

Optimal TMD-based Control Strategy for Human-induced Vibrations

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ABSTRACT

Civil engineering structures such as office floors and singular footbridges may present vibration modes locally spatially distributed with closely spaced natural frequencies. A set of tuned mass dampers (TMDs) may control several vibration modes at the same time and make the control system more robust. However, under these circumstances, Den Hartog's theory may not be applicable. In this paper, an optimum algorithm to design a set of TMDs is proposed to cancel effectively human-induced vibrations for floor structures. A comparative study is carried out in terms of the same performance indicator to find the optimum system, considering different configurations by Den Hartog's classical approach and this optimal algorithm. This proposed algorithm will contribute to motivate future researches on passive vibration control (PVC) for floor vibrations with closely spaced natural frequencies.

Keywords: Structural control, Passive vibration control, Tuned mass damper, Human-induced vibration, Floor vibration.

1. INTRODUCTION

The current trend towards lighter and more slender structures has resulted in structures with less inherent damping and lower natural frequencies, which are sometimes prone to excessive human-induced vibrations. Examples of significant vibrations induced by human motions have been reported in footbridges and open-plan office floors, amongst other structures. Passive, semi-active and active vibration control systems have been proposed to mitigate vibrations [1]. Passive vibration control (PVC) improves the vibration response of a structure by increasing its damping (i.e. energy dissipation) and/or stiffness (i.e. energy storage) through the application of forces generated in response to the movement of the structure [2] without requiring any external power source. That is, no control forces are applied directly to the structural system [3].

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Tuned mass dampers (TMDs) have been extensively used in PVC to reduce human-induced vibrations due to the fact that they are simple, low cost, and low maintenance mechanical devices, which can be installed without interrupting operational or human activities in the structure. However, as compared with active control device, one TMD (i.e., only one mass, spring and damper) is designed to a particular vibration mode and its effectiveness is highly sensitive to variations of the modal parameters of the vibration modes. Therefore, the use of a set of TMDs might be motivated by the following reasons: i) to control at the same time several vibration modes prone to be excited by human loading [4], and ii) to make the control system more robust to variations or uncertainties in modal parameters [5].

Structures, such as office floors or singular footbridges, may present several vibration modes locally spatially distributed with closely spaced natural frequencies. Under these circumstances, Den Hartog's theory [6] may not be applicable since a TMD designed for a particular vibration mode affects the response due to other vibration modes. That is, there exists a coupling between all the vibration modes of the structure and the used TMDs designed for the selected vibration modes [4]. To control these modes effectively, the number of TMDs must be equal or larger than the number of the vibration modes taken into consideration. In addition, the optimal location and optimal parameters of each TMD depend on the dimensions and supporting conditions of the floor structure and the number of controlled vibration modes [7].

This work builds on the framework presented in [8] for active control of human-induced vibration and considers the design of a set of TMDs for a multi-mode structure as a feedback control system ([9] and [10]). In [8], an optimization process that minimises a functional, which depends on the amplitude and duration of vibration all throughout the structure, is presented. Thus, the framework of [8] is used in this paper to design a set of TMDs to cancel effectively human-induced vibrations in an example of a floor structure. The first step of the optimization process consists of finding the optimum TMD system for a predefined set of possible locations and the second step is to select the optimum locations.

The optimum algorithm is tested by considering a rectangular plate, where supports are placed at the four corners and at midpoints of the longer spans. This is a typical example of simple floor configurations in which closely-spaced natural frequencies appear. This illustrative example shows the importance of considering the set of TMDs and significant vibration modes to improve the vibration reduction. The design problem of finding the optimal set of TMDs is modelled by state-space representation of the structure example, which is excited by heel-drop perturbations and controlled by one of the following solutions: i) only one TMD designed by Den Hartog's theory for the vibration mode most likely to vibrate, ii) only one TMD designed by the proposed algorithm, iii) two TMDs (each one for one of the two vibration modes most likely to vibrate) designed by Den Hartog's theory, and iv) two TMDs designed by the proposed algorithm. The comparison of these control designs in terms of the same performance index illustrates the contribution of this work to find the optimum TMD system. In addition, this work shows that the design of a set of TMDs by the proposed optimization algorithm process will contribute to motivate future research on PVC for floor vibrations with closely spaced natural frequencies.

2. TMD-BASED CONTROL STRATEGY

2.1. General control strategy

Fig. 1 shows the block diagram of general control strategy in which the floor structure and TMD system with p devices have been represented using state-space models.

