A P System Modeling an Ecosystem Related to the Bearded Vulture

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Summary. The Bearded Vulture is one of the rarest raptors in Europe and it is an endangered species. In this paper, we present a model of an ecosystem related with the Bearded Vulture which is located in the Catalan Pyrenees, by using P systems. The population dynamics constituted by the Bearded Vulture (that feeds almost exclusively on bones) and other five subfamilies that provide the bones they feed on, is studied.

P systems provide a high level computational modeling framework which integrates the structural and dynamical aspects of ecosystems in a comprehensive and relevant way. P systems explicitly represent the discrete character of the components of an ecosystem by using rewriting rules on multisets of objects which represent individuals of the population and bones. The inherent stochasticity and uncertainty in ecosystems is captured by using probabilistic strategies.

In order to give an experimental validation of the P system designed, we have constructed a simulator that allows us to analyze the evolution of the ecosystem with different initial conditions.

1 Introduction

An ecosystem is a natural unit consisting of all plants, animals and microorganisms (biotic factors) in an area functioning together with all of the non-living physical (abiotic) factors of the environment.

Animal species are interconnected in a food chain in which some species depend on others [6], [16]. The variations of different biomass affect the composition of the population structures [15]. In mountain ecosystems where livestock activities are developed in a traditional way, the relationship among ungulates and their predators has been disrupted by the presence of domestic animals [5]. Animals located at the top of the ecological pyramid are influenced by their presence and number. The abandonment of corpses on mountains is a major source of food for necrophagous animals [8].

The study of population ecology and how the species interact with the environment is one of the aspects of the conservation biology with more and more interest for managers and conservationists [2]. A widespread tool used are the ecological models, based on mathematical representations of ecological processes [13]. Ecosystems are dynamic entities composed of the biological community and the abiotic environment. An ecosystem's abiotic and biotic composition and structure is determined by the state of a number of interrelated environmental factors. Changes in any of these factors, such as nutrient availability, temperature, light intensity, grazing intensity, and species population density, will result in dynamic changes in the nature of these systems.

Scientists have recognized that organisms can be organized according to several different functional levels. The functional level known as species refers to a group of organisms that are similar in morphology and physiology and which have the ability to interbreed. A population is formed by all the different organisms of a single species occupying a specific area on the Earth. A community is defined as all the populations of different species inhabiting a particular region of the Earth. The most complex functional level of organization is the ecosystem. An ecosystem consists of the community and its relationship with the abiotic factors of the environment.

Ecosystems are primarily governed by stochastic events, the reactions they provoke on non-living materials and the responses by organisms to the conditions surrounding them. Thus, an ecosystem results from the sum of myriad individual responses of organisms to stimuli from non-living and living elements from the environment. As the number of species in an ecosystem increases, the number of stimuli also does. Ever since life began, organisms have survived continuous change through natural selection of successful feeding, reproductive and dispersal behavior. Ecosystems can be of any size. An ecosystem can be as large as a desert or a lake or as small as a tree or a puddle. A terrarium can be described as an artificial ecosystem.

Ecosystem models, or ecological models, are mathematical representations of ecosystems. Given the complexity of the problems to be modeled, simplifications need to be made in order to achieve a good numerical approach. Ecosystem models are a development of theoretical ecology that describes the major dynamics of ecosystems. Ecosystem models aim not only to integrate the understanding of these systems but also to be able to predict their behavior (in general terms, or in response to particular changes). Due to the complexity of ecosystems in terms of numbers of species/ecological interactions, ecosystem models tend to simplify the systems they are studying down to a limited number of pragmatic components. These may be particular species of interest, or may be broad functional types such as autotrophs, heterotrophs or saprotrophs. In biogeochemistry, ecosystem models usually include representations of non-living “resources” such as nutrients, which are consumed by living components of the model.

The process of simplification reduces an ecosystem to a small number of state variables. Depending upon the system under study, these may represent ecological components in terms of numbers of discrete individuals or quantify the component continuously as a measure of the total biomass of all organisms of that type, often using a common model currency. The components are then linked together by mathematical functions that describe the nature of the relationships between them. For instance, in models which include predator-prey relationships, the two components are usually linked by some function that relates total prey captured to the populations of both predators and prey. Deriving these relationships is often extremely difficult given habitat heterogeneity, the details of component behavioral ecology (including issues such as perception, foraging behavior), and the difficulties involved in unobtrusively studying these relationships under field conditions. Typically, relationships are derived statistically or heuristically. For example, some standard functional forms describing these relationships are linear, quadratic, hyperbolic or sigmoid functions.

Besides establishing the components to be modeled and the relationships between them, another important factor in ecosystem model structure is the representation of space used. Historically, models have often ignored the confusing issue of space by using zero-dimensional approaches such as ordinary differential equations. As computing power increases in, models which incorporate space are being increasingly used, for example, based on partial differential equations or cellular automata. The inclusion of the factor of space allows dynamics not present in non-spatial frameworks to be considered, and sheds light on processes that lead to the formation of patterns in ecological systems. One of the earliest and best-known ecological models is the predator-prey model described by Alfred J. Lotka (1925) and Vito Volterra (1926). This model is composed of a pair of ordinary differential equations where one represents a prey species and the other its predator.

