Solving SAT with Active Membranes and Pre-Computed Initial Configurations

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Summary. In this paper we provide algorithms for solving the SAT problem using P systems with active membranes with neither polarization nor division rules. The semi-uniform solutions are given under the assumption that initial configurations (either alphabet or structure) of exponential size are pre-computed by well-defined P systems (P systems with replicated rewriting and P systems with active membranes and membrane creation, respectively) working in polynomial time. An important observation is that we specify how the pre-computed initial configurations are constructed.

1 Introduction

Membrane computing is inspired by the architecture and the behaviour of living cells. Various classes of membrane systems (also called P systems) have been defined in [9], while several applications of these systems are described in [3]. Membrane systems are characterised by three features: (i) a membrane structure consisting of a hierarchy of membranes (which are either disjoint or nested), with an unique top membrane called the *skin*; (ii) multisets of objects associated with membranes; (iii) rules for processing the objects and membranes. When membrane systems are seen as computing devices, two main research directions are usually considered: computational power in terms of the classical notion of Turing computability (e.g., [1]), and efficiency in algorithmically solving NP-complete problems in polynomial time (e.g., [2]). Thus, membrane systems define classes of computing devices which are both powerful and efficient.

Under the assumption that $\mathbf{P} \neq \mathbf{NP}$, efficient solutions to **NP**-complete problems cannot be obtained without introducing features which enhance the efficiency of the system ways to exponentially grow the workspace during the computation, nondeterminism, and so on). For instance, some pre-computed resources are used in [4, 6].

In this paper we consider P systems with active membranes [7], and show that they can provide simple semi-uniform solutions to the SAT problem without using neither polarization nor division, but using exponential size pre-computed initial configurations (either alphabet or structure). An important observation is that we specify how our pre-computed initial configurations are constructed in a polynomial number of steps by additional well-defined P systems (P systems with replicated rewriting and P systems with active membranes and membrane creation, respectively).

2 Preliminaries

We consider polarizationless P systems with active membranes [7]. The original definition also includes division rules, rules that are not needed here.

Definition 1. *A* polarizationless P system with active membranes *is a tuple*

$$\Pi = (\Gamma, \Lambda, \mu, w_1, \dots, w_d, R), \text{ where:}$$

- $d \geq 1$ *is the* initial degree*;*
- Γ *is a finite non-empty alphabet of* objects*;*
- Λ *is a finite* set of labels *for membranes;*
- μ *is a* membrane structure *(i.e., a rooted unordered tree, usually represented by nested brackets) in which each membrane is labelled by an element of* Λ *in a one-to-one way;*
- $w_1, \dots, w_d$ *are strings over* Γ*, describing the* initial multisets *of objects placed in a number of* d *membranes of* μ*;*
- R *is a finite* set of rules *over* Γ*:*
  1. $[a \to w]_h$ — object evolution rules
     An object a *is rewritten into the multiset* w*, if* a *is placed inside a membrane labelled by* h*.*
  2. $a[\ ]_h \to [b]_h$ — send-in communication rules
     An object a *is sent into a membrane labelled by* h*, becoming* b*.*
  3. $[a]_h \to b[\ ]_h$ — send-out communication rules
     An object a*, placed into a membrane labelled by* h*, is sent out of membrane* h *and becomes* b*.*
  4. $[a]_h \to b$ — dissolution rules
     A membrane h *containing an object* a *is disslved, while object* a *is rewritten to* b*.*

Each configuration $\mathcal{C}_i$ of a P system with active membranes and input objects is described by the membrane structure, together with the multisets of objects located in the corresponding membranes. The initial configuration of such a system is denoted by $\mathcal{C}_0$. An evolution step $\mathcal{C}_i \Rightarrow \mathcal{C}_{i+1}$ from a configuration $\mathcal{C}_i$ to a new configuration $\mathcal{C}_{i+1}$ is done according to the following principles:

- Each object is involved in at most one rule per step, while each membrane could be involved in several rules.
- The application of rules is maximally parallel: all rules that can be applied are applied.

- When several conflicting rules could be applied at the same time, a nondeterministic choice is performed; this implies that multiple configurations can be reached as the result of an evolution step.
- In each evolution step, all evolution rules are applied inside the most inner membranes, followed by all communication rules involving the membranes themselves. This process is then repeated to the membranes containing them, and so on towards the skin membrane.
- Objects sent out from the skin membrane represent the computation result.