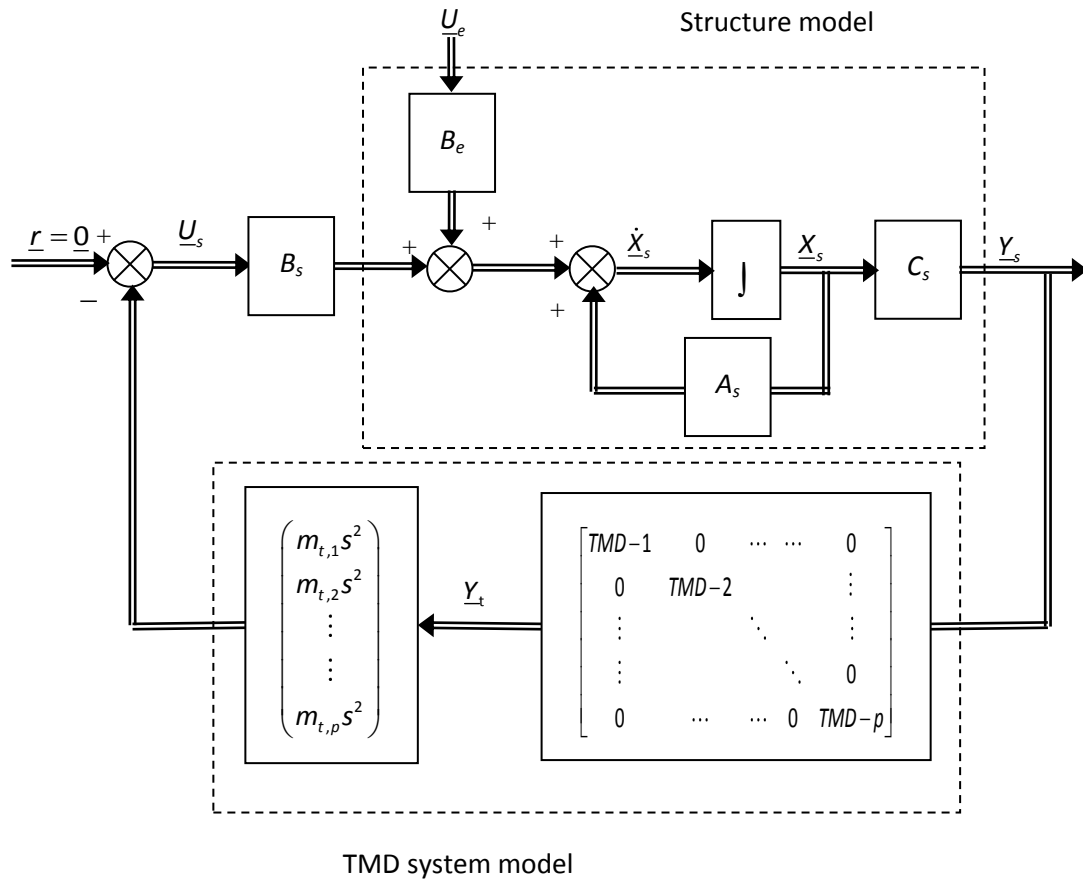


Figure 1. General control strategy.

2.2. State-space model of the floor structure

A distributed parameter system (like a floor structure) can be discretized (using the finite element method, FEM) such that mass, damping and stiffness properties are lumped at n degrees of freedom. The dynamic behaviour is represented by n -coupled second-order differential equations that can be expressed in matrix form as

$$M\ddot{\underline{u}}(t) + D\dot{\underline{u}}(t) + K\underline{u}(t) = \underline{F}(t) \tag{1}$$

where $\underline{u}(t) = [u_1, u_2, \dots, u_n]^T$ is the displacement vector, $\underline{F}(t) = [F_1, F_2, \dots, F_n]^T$ is the force vector and M, D and $K \in \mathbb{R}^{n \times n}$ are the mass, damping and stiffness matrices. Using the modal analysis and the mode superposition method (or separation of variables), the displacement vector is expressed as a linear combination of a generalised coordinates (usually known as normal or modal coordinates) $\underline{\eta}(t) = [\eta_1, \dots, \eta_m]^T$, m being the number of vibration modes considered into the analysis. That is,

$$\underline{u}(t) = \Phi \underline{\eta}(t) \quad (2)$$

where $\Phi \in \mathbb{R}^{n \times m}$ is the modal transformation matrix which contains the modal shapes in columns,

$$\Phi = [\underline{\phi}_1 \ \dots \ \underline{\phi}_m], \text{ with } \underline{\phi}_i \in \mathbb{R}^{n \times 1} \text{ and } i = 1, \dots, m. \quad (3)$$

