

ON THE EXISTENCE AND MULTIPLICITY OF POSITIVE SOLUTIONS FOR SOME INDEFINITE NONLINEAR EIGENVALUE PROBLEM

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ABSTRACT. This paper concerns with the existence, uniqueness and/or multiplicity, and stability of positive solutions of an indefinite weight elliptic problem with concave or convex nonlinearity. We use mainly bifurcation method to obtain our results.

1. INTRODUCTION AND MAIN RESULT

In this work we analyze the positive solutions of the weight elliptic problem

$$(1.1) \quad \begin{cases} \mathcal{L}u = \lambda m(x)f(u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded domain of \mathbb{R}^N with regular boundary $\partial\Omega$; $m \in C(\overline{\Omega})$ changes sign, $f : \mathbb{R}_+ \mapsto \mathbb{R}$ satisfies some assumptions that will be detailed below, $\lambda \in \mathbb{R}$ that will be regarded as a bifurcation parameter and \mathcal{L} is a second order uniformly elliptic operator of the form

$$(1.2) \quad \mathcal{L}u := - \sum_{i,j=1}^N D_i(a_{ij}(x)D_ju) + \sum_{i=1}^N b_i(x)D_iu,$$

with $a_{ij} = a_{ji} \in C^1(\overline{\Omega})$ and $b_i \in C^1(\overline{\Omega})$.

When $m \equiv 1$, (1.1) was treated in [2] and [10] showing that there exists at most a positive solution if f is concave but may have multiple positive solutions in the case f convex.

When m changes sign the study of (1.1) is more difficult. In order to state the results we need to introduce some notation. Firstly, we can suppose without loss of generality that $\lambda > 0$ (similar results are obtained if $\lambda < 0$) and $f'(0) = 1$ (see Remark 1.2 where we show how to overcome this restriction). Since the principal eigenvalue of \mathcal{L} is positive (observe that positive constants are supersolutions of \mathcal{L} , see [15]), it is well known (cf. [14]) that the linear eigenvalue problem with indefinite weight function

$$(1.3) \quad \begin{cases} \mathcal{L}u = \lambda m(x)u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

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admits two principal eigenvalues (i.e. eigenvalue having positive eigenfunction) such that $\lambda_- < 0 < \lambda_+$.

When $\mathcal{L} = -\Delta$, $f(0) = 0$, f is concave, nonnegative and $f(1) = 0$, Brown and Hess [6] proved that (1.1) possesses exactly one positive solution (λ, u) such that $\|u\|_\infty \leq 1$ if $\lambda > \lambda_+$ and it has no positive solution if $\lambda \in [0, \lambda_+]$.

Recently, Brown and Ko [7] studied (1.1) when $f(u) = f_1(u) = u - u^p$ (concave) and $f(u) = f_2(u) = u + u^p$ (convex), $p > 1$ (in fact, they consider a more general boundary condition). They showed that from the trivial solution $u = 0$ bifurcates a unbounded in $\mathbf{R} \times C(\overline{\Omega})$ continuum (maximal closed and connected set) of positive solutions \mathcal{C} at $\lambda = \lambda_+$. In the case $f = f_1$, the solutions $u \in \mathcal{C}$ satisfy that $\|u\|_\infty < 1$, and the projection of \mathcal{C} onto \mathbf{R} is $(\lambda_+, +\infty)$. Moreover, for $\lambda \in (\lambda_-, \lambda_+) \setminus \{0\}$ and $p < (N+2)/(N-2)$ they proved the existence of positive solution (different from the ones arising from bifurcation) by using variational methods.

In the convex case, $f = f_2$, \mathcal{C} goes to the left. Assuming $p < (N+2)/(N-1)$ and some restrictions on m in order to obtain a priori bounds for $\lambda \neq 0$, \mathcal{C} approaches infinity as $\lambda \rightarrow 0^+$. They also derived that there is no positive solution in the particular case $\lambda = \lambda_+$ but did not provide information for $\lambda > \lambda_+$.

We generalize and improve these results in different ways. We consider a not necessarily self-adjoint operator and more general reaction function. In the case f concave, we prove that the solutions obtained by variational methods in [7] constitute in fact a branch bifurcating from infinity at $\lambda = 0$ whose projection on the λ axis is $(0, +\infty)$ and we deduce the existence of a second positive solution for $\lambda \in (\lambda_+, +\infty)$. On the other hand, when f is convex, we show the existence of positive solution if $\lambda \in (0, \lambda_+)$, and in the particular case $f = f_2$ we prove that there is no positive solution for $\lambda \geq \lambda_+$.

