Science and Maths by inquiring about the image size in a camera obscura -

A. M. Criado, A. García-Carmona
Universidad de Sevilla, Spain

1 Introduction and objectives

Our line of research[1] is based on IBL strategies applied to practical work as part of science teacher education. The present communication describes an activity carried out by prospective primary teachers as part of the unit "How can one learn through inquiry? In the context of a school inquiry into the camera obscura, it is shown how the students are encouraged to review their hypotheses using qualitative arguments, and a proposal is made for them to go deeper into that review using more quantitative approaches. The initial goal was for them to reach a result that possibly contradicts the first hypotheses. This expressly fosters the process of reformulation and the search for new explanations that can reconcile theoretical predictions with experiment. All this takes place within an activity of school IBL about the characteristics of the projected image in a camera obscura in which the students will find that, in some cases, one does not appreciate the expected change in size of the image when the depth of the camera is increased.

2 Methodological aspects (implementation and instruments)

As an example of IBL being initiated due to the existence of a question without an immediate answer, the students are grouped into teams of 4 to inquire into the following problem:

"What will be the characteristics of the image seen in the camera obscura?"

To delimit the problem, and focus the students' attention on the variables of interest, they were asked about the lighting, sharpness, and image size. We shall here focus on how this last variable (image size) depends on several characteristics.

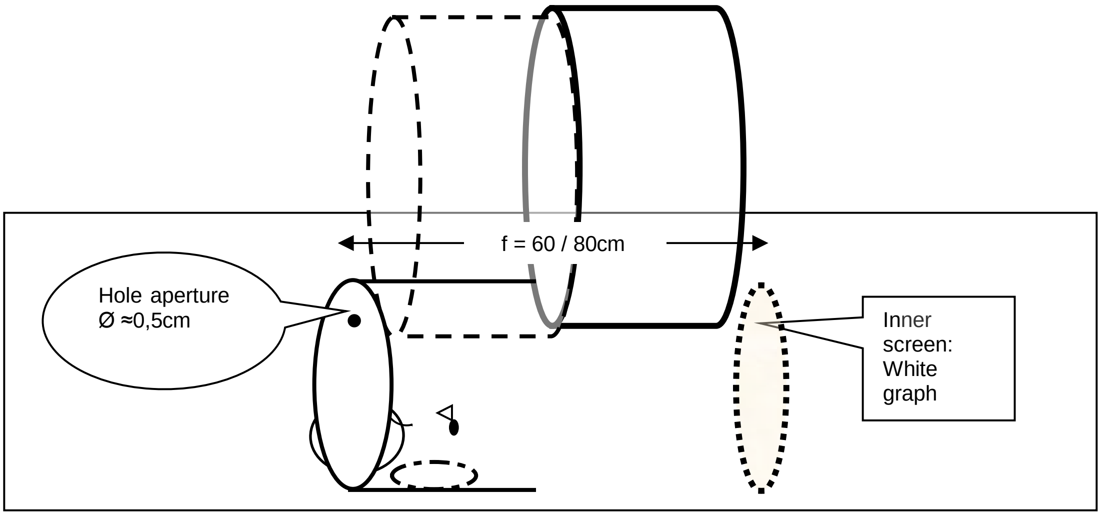

Figure 1: An observer, holding the camera made of black cardboard in their hands, has put their head inside.

The class IBL activity consists of the following phases:

I. The proposal put to the students is for them to construct a camera obscura (Figure 1) for which they are given precise instructions, (Criado et al., 2007), reinforced with the help of an educational video[2].

II. They are next asked to formulate their hypotheses in writing. This orients them towards considering factors that may affect the image's characteristics. They then consult a document on how the camera obscura works (Criado at al., 2007), revise their hypotheses, and prepare a table for data collection.

III. They then make empirical observations with the camera, and log the data in a table. They are asked to include observations made outdoors, viewing nearby buildings and a classmate.

IV. With the data they collected, each group writes up a report, completing the tables by relating the hypotheses to the results (both definitive and inconclusive), and ending with a summary of the conclusions.

V. The groups' results and conclusions are pooled, the differences found are discussed and analysed, and a revision is proposed for those which are most divergent.

VI. For the case of how the depth of the camera affects the image size, the students are provided with another document (Figure 2) as support in their revision of their hypotheses, and a new brief experimental session is proposed to make the necessary checks.

3 Results

The students' hypotheses

In their first response to the problem, the students' commonest beliefs about what factors will influence the size of the image were, in this order: (i) the distance of the object, (ii) the depth of the camera, and (iii) the aperture. Once they had written down their hypotheses, they consulted the documentation (which included Figure 2a connecting image size with depth of the camera). They then revised the hypotheses before going on to test them empirically. Their comprehension of the figure either supported their predictions or helped with their modification. The presumption that the diameter of the aperture might affect the image size had to be discussed in terms of an imperceptible influence.