Thus, $\underline{\phi}_i$ are the base vectors and η_i are the coordinates of the modal model. The substitution of Eq. (2) into Eq. (1) and pre-multiplying by Φ^T , yield a set of m -decoupled second-order differential equations. Its matrix representation considering mass-normalised mode shapes is as follows

$$I \ddot{\underline{\eta}}(t) + \Sigma \dot{\underline{\eta}}(t) + \Lambda \underline{\eta}(t) = \Phi^T \underline{F}(t) \quad (4)$$

where I is the identity matrix, $\Sigma = \text{diag}(2\zeta_1\omega_1, \dots, 2\zeta_m\omega_m)$, $\Lambda = \text{diag}(\omega_1^2, \dots, \omega_m^2)$ and $\Phi^T \underline{F}(t)$ are respectively the modal participation factors, ζ_i and ω_i are the damping ratio and natural frequency associated to the i -vibration mode.

Consider a state-space representation of Eq. (4) in which subscripts s and e indicate structure and excitation, respectively,

$$\dot{\underline{X}}_s(t) = A_s \underline{X}_s(t) + B_s \underline{U}_s(t) + B_e \underline{U}_e(t); \underline{Y}_s(t) = C_s \underline{X}_s(t) \quad (5)$$

in which the state vector is $\underline{X}_s = [\eta_1, \dots, \eta_m, \dot{\eta}_1, \dots, \dot{\eta}_m]^T$, the system matrix

$$A_s = \begin{bmatrix} \mathbf{0}_{m \times m} & I_{m \times m} \\ -\Lambda & -\Sigma \end{bmatrix} \quad (6)$$

the control input (i.e. force due to the acceleration of the TMD mass) $\underline{U}_s = [F_{F1}, \dots, F_{Fp}]^T$ for p TMDs, the input matrix

$$B_s = \begin{bmatrix} \mathbf{0}_{m \times p} \\ \Phi_a \end{bmatrix}, \text{ with } \Phi_a = \begin{bmatrix} \phi_{1,F1} & \dots & \phi_{1,Fp} \\ \vdots & \ddots & \vdots \\ \phi_{m,F1} & \dots & \phi_{m,Fp} \end{bmatrix} \quad (7)$$

where $\Phi_a \in \mathbb{R}^{m \times p}$ and $\phi_{i,Fj}$ is the value of the modal shape i at the position of the TMD j . The output vector is $\underline{Y}_s = [y_{s,1} \ \dots \ y_{s,p}]$, in which is assumed that the displacement is the quantity measured at p points where the set of TMDs is placed. The output matrix is as follows

$$C_s = [\Phi_s \ \mathbf{0}_{p \times m}], \text{ with } \Phi_s = \begin{bmatrix} \phi_{1,S1} & \dots & \phi_{m,S1} \\ \vdots & \ddots & \vdots \\ \phi_{1,Sp} & \dots & \phi_{m,Sp} \end{bmatrix} \quad (8)$$

where $\Phi_s \in \mathbb{R}^{m \times p}$ and $\phi_{i,Sj}$ is the value of the modal shape i at the position of the TMD j . Finally, B_e is perturbation input matrix and $\underline{U}_e(t)$ is the perturbation input.

From the state-space representation of Eq. (5), the characteristic equation is given by [11]

$$|\lambda_s I - A_s| = 0 \tag{9}$$

λ_s being the open-loop poles of the structure (or eigenvalues of A_s).

2.3. TMD dynamic behaviours

TMD is a PVC device mounted in structures to reduce the amplitude of mechanical vibrations by dissipating energy and changing the value of the natural frequencies of the structures. A TMD or a set of TMDs (see Fig. 2) consist of a mass, a damper and a spring attached to a structure. The TMD design consists of tuning these mechanical parameters and locates the position of the TMD to reduce mechanical vibrations. In Den Hartog’s classical approach, the optimal TMD parameters are those that minimise the maximum value of the resonant peak at a point for a structure with vanishing damping. Generally, this assumption can be adopted and the TMD is installed where occurs the maximum displacement of the controlled vibration mode.

