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Upper bound to the effective area of concrete in tension

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ABSTRACT

The effective tension area of concrete is a well known parameter in structural concrete. It is involved in several phenomena that affect the design of structural concrete elements, such as cracking, shear deformation or flexural deformation. In this work, the authors put forward a necessary change in the definition of the effective tension area of concrete provided by some groups of Standards.

Keywords: Tension stiffening, effective area of concrete.

1. INTRODUCTION

The uncracked concrete between adjacent cracks contributes to the stiffness of the composite material (i.e. reinforced concrete) due to the bond between the reinforcing bars and the concrete. This effect, called tension stiffening ([1]), significantly effects the response of the RC members. In fact, actual Standards include it in load-deflection and serviceability verifications.

The tension-stiffening model considered in Model Code [2] and Eurocode 2 [3] introduces the tension stiffening effect increasing the stiffening of the reinforcement by means of the presence of a surrounding area of concrete working in tension for strain levels greater than the cracking strain of concrete ([4]). This surrounding area is the effective tension area of concrete, $A_{c,eff}$.

The value of $A_{c,eff}$ is given by Eurocode 2 (§7.3.4) as function of: a) the type of structural element such as beams, slabs, … b) the position of the tensile reinforcement defined by its centroid and c) the neutral fiber depth.

This paper questions the proposed expression by Eurocode 2 for the dependence of the effective tension area of concrete on the neutral fiber depth. Moreover, in a monotonically growing process of loading, the neutral fiber depth (x) decreases, and, in line with the proposed formula, this implies an increase in the effective tension area of concrete during the loading process, something which is physically impossible. Several examples are shown below.

The notation used in this work is in accordance with Eurocode 2.

2. CONSTITUTIVE equATIONS FOR CONCRETE AND STEEL.

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2.1. Concrete

The non-linear relation between compressive stress, σ_c, and shortening strain, ε_c, proposed by EC2 - §3.1.5(1) - for concrete under short term uniaxial loading is used.

There are several expressions available for the post-cracking relation of the tensile stress, σ_{ct}, and tensile strain in concrete, ε_c. In this paper, the expression proposed by Hernández-Montes et al. [5] has been considered (Eq. (1)). The reason for doing so is that this expression has been derived from the formulation given in the Model Code 2010 with no additional assumptions:

$$\sigma_{ct}[\varepsilon_c] = -\frac{\rho}{2}E_s\varepsilon_c + \sqrt{\left(\frac{\rho}{2}E_s\varepsilon_c\right)^2 + f_{ctm}^2(1+n\rho)} \qquad (1)$$

where

$$\rho = \frac{A_s}{A_{c,eff}}$$

The remaining the notation is in accordance with Eurocode 2. In fact, As is the area of tension reinforcement and $A_{c,eff}$, the effective area, for rectangular sections is the area of concrete surrounding the tension reinforcement of depth, $h_{c,ef}$, where $h_{c,ef}$ is the lesser of 2.5(h-d), (h-x)/3 or h/2 (see Fig. 1).