Volterra originally devised the model to explain fluctuations in fish and shark populations observed in the Adriatic Sea following the First World War when fishing had been curtailed. However, the equations have subsequently been applied more generally. Although simple, they illustrate some of the salient features of ecological models: the considered biological populations grow, interact with other populations (either as predators, prey or competitors), and suffer mortality.

The objective of this paper is to design a model that studies the evolution of an ecosystem located in the Pyrenees, taking advantage of the capacity the P Systems to work in parallel. The ecosystem includes six species: the Bearded Vulture (Gypaetus barbatus) as scavenger (predator) species and the Pyrenean Chamois (Rupicapra pyrenaica), Red Deer (Cervus elaphus), Fallow Deer (Dama

dama), Roe Deer (Capreolus capreolus) and Sheep (Ovis capra) as carrion (prey) species.

The paper is structured as follows. In the next section, basic concepts of the ecosystem to be modeled are introduced. The most outstanding aspects of each species are detailed as well as interactions among them. In Section 3, a probabilistic P system is presented in order to describe the ecosystem. A simulator of that probabilistic P system is designed in Section 4, in order to study the dynamics of the ecosystem. Section 5 is devoted to the analysis of the results produced by the simulator. Finally, conclusions are presented in the last section.

2 Modeling the Ecosystem

The ecosystem to be modeled is located in the Catalan Pyrenees, in the Northeast of Spain. This area contains a total of 35 territories that constitutes the 34.3% of the population of the Bearded Vulture Spanish population in 2007 (n = 102). See Figure 1 [8].

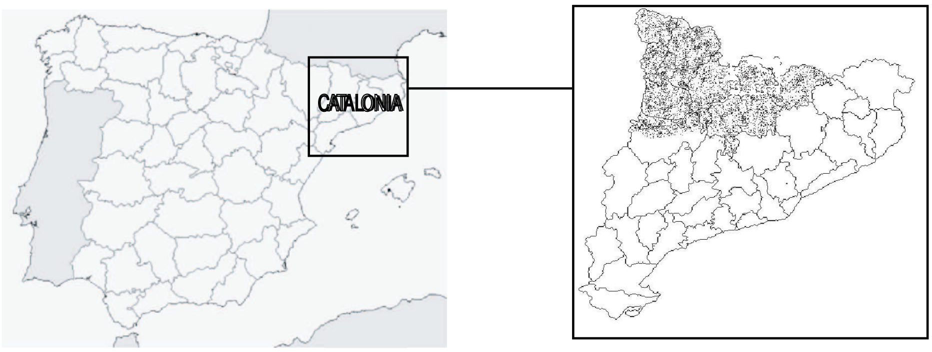

Fig. 1. Regional distribution of the Bearded Vulture in the Catalan Pyrenees (NE Spain).

The ecosystem is composed of six species: the Bearded Vulture (predator species) and the Pyrenean Chamois, Red Deer, Fallow Deer, Roe Deer, and Sheep (prey species). The last five prey species belong to the bovid family, they are herbivores and their bone remains form the basic source of nourishment for the Bearded Vulture in the Pyrenees.

The Bearded Vulture (Gypaetus barbatus) is a cliff-nesting and territorial large scavenger distributed in mountains ranges in Eurasia and Africa. It is one of the rarest raptors in Europe (150 breeding pairs in 2007), it inhabits areas of high altitude (1500-4000 m), though it can be seen in areas of lower altitude (500-800 m) in the winter when high mountains are covered with snow. The mean lifespan of wild Bearded Vultures is 21.4 years [4]. The mean age of first breeding is 8.1 years, whereas the mean age of first successful breeding is 11.4 years [1]. Egg-laying take places during December-February and, after 52-54 days of incubation and around 120 days of chick-rearing, the chick abandons the nest between June-August [12]. The clutch size in this species is usually two eggs, but only one chick survives as a consequence of sibling aggression [11]. Bearded Vultures are fertile from the age of eight, when they become adult, and they cease to be fertile at the age of twenty. The female's annual fertility rate in Catalonia during the last five years is estimated around 38% [8], the female lays two eggs and incubates them for 55-57 days. However, as with most birds of prey, only one of the youngsters fledges.

The Bearded Vulture is the only vertebrate animal that feeds almost exclusively on bone remains. Its main food source is bone remains of dead small and medium-sized animals. It searches for food either alone or in pairs. In the Pyrenees the bone remains of Pyrenean Chamois, Red Deer, Fallow Deer, Roe Deer, and Sheep form 67% of the vulture's food resources of nourishment, and the other 33% includes the bone remains of small size mammals (e.g., dogs, cats), badgers, least weasel, large mammals (cows, horses), medium size mammals (e.g., wild boars) and birds [8]. A pair of Bearded Vultures needs an average 341 Kg of bones per year [10], [9].