Fig. 2 shows the model to simulate a floor structure equipped with a TMD system (p TMDs). In Fig. 2, subscripts s and t indicate structure and TMD respectively, $m_{t,j}$, $k_{t,j}$, $c_{t,j}$, $y_{t,j}$ are the mass, stiffness, damping and vertical displacement of the j -TMD respectively, while $y_{s,j}$ being the local vertical displacement (the displacement at this particular position) of the floor where the j -TMD is placed. The dynamics of each TMD is as follows:

$$m_{t,j}\ddot{y}_{t,j} + c_{t,j}\dot{y}_{t,j} + k_{t,j}y_{t,j} - c_{t,j}\dot{y}_{s,j} - k_{t,j}y_{s,j} = 0 \tag{10}$$

The vector $\underline{U}_s(t)$ is obtained by multiplying the vector \underline{Y}_t by $[m_{t,1}, \dots, m_{t,p}]$ and derivating two times.

In Den Hartog’s theory, the optimal $k_{t,j}$ (Eq. (13)), $c_{t,j}$ (Eq. (14)) can be obtained easily from Eqs. (10) and (11), once the $m_{t,j}$ is selected using mass ratio μ_j (Eq. (12)), if the j -TMD and the structure are separated from the others as a two-degree-of-freedom system.

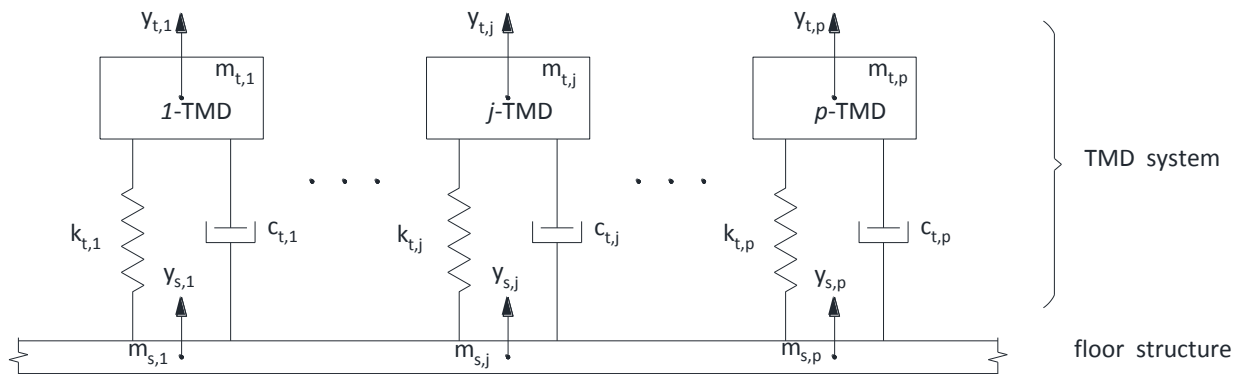


Figure 2. Floor structure and a TMD system.

$$m_{s,j}\ddot{y}_{s,j} + (c_{t,j} + c_{s,j})\dot{y}_{s,j} + (k_{t,j} + k_{s,j})y_{s,j} - c_{t,j}\dot{y}_{t,j} - k_{t,j}y_{t,j} = U_{e,j}(t) \quad (11)$$

$$\mu_j = \frac{m_{t,j}}{m_{s,j}} \quad (12)$$

$$k_{t,j} = m_{t,j}\omega_{t,j}^2 = m_{t,j} \cdot (2\pi f_{t,j})^2, \text{ with } f_{t,j} = \frac{1}{1 + \mu_j} f_{s,j} \quad (13)$$

$$\zeta_{t,j} = 2 \cdot m_{t,j} \omega_{t,j} \zeta_{t,j}, \text{ with } \zeta_{t,j} = \sqrt{\frac{3\mu_j}{8(1 + \mu_j)^3}} \quad (14)$$

where $\omega_{t,j}, f_{t,j}, \zeta_{t,j}$ are respectively the angular frequency, natural frequency, damping ratio and of the j -TMD, $U_{e,j}(t)$, $m_{s,j}$ and $f_{s,j}$ are respectively the perturbation at this position of the structure, the modal mass and natural frequency of the structure for this vibration mode.

However, this approach may not be applicable for floor vibrations with closely-spaced natural frequencies. Because there exists a coupling between all the vibration modes of the structure and the used TMD designed for the selected vibration mode [4]. To control these modes effectively, the number of TMDs must be equal or larger than the number of the vibration modes taken into consideration.