The empirical verification and pooling of the groups' results

The students observed in practice that they did not always get the results they anticipated from the document they were given to consult. In pooling their results, there was no unanimity: some groups state they clearly saw the change in size of the image, and others not.

Looking deeper into their conditions of observation, different situations emerged. Some had made a camera that could only be extended 5 or 6 cm, while others to 20cm following the instructions. In other cases, one had to distinguish between observing a classmate 2m away and a far more distant building.

Revision of the hypotheses

The instructor encouraged the students to put forward an explanation of these results. Someone always realized that certain conditions must be met for extending the camera to produce a perceptible increase in image size. With blackboard drawings made by volunteers, the importance of the proportions between the dimensions of the variables involved was discussed. It was then understood, for example, that the change in image size will be small if the object is the building that is fairly far away. Following this pooling of results, to extend the information provided for their consideration, the students were provided with the qualitative description in the second consultation document (Figure 2b).

New empirical evidence, following review of the hypotheses

With this information, the groups were able to do their last empirical tests and thus verify the evident change in image size when they watch a classmate who changes their position from 2 to 4 m distant from the camera. But with the latter distance, if the depth of the camera is altered by only 5 or 6 cm, no changes are appreciated in the image size.

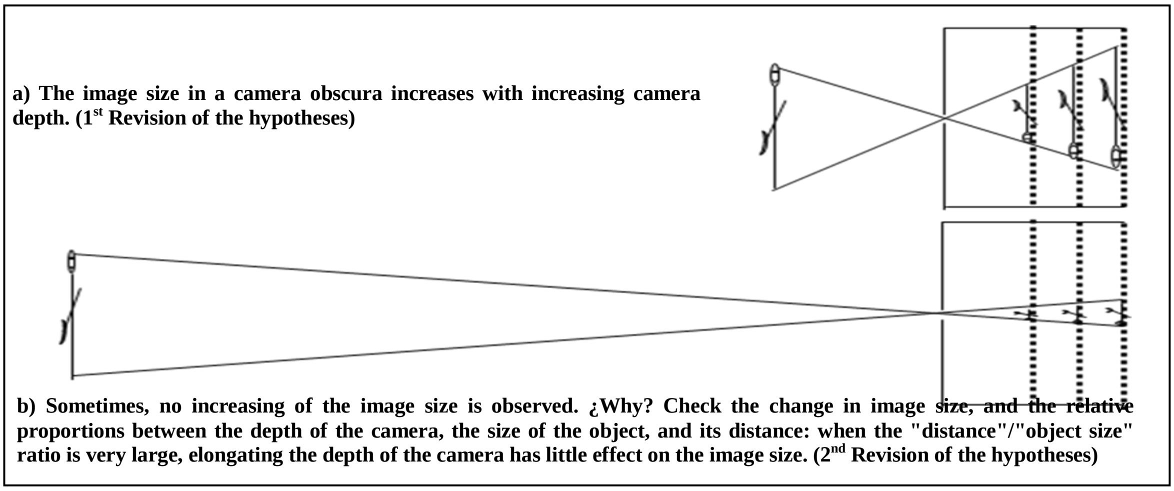

Figure 2: Consultation documents (a) and (b) on changes in image size on changing the depth.

4 Quantitative inquiry

The next problem proposed to the students was for them to record specific numerical data, and to predict under what conditions a change in the size of the image would be observed on modifying the depth of the camera. Depending on the difficulties that the students showed in this respect, the problem was further delimited with a progressive contribution of further information until they were able to reach a conclusion independently.

In a *first phase*, they were given Figure 3 with the relevant variables of the problem[3], leaving the angles unmarked until the next phase. In this *second phase*, they were required to use their knowledge of symmetry, trigonometry, similarity relations, etc., so as to be able to conclude that the parameters they had discussed before satisfied the following relationships:

$$H / D = h / f = \Delta h / \Delta f \qquad \text{(I)}$$

In a *third phase*, it was indicated to them that if the concentric rectangles on the camera screen have a 1-cm spacing then one can set the minimum appreciable size difference to that value. I.e., for the dimensions of our camera (depth, $f = 0.50$ m, and its increment, $\Delta f = 0.30$ cm), to be able to appreciate a 1-cm increase in the size of the observed object, one must have that:

$$\Delta h \geq 0.01\text{m}$$

The problem to solve now is therefore:

"What relationship must there be between the distance and the size of the object?"