During the dispersal period (from fledgling until the bird become territorial at 5-6 years), the non-adult Bearded Vultures birds cover large distances surveying different areas. For example, the averaged surface covered by four young monitored after fledging was 4932 km^2 (range 950-10294 km^2, [14]). They may return to breed in the area where they were born but it is not frequent. The choice of the area where they settle down definitively is based on different parameters such as the availability of food and the vulture population density. Breeding mature birds are territorial and the approximate home ranges obtained for eight pairs studied varied between 250 km^2 and 650 km^2. They almost never leave their territory to settle in neighboring mountainous areas even if these are close by, and this fact makes it more difficult to estimate the growth of its population. Studies by Margalida estimate the average annual growth in the population of Bearded Vultures in the Pyrenees to be 4-53% of the existing population, and that the average floating population is 20 principally remains in feeding stations situated in the central Pyrenees (Aragon) birds.

The natural behavior of the five bovid species is similar as they are all herbivores and they all reach the size of the adult animal when they are one year old. In general, they arrive at the sexual maturity within two years from birth. Chamois and the Red Deer have a longer life expectancy than Fallow Deer and Roe Deer. The natural mortality rates are similar in all five species, in the first year of life it is calculated to be 50% and 6% during the remaining years. In spite of the great degree of similarity among these five species, there are differences among them, some are of natural origin and other are induced by human action. It is essential to bear them in mind in order to define a P system that can reliably simulate the ecosystem.

Red Deer are very much appreciated by the hunters, not for their meat but as trophies and so only males are hunted, and this causes the natural evolution of the population to be modified. The hunter only takes the head as a trophy leaving the animal's body on the field, and so the carcass is eaten by other species and the bones may then be eaten by the Bearded Vulture.

Fallow Deer and Roe Deer live in areas that are difficult to reach and for this reason the Bearded Vulture cannot take advantage of the bones of all of the dead animals. Studies estimate that the Bearded Vulture only uses 20% of the bones available on the field.

As sheep are domestic animals, humans exert a high level of control over the sheep populations. The size and growth of the sheep population is limited by the owners of the flocks. The natural average life expectancy of sheep is longer that their actual life expectancy in the field because when its fertility rate decreases at the age of eight, they are taken out of the habitat. Most of the lambs are sold to market and so they are taken out of the habitat too in the first year of life. Only 20% of the lambs, mostly females, are left on the field and they are used to replace sheep that have died naturally and the old ones that have been removed from the flock.

As the Bearded Vulture is an endangered species, there are many projects that study its behavior and the way it is affected by its environment. Thanks to these studies there is available a large amount of information which is required to define the P System and to validate the results obtained.

In this study, the feeding of the Bearded Vulture is dependent on the evolution of the P System. However, the P System does not consider that the availability of food limits the feeding of the herbivores, and so the growth of the vegetation is not modeled.

Taking all of this background information into consideration, the following data was required for each species:

- $k_{i,1}$: age at which adult size is reached. This is the age at which the animal eats like the adult does, and at which if the animal dies, the amount of biomass it leaves is similar to the ole left by an adult. Moreover, at this age it will have surpassed the critical early phase during which the mortality rate is high.
- $k_{i,2}$: age at which it starts to be fertile.
- $k_{i,3}$: age at which it stops being fertile.
- $k_{i,4}$: average life expectancy.
- $k_{i,5}$: fertile ratio (number of descendants by fertile female).
- $k_{i,6}$: population growth.
- $k_{i,7}$: mortality ratio in first years ($age < k_{i,1}$) in which biomass in the form of bones is not left on the field.
- $k_{i,8}$: mortality ratio in first years ($age < k_{i,1}$) in which biomass in the form of bones is left on the field.
- $k_{i,9}$: mortality ratio in adult animals ($age \geq k_{i,11}$) in which biomass in the form of bones is not left on the field.

- $k_{i,10}$: mortality ratio in adults animals ($age \geq k_{i,1}$) in which biomass in the form of bones is left on the field.
- $k_{i,11}$ is equal to 1 if the animal dies at the age of $k_{i,4}$ leaving biomass, and it is equal to 0 if the animal dies at the age of $k_{i,4}$ without leaving bones.
- $k_{i,12}$: amount of bones from young animals ($age < k_{i,1}$).
- $k_{i,13}$: amount of bones from adult animals ($age \geq k_{i,1}$).
- $k_{i,14}$: percentage of females in the population.
- $k_{i,15}$: type of food.
- $k_{i,16}$: amount of food necessary per year and animal (1 unit is equal 0.5 kg of bones).

