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Multi-criteria minimum cost spanning tree games

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Abstract

The minimum cost spanning tree game (mcst-game) is a well-known model within operations research games that has been widely studied in the literature. In this paper we introduce the multi-criteria version of the mcst-game as a set-valued TU-game. We prove that the extension of Bird's cost allocation rule provides dominance core elements in this game. We also give a family of core solutions that are different from the previous one; these solutions are based on proportional allocations obtained using scalar solutions of the multi-criteria spanning tree problem. Besides, we prove necessary and sufficient conditions ensuring that the preference core of this game is not empty.

Keywords: Cooperative game theory; Multi-criteria optimization; Minimum cost spanning tree

1. Introduction

Optimization problems in which one or several decision-makers that consider one or several objective functions, analyze how to act in an optimal way constitute the essence of operations research models. Optimization theory analyzes situations in which one decision-maker faces an optimization problem with one or several criteria. If several decision-makers interact conventional game theory is a suitable framework. (See Owen, 1995 for further details.) Finally, when several decision-makers each one controlling several criteria interact it appears multi-criteria game theory. A methodological approach to cooperative games with vector-valued payoff can be seen in Fernández et al. (2002).

Traditionally, operations research focus on choosing the optimal alternatives and game theory focus on models of competition and cooperation. Nevertheless, recent developments in both disciplines have shown strong interplay between then. These models are called Operations Research Games (see Borm et al., 2001). In these models apart from the inherent optimization problem it arises the natural question of how to allocate the joint cost/benefit among the individual decision-makers. Recently, a new issue in game theory has been to consider the multi-criteria operations research games, see for instance Nishizaki and Sakawa (2001) and Fernández et al. (2001).

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In this paper we concentrate on the minimum cost spanning tree (mcst) game. These games arise from analyzing the problem of allocating the costs of a spanning tree in a graph among the users which are located at the nodes of a graph, with one node reserved for a common supplier which is not to participate in the cost sharing. This problem was first introduced by Claus and Kleitman (1973). Bird, in 1976, suggested a game theoretic approach to the problem and proposed a cost allocation scheme that consists of assigning to each player (node) the cost of the edge incident upon the node on the unique path from the source to the mentioned node in a minimum cost spanning tree. Since there can be more than one minimum cost spanning tree for a graph, this way of dividing the costs need not lead to a unique cost allocation. Later on, Granot and Huberman (1981) showed that the allocations arising from Bird's cost allocation scheme are always in the core of the mcst-game (see Curiel, 1997).

Situations in which the cost associated to an edge is a vector instead of a single number yields to multi-criteria minimum cost spanning tree games that we analyze in this paper.

The paper is organized as follows. Section 2 is devoted to define the Pareto-minimum cost spanning tree game as a set-valued transferable utility (TU) game. We include the necessary concepts about Graph Theory. In Section 3 we analyze two core concepts for set-valued TU-games. We prove that the extension of Bird's cost allocation rule provides dominance core elements in this game. We also give a family of core solutions that are different from the previous one.

2. The game

In general, a set-valued TU-game is a pair (N, V), where $N = \{1, 2, \dots, n\}$ is the set of players and V is a function which assigns to each coalition $S \subseteq N$ a compact subset $V(S)$ of $\mathbb{R}^k$, the *characteristic set* of coalition S, such that $V(\emptyset) = 0$.

Vectors in $V(S)$ represent the worth that the members of coalition S can guarantee by themselves. Notice that the characteristic function in these games are set-to-set maps instead of the usual set-to-point maps.

Consider a set of N users of some good that is supplied by a common supplier 0 ($N_0 = N \cup \{0\}$). There is a multi-criteria cost associated to the distribution system that has to be divided among the users. This situation can be formulated as a set-valued game with N players and a characteristic function that associates to each coalition S a set $V(S)$ that represent the Pareto-minimum cost of constructing a distribution system among the users in S from the source 0.

Let $G = (N_0, E)$ be the complete graph with set of nodes N_0 and set of edges (links) denoted by E. There is a vector of costs associated with the use of each link. Let $e^{ij} = e^{ji} = (e^{ij}_1, e^{ij}_2, \dots, e^{ij}_k)$ denote the vector-valued cost of using the link $\{i, j\} \in E$. A tree is a connected graph which contains no cycles. A spanning tree for a given connected graph is a tree, with set of nodes equal to the set of nodes of the given graph, and set of edges a subset of the set of edges of the given graph connected and without cycles. A Pareto-minimum cost spanning tree for a given connected graph, with costs on the edges, is a spanning tree which has Pareto-minimum costs among all spanning trees (see Ehrgott, 2000).