A *halting evolution* of such a system Π is a finite sequence of configurations $\overrightarrow{\mathcal{C}} = (\mathcal{C}_0, \ldots, \mathcal{C}_k)$, such that $\mathcal{C}_0 \Rightarrow \mathcal{C}_1 \Rightarrow \ldots \Rightarrow \mathcal{C}_k$, and no rules can be applied any more in $\mathcal{C}_k$. A *non-halting* evolution $\overrightarrow{\mathcal{C}} = (\mathcal{C}_i \mid i \in \mathbb{N})$ consists of infinite evolution $\mathcal{C}_0 \Rightarrow \mathcal{C}_1 \Rightarrow \ldots$, where the applicable rules are never exhausted.

3 Solving the SAT Problem with Active Membranes

At the beginning of 2005, Gh. Păun wrote:

"My favourite question (related to complexity aspects in P systems with active membranes and with electrical charges) is that about the number of polarizations. Can the polarizations be completely avoided? The feeling is that this is not possible - and such a result would be rather sound: passing from no polarization to two polarizations amounts to passing from non-efficiency to efficiency."

This conjecture (problem F in [8]) can be formally described in terms of membrane computing complexity classes as follows:

$$P = PMC_{\mathcal{AM}^0(+d,-n,+e,+c)}$$

where

- $PMC_{\mathcal{R}}$ indicates that the result holds for P systems with input membrane;
- $+d$ indicates that dissolution rules are permitted;
- $-n$ indicates that only division rules for elementary membranes are allowed;
- $+e$ indicates that evolution rules are permitted;
- $+c$ indicates that communication rules are permitted.

The SAT problem checks the satisfiability of a propositional logic formula in conjunctive normal form (CNF). Let $\{x_1, x_2, \ldots, x_n\}$ be a set of propositional variables. A formula in CNF is of the form $\varphi = C_1 \wedge C_2 \wedge \cdots \wedge C_m$ where each C_i, $1 \leq i \leq m$ is a disjunction of the form $C_i = y_1 \vee y_2 \vee \cdots \vee y_r$ $(r \leq n)$, where each y_j is either a variable x_k or its negation $\neg x_k$.

We present some attempts to solve this conjecture by providing algorithms solving the SAT problem using P systems with active membranes with neither polarizations nor division, but using exponential pre-computed initial configurations constructed by additional P systems in polynomial time.

3.1 Solving SAT Problem by Using a Pre-Computed Alphabet

In this section, we propose a polynomial time solution to the SAT problem using the polarizationless P systems with active membranes, without division, but with a pre-computed alphabet. For any instance of SAT we construct effectively a system of membranes that solves it. Formally, we prove the following result:

Theorem 1. *The SAT problem can be solved by a* **polarizationless** *P system with active membranes and* **without division***, but with an exponential alphabet pre-computed in linear time with respect to the number of variables and the number of clauses, i.e.,*

$$P = PMC_{\mathcal{AM}^0(+d,+e,+c,pre(\alpha))} \ .$$

where $pre(\alpha)$ *indicates that a pre-computed alphabet is permitted.*

Proof. Let us consider a propositional formula

$$\varphi = C_1 \wedge C_2 \wedge \cdots \wedge C_m$$

where each $C_i, 1 \leq i \leq m$ is a disjunction of the form

$$C_i = y_1 \vee y_2 \vee \cdots \vee y_r \ (r \leq n),$$

where each y_j is either a variable x_k or its negation $\neg x_k$.

We construct a P system with active membranes able to check the satisfiability of φ. The P system is given by $\Pi = (\Gamma, \Lambda, \mu, w_1, \ldots, w_d, R)$, where:

- $V = \{z_i \mid 0 \leq i \leq max\{m,n\}\} \cup$
  $\cup \{s_i \mid i = t_1 \ldots t_n, t_j \in \{0,1\}$ and $1 \leq j \leq n\} \cup \{yes, no\}$.
  The alphabet $\{s_i \mid i = t_1 \ldots t_n, t_j \in \{0,1\}$ and $1 \leq j \leq n\}$ to be placed inside the input membrane 0 can be generated, starting from an object s, using the rules:
  - $s \to s_0 s_1$;
  - $s_i \to s_{i0} s_{i1}$, for $i = t_1 \ldots t_k$ where $t_j \in \{0,1\}$ and $1 \leq j \leq k < n$.