2.4. Human vibration perception

2.4.1 Frequency weighting

The vibration that can be perceived by a human depends on the direction of incidence to the human body and the frequency content of the vibration (for a given amplitude), among other factors. As such, the variation of sensitivity of frequency for a body position can be taken into account by attenuating or enhancing the system response for frequencies where perception is less or higher sensitive, respectively. The degree to which the response is attenuated or enhanced is referred to as frequency weighting. Thus, frequency weighting functions are applied in order to account for the different acceptability of vibrations for different directions and body positions. ISO 2631 [12] and BS 6841 [13] provide details for frequency and direction weighting functions that can be applied which are all based on the barycentric coordinate system shown in Fig. 3. These have been included in current floor design guidelines such as the SCI guidance [14]. According to ISO 2631, for z-axis vibration and standing and seating, the frequency weighting function is W_k . This curve and its asymptotic definition are graphed in Fig. 4. Thus, the frequency weighted state vector is obtained as follows in frequency domain

$$\underline{X}_{s,w}(s) = \underline{X}_s(s) \cdot W_k(s) \quad (15)$$

in which $W_k(s)$ is the transfer function (or Fourier transform) of the frequency weighting function. Eq. (15) can also be expressed in time domain as

$$\underline{X}_{s,w}(t) = \underline{X}_s(t) * w_k(t) \quad (16)$$

where (*) denotes the convolution process and $w_k(t)$ is the impulse response function of $W_k(s)$.

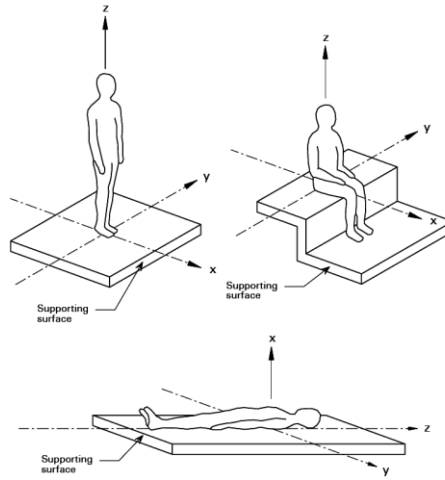


Figure 3. Directions for vibration according to ISO 2631 [12] and BS 6841 [13] (after [14]).

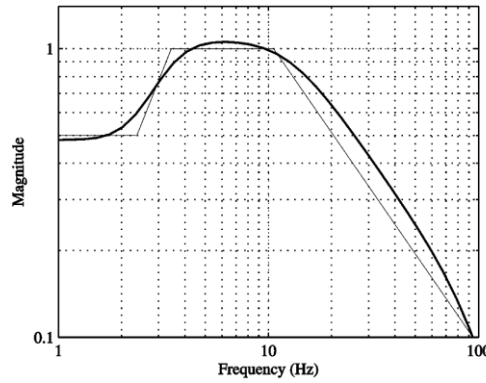


Figure 4. W_k frequency weighting function (thicker curve) and its asymptotic definition (thinner curve) [12].

2.4.2 Time weighting

As has been mentioned before, the human comfort to vibration is directly related to the duration of sustained vibration. Thus, persistent vibrations should be penalised in the control design, giving more importance to transient vibration of long-duration than those of short-duration. As it was commented before, the control design proposed in this work consists of minimizing a PI that depends on the energy of the system after an initial condition. Therefore, exponential time weighting will be suitable for this application. The time weighted state \hat{X}_s is computed from the state vector \underline{X}_s as follows

$$\hat{X}_s(t) = e^{\alpha t} \cdot \underline{X}_s(t), \text{ with } \alpha \geq 0 \quad (17)$$

Note that the exponential time weighting adds a constraint in the relative stability of the controlled system. Note also that persistent states are more penalised as α is increased. Finally, if the state vector is time weighted by Eq. (17) and frequency weighted by Eq. (16), the time and frequency weighted state is as follows

$$\hat{X}_{s,w}(t) = \hat{X}_s(t) * w_k(t) \quad (18)$$

2.5. Design algorithm

The purpose of this section is to provide a procedure to find the optimal locations and stiffness (or natural frequencies) of a given number of TMDs, as is illustrated in Fig. 1.