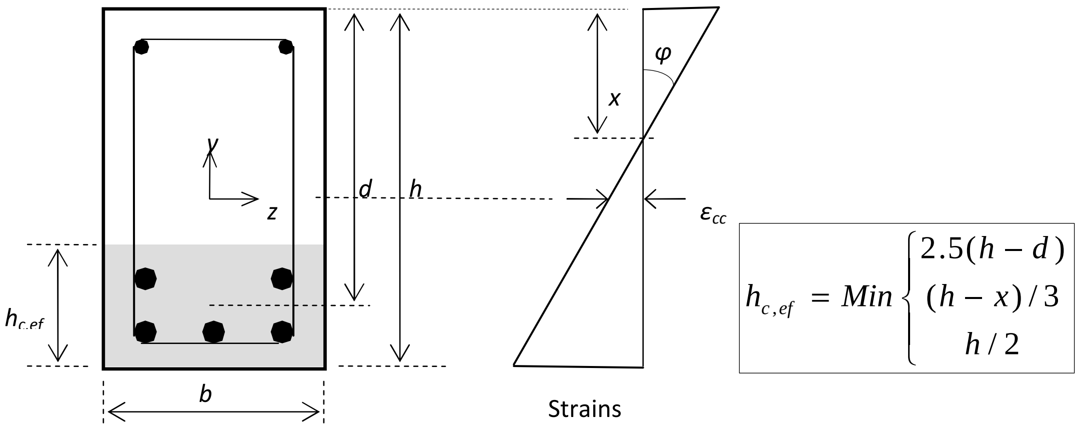

Figure 1. *Effective height of concrete in tension and notation.*

Expression in Eq. (1) is reinforced with the following observation by Wu & Gilbert [6]. After an exhaustive test campaign, they concluded that the tensile strength of concrete disappears when the deformation in steel reaches the value corresponding to yield, ε_y. Taking into account this evidence, Eq. (1) has been corrected by introducing a linear variation from the apparent yield strain of the steel to the value of 0 for ε_y. The value of the apparent yield strain of the steel, ε_{ced}, is obtained, as suggested by Gil-Martín *et al. [7]*, by imposing equilibrium in a short segment of an RC tension member, between a cracked section (where the reinforcement has yielded) and a generic section where the reinforcement stress is equal to the average stress (Fig. 2).

$$\underbrace{A_s E_s \varepsilon_y}_{at\ crack} = \underbrace{A_s E_s \varepsilon_{ced} + A_{c,eff}\sigma_{ct}[\varepsilon_{ced}]}_{average\ section} \rightarrow \varepsilon_{ced} \tag{2}$$

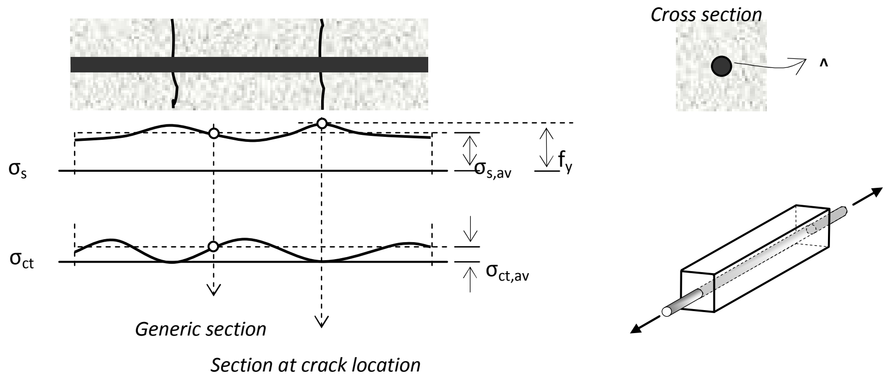

Figure 2. *Apparent yield strain of the steel.*

The complete concrete model is shown in Fig. 3 for concrete with a characteristic strength of 30 MPa and steel with a characteristic strength 500 MPa.

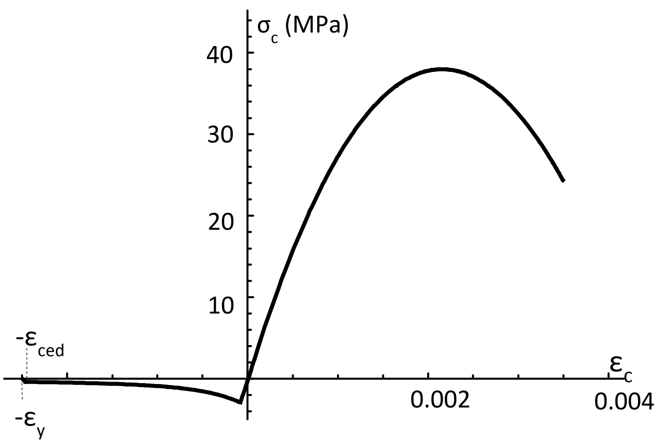

Figure 3. *Non-lineal strain-stress response of concrete for specimen represented in Fig. 5 with f_{ck}=30 MPa, f_{yk}=500 MPa*.

2.3. Steel

The considered strain-stress curve for reinforcement steel is bilinear with no strain hardening. No p-delta effects are considered in the reinforcement due to the presence of transversal reinforcement, Gil-Martin et al. [8]. The steel model has been corrected to account for the presence of the reinforcement embedded in the concrete cross-section, which is due to the fact that concrete stress is considered to operate over the gross section, not accounting for the presence of the reinforcement. The model of steel is represented in Fig. 4.