  Thus all the possible assignments for the n variable $\{x_1, x_2, \ldots, x_n\}$ are created. The rules are applied until the length k of i in the second rule equals n. For example, s_{100} over $\{x_1, x_2, x_3\}$ represents the assignment $x_1 = 1$, $x_2 = 0$ and $x_3 = 0$ (1 stands for $true$, while 0 stands for $false$). The input alphabet can be computed in linear (polynomial) time by using an additional device, for instance P systems with replicated rewriting [5].
- $\Lambda = \{0, c_1, \ldots, c_m, h\}$, with $c_i = z_1 \ldots z_n$, $1 \leq i \leq m$ where
  - $z_j = 1$ if x_j appears in C_i;
  - $z_j = 0$ if $\neg x_j$ appears in C_i;
  - $z_j = \star$ if neither x_j nor $\neg x_j$ appear in C_i.

  For example $c_1 = 1 \star 0$ over the set of variables $\{x_1, x_2, x_3\}$ represents the disjunction $c_1 = x_1 \vee \neg x_3$.
- $\mu = [[[\cdots[[[\ ]_0]_{c_1}]_{c_2}\cdots]_{c_{m-1}}]_{c_m}]_h$.
- $w_0 = z_0$.
- $w_i = \lambda$, for all $i \in \Lambda \backslash \{0\}$.
- The set R contains the following rules:

1. $[z_0]_0 \rightarrow z_0$
   After the input is placed inside membrane 0, membrane 0 is dissolved, and its content is released in the upper membrane labelled with c_1.
2. $[s_i]_{c_j} \rightarrow s_i[\ ]_{c_j}$
   if i and j have at least one position with the same value (either 0 or 1);
   $[s_i]_{c_m} \rightarrow yes$
   if i and m have at least one position with the same value (either 0 or 1).

   An assignment s_i is sent out of a membrane c_m if there is at least one position in i and j that is equal, namely an assignment to a variable x_k such that it makes C_j true. Once an object *yes* is generated, another object *yes* cannot be created because membrane c_m was dissolved and the rule $[s_i]_{c_m} \rightarrow yes$ cannot be applied. For example, if $c_1 = 1 \star 0$ and s_{101} (as described above), then this means that s_{101} satisfies the clause coded by $c_1 = 1 \star 0$ since both have 1 on their first position, and this is enough to make true a disjunction.
3. $[z_0 \rightarrow z_1]_{c_1}$
   $[z_i]_{c_i} \rightarrow [\ ]_{c_i} z_{i+1}$, for $1 \leq i \leq m-1$
   $[z_m]_{c_m} \rightarrow no$
   The object z_0 waits a step after membrane 0 is dissolved in order to allow the other objects s_i to go through the c_j membranes. The object z_i then is communicated through the c_j membranes. Once z_m reached the membrane c_m, if membrane c_m still exists (i.e., the rule $[s_i]_{c_m} \rightarrow yes$ was not applied), then the answer *no* is generated. Once an object *yes* or *no* is generated, other objects *yes* or *no* cannot be created because membrane c_m was dissolved, and neither rule $[s_i]_{c_m} \rightarrow yes$ nor $[z_m]_{c_m} \rightarrow no$ can be applied.
4. $[yes]_h \rightarrow yes[\ ]_h$
   $[no]_h \rightarrow no[\ ]_h$
   The answer *yes* or *no* regarding the satisfiability is sent out of the skin.

3.2 Solving SAT Problem Using a Pre-Computed Initial Structure

In this section, we propose a polynomial time solution to the SAT problem using the polarizationless P systems with active membranes and without division, but with a pre-computed structure. For any instance of SAT we construct effectively a system of membranes that solves it. We also enforce another principle needed to perform an evolution step: each membrane can be subject to at most one communication rule per step. This principle is needed when generating all possible assignments to be verified. Formally, we prove the following result:

Theorem 2. *The SAT problem can be solved by a* **polarizationless** *P system with active membranes and* **without division***, but with an initial exponential structure pre-computed in linear time with respect to the number of variables and the number of clauses, i.e.,*

$$P = PMC_{\mathcal{AM}^0(+d,+e,+c,pre(\mu))} \ .$$

where $pre(\mu)$ indicates that a pre-computed structure is permitted.