2.5.1. TMD design as a feedback loop

It is now demonstrated that each TMD can be considered as a control system working as a feedback controller. For a given mass and damping, the stiffness is the only parameter to be designed for a prescribed location. Thus, the proposed algorithm finds simultaneously the optimal location for each TMD and the optimal natural frequency (equivalent to design the stiffness since the mass is fixed) through a feedback loop. From Fig. 1, the TMD system model depends mainly on the diagonal matrix which contains p TMD transfer functions, which can be explained clearly by the transfer function of the j -TMD.

$$TMD_{-j}(s) = \frac{y_{t,j}(s)}{y_{s,j}(s)} = \frac{c_{t,j}s + k_{t,j}}{m_{t,j}s^2 + c_{t,j}s + k_{t,j}} = \frac{2\zeta_{t,j}\omega_{t,j}s + \omega_{t,j}^2}{s^2 + 2\zeta_{t,j}\omega_{t,j}s + \omega_{t,j}^2} \quad (19)$$

2.5.2. Design process

The design process is based on the minimisation of a performance index (PI) related to the dissipation energy of the whole structure due to the PVC action for a given excitation. The PI is calculated using the time and frequency weighted structure states as follows (see Eq. (18) and Fig. 4)

$$J(K, \underline{z}) = \frac{1}{2} \int_0^{t_f} \hat{\underline{X}}_{s,w}^T(t) Q \hat{\underline{X}}_{s,w}(t) \quad (20)$$

in which the weighting matrix $Q \in \mathbb{R}^{2m \times 2m}$ is a positive definite matrix, $\underline{z} \in \mathbb{R}^p$ is the position of the TMDs (obviously, \underline{z} must be included into the spatial domain of the structure) and t_f is the final time used to compute the PI. This time should be sufficiently long such that the system energy is totally dissipated due to the control action. The weighting matrix is taken as [15]

$$Q = \begin{bmatrix} \omega_1 \phi_{1,\max}^2 & 0 & \dots & 0 \\ 0 & \ddots & & \\ \vdots & & \omega_m \phi_{m,\max}^2 & \vdots \\ \vdots & & \ddots & \phi_{1,\max}^2 \\ 0 & \dots & & \ddots & 0 \\ & & & & \phi_{m,\max}^2 \end{bmatrix} \quad (21)$$

in which $\phi_{i,\max}$ is the maximum value of the i -th eigenvector $\underline{\phi}_i$. Note that the displacement states are weighted by the natural frequencies, making thus the displacement states comparable to the velocity states.

The design process proposed for the TMDs location and the diagonal matrix can be divided into the following steps:

Step 1: Obtain a model of the structure considering m vibration modes defined at n points spatially distributed along the structure.

Step 2: Obtain the eigenvalues of open-loop structure (Eq. (9)), choose parameter α as

$$\alpha \leq \min_i \left| \operatorname{Re}(s_i) \right|, \quad \forall i = 1, \dots, 2m \quad (22)$$

choose the final time used to compute the PI as

$$t_f \geq 10/\alpha \quad (23)$$

decide the frequency weighting function to be used [12], the number of TMDs, p , and the excitation. As a perturbation, a model of a heel-drop at all the points of the floor structure can be considered.

A heel-drop impact is a convenient excitation to evaluate transient response because it is easy to implement analytically and experimentally. Physically, the standard heel-drop impact is the force created by an 86.1 kg person standing on their toes and dropping their heels to impact the floor. Analytically, this force is estimated by a 2670 N decreasing function with a 50 ms duration [16].

Step 3: For each possible combination of positions of the p TMDs, find the optimal diagonal matrix by minimising the PI (Eq.(20)), but updated for exponentially weighted states. Mathematically, the problem may be established as

$$J(\underline{Z}) = \min_K J(K, \underline{Z}) \quad (24)$$

where the vector K is formed by the frequencies of the TMDs in the form of $[f_{t,1}, \dots, f_{t,p}]$.