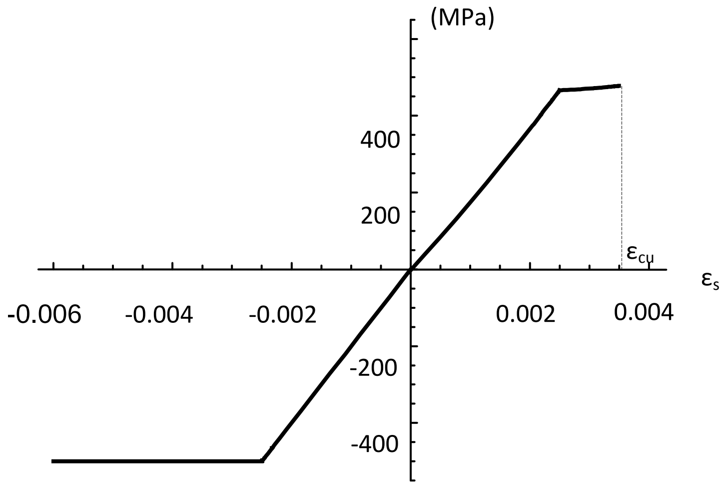

Figure 4. : *Strain-stress curve of steel for specimen represented in Fig. 5 with f_{ck}=30 MPa, f_{yk}=500 MPa.*

3. EQUILIBRIUM.

There are two accepted hypotheses at cross-section level for the study of reinforced concrete elements: a. Adherence concrete - steel: steel and concrete placed at the same fiber of the cross-section presents the same strain; and b. Plane section hypothesis (PSH): plane sections remain plane after deformation. As shown in Fig. 1, the PSH hypothesis implies that the deformation of any fiber located at a distance y of the center of gravity of the gross section can be expressed as a function of the curvature (ϕ) and of the strain of the center of gravity (ε_{cc}).

$$\varepsilon[y,\varphi,\varepsilon_{cc}] = \varphi y + \varepsilon_{cc} \tag{3}$$

Imposing the equilibrium of axial force and of flexural moment at the center of gravity of the gross section, the response of the reinforced concrete section, in terms of ϕ and ε_{cc}, to the loadings N_{Sd} and M_{Sd} can be obtained from:

$$N_{Sd} = \int_{y\min}^{y\max} \sigma_c[\varepsilon[y,\varphi,\varepsilon_{cc}]]bdy + \sum_{i=1}^{n} A_{\phi i}\sigma_s[\varepsilon[y_{\phi i},\varphi,\varepsilon_{cc}]] \tag{4}$$

$$M_{Sd} = \int_{y\min}^{y\max} \sigma_c[\varepsilon[y,\varphi,\varepsilon_{cc}]]bydy + \sum_{i=1}^{n} A_{\phi i}y_{\phi i}\sigma_s[\varepsilon[y_{\phi i},\varphi,\varepsilon_{cc}]] \tag{5}$$

where *b* is the width of the cross section, in the case of non-rectangular sections, *b* is function of *y: b[y]*. $A_{\phi i}$ and $y_{\phi i}$ are the area of the cross section and the y-coordinate of bar ϕ_i.

In the former expression $\sigma_c[\varepsilon]$ and $\sigma_s[\varepsilon]$ are the constitutive equations of concrete and steel represented in Fig. 3 and 4, respectively.

Eqs. (4) and (5) can be solved estimating values of the curvature (φ), for a given value of N_{Sd}. For each value of the curvature, ε_{cc} can be obtained from the equation 4. The value of M_{Sd} is obtained from Equation 5 for each pair (φ , ε_{cc}). For a constant axial load N_{Sd}, values of the stress in concrete, the stress in steel and the depth of the neutral fiber (*x*), can be obtained from monotonic values of the flexural moment M_{Sd}.

4. EXAMPLE.

Fig. 5 shows a 0.3×0.5 m cross section of a beam having concrete with a characteristic compressive strength of 30 MPa and a reinforcement characteristic yield strength of 500 MPa. The longitudinal reinforcement consists of two bars of 12 mm diameter and 5 bars of 20 mm diameter, positioned as shown in Fig. 5. Transverse reinforcement corresponds to stirrups 10 mm diameter at 150 mm.