Definition 2.1. A Pareto-minimum cost spanning tree game, associated to the complete graph $G = (N_0, E)$, is a pair (N, V) where N is the set of player and V is the characteristic function defined by:

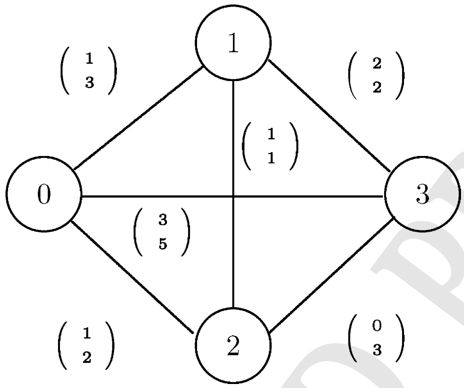
1. $V(\emptyset) = \{0\}$,
2. For each non-empty coalition $S \subseteq N$,

$$V(S) = \underset{T_{S_0}\,:\,\text{spanning tree}}{v-\min} \sum_{\{i,j\} \in E_{T_{S_0}}} e^{ij},$$

where $E_{T_{S_0}}$ is the set of edges of the spanning tree, T_{S_0}, that contains $S_0 = S \cup \{0\}$; and $v-\min$ stands for Pareto-minimization.

Remark that the resulting spanning tree T_{S_0} must contain S_0 but it may also contain some additional nodes.

Example 2.1. Consider the complete graph below.

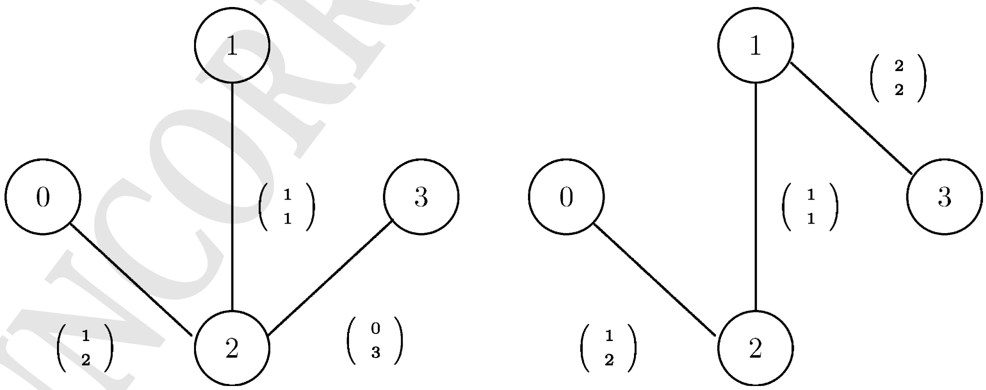


The bi-criteria Pareto-minimum cost spanning tree game associated to the graph is:

S	{1}	{2}	{3}{2,3}	{1,2}	{1,3}	N
$V(S)$	$\left\{\begin{pmatrix}1\\3\end{pmatrix}\right\}$	$\left\{\begin{pmatrix}1\\2\end{pmatrix}\right\}$	$\left\{\begin{pmatrix}1\\5\end{pmatrix}\right\}$	$\left\{\begin{pmatrix}2\\3\end{pmatrix}\right\}$	$\left\{\begin{pmatrix}3\\5\end{pmatrix},\begin{pmatrix}2\\6\end{pmatrix}\right\}$	$\left\{\begin{pmatrix}2\\6\end{pmatrix},\begin{pmatrix}4\\5\end{pmatrix}\right\}$

Note that $V(\{3\})$ is $(1,5)^t$ because the Pareto-minimum spanning tree that contains the nodes 0 and 3 is the subgraph induced by the edges {0,2} and {2,3}.

There are two Pareto-minimum cost spanning trees in the complete graph G for the grand coalition N. The first one correspond to $(2,6)^t \in V(N)$ and the second one correspond to $(4,5)^t \in V(N)$:



The interesting question that arises when a multi-criteria mcst-game, or in general a set-valued TU-game, is played is how to allocate fairly an achievable vector $z^N \in V(N)$ among the players.