When an animal dies, the weight of bones which it leaves is around 20% of its total weight. Table 1 shows the average weight of each animal as well as the weight of bones they leave. In the case of Fallow Deer and Roe Deer, the value of the weight of bones must then be multiplied by 0,2 (20%) which is the proportion of bones that are accessible for the Bearded Vulture.

Table 1. Bones

Species	Weigh Male	Weigh Female	Percentage Female	Average weigh	Biomass: bons adult	Biomass: bons young	Kg accessible by B.Vulture (adult/young)
B.Vulture	5	6.5	60	2	-	-	-
Chamois	28	32	50	30	6	3	6/3
Red Deer Female	-	75	-	75	15	7.5	15/7.5
Red Deer Male	120	-	-	120	24	12	24/12
Fallow Deer	63	42	80	46	9	4.5	2/1
Roe Deer	27	23	66	24	5	2.5	1/0.5
Sheep	42	35	97	35.2	7	3.5	7/3.5

33% of the Bearded Vulture's nutrition is formed by bone remains of other species that belong to the ecosystem but which are not studied in this model, and so it was necessary for the P system to include an annual contribution from the ecosystem. This contribution was calculated to be 12500 kg bones by considering the size of the Bearded Vulture population and its weight. By subtracting the 3500 kg of food in form of bones that is consumed by the floating population of Bearded Vultures, the annual external contribution is calculated to be 9000 kg of bones. There are seven feeding stations in Catalonia which provide around 10500 kg of bone remains annually. These artificial feeding sites have not been considered in the study and most of the floating birds feed at these sites.

The required information about each species is shown in Table 2.

Table 2. Constants

Specie	i	$k_{i,1}$	$k_{i,2}$	$k_{i,3}$	$k_{i,4}$	$k_{i,5}$	$k_{i,6}$	$k_{i,7}$	$k_{i,8}$	$k_{i,9}$	$k_{i,10}$	$k_{i,11}$	$k_{i,12}$	$k_{i,13}$	$k_{i,14}$	$k_{i,15}$	$k_{i,16}$
B.Vulture	1	1	8	20	21	0	3	6	0	7	0	0	0	0	50	bones	341
Chamois	2	1	2	18	18	75	0	0	55	0	6	1	6	12	55	grass	0
Red Deer Female	3	1	2	13	17	50	0	0	50	0	6	1	15	30	100	grass	0
Red Deer Male	4	1	2	18	20	0	0	0	50	0	36	1	24	48	0	grass	0
Fallow Deer	5	1	2	11	12	50	0	0	50	0	7	1	2	4	75	grass	0
Roe Deer	6	1	2	9	10	90	0	0	55	0	5	1	1	2	67	grass	0
Sheep	7	1	2	7	7	75	0	59	15	0	4	0	7	14	96	grass	0

3 A P System Based Model of the Ecosystem

In this section we present a model of the ecosystem described in Section 2 by means of probabilistic P systems. We will study the behavior of this ecosystem in diverse initial conditions.

First, we define the P systems based framework (probabilistic P systems), where additional features such as two electrical charges which describe specific properties in a better way, are used.

Definition 1. *A probabilistic P system of degree n is a tuple*

$$\Pi = (\Gamma, \mu, \mathcal{M}_0, \dots, \mathcal{M}_{n-1}, R)$$

where:

- *Γ is the alphabet (finite and nonempty) of objects (the working alphabet).*
- *μ is a membrane structure, consisting of n membranes, labeled $0, 1, \dots, n-1$. The skin membrane is labeled by 0. We also associate electrical charges with membranes from the set $\{0, +\}$, neutral and positive.*
- *$\mathcal{M}_0, \dots, \mathcal{M}_{n-1}$ are strings over Γ, describing the multisets of objects initially placed in the n regions of μ.*
- *R is a finite set of evolution rules. An evolution rule is of the form $r: u \xrightarrow{k} v$, where u, v are a multiset over Γ and $k \in [0,1]$ is a real number between 0 and 1 associated with the rule.*

We assume that a global clock exists, marking the time for the whole system (for all compartments of the system); that is, all membranes and the application of all rules are synchronized.

The n-tuple of multisets of objects present at any moment in the *n regions* of the system constitutes the *configuration* of the system at that moment. The tuple $(\mathcal{M}_0, \dots, \mathcal{M}_{n-1})$ is the initial configuration of the system.

We can pass from one configuration to another one by using the rules from R as follows: at each transition step the rules to be applied are selected according to the probabilities assigned to them, and all applicable rules are simultaneously applied and all occurrences of the left–hand side of the rules are consumed, as usual.