Specifically, we show the following result. Assume that

$$\Omega_\pm := \{x \in \Omega : m^\pm > 0\}$$

are open and regular sets, where m^\pm represent the positive and negative part of m respectively; and suppose that $m^\pm(x) \approx [\text{dist}(x, \partial\Omega_\pm)]^\gamma$ for x close to $\partial\Omega_\pm$ and some $\gamma_\pm \geq 0$. Throughout this paper we are going to work with a smooth function $f : \mathbf{R}_+ \mapsto \mathbf{R}$, $f(0) = 0$, $f'(0) = 1$ and verifying:

$$(f1) \quad f'' < 0, \quad \lim_{s \rightarrow +\infty} \frac{f(s)}{s^p} = \alpha < 0,$$

or

$$(f2) \quad f'' > 0, \quad \lim_{s \rightarrow +\infty} \frac{f(s)}{s^p} = \beta > 0,$$

for some p such that

$$(1.4) \quad 1 < p < \min \left\{ \frac{N+1+\gamma_\pm}{N-1}, \frac{N+2}{N-2} \right\}.$$

Our main result is the following

Theorem 1.1. *Suppose (1.4).*

- a) *Assume that f satisfies (f1). Then (1.1) has positive solution if, and only if, $\lambda \neq 0$. Moreover, for $\lambda \in (-\infty, \lambda_-) \cup (\lambda_+, +\infty)$ there exist at least two positive solutions, one of them linearly asymptotically stable.*
- b) *Assume that f satisfies (f2). Then (1.1) has positive solution if $\lambda \in (\lambda_-, 0) \cup (0, \lambda_+)$.*

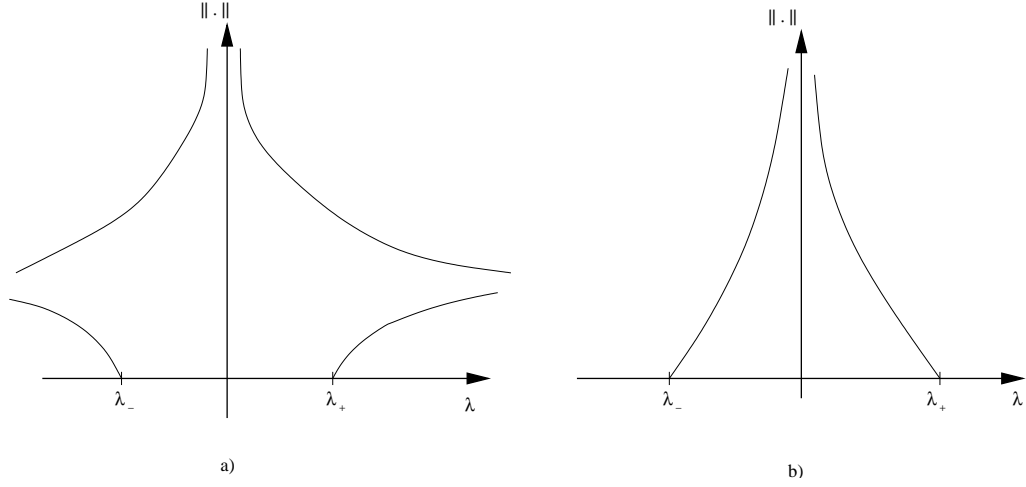


FIGURE 1. Bifurcation diagram for (1.1)

Furthermore, in any case for each $(\lambda_n, u_n)_{n \in \mathbb{N}}$ sequence of positive solution of (1.1) such that $\lambda_n \rightarrow 0$, then $\|u_n\|_\infty \rightarrow \infty$ as $n \rightarrow \infty$.

See Figure 1, where we have summarized the information of this result.

Remark 1.2. As we said before we can assume that $f'(0) = 1$. Indeed, if f satisfies (f1) and $f'(0) < 0$, then (1.1) is equivalent to $\mathcal{L}u = \lambda m_1(x)h(u)$ with $m_1 = -m$ and $h(u) = -f(u)$ and so verifying (f2). So, Theorem 1.1 b) applies in this case. On the other hand, if f satisfies (f2) and $f'(0) < 0$, then (1.1) is equivalent to $\mathcal{L}u = \lambda m_2(x)j(u)$ with $m_2 = -m$ and $j(u) = -f(u)$. In this case, we can apply Theorem 1.1 a).