Step 4: When the optimal matrix gain for each possible combination of TMDs position is obtained, the combination of positions \underline{Z}^* , together with its corresponding matrix, that provides the minimum PI is the solution searched

$$J^* = \min_{\underline{Z}} J(\underline{Z}) \quad (25)$$

3. DESIGN EXAMPLE

3.1. Description of a floor structure model

The study is undertaken for a rectangular isotropic plate of dimension 12×8 m and a depth of 0.25 m, whose general view is shown in Fig. 5. The plate is supported by 6 simple supports, 4 of them at the corners and 2 at the mid-points of the longer spans. The material properties considered are: modulus of elasticity $E = 20 \times 10^9$ N/m², Poisson's ratio $\nu = 0.15$ and density $\rho = 3000$ kg/m³. The density has been increased from 2500 kg/m³ (the characteristic value for reinforced or pre-stressed concrete) to 3000 kg/m³ in order to include a portion of the imposed load and the total dead load [14]. Fig. 6 shows the 1st and 2nd vibration mode shapes of the structure and the obtained natural frequencies. The damping ratios for all the modes have been taken as 0.01. This value is representative of partially fully fitted out floors. Note that the structure model can be obtained experimentally, through

experimental modal analysis, or numerically, through the FEM. Current experimental modal identification procedures use a state-space realisation, such as [17].

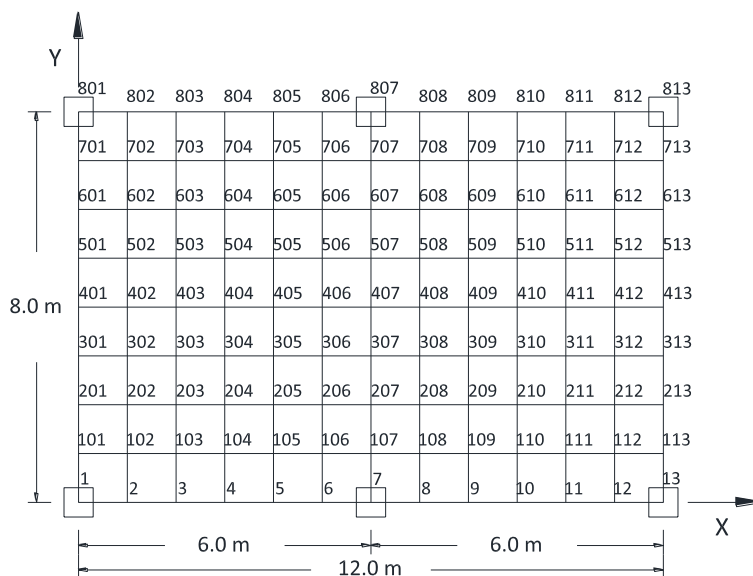


Figure 5. Plane layout and the mesh grid of the plate simulated.

For the particular example considered here, the structure model is obtained through FEM. Fig. 5 shows the mesh grid used. Isotropic shell elements are equidistant, and 1 meter has been used. A coarse grid has been used in order to make in order to make this grid similar to a possible experimental grid. The research is mainly focused on the first two vibration modes, whose natural frequencies are 4.4820Hz and 5.5379Hz respectively. Not only the natural frequencies are closely spaced, but also the positions of maximum displacement are only at a distance of 3 meters, the coordinates of which being (9m,4m) and (12m,4m).

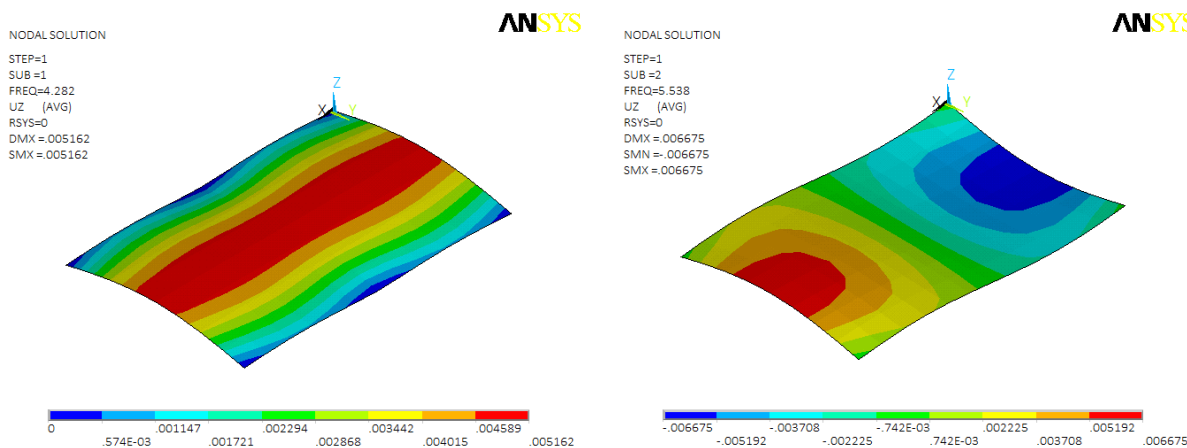


Figure 6. First two vibration mode shapes of the floor.