3.1 The model

Our model consists in the following probabilistic P system of degree 2 with two electrical charges:

$$\Pi = (\Gamma, \mu, \mathcal{M}_0, \mathcal{M}_1, R)$$

where:

- In the alphabet Γ we represent the six species of the ecosystem (index i is associated with the species and index j is associated with their age, and the symbols X, Y and Z represent the same animal but in different state); it also contains the auxiliary symbols B and C.

$$\Gamma = \{X_{ij},\ Y_{ij},\ Z_{ij} :\ 1 \le i \le 7,\ 0 \le j \le k_{i,5}\} \cup \{\ B,\ C\}$$

- In the membrane structure we represent two regions, the skin (where animals reproduce) and an inner membrane (where animals feed and die): $\mu = [\ [\ ]_1\ ]_0$ (neutral polarization will be omitted)
- In $\mathcal{M}_0$ and $\mathcal{M}_1$ we specify the initial number of objects present in each regions (encoding the initial population and the initial food).
  - $\mathcal{M}_0 = \{X_{ij}^{q_{ij}} : 1 \le i \le 7,\ 0 \le j \le k_{i,}\}$, where the multiplicity q_{ij} indicates the number of animals, of species i whose age is j that are initially present in the ecosystem.
  - $\mathcal{M}_1 = \{C\ B^{18000}\}$, where the object B represent 0.5 kg of bones, and 9000 kg is the external contribution of bones to the P system corresponding to the 33% of feeding that come from animals do not modeled in the P system.
- The set R of evolution rules consists of:
  - Reproduction-rules

    Adult males:

    $r_0 \equiv [X_{ij} \xrightarrow{1-k_{i,14}} Y_{ij}]_0, 1 \le i \le 7, 0 \le j \le k_{i,4}$

    Adult females that reproduce:

    $r_1 \equiv [X_{ij} \xrightarrow{k_{i,5}k_{i,14}} Y_{ij}Y_{i0}]_0,\ 1 \le i \le 7,\ k_{i,2} \le j < k_{i,3}$

    Adult females that do not reproduce:

    $r_2 \equiv [X_{ij} \xrightarrow{(1-k_{i,5})k_{i,14}} Y_{ij}]_0,\ 1 \le i \le 7,\ k_{i,2} \le j < k_{i,3}$

    Young animals that do not reproduce:

    $r_3 \equiv [X_{ij} \rightarrow Y_{ij}]_0,\ 1 \le i \le 7,\ k_{i,3} \le j < k_{i,2}$
  - Young animals mortality rules:

    Those which survive:

$r_4 \equiv Y_{ij}[\ ]_1 \xrightarrow{1-k_{i,7}-k_{i,8}} [Z_{ij}]_1 :\ 1 \le i \le 7,\ 0 \le j < k_{i,1}$

Those which die and leaving bones:

$r_5 \equiv Y_{ij}[\ ]_1 \xrightarrow{k_{i,8}} [B^{k_{i,12}}]_1 :\ 1 \le i \le 7,\ 0 \le j < k_{i,1}$

Those which die and do not leave bones:

$r_6 \equiv Y_{ij}[\ ]_1 \xrightarrow{k_{i,7}} [\ ]_1 :\ 1 \le i \le 7,\ 0 \le j < k_{i,1}$

– Adult animals mortality rules:

Those which survive:

$r_7 \equiv Y_{ij}[\ ]_1 \xrightarrow{1-k_{i,9}-k_{i,10}} [Z_{ij}]_1 :\ 1 \le i \le 7,\ k_{i,1} \le j < k_{i,4}$

Those which die leaving bones:

$r_8 \equiv Y_{ij}[\ ]_1 \xrightarrow{k_{i,10}} [B^{k_{i,13}}]_1 :\ 1 \le i \le 7,\ k_{i,1} \le j < k_{i,4}$

Those which die and do not leave bones:

$r_9 \equiv Y_{ij}[\ ]_1 \xrightarrow{k_{i,9}} [\ ]_1 :\ 1 \le i \le 7,\ k_{i,1} \le j < k_{1,4}$

Animals that die at an average life expectancy:

$r_{10} \equiv Y_{ij}[\ ]_1 \rightarrow [B^{k_{i,13} \cdot k_{i,11}}]_1 :\ 1 \le i \le 7,\ j = k_{i,4}$

– Feeding rules:

$r_{11} \equiv [Z_{ij} B^{k_{i,16}}]_1 \rightarrow X_{ij+1}[\ ]_1^+ :\ 1 \le i \le 7,\ 0 \le j \le k_{i,4}$

– Rules of mortality due to lack of food, and the elimination from the system of bones that are not eaten by the Bearded Vulture:

Elimination of remaining bones:

$r_{12} \equiv [B]_1^+ \rightarrow [\ ]_1$

External contribution that represent the bones:

$r_{13} \equiv [C]_1^+ \rightarrow [CB^{18000}]_1$

Adult animals that die because they have not enough food:

$r_{14} \equiv [Z_{ij}]_1^+ \rightarrow [B^{k_{i,13} \cdot k_{i,11}}]_1 :\ 1 \le i \le 7,\ k_{i,1} \le j \le k_{i,4}$

Young animals that die because they have not enough food:

$r_{15} \equiv [Z_{ij}]_1^+ \rightarrow [B^{k_{i,12} \cdot k_{i,11}}]_1 :\ 1 \le i \le 7,\ j < k_{i,1}$

Figure 2 gives a schematic view of how the P system works.