In order to prove the above result, we include (1.1) in the more general equation

$$(1.5) \quad \begin{cases} \mathcal{L}u = \mu m(x)u + \lambda m(x)g(u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

with λ fixed, $g(u) = f(u) - u$, adding $\mu \in \mathbb{R}$ as parameter and look for solutions to (1.5) for $\mu = \lambda$. This equation is interesting in itself and it has attracted a great deal of attention during last years (see for example [1], [3], [4], [5], [13] and [16]) when $m \equiv 1$ in the first term on the right-hand side of (1.5) and in [9] with the right-hand side of the form $\mu h(x)u + g(x)u^p$ and restrictive conditions on h and g which are not satisfied in our case. We give a complete information of the bifurcation diagrams of (1.5) and deduce Theorem 1.1 from them.

An outline of the work is as follows: in Sections 2 we study (1.5) when f satisfies (f1) or (f2) and prove the main result of this paper. In the last Section we study the particular case $f(u) = u + u^p$.

2. STUDY OF THE EQUATION (1.5)

In this Section we study the equation (1.5) giving a description of its bifurcation diagrams. We define $\Pi : \mathbb{R} \times C(\bar{\Omega}) \mapsto \mathbb{R}$ the projection map onto \mathbb{R} , i.e. $\Pi(\mu, u) = \mu$

Proposition 2.1. *Assume (1.4) and fix $\lambda > 0$.*

- a) If f satisfies (f1), then (1.5) possesses a positive solution if $\mu > \lambda_-$. Moreover, from the trivial solution $u = 0$ emanate two unbounded in $\mathbf{R} \times C(\overline{\Omega})$ continua of positive solutions $\mathcal{C}_+ := \{(\mu, u_\mu)\}$ and $\mathcal{C}_- := \{(\mu, w_\mu)\}$ at $\mu = \lambda_+$ and $\mu = \lambda_-$, respectively. Both continua bifurcate to the right and $\Pi(\mathcal{C}_-) \supset (\lambda_-, +\infty)$, $\Pi(\mathcal{C}_+) = (\lambda_+, +\infty)$. Finally, for $\mu > \lambda_+$, u_μ is linearly asymptotically stable and $u_\mu \neq w_\mu$.
- b) If f satisfies (f2), then (1.5) possesses a positive solution if $\mu < \lambda_+$. Moreover, from the trivial solution $u = 0$ emanate two unbounded in $\mathbf{R} \times C(\overline{\Omega})$ continua of positive solutions $\mathcal{C}_+ := \{(\mu, w_\mu)\}$ and $\mathcal{C}_- := \{(\mu, u_\mu)\}$ at $\mu = \lambda_+$ and $\mu = \lambda_-$, respectively. Both continua bifurcate to the left and $\Pi(\mathcal{C}_-) = (-\infty, \lambda_-)$, $\Pi(\mathcal{C}_+) \supset (-\infty, \lambda_+)$. Finally, for $\mu < \lambda_-$, u_μ is linearly asymptotically stable and $u_\mu \neq w_\mu$.

Proof. Firstly, we prove part a). We begin by showing that if (μ, u) is a positive solution of (1.5) then there exists $\mu_0 < 0$ (in fact, $\mu_0 < \lambda_-$) such that

$$(2.1) \quad \mu > \mu_0.$$

Since (μ, u) is a positive solution of (1.5) then $\lambda_1^\Omega(\mathcal{L} - \mu m - \lambda m \frac{g(u)}{u}) = 0$, where $\lambda_1^D(\mathcal{L} + q)$ stands for the principal eigenvalue of $\mathcal{L} + q$ in a domain $D \subset \mathbf{R}^N$ with $q \in L^\infty(D)$ subject to homogeneous Dirichlet boundary condition and recall that $g(u) = f(u) - u$. Using that f is concave and the monotony properties of the principal eigenvalues with respect to the domain and the potential q , it follows that

$$0 = \lambda_1^\Omega(\mathcal{L} - \mu m - \lambda m \frac{g(u)}{u}) < \lambda_1^{\Omega^-}(\mathcal{L} - \mu m),$$

and so (2.1) (see for instance Remark 6.3 in [15]).