3.2. Comparison among different TMD systems

In this part, different control systems are compared in terms of the aforementioned functional J . The floor model is excited by heel-drop perturbations at all the nodes of the floor and the following solutions are considered: i) only one TMD designed by Den Hartog's theory for the vibration mode at 4.2820 Hz, ii) only one TMD is designed by the proposed algorithm, iii) two TMDs (one for mode at 4.2820 Hz and the other for mode at 5.5379 Hz) designed by Den Hartog's theory, and iv) two TMDs designed by the proposed algorithm.

For all the cases, the mass of each TMD is fixed as 700kg, which makes the mass ratio of TMD to 1st and 2nd vibration mode 0.0187 and 0.0312, respectively, which are quite common for real-life TMDs. All the damping ratios of TMDs are fixed as a common value 0.10. When the damping and mass value are set, the natural frequency, which is quadratic proportional to the stiffness, is computed as a close-loop system and the optimal positions are obtained meanwhile.

In the classical approach, however, the TMD is designed only to control one particular (1st or 2nd) vibration mode, thus being placed at the maximum displacement of corresponding mode. Once the mass (or mass ratio) is selected, other key parameters like stiffness and damping can be calculated using Eq. (13) and Eq. (14).

Table 1 shows the comparison between Den Hartog's classical approach and the proposed algorithm when only one TMD is designed for the system, where the input is chosen as heel-drop impacts [15]. Also, the PIs of the structure without control system are listed in *Table 1*. Firstly only the first vibration of the structure is considered and then four modes are taken into account. As is shown, in both approaches the control system varies in the TMD frequency and position with the number of controlled modes. A smaller value of PI indicates a better expected effect of the control system, which can explain why the TMD system becomes less efficient when the number of controlled modes increases. In classical approach, no matter how many modes are expected to be controlled, the TMD stays the same, which is designed to control particularly the 1st mode.

Table 2 shows the cases when two TMDs are designed for the TMD systems. As can be seen from *Table 1 and Table 2*, the proposed algorithm always provides smaller PIs than Den Hartog's classical approach and the PIs improve when more modes are taken into consideration. In addition, when considering four vibration modes, increasing the number of TMDs improves the performance of TMD system, both in Den Hartog's classical approach and proposed optimal approach.

Table 1. Control systems consisting of 1 TMD

Structural modes considered	1 mode	4 modes
Optimal approach		
PI	$3.12 \cdot 10^{-3}$	$3.30 \cdot 10^{-3}$
Position	(3,4)	(6,4)
TMD frequency	4.2939 Hz	4.3094 Hz
Classical approach		
PI	$3.20 \cdot 10^{-3}$	$4.03 \cdot 10^{-3}$
Position	(3,4)	(3,4)
TMD frequency	4.2036 Hz	4.2036 Hz
Structure without control		
PI	$182.55 \cdot 10^{-3}$	$182.67 \cdot 10^{-3}$

Table 2. Control systems consisting of 2 TMDs

Structural modes considered	2 modes	4 modes
Optimal approach		
PI	$2.09 \cdot 10^{-3}$	$2.18 \cdot 10^{-3}$
Positions	(9,4), (4,4)	(7,4), (4,4)
TMD frequencies	4.0613 Hz, 4.5305 Hz	4.0504 Hz, 4.5994 Hz
Classical approach		
PI	$2.79 \cdot 10^{-3}$	$2.92 \cdot 10^{-3}$
Positions	(3,4), (12,4)	(3,4), (12,4)
TMD frequencies	4.2036 Hz, 5.3705 Hz	4.2036 Hz, 5.3705 Hz

4. CONCLUSIONS

This paper has proposed an optimal TMD-based strategy for human-induced vibrations, in which an optimal set of TMDs can be designed by minimising the PI (representative of the dissipation energy). The design process considers also the time and frequency weighting for the states. Different TMD

systems are designed using this optimal approach and compared with those by Den Hartog's classical theory, which demonstrates the advantage of this approach over the classical theory.

This proposed optimization algorithm process will contribute to motivate future research on PVC for floor vibrations with closely spaced natural frequencies. The authors are now working on FEM to assess the TMD system designed by this approach, and also plan to apply this methodology to real-life structures, particularly in-service open-plan office floors.

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