For set-valued TU-games an allocation consists of a matrix $X \in \mathbb{R}^{k \times n}$ whose rows are allocations of the criteria. The ith column, $X^i = (x_1^i, x_2^i, \ldots, x_k^i)^t$ represents the payoffs of ith player for each criteria and the jth row, $X_j = (x_j^1, x_j^2, \ldots, x_j^n)$ is an allocation among the players of the total amount obtained with respect to criterion j. The sum $X^S = \sum_{i \in S} X^i$ is the overall payoff obtained by coalition S.

The matrix X is an allocation of the game (N, V) if $X^N = \sum_{i \in N} X^i \in V(N)$. The set of all the allocations of the game is denoted by $I^*(N, V)$.

3. Core concepts

Apart from efficiency and using the individual and collective rationality principles, in what follows we are going to establish core concepts for multi-criteria mcst-games.

3.1. Preference core

It is reasonable to think that coalitions only accept allocations if they pay less than any of the worths given by the characteristic set. To simplify the presentation, by $X^S \leqslant V(S)$ we will be denoted that $X_j^S \leqslant z_j^S$, $\forall j = 1, 2, \ldots, k$, $\forall z^S \in V(S)$.

This assumption leads us to introduce the concept of preference core.

Definition 3.1. The preference core of a game (N, V) is the set of allocations, $X \in I^*(N, V)$, such that $X^S \leqslant V(S) \quad \forall S \subset N$. We will denote this set as $C(N, v; \leqslant)$.

In order to characterize the non-emptiness of the preference core, consider a vector $\bar{z} \in \mathbb{R}^k$, not necessarily in $V(N)$, and the following k scalar games:

Definition 3.2. The scalar l-component minimum cost spanning tree game $(l = 1, 2, \ldots, k)$ associated to $\bar{z}$ is a pair $(N, v_l^{\bar{z}})$ where N is the set of player and $v_l^{\bar{z}}$ is the characteristic function defined by:

1. $v_l^{\bar{z}}(\emptyset) = 0$.
2. For each non-empty coalition $S \subset N$,

$$v_l^{\bar{z}}(S) = \min_{T_{S_0}:\ \text{spanning tree}} \sum_{\{i,j\} \in E_{T_{S_0}}} e_l^{ij},$$

where $E_{T_{S_0}}$ is the set of edges of the spanning tree, T_{S_0}, that contains the set of nodes $S_0 = S \cup \{0\}$.

3. $v_l^{\bar{z}}(N) = \bar{z}_l$.For each non-empty coalition, $S \subset N$, $v_l^{\bar{z}}(S)$ is the solution of the problem:

$$\begin{array}{ll} \min & z_l^S, \\ \text{s.t.:} & z^S \in V(S) \end{array},$$

where z_l^S, $l = 1, 2, \ldots, k$, is the lth component of vector z^S.

Notice that for a fixed coalition S, if an allocation X of the mcst-game, (N, V), verifies $X^S \leqslant V(S)$ then $X^S \leqslant z^*(S)$, where $z^*(S) = (v_1^{\bar{z}}(S), v_2^{\bar{z}}(S), \ldots, v_k^{\bar{z}}(S))$ denote the k-dimensional vector whose components are, respectively, the solutions of the above problems. Conversely, if $X^S \leqslant z^*(S)$ then $X^S \leqslant V(S)$.

A necessary and sufficient condition for the non-emptiness of the preference core is given in the next result. This condition is based on the balancedness concept of standard scalar cooperative games. For a definition of balanced games the reader is referred to Owen (1995).

Theorem 3.1. *The preference core is non-empty if and only if there exists at least one* $z^N \in V(N)$ *such that all the scalar l-component games* $(N, v_l^{z^N})$ *are balanced.*

Proof. If every scalar l-component game $(N, v_l^{z^N})$ is balanced, consider any allocation, X_l, in the core of $(N, v_l^{z^N})$, $l = 1, 2, \ldots, k$. Then, the $k \times n$-matrix X whose rows are X_l, $l = 1, 2, \ldots, k$, is an allocation associated with z^N. Moreover, for each $S \subset N$, $X^S \leqslant z^*(S)$ and $X^S \leqslant V(S)$.