Now, from the Crandall and Rabinowitz Theorem (cf. [11]) there exist $\varepsilon > 0$ and two differentiable curves of positive solutions of the form $(\mu_\pm(s), u_\pm(s))$ such that $\mu_\pm(s) = \lambda_\pm + \mu_\pm^1(s)$ and $u_\pm(s) = s(\varphi_1^\pm + v_\pm(s))$ for $s \in (-\varepsilon, \varepsilon)$ with $v_\pm(0) = 0$, $\mu_\pm^1(0) = 0$ and where φ_1^\pm are the positive eigenfunctions of (1.3) associated to λ_\pm . Substituting these expressions in (1.5) and using that $\mathcal{L}\varphi_1^\pm = \lambda_\pm m(x)\varphi_1^\pm$, we deduce that

$$(\mathcal{L} - \lambda_\pm m)v_\pm(s) = \mu_\pm^1(s)m(x)(\varphi_1^\pm + v_\pm(s)) + \lambda m(x)\left(\frac{f(u_\pm(s))}{s} - \varphi_1^\pm - v_\pm(s)\right),$$

or equivalently,

$$(2.2) \quad \frac{1}{\pm\lambda_\pm}v_\pm - T_\pm v_\pm = \frac{1}{\lambda_\pm}T_\pm(\mu_\pm^1(s)(\varphi_1^\pm + v_\pm(s)) + \lambda\left(\frac{f(u_\pm(s))}{s} - \varphi_1^\pm - v_\pm(s)\right)),$$

where $T_\pm : E := C_0(\overline{\Omega}) \mapsto E$; $T_\pm := (\mathcal{L})^{-1}M_\pm$ and $M_\pm : E \mapsto E$ denotes the multiplication operator induced by the function $\pm m$. Considering E ordered by its positive cone P_E , it is easy to see that φ_1^\pm are the positive eigenfunctions of T_\pm and that the spectral radius $r(T_\pm) = r(T_\pm^*) = \frac{1}{\pm\lambda_\pm}$, where T_\pm^* stands for the adjoint of T_\pm in the sense of L^2 . Denote $\Phi_\pm^* \in P_{E^*}$ the positive eigenfunctions of T_\pm^* , the Fredholm alternative applied to (2.2) concludes that

$$(2.3) \quad \lim_{s \rightarrow 0} \frac{\mu_\pm^1(s)}{s} = -\lambda \frac{f''(0)}{2} \frac{\int_\Omega (\varphi_1^\pm)^2 \Phi_\pm^*}{\int_\Omega \varphi_1^\pm \Phi_\pm^*},$$

and so,

$$(2.4) \quad \text{sign}\left(\lim_{s \rightarrow 0} \frac{\mu_{\pm}^1(s)}{s}\right) \neq \text{sign}(f''(0)).$$

Therefore, the bifurcation directions are both supercritical in the case f concave.

We now analyze the global behavior of these curves bifurcating from $(\lambda_{\pm}, 0)$. By the Rabinowitz global bifurcation theorem (cf. [17]) there exist two continua $\mathcal{C}_+ := \{(\mu, u_{\mu})\}$ and $\mathcal{C}_- := \{(\mu, w_{\mu})\}$ of solutions bifurcating from $(\lambda_+, 0)$ and $(\lambda_-, 0)$, respectively. Firstly, we study \mathcal{C}_+ . By (f1), there exists $s_{\mu} > 0$ such that $(\mu - \lambda)s_{\mu} + \lambda f(s_{\mu}) = 0$ and for $s < s_{\mu}$ we have that $\lambda f(s) + (\mu - \lambda)s > 0$. Consider the family $\bar{u}(\mu) := s_{\mu}$ of supersolutions of (1.5). Observe that \bar{u} is not a solution and that for $\mu > \lambda_+$ and close to it, $u_{\mu} < \bar{u}(\mu)$. Consequently, we can apply Theorem 2.2 in [12] to conclude that

$$(2.5) \quad u_{\mu} < s_{\mu}$$

for all $\mu \geq \lambda_+$. We are going to prove that u_{μ} is asymptotically stable, i.e. that

$$(2.6) \quad \lambda_1^{\Omega}(\mathcal{L} - \mu m - \lambda m g'(u_{\mu})) > 0.$$

Indeed, taking $\psi := \lambda f(u_{\mu}) + (\mu - \lambda)u_{\mu}$, we have that $\psi > 0$ by (2.5) and that

$$(\mathcal{L} - \mu m - \lambda m g'(u_{\mu}))\psi = -\lambda f''(u_{\mu}) \sum_{i,j=1}^N a_{ij} D_i(u_{\mu}) D_j(u_{\mu}) > 0,$$

whence we obtain that ψ is a strict supersolution of $\mathcal{L} - \mu m - \lambda m g'(u_{\mu})$, and hence (2.6).