To address this, the students can construct a table on a spreadsheet program such as Excel, with the actual values of the distances (D), of the characteristic parameters of the camera (f and Δf), and of the half height (h) and its increment (Δh) in the image. They can then make the checks needed to test their hypotheses. Tables 1 and 2 are two examples of how this can be done. With the problem now reduced to the equality of the first and last fractions of Equation (I), one reaches the conclusions drawn from the values in Tables 1 and 2. I.e., for the change in the image to be perceptible when the camera is lengthened by 0.30 m, the ratio between the size (H) and the object's distance (D) must be:

$$D \leq 30 \cdot H$$

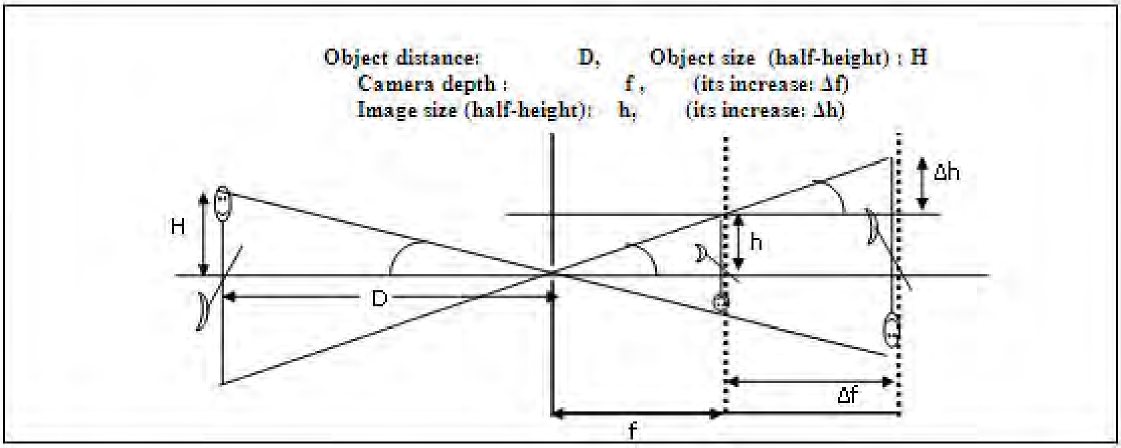

Figure 3: Parameters involved in the camera obscura image size.

Table 1: Calculation of the distance (D)/height (H) ratio of an object, so that an elongation (Δf = 0.30 m) of the camera produces a noticeable change (Δh ≥ 0.01 m) in the image. The object is a person whose torso is H = 0.5 m.

D(m)	**h(m) Image height when f = 0.50 h= H · f/D**	**h(m) Image height when f = 0.80 h= H · f/D**	**Δh(m) calculated as h(0.8)-h(0.5)**	**Δh(m) calculated as Δh=Δf · H / D**	**D / H**
2	0.125	0.200	0.075	0.075	4.000
4	0.063	0.100	0.038	0.038	8.000
10	0.025	0.040	0.015	0.015	20.000
15	**0.017**	**0.027**	**0.010**	**0.010**	**30.000**
20	0.013	0.020	0.008	0.008	40.000

Table 2: Calculation of the distance (D)/height (H) ratio of an object, so that an elongation (Δf = 0.30 m) of the camera produces a noticeable change (Δh ≥ 0.01 m) in the image. The object is a one-storey building (H = 4m).

D(m)	h(m) Image height when f = 0.50 h= H · f/D	h(m) Image Height when f = 0.80 h= H · f/D	Δh(m) calculated as h(0.8)-h(0.5)	Δh(m) calculated as Δh=Δf · H / D	D / H
10	0.200	0.320	0.120	0.120	2.500
50	0.040	0.064	0.024	0.024	12.500
100	0.020	0.032	0.012	0.012	25.000
119	0.0168	0.0269	0.0101	0.0101	29.7500
120	**0.0167**	**0.0267**	**0.0100**	**0.0100**	**30.0000**
121	0.0165	0.0264	0.0099	0.0099	30.2500
140	0.0143	0.0229	0.0086	0.0086	35.0000

5 Final assessment and expectations of the proposal

Although we have only presented the characteristics of the study with prospective primary teachers, the proposal can also be implemented for secondary education. In this activity, students learn in a natural integrated form: conceptual and procedural content of Optics and Mathematics; the use of a spreadsheet; the development of attitudinal IBL objectives; and the design of technological artefacts. In sum, they learn the practice school IBL processes, and, most importantly, they acquire a good attitude to inquiry in the classroom.

Endnotes

[1] This work is associated with the Curricular Project Investigating Our World [Investigando Nuestro Mundo, INM6 -12], designed by the GAIA research group (HUM133).

[2] Domestic recording (5 minutes) of the series Beakman's World, TV Series Directed by Jay Dubin.

[3] In the figures, there are various simplifications that do not conform to reality (such as the object and aperture located on the optical axis). Therefore, the conclusions will only be a rough approximation to what one sees in practice.

References

Criado, A.M., del Cid, R & Garcia-Carmona, A. (2007) “La Cámara Oscura en la Clase de Ciencias: Fundamento y Utilidades Didácticas”. Revista Eureka sobre Enseñanza y Divulgación de las Ciencias, Vol. 04 (1), pp. 123-140. http://reuredc.uca.es/index.php/tavira.

Criado, A.M. & García-Carmona, A. (2011) “Investigando las máquinas y artefactos”. Sevilla: Díada.