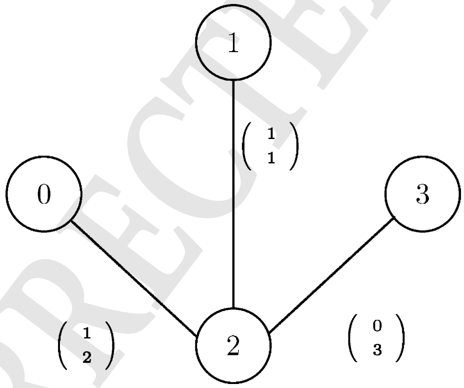
Conversely, let X be an allocation in the preference core such that $X^N = z^N \in V(N)$. Then $X^S \leqslant V(S)$, $\forall S \subset N$ and $X_l^S \leqslant v_l^{z^N}(S)$, $\forall S \subset N$, $\forall l = 1, 2, \ldots, k$. Therefore, X_l is an allocation in the core of the game $(N, v_l^{z^N})$. □

In scalar mcst-game there exists a simple rule to allocate costs among the users in the game. This allocation, called Bird rule (Bird, 1976), is given by:

"Each player supports the cost of the edge incident upon it on the unique path between 0 and the player's node, in the corresponding minimum spanning tree."

This rule can be extended to the multi-criteria msct-game by allocating to each player the cost vector of the edge incident upon it on the unique path between 0 and the player's node, in the corresponding Pareto-minimum spanning tree.

Example 2.1 (*Continued*). In the example above, we can allocate $(2,6)^t \in V(N)$ by the matrix $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 3 \end{pmatrix}$ that is in the preference core. This allocation has been obtained applying Bird's rule to the Pareto-minimum tree given in the following figure.



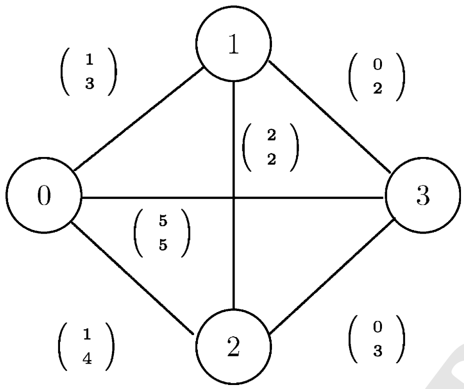
Nevertheless we can not divide among the players the vector $z^N = (4, 5)^t \in V(N)$ by an allocation in the preference core because the game $(N, v_1^{z^N})$ given by:

S	{1}	{2}	{3}{2,3}	{1,2}	{1,3}	N
$v_1^{z^N}(S)$	1	1	1	2	2	4

is not balanced.

Unfortunately extended Bird's cost allocation scheme is not, in general, a way to obtain allocations in the preference core as we show in the following example.

Example 3.1. Consider the complete graph below.



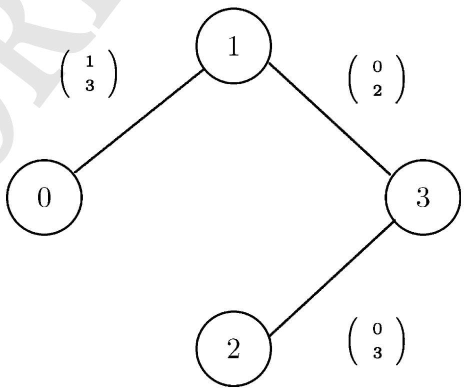
The bi-criteria minimum cost spanning tree game associated with the graph is:

S	$\{1\}$	$\{2\}$	$\{3\}\{1,3\}$	$\{1,2\}$	$\{2,3\}$	N
$V(S)$	$\left\{\begin{pmatrix}1\\3\end{pmatrix}\right\}$	$\left\{\begin{pmatrix}1\\4\end{pmatrix}\right\}$	$\left\{\begin{pmatrix}1\\5\end{pmatrix}\right\}$	$\left\{\begin{pmatrix}2\\7\end{pmatrix},\begin{pmatrix}3\\5\end{pmatrix},\begin{pmatrix}1\\8\end{pmatrix}\right\}$	$\left\{\begin{pmatrix}1\\7\end{pmatrix}\right\}$	$\left\{\begin{pmatrix}3\\7\end{pmatrix},\begin{pmatrix}1\\8\end{pmatrix}\right\}$

Consider $z^N = (1,8)^t \in V(N)$. Applying Definition 3.2 we obtain that the scalar component games $(N, v_l^{z^N})$, $l = 1,2$ are given by:

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	N
$v_1^{z^N}(S)$	1	1	1	2	1	1	1
$v_2^{z^N}(S)$	3	4	5	5	5	7	8

Notice that both games are balanced (Owen, 1995). Therefore the vector $(1,8)^t$ can be divided among the three players by allocations in the preference core. For instance, $X = \begin{pmatrix}1 & 0 & 0\\ 2 & 3 & 3\end{pmatrix} \in C(N, V; \leqslant)$. However Bird's tree allocation, $X = \begin{pmatrix}1 & 0 & 0\\ 3 & 3 & 2\end{pmatrix}$ associated with the following Pareto-minimum spanning tree



is not in the preference core because the coalition {1,2} obtains $X^{\{1,2\}} = (1,6)^t$ and $X^{\{1,2\}} \leqslant V(\{1,2\})$ does not hold.

3.2. Dominance core

To impose that coalitions only accept allocations in which they pay less than any of the worths given by the characteristic set is too strong. Now suppose that each coalition S will not accept to pay a total cost greater than any of the guaranteed costs in $V(S)$. This will be denoted in the following by $X^S \ngeq V(S)$ and means $X^S \ngeq z^S \quad \forall z^S \in V(S)$, that is, there does not exist $z^S \in V(S)$ such that $X_j^S \geqslant z_j^S \; \forall j = 1, 2, \ldots, k$, $X^S \neq z^S$.

Definition 3.3. The dominance core of (N, V) is the set of allocations, $X \in I^*(N, V)$, such that $X^S \ngeq V(S) \quad \forall S \subset N$. We will denote this set as $C(N, V; \ngeq)$.

Bird's cost allocation scheme leads always to an element in the scalar core. In the following result we prove that any vectorial Bird's cost allocation belongs to the dominance core.

Theorem 3.2. *Let T_N be a Pareto-minimum cost spanning tree of a complete graph with associated cost vector $z^N \in V(N)$. Then the corresponding vectorial Bird's cost allocation is in the dominance core.*

Proof. Let X be the vectorial Bird's allocation of the Pareto-minimum cost spanning tree T_N. It is clear that $X^N = z^N \in V(N)$ and therefore $X \in I^*(N, V)$. For a non-empty coalition $S \subset N$ let T_S be a Pareto-minimum cost spanning tree on the graph G that contains $S \cup \{0\}$. Construct a spanning tree $\widehat{T}_N$ for N_0 as follows. Add all the nodes in $N \setminus S$ to T_S and for each $i \in N \setminus S$ add the edge incident upon i on the unique path from 0 to i in T_N. Then $\widehat{T}_N$, constructed in this way is a spanning tree for N_0. So, if $z^S(T_S) \in V(S)$ is the vector of costs associated to T_S, $z(\widehat{T}_N) = z^S(T_S) + X^{N \setminus S}$ is the vector of costs associated to the spanning tree $\widehat{T}_N$. Then $z^N \ngeq z(\widehat{T}_N)$, indeed, $z^N = X^N = X^S + X^{N \setminus S} \ngeq z^S(T_S) + X^{N \setminus S}$. Then $X^S \ngeq z^S(T_S)$. As T_S is any minimum cost spanning tree on G for S_0, we can conclude that $X^S \ngeq V(S)$. □

Example 2.1 (*Continued*). In this example, we show that vector $(4, 5)^t \in V(N)$ can not be allocated among the players by an allocation in the preference core. Nevertheless, as we have seen above, it can be allocated by Bird's cost allocation that is an element of the dominance core: $\begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 2 \end{pmatrix} \in C(N, V; \ngeq)$.

Apart from Bird's cost allocations, there are many other allocations in the dominance core. The question that arises is whether we can provide a method to obtain, easily, some of them and whether all the vectors of costs in $V(N)$ can be allocated with this method.

A way to deal with this problems is using topological orders in $\mathbb{R}^k$. As was shown in Ehrgott (2000), every Pareto optimal spanning tree of a graph is the conventional mcst using the appropriate topological order. Unfortunately, any topological order may not result in a Pareto optimal tree. Nevertheless, restricting to topological orders induced by an increasing linear utility function, the mcst obtained from the weighted graph is a Pareto optimal tree.

In what follows we are going to consider only topological orders defined by an strictly increasing linear utility function $u : \mathbb{R}^k \to \mathbb{R}$. In this situation a player or coalition prefers a vector