It is clear now that \mathcal{C}_+ is unbounded and that $\Pi(\mathcal{C}_+) = (\lambda_+, \infty)$. Indeed, at $\mu = \lambda_+$ the direction is supercritical. By (2.6) this continuum can be prolonged indefinitely to the right (i.e. \mathcal{C}_+ can not bend back). Finally, by (2.5) if $(\mu, u_{\mu}) \in \mathcal{C}_+$ then u_{μ} is bounded, and so it follows the above claim.

We analyze now \mathcal{C}_- . By (2.3) and (2.4), \mathcal{C}_- goes to the right at $\mu = \lambda_-$. By Crandall and Rabinowitz theorem, \mathcal{C}_- does not reach $(\lambda_+, 0)$ because in a neighborhood of $(\lambda_+, 0)$ \mathcal{C}_- entirely consists of $(\mu_+(s), u_+(s)), s > 0$. On the other hand, by (2.6) it follows that in each neighborhood of (μ_0, u_{μ_0}) in $\mathbb{R} \times C(\bar{\Omega})$ with $\mu_0 > \lambda_+$ the unique positive solutions of (1.5) are of the form (μ, u_{μ}) (see Proposition 20.6 in [2]), and so \mathcal{C}_- can not finish in \mathcal{C}_+ .

Finally, by (1.4) it follows from Theorem 4.3 in [3] that for μ in a compact interval of \mathbb{R} the solutions (μ, u) of (1.5) are bounded in $\mathbb{R} \times C(\bar{\Omega})$. So, $\Pi(\mathcal{C}_-)$ is unbounded. This concludes the proof of part a).

Since the proof of part b) differs slightly of the above one, so we only sketch it. Let (μ, u) be a positive solution of (1.5). Since f is convex it follows that

$$0 = \lambda_1^{\Omega}(\mathcal{L} - \mu m - \lambda m \frac{g(u)}{u}) < \lambda_1^{\Omega+}(\mathcal{L} - \mu m),$$

and so there exists $\mu_1 > 0$ (in fact, $\mu_1 > \lambda_+$) such that $\mu < \mu_1$.

By (2.3) and (2.4), we get that the bifurcation directions are in this case subcritical. On the other hand, by (f2) for negative μ , there exists $t_{\mu} > 0$ such that $(\mu - \lambda)t_{\mu} + \lambda f(t_{\mu}) = 0$ and for $t < t_{\mu}$ we have that $\lambda f(t) + (\mu - \lambda)t < 0$. With a similar reasoning to the part a), we can prove that $u_{\mu} < t_{\mu}$ and taking now as strict supersolution $\psi := -\lambda f(u_{\mu}) + (\lambda - \mu)u_{\mu}$, we infer that u_{μ} is linearly asymptotically stable. The rest of the proof follows analogously. \square

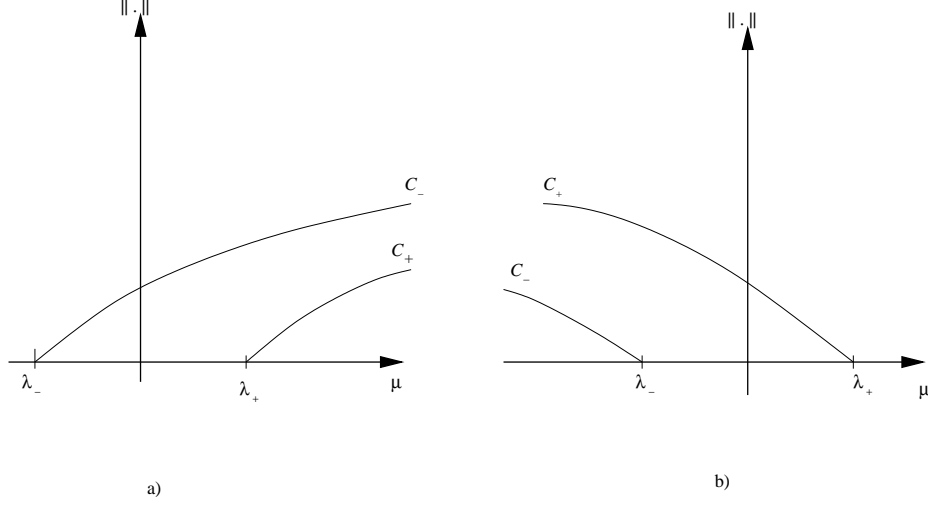


FIGURE 2. Bifurcation diagram of (1.5)

Again, we have summarized the results of the above Proposition in the Figure 2.