4 A Simulator

In order to study the dynamics of the species that belong to the ecosystem, we have designed a simulator written in $C++$ language. This program runs on a PC.

In the simulation, the objects that encode the species and the age are represented by two vectors which are related through the number assigned to each animal of the ecosystem. The objects of the P system evolve in a random way; this

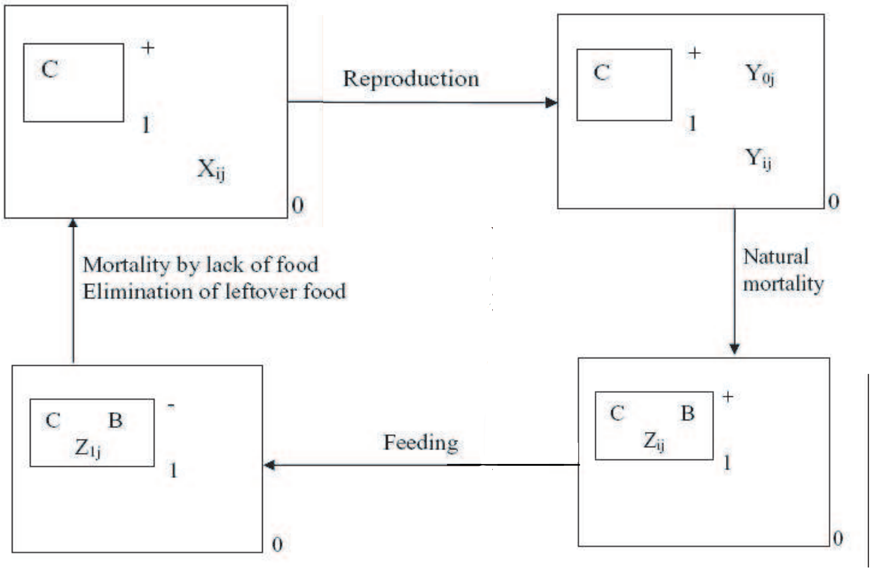

Fig. 2. Schema

stochasticity is implemented by generating random numbers between 1 and 100, according to an uniform distribution. One of the generated numbers is assigned to each animal. Then, the animal evolves according to the assigned number and the constant probability. For example, when the probability of surviving is 70%, the animal will die if the assigned number is greater than 70.

The input of the program consists of the parameters of each species that are considered in the P system and the number of animals of each species and age that are present at time zero. The output is the number of animals and age of each species that are present every year after completing the following processes: reproduction, mortality and feeding.

In nature an ecosystem is governed by nondeterminism, and this implies a complex mathematical model. Nevertheless all the processes that are carried out have an important degree of randomness. This randomness can be predicted and can be quantified at every moment and situation of the ecosystem.

The program has been structured in four modules:

- *Reproduction.* The inputs are the age at which each species begins to be fertile, the age at which it stops being fertile, the fertility rate, and the proportion of females of the species. This module also requires the total number of existing animals and the distribution of these animals in terms of species and ages. The output of this module is the number and age of animals of each species.

- *Mortality.* The inputs are the mortality rate based on the age, the average life expectancy of each species, and finally the weight of bones left by the dead animal which is dependent on its age. Once again this module also requires the total number of animals and their distribution in terms of species and ages. As with the other modules, the output of this module is the number and age of animals of each species when the process is completed. Another output of this module is the amount of food that is generated in terms of the weight of bones produced that provide the Bearded Vulture's basic source of nourishment.
- *Feeding.* The inputs are the amount of food available in the ecosystem and the annual amount of food that is necessary for the animal to survive in suitable conditions, in other words, conditions under which the animals are not debilitated and do not suffer the consequent effects on their capabilities. As was seen in the previous modules, inputs are generated by the P system itself as it quantifies objects representing the number of animals of each existing species and age. Once again, the output of this module is the number and age of animals of each species.
- *Elimination* of unused leftover food and the animal mortality from insufficient feeding. The input of this module is part of output of the feeding module. The aim of this process is to eliminate the number of animals that were not able to find the necessary amount of food for their survival, and also to consider the amount of leftover food that is degraded with time and that therefore stops having a role in the model. The animals that die due to a lack of food are transformed into bones that can then be eaten by the Bearded Vulture. The output of this module is an amount of food in form of bones that is available to the Bearded Vulture.

The unit of reference used in this study is the year, that is, the food consumed throughout an annual period is given at one single point in time, and with one application of each rule. The mortality of animals in an ecosystem is also a process that is carried out in a continuous way, throughout the year. However, reproduction is an activity that takes place at a specific time of the year, and moreover, it takes place at the same time for all of the species considered in this study. It will be necessary to verify if the one year unit time which was chosen is correct or whether a shorter inferior unit of time should be used in the P system. It was also necessary to check the robustness of the proposed model and to do this, it was ran a second time with a modified order of application of the four processes modules. Given independence of the four modules that form the P system, it would be a simple exercise to run probability experiments with each module.