Proof. Let us consider a propositional formula

$$\varphi = C_1 \wedge C_2 \wedge \cdots \wedge C_m$$

where each $C_i, 1 \leq i \leq m$ is a disjunction of the form

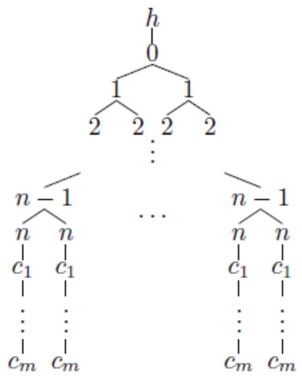
$$C_i = y_1 \vee y_2 \vee \cdots \vee y_r \ (r \leq n),$$

where each y_j is either a variable x_k or its negation $\neg x_k$.

We construct a P system with active membranes able to check the satisfiability of φ. The P system is given by $\Pi = (\Gamma, \Lambda, \mu, w_1, \ldots, w_d, R)$, where:

- $V = \{a_i, t_i, t'_i, f_i, f'_i \mid 1 \leq i \leq n\} \cup \{z_i \mid 0 \leq i \leq 4 \times n + 2 \times m\} \cup \{yes, no\}$.
- $\Lambda = \{0, \ldots, n, c_1, \ldots, c_m, h\}$, $1 \leq i \leq m$.
- $\mu = [[[[\ldots]_2[\ldots]_2]_1[[\ldots]_2[\ldots]_2]_1]_0]_h$, where
  - each membrane i contains two membranes $i+1$ for $0 \leq i \leq n-1$;
  - each membrane n contains a membrane structure $[[\ldots[\ ]_{c_m}\ldots]_{c_1}]_{c_0}$;
  - membrane 0 is the input membrane.

  Graphically, the membrane structure μ can be represented as a tree:



  This membrane structure can be generate in linear (polynomial) time with respect to the number of variables and the number of clauses. This is done by using an additional device that starts from a membrane structure $[[\ ]_0]_h$, with object 0 placed inside membrane 0 and rules of the form:
  - $[i \rightarrow (i+1)'\ (i+1)']_i$, for $0 \leq i \leq n-1$
  - $i' \rightarrow [i]_i$, for $1 \leq i \leq n$
  - $n \rightarrow [c_2]_{c_1}$
  - $c_k \rightarrow [c_{k+1}]_{c_k}$, for $2 \leq k \leq m-1$
  - $c_m \rightarrow [\ ]_{c_m}$.
- $w_0 = a_1 z_0$.
- $w_i = \lambda$, for all $i \in \Lambda \backslash \{0\}$.
- The set R contains the following rules:
  1. $[z_i \rightarrow z_{i+1}]_0$, for all $0 \leq i < 4 \times n + 2 \times m$
     These rules count the time needed for producing the truth assignments for the n variables inside the membranes labelled by n ($3 \times n$ steps), then to

dissolve the membranes labelled by c_j, $1 \leq j \leq m$ ($2 \times m$ steps), and for an y object to reach the membrane labelled by 0 (n steps).

2. $[a_i \to t_i f_i]_{i-1}$, for $1 \leq i \leq n$
$t_i[\ ]_i \to [t_i]_i$, for $1 \leq i \leq n$
$f_i[\ ]_i \to [f_i]_i$, for $1 \leq i \leq n$
$[t_i \to t'_i t'_i a_{i+1}]_i$, for $1 \leq i \leq n-1$
$t'_i[\ ]_k \to [t_i]_k$, for $i+1 \leq k \leq n$
$[t_i \to t'_i t'_i]_k$, for $i+1 \leq k \leq n-1$
$[f_i \to f'_i f'_i a_{i+1}]_i$, for $1 \leq i \leq n-1$
$f'_i[\ ]_k \to [f_i]_k$, for $i+1 \leq k \leq n$
$[f_i \to f'_i f'_i]_k$, for $i+1 \leq k \leq n-1$
In membranes n we create all possible assignments for the n variable $\{x_1, x_2, \ldots, x_n\}$. It starts from an object a_1 placed initially in membrane labelled by 0. Each a_i is used to create t_i and f_i that are then send in one of the two membranes labelled by i placed in membrane $i-1$. In fact each membrane i receives either t_i or f_i, and this is possible because a membrane can be involved in only one communication rule of an evolution step. After an object t_i or f_i reaches a membrane i, it generates two new copies of it to be sent inside membranes $i+1$ together with an object a_{i+1} that is used then to construct the assignments over variable x_{i+1}.
3. $t_i[\ ]_{c_j} \to [t_i]_{c_j}$, if x_i appears in C_j
$[t_i]_{c_j} \to t_i$, for $1 \leq i \leq n$, $1 \leq j < m$
$[t_i]_{c_m} \to y$, for $1 \leq i \leq n$
$f_i[\ ]_{c_j} \to [t_i]_{c_j}$, if $\neg x_i$ appears in C_j
$[f_i]_{c_j} \to f_i$, for $1 \leq i \leq n$, $1 \leq j \leq m$
$[f_i]_{c_m} \to y$, for $1 \leq i \leq n$.