We are ready to prove the main result.

Proof of Theorem 1.1. Consider $\lambda > 0$ and f satisfies (f1), similarly it can be shown the other case. If $\lambda \in (0, \lambda_+]$, then there exists (see Figure 2 a)) the solution w_λ of (1.5) and so of (1.1). If $\lambda \in (\lambda_+, +\infty)$, then there exist at least two positive solutions $w_\lambda \neq u_\lambda$, with u_λ linearly asymptotically stable.

Assume now that there exist a sequence $(\lambda_n, u_n)_{n \in \mathbb{N}}$ of positive solution with $\lambda_n \rightarrow 0$ and $\|u_n\|_\infty \leq K$ for some $K > 0$. Since there does not exist positive solution of (1.1) for $\lambda = 0$, we obtain that $\|u_n\|_\infty \rightarrow 0$. We see that this is impossible. Indeed, we define

$$w_n = \frac{u_n}{\|u_n\|_\infty},$$

then w_n is uniformly bounded and, by passing to a suitable sequence again denoted by w_n , $w_n \rightarrow w^*$ as $n \rightarrow \infty$ for some $w \in C(\bar{\Omega})$ with $\|w^*\|_\infty = 1$. But,

$$\mathcal{L}w_n = \lambda_n m(x) \frac{f(u_n)}{\|u_n\|_\infty},$$

and so $\mathcal{L}w^* = 0$, which is an absurd. This completes the proof. \square

Remark 2.2. (1) Note that the existence of \mathcal{C}_+ (resp. \mathcal{C}_-) in the case f concave (resp. convex) is independent to the value of p .

(2) Other conditions can be imposed on p and m to establish a priori bounds for the positive solutions of (1.5) for compact interval of \mathbb{R} , see [3], [4] and [8].

3. CASE \mathcal{L} SELF-ADJOINT AND $f(u) = u + u^p$

In the particular case $f(u) = u + u^p$ and $b_i \equiv 0$ in (1.2), we can complete the information of Theorem 1.1 b). Indeed, we are going to show that there exist

positive solutions if, and only if, $\lambda \in (\lambda_-, \lambda_+)$. Let $\lambda \leq \lambda_- < 0$ be (the case $\lambda > \lambda_+$ can be treated analogously). Then it is well-known that $\lambda_1^\Omega(\mathcal{L} - \lambda m) \leq 0$. Let φ_1 be the positive eigenfunction associated to $\mathcal{L} - \lambda m$, i.e.,

$$\mathcal{L}\varphi_1 - \lambda m\varphi_1 = \lambda_1^\Omega(\mathcal{L} - \lambda m)\varphi_1.$$

Multiplying this equation by φ_1^p and using the Green identity, we get

$$p \int_{\Omega} \varphi_1^{p-1} \sum_{i,j=1}^N a_{ij} D_i(\varphi_1) D_j(\varphi_1) = \lambda \int_{\Omega} m(x) \varphi_1^{p+1} + \lambda_1^\Omega(\mathcal{L} - \lambda m) \int_{\Omega} \varphi_1^{p+1},$$

and so,

$$(3.1) \quad \int_{\Omega} m(x) \varphi_1^{p+1} < 0.$$

Using the Picone's inequality (see for example [5] or Lemma 4.1 in [16]) we obtain

$$\int_{\Omega} \left(\frac{\varphi_1}{u} \right)^p [\mathcal{L}u\varphi_1 - \mathcal{L}\varphi_1 u] < 0$$

and hence

$$\lambda \int_{\Omega} \varphi_1^{p+1} m(x) < \lambda_1^\Omega(\mathcal{L} - \lambda m) \int_{\Omega} \varphi_1^{p+1} / u^{p-1}$$

which contradicts (3.1).

Remark 3.1. This same argument can be used to prove that (1.5) possesses a positive solution if, and only if, $\mu > \lambda_-$ (resp. $\mu < \lambda_+$) if $f = u - u^p$ (resp. $f = u + u^p$).

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