5 Results and Discussions

In order to validate the P system, it is necessary to check its robustness. Considering the year as unit time it was necessary to discretize feeding and mortality variables.

According to the design of the P system, reproduction rules have a higher priority than mortality ones. Then, the robustness of the model regarding the change of that priority is analyzed. For that reason, two variants of the simulator have been studied changing the order of the corresponding modules (in the P system we change variable X by variable Y in the initial multiset $\mathcal{M}_0$).

The data used to verify the robustness of the P system correspond to the present situation (2008) in the Catalan Pyrenees of the ecosystem formed by the Bearded Vulture, Chamois, Red Deer, Fallow Deer, Roe Deer and Sheep (see Table 3).

Table 3. Number of animals, at the moment, in the Pyrenean Catalan

Species	Number
Bearded Vulture	74
Chamois	12000
Red deer female	4400
Red deer male	1100
Fallow deer	900
Roe deer	10000
Sheep	200000

As shown in Table 3, the data of the current number of animals in the Catalan Pyrenees does not indicate the ages of animals. An age distribution has been generated considering the different constants that affect the animals throughout their life. These constants are fertility rate, mortality rate and percentage of females in the population. The age distribution obtained is shown in Table 4.

In both cases, the simulator was ran 10 times until it covered a period of 20 years.

In these figures, solid lines and dashed lines represent the population dynamics when the simulator modules are applied following the *orders* reproduction–mortality–feeding and mortality–feeding–reproduction, respectively. Taking into account that the P system behavior is similar in both cases, it can be deduced that our model is robust with regard to the properties considered.

The very important factor of population density was not considered in the model of the ecosystem. This is the reason why the population of some of the species grew in an exponential way reaching values which cannot be obtained in the ecosystem. It is well-known, for example, that when a population of Red Deer reaches a level of 15000 animals, a regulation process starts that implies a drastic decrease of the population down to 1000 individuals.

6 Conclusions

A probabilistic P System modeling an ecosystem related with the Bearded Vulture which is located in the Catalan Pyrinees, has been presented.

Table 4. Number of animals for age

Age	B. Vulture	Chamois	Red deer female	Red deer male	Fallow deer	Roe deer	Sheep
1	0	988	978	254	125	1210	31246
2	0	987	780	192	110	1207	30310
3	0	890	625	154	103	1207	29400
4	0	889	500	124	95	1207	28519
5	0	889	400	99	89	1085	27663
6	0	795	240	60	83	1083	26834
7	0	792	195	48	77	1083	26028
8	6	690	155	38	71	959	0
9	6	689	123	30	52	959	0
10	6	592	97	24	50	0	0
11	6	592	78	20	45	0	0
12	5	592	62	16	0	0	0
13	5	497	50	12	0	0	0
14	5	497	40	10	0	0	0
15	5	496	32	8	0	0	0
16	5	395	25	6	0	0	0
17	5	394	20	5	0	0	0
18	5	336	0	0	0	0	0
19	5	0	0	0	0	0	0
20	5	0	0	0	0	0	0
21	5	0	0	0	0	0	0

By using this P system has been possible to study the dynamics of the ecosystem modifying the framework in order to calculate how the ecosystem would evolve if different biological factors were modified either by nature or through human intervention.

A simulator of the P system has been designed and the robustness of the model with respect the order of application of the different kinds of rules, has been shown.

The P system does not consider levels of population density and, as a consequence, an exponential growth of populations of species was obtained. In a future work this factor and other parameters (e.g., the amount of food of the herbivores species, the climatic changes in the ecosystem, etc.) should be considered.