An assignment t_i (f_i) is sent into a membrane c_j if there is an assignment to a variable x_k ($\neg x_k$) such that it makes C_j true. Once all membranes labelled by c_i are dissolved inside a membrane labelled by n, an object y is generated.
4. $[y]_k \to [\ ]_k y$, for $k \in \Lambda \backslash \{0, h\}$
$[y]_0 \to yes$
$[z_{4\times n+2\times m}]_0 \to no$.
The object z_0 waits for $4 \times n + 2 \times m$ steps in order to allow dissolving the membrane labelled by 0 if this still exists (i.e., the rule $[y]_0 \to yes$ was not applied), then the answer *no* is generated. Once an object *yes* or *no* is generated, other objects *yes* or *no* cannot be created because membrane c_m was dissolved, and neither rule $[y]_0 \to yes$ nor $[z_{4\times n+2\times m}]_0 \to no$ can be applied.
5. $[yes]_h \to yes[\ ]_h$
$[no]_h \to no[\ ]_h$.
The answer *yes* or *no* regarding the satisfiability is sent out of the skin.

Example 1. We illustrate this algorithm and the evolution of a system Π constructed for the propositional formula $\psi = (x_1 \vee x_2) \wedge (\neg x_1 \vee \neg x_2)$.

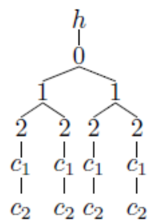
Thus, m=n=2. The initial configuration of the systems, constructed by an additional device that starts from a membrane structure $[[\ ]_0]_h$, with object 0 placed inside membrane 0 and rules of the form:

- $[0 \to 1'\ 1']_0$ and $[1 \to 2'\ 2']_1$
- $1' \to [1]_1$ and $2' \to [2]_2$
- $2 \to [c_2]_{c_1}$ and $c_2 \to [\ ]_{c_2}$.

The obtained structure is

$$[[[[[\ ]_{c_2}]_{c_1}]_2[[[\ ]_{c_2}]_{c_1}]_2]_1[[[[\ ]_{c_2}]_{c_1}]_2[[[\ ]_{c_2}]_{c_1}]_2]_1 a_1 z_0]_0]_h$$

Graphically, the membrane structure μ can be represented as a tree:



Using the set R of rules $1 \div 5$, the computation proceeds as follows:

$[[[[[\ ]_{c_2}]_{c_1}]_2[[[\ ]_{c_2}]_{c_1}]_2]_1[[[[\ ]_{c_2}]_{c_1}]_2[[[\ ]_{c_2}]_{c_1}]_2]_1 a_1 z_0]_0]_h$
$\Rightarrow [[[[[\ ]_{c_2}]_{c_1}]_2[[[\ ]_{c_2}]_{c_1}]_2]_1[[[[\ ]_{c_2}]_{c_1}]_2[[[\ ]_{c_2}]_{c_1}]_2]_1 t_1 f_1 z_1]_0]_h$
$\Rightarrow [[[[[\ ]_{c_2}]_{c_1}]_2[[[\ ]_{c_2}]_{c_1}]_2 t_1]_1[[[[\ ]_{c_2}]_{c_1}]_2[[[\ ]_{c_2}]_{c_1}]_2 f_1]_1 z_2]_0]_h$
$\Rightarrow [[[[[\ ]_{c_2}]_{c_1}]_2[[[\ ]_{c_2}]_{c_1}]_2 t_1' t_1' a_2]_1[[[[\ ]_{c_2}]_{c_1}]_2[[[\ ]_{c_2}]_{c_1}]_2 f_1' f_1' a_2]_1 z_3]_0]_h$
$\Rightarrow [[[[[\ ]_{c_2}]_{c_1} t_1]_2[[[\ ]_{c_2}]_{c_1} t_1]_2 t_2 f_2]_1[[[[\ ]_{c_2}]_{c_1} f_1]_2[[[\ ]_{c_2}]_{c_1} f_1]_2 t_2 f_2]_1 z_4]_0]_h$
$\Rightarrow [[[[[\ ]_{c_2}]_{c_1} t_1 t_2]_2[[[\ ]_{c_2}]_{c_1} t_1 f_2]_2]_1[[[[\ ]_{c_2}]_{c_1} f_1 t_2]_2[[[\ ]_{c_2}]_{c_1} f_1 f_2]_2]_1 z_5]_0]_h$
$\Rightarrow [[[[[\ ]_{c_2} t_1]_{c_1} t_2]_2[[[\ ]_{c_2} t_1]_{c_1} f_2]_2]_1[[[[\ ]_{c_2} t_2]_{c_1} f_1]_2[[[\ ]_{c_2}]_{c_1} f_1 f_2]_2]_1 z_6]_0]_h$
$\Rightarrow [[[[\ ]_{c_2} t_1 t_2]_2[[\ ]_{c_2} t_1 f_2]_2]_1[[[\ ]_{c_2} t_2 f_1]_2[[[\ ]_{c_2}]_{c_1} f_1 f_2]_2]_1 z_7]_0]_h$
$\Rightarrow [[[[\ ]_{c_2} t_1 t_2]_2[[f_2]_{c_2} t_1]_2]_1[[[f_1]_{c_2} t_2]_2[[[\ ]_{c_2}]_{c_1} f_1 f_2]_2]_1 z_8]_0]_h$
$\Rightarrow [[[[\ ]_{c_2} t_1 t_2]_2[y t_1]_2]_1[[y t_2]_2[[[\ ]_{c_2}]_{c_1} f_1 f_2]_2]_1 z_9]_0]_h$
$\Rightarrow [[[[\ ]_{c_2} t_1 t_2]_2[t_1]_2 y]_1[[t_2]_2[[[\ ]_{c_2}]_{c_1} f_1 f_2]_2 y]_1 z_{10}]_0]_h$
$\Rightarrow [[[[\ ]_{c_2} t_1 t_2]_2[t_1]_2]_1[[t_2]_2[[[\ ]_{c_2}]_{c_1} f_1 f_2]_2]_1 y y z_{11}]_0]_h$
$\Rightarrow [[[\ ]_{c_2} t_1 t_2]_2[t_1]_2]_1[[t_2]_2[[[\ ]_{c_2}]_{c_1} f_1 f_2]_2]_1 y z_{12} yes]_h$
$\Rightarrow [[[\ ]_{c_2} t_1 t_2]_2[t_1]_2]_1[[t_2]_2[[[\ ]_{c_2}]_{c_1} f_1 f_2]_2]_1 y z_{12}]_h yes$

It can be noticed that even the object z has now the subscript $4 \times n + 2 \times m = 4 \times 2 + 2 \times 2 = 12$, it cannot generate a *no* object because membrane labelled by 0 was already dissolved by an y object in the previous step. Also, even another y object reached the membrane labelled by 0, it cannot generate an *yes* object because membrane labelled by 0 was already dissolved by another y object in a previous step.

4 Conclusion

In this paper we deal with a question presented by Păun in 2005: *Can the polarizations be completely avoided?* (related to complexity aspects of P systems with active membranes and with electrical charges). We answer positively to this question: we do not use polarizations in solving the SAT problem, but use a pre-computed initial configuration involving either exponential alphabet or exponential structure.

We proved $P = PMC_{\mathcal{AM}^0(+d,+e,+c,pre(\alpha))}$ and $P = PMC_{\mathcal{AM}^0(+d,+e,+c,pre(\mu))}$ by providing two algorithms for solving the SAT problem using **polarizationless** P system with active membranes and **without division**. For the former equality, the provided algorithm is using an exponential alphabet pre-computed in linear time by a P system with replicated rewriting, while the later one is using an initial exponential structure pre-computed in linear time with respect to the number of variables and the number of clauses by P systems with membrane creation.

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