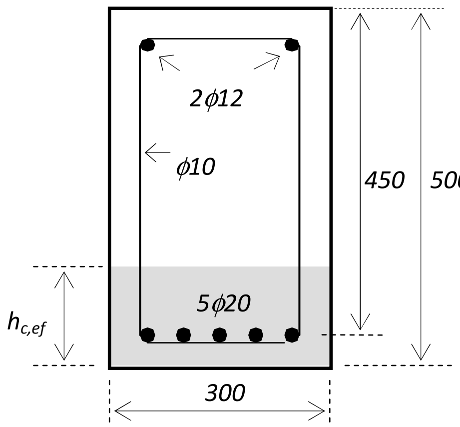

Figure 5. *Example of beam cross section.*

For no axial load, i.e. $N_{Sd} = 0$, and for values of the curvature (φ) between $2 \cdot 10^{-8}$ and $2 \cdot 10^{-5}$ the system of Eqs. (4) and (5) has been solved using Mathematica©. The bisection method has been used with a tolerance for axial load equal to 10^{-1} N.

The value of $h_{c,ef}$, given in Fig. 1, is affected by the neutral fiber depth. Using an iterative procedure, the neutral fiber depth (*x*) is obtained from Eqs. (4) and (5), see Fig. 6.

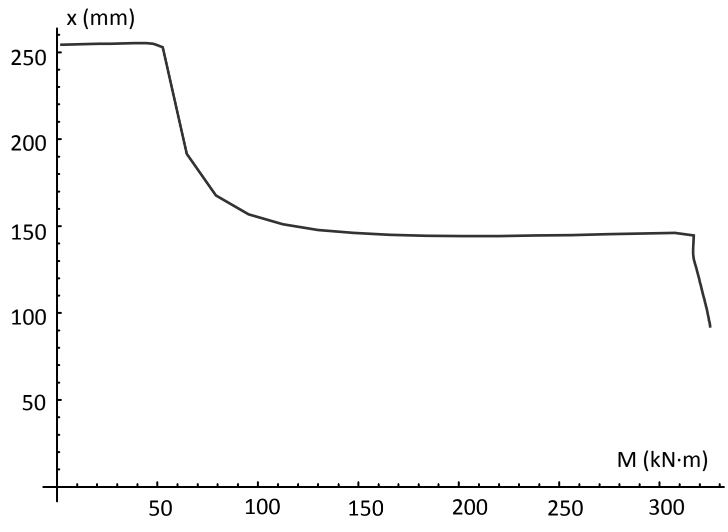

Figure 6. *Evolution of the neutral fiber depth, x.*

Fig. 6 shows that the neutral fiber depth, *x*, decreases with increasing flexural bending moment. In such circumstances, the value of the effective height of concrete in tension, $h_{c,ef}$, given in Fig. 1 (Eurocode2, §7.3.4(2)), might increase during the load history, as is shown in Fig. 7. Notice that the curve h/2=250 mm, related to one of the values of $h_{c,ef}$, is not represented.

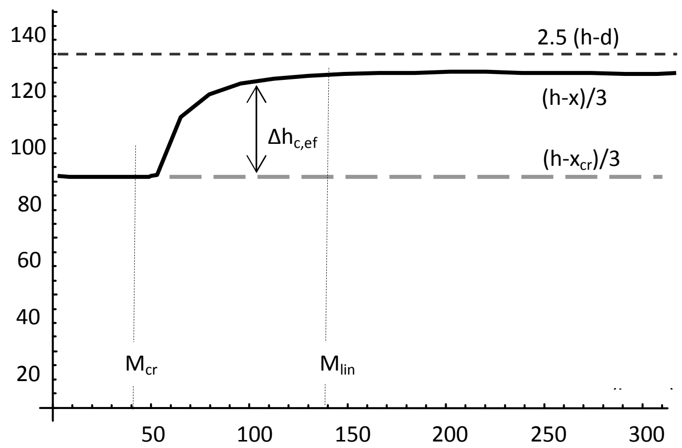

Figure 7. *Effective height of concrete in tension for the rectangular cross-section represented in Fig. 1.*

In Fig. 7, values of the cracking moment, M_{cr}, and value of flexural moment for which the behavior of the concrete is no longer lineal (i.e. the maximum compressive stress in concrete reaches $0.4f_{cm}$), M_{linear}, have also been represented.

The increase of the effective area or effective height of concrete in tension in Fig. 7 goes against the physical phenomenon of concrete degradation associated with bond decay at higher tensile strains. So an upper limit should be imposed to the condition (h-x)/3. If the value $h_{c,ef}$ is defined as less than 2.5(*h-d*) or (*h-x*)/3 with *x* corresponding to M_{cr} (Fig. 7) this incongruence can be avoided.