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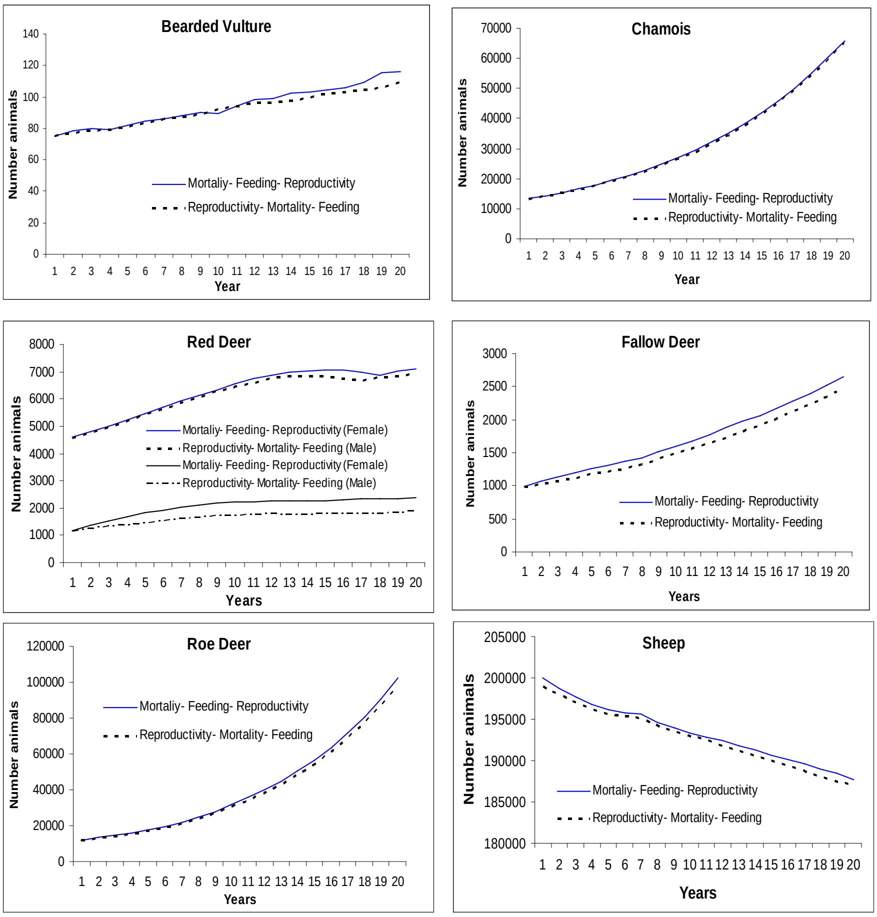

Fig. 3. Robustness of the ecosystem

References

1. R.J. Antor, A. Margalida, H. Frey, R. Heredia, L. Lorente, J.A. Sesé. Age of first breeding in wild and captive populations of Bearded Vultures (Gypaetus barbatus). *Acta Ornithologica*, **42** (2007), 114–118.
2. M. Begon, J.L. Harper, C.R. Townsend. *Ecology: Individuals, Populations and Communities*. Blackwell Scientific Publications Inc., Oxford, UK, 1988.
3. D. Besozzi, P. Cazzaniga, D. Pescini, G. Mauri. Modelling metapopulations with stochastic membrane systems. *BioSystems*, **91**, 3 (2007), 499–514.
4. C.J. Brown. Population dynamics of the bearded vulture Gypaetus barbatus in southern Africa. *African Journal of Ecology*, **35** (1997), 53–63.

5. C. Chocarro, R. Fanlo, F. Fillat, P. Marín. Historical evolution of natural resource use in the central Pyrenees of Spain. *Mountain Research And Development* , **10** (1990), 257–265.
6. P.H. Harvey, A. Purvis. Understanding the ecological and evolutionary reasons for life history variation: mammals as a case study. In McGlade J., ed. *Advanced ecological theory: principles and applications*. Blackwell Science Publications, Oxford, 1999, pp. 232–247.
7. J. Del Hoyo, A. Elliott, J. Sargatal. *Handbook of the Birds of theWorld, vol. 2.*. Lynx Edicions, Barcelona, 1994.
8. A. Margalida, D. García, A. Cortés-Avizanda. Factors influencing the breeding density of Bearded Vultures, Egyptian Vultures and Eurasian Griffon Vultures in Catalonia (NE Spain): management implications. *Animal Biodiversity and Conservation*, **30**, 2 (2007), 189–200.
9. A. Margalida, S. Mañosa, J. Bertran, D. García. Biases in studying the diet of the Bearded Vulture. *The Journal of Wildlife Management*, **71**, 5 (2006),1621–1625.
10. A. Margalida, J. Bertran, J. Boudet. Assessing the diet of nestling Bearded Vultures: a comparison between direct observation methods, *Journal of Field Ornithology*, **76**, 1 (2005), 40–45.
11. A. Margalida, J. Bertran, J. Boudet, R. Heredia. Hatching asynchrony, sibling aggression and cannibalism in the Bearded Vulture (Gypaetus barbatus). *Ibis*, **146**, (2004) 386–393.
12. A. Margalida, D. García, J. Bertran, R. Heredia. Breeding biology and success of the Bearded Vulture Gypaetus barbatus in the eastern Pyrenees. *Ibis*, **145**, (2003) 244–252.
13. H. McCallum. *Population Parameters: Estimation for Ecological Models*. Blackwell Science Publications, Oxford, 2000.
14. C. Sunyer. El periodo de emancipación en el Quebrantahuesos: consideraciones sobre su conservación. In R. Heredia, B. Heredia, eds. *El Quebrantahuesos (Gypaetus barbatus) en los Pirineos* Colección Técnica. Madrid, ICONA, 1991, pp. 47–65.
15. D. Tilman. *Resource Competition and Community Structure*. Princeton University Press, Princeton, New Jersey, 1982.
16. A. Watson. *Animal Populations in Relation to Their Food Resources*. Blackwell Scientific Publications, Oxford, 1970.