing fast heuristics for operating room planning. Proceedings of the $13^{\text {th }}$ International Conference on Project Management and Scheduling (PMS), Leuven, April 1-4, 2012.
- Dios, M., Fernández-Viagas, V, Molina-Pariente, J.M., Andrade-Pineda, J.L., Framinan, J.M. ASSYST®: Herramienta para el soporte a la toma de decisiones en
planificación quirúrgica. XV Congreso Nacional de Informática de la Salud, Madrid, March 20-22, 2012.
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- Molina-Pariente, J.M., Framinan, J.M., Perez-Gonzalez, P. New heuristics for the Operating Room Planning Problem. Proceedings of the International Conference on Industrial Engineering and Systems Management (IESM), Metz, May 25-27, 2011.
- Molina-Pariente, J.M., Framinan, J.M., Perez-Gonzalez, P. Approximate Methods for Solving the Operating Room Planning Problem. Proceedings of the $24^{\text {th }}$ European Conference in Operational Research (EURO XXIV), Lisbon, July 11-14, 2011.
- Molina-Pariente, J.M., Framinan, J.M., Perez-Gonzalez, P., Andrade-Pineda, J.L. Algoritmos Aproximados para la Resolución de la Planificación de Intervenciones Quirúrgicas. Proceedings of the $4^{\text {th }}$ International Conference on Industrial Engineering and Industrial Management, San Sebastian, September 8-10, 2010.
- Molina-Pariente, J.M., Framinan, J.M., Gonzalez-Rodriguez, P.L., Andrade-Pineda, J.L. Planificación Quirúrgica: Revisión de la Literatura. Proceedings of the $3^{\text {rd }}$ International Conference on Industrial Engineering and Industrial Management, Barcelona, September 2-4, 2009.
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- Molina-Pariente, J.M., Framinan, J.M. Testing Planning Policies for Solving the Elective Case Scheduling Phase: a Real Application. Proceedings of the $35^{\text {th }}$ International Conference on Operational Research Applied to Health Services (ORAHS), Leuven, July 12-17, 2009.
- Molina-Pariente, J.M., Framinan, J.M. Policies and Decision Models for Solving Elective Case Operating Room Scheduling. Proceedings of the $39^{\text {th }}$ International Conference on Computers \& Industrial Engineering (CIE-39), Troyes, July 6-9, 2009.


### 8.3 Future research lines

In this section we present some research issues that need to be further addressed for enhancing the real-life application of the proposed decision models and solution procedures. In addition, we discuss future research lines to improve the efficiency of the operating theatre resources and the quality of the healthcare services.

1) In this Thesis, the OR planning and scheduling problem is analyzed and solved considering only the perioperative stage, given the fact that the resources that commonly represent bottleneck at most hospital are the ORs and surgeons. However, the unavailability of other operating theatre resources could negatively influence the surgical schedule, causing delays or cancellations. Therefore, the integration of preoperative (ward) and post-operative (post anesthesia care unit, intensive care unit and wards) resources in the OR planning and scheduling problem, along with the consideration of stochastic issues represents an interesting future research line.
2) In order to improve the efficiency of the operating theatre resources and the quality of the healthcare services, new research lines would be focused on the integration of tactical and operational decision levels, as are:
2.1) the construction of efficient master surgical schedules and surgical schedule, integrating simultaneously all surgical resources and waiting lists of surgical specialties,
2.2.) the determination of the pool of sharable surgical resources (surgeons, nurses, etc.) to reach the goals defined in the surgical specialty (i.e. how much surgeon time
is allocate to do consultations and to perform surgeries during a given planning horizon), and
2.3) the development of solution procedures that allow to decision makers use planning horizons shorter than those decision makers use in real-life applications (typically, a year) for solving the problem.
3) Finally, in order to ensure patient safety and to minimize risks, an interesting direction would be to analyze, in the construction of the surgical schedule, the tradeoff between the efficiency and the formation of stable functional surgical teams.

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## Acronyms

## A

ANOVA ANalysis Of Variance
ARPD Average relative Percentage Deviation

## B

Be Bernoulli distribution

BF Best Fit algorithm

BP Bin Packing

## C

C Composite heuristic
CGBH Column-Generation-Based Heuristic procedure

CPU Computing Processing Time
CV Coefficient of Variation

D
DEC DECreasing sorting criterion

DOE Design Of Experiments

DPA Data Protection Act

DPH Dynamic Programming Heuristic
DSS Decision Support System

## F

FF Fist Fit algorithm

## H

HILL HILL sorting criterion
HILO HILO sorting criterion

HM Hungarian Method
HS Hybrid Swapping method
HSG Hybrid Swapping Global method

## I

IC Iterated Constructive method

ICU Intensive Care Unit
IDE Integrated Development Environment
IGLS Iterative Greedy Local Search method

ILP Integer Linear Programming
INC INCreasing sorting criterion

## L

LF Level Fit algorithm
LN LogNormal distribution

LOHI LOHI sorting criterion

## M

MT Multiple-Tuple method
MTBT Maximum Time Before Treatment

MS Multi-Start method

N
$\mathrm{N} \quad$ Normal distribution

NC Not Considered in the literature

NF Next Fit algorithm

## 0

OF Objective Function

OFF OFF-line method

OR Operating Room

## P

PACU Post Anesthesia Care Unit

PIII Pearson III distribution

PS Pair-wise Swapping method

PSG Pair-wise Swapping Global method

## R

R Random generation

| RDI | Relative Deviation Index |
| :--- | :--- |
| REI | Random Extraction-Insertion algorithm |
| RPD | Relative Percentage Deviation |

## S

SA Simulated Annealing method

SA $_{C} \quad$ Simulated Annealing method with constant temperature
SAA Sample Average Approximation
$\mathrm{S}_{\mathrm{C}} \quad$ Sorting Criterion
$\mathrm{S}_{\mathrm{I}} \quad$ Sorting Indicator

ST Single-Tuple method

T

TABOO TABOO search method

TS Triplet-wise Swapping method
TSG Triplet-wise Swapping Global method

U

U Uniform distribution

V

VALLEY VALLEY sorting criterion