Fig. 8 shows two M-ϕ curves, one considering $h_{c,ef}$ as proposed by Eurocode2, in a continuous line, and the other considering $(h-x_{cr})/3$ instead of $(h-x)/3$, represented by the dashed line. This figure shows that in the linear range, in which the design for serviceability is evaluated, the increase of effective height, $\Delta h_{c,ef}$ in Fig. 7, as is defined in Eurocode2 leads to smaller deflections.

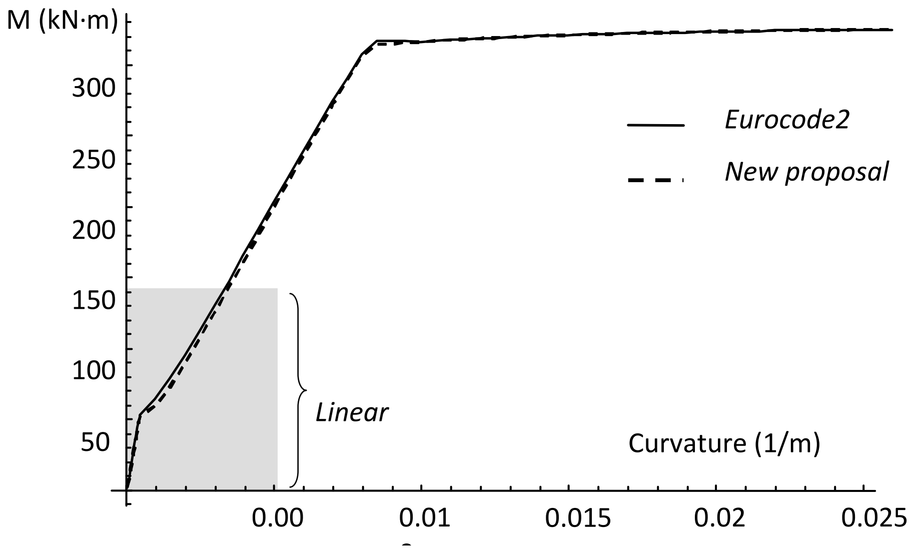

Figure 8. *Moment-curvature diagrams.*

5. CONCLUSIONS.

Usually a linear stress-strain relationship is assumed for the concrete under service conditions, provided that the concrete stress does not exceed 40% of the compressive strength, according to Eurocode2. Serviceability design is essential to users of the structures and it is based on sectional analysis, assuming an effective tension area of concrete (the product of width times the effective height). Regulations, such as Eurocode2 or Model Code 2010, propose an effective height as the lesser of three values, one of which depends on the neutral fiber depth. In order to obtain physically possible results, the value of the neutral fiber depth should have a lower bound; possibly being the one corresponding to the cracking moment.

REFERENCES

[1] Vecchio F.J. and Collins M.P. (1986). "The Modified Compression Field Theory for Reinforced Concrete Elements Subjected to Shear". Journal of the American Concrete Institute, V.83, No.2, 219-231

[2] Fib Model Code for Concrete Structures 2010. October 2013. Ernst and Sohn.

[3] Eurocode 2. Design of concrete structures—Part 1: General rules and rules for buildings prEN 1992-1-1 (July 2004). European Committee for Standardization, Brussels.

[4] ACI 435R-95. Control of Deflection in Concrete Structures. Appendix B. 2003.

[5] Hernández-Montes E., Cesetti A.., Gil-Martín L.M. (2013). "Discussion of "An efficient tension-stiffening model for nonlinear analysis of reinforced concrete members" by Renata S.B. Stramandinoli, Henriette L. La Rovere". Engineering Structures. Vol.: 48. Pp: 763-764. (2013)

[6] Wu H.Q. & Gilbert R.I. (2008)."An experimental study of tension stiffening in reinforced concrete tension members under short-term and long-term service loads". UNICIV Report No.R-449, School of Civil & Environmental Engineering, Sydney NSW.

[7] Gil Martín L.M., Hernández Montes E., Aschheim M., Pantazopoulou S. (2012). “A simpler compression field theory for structural concrete”. Annual Review of Structural Concrete (Politecnico di Milano). Vol.:31, Pp: 11-41.

[8] Gil-Martín, L.M., Hernández-Montes, E., Aschheim, M. and Pantazapoulou S. (2006). “Slenderness effects on the simulated response of longitudinal reinforcement in monotonic compression”. Structural Engineering and Mechanics. Vol: 23. Pp: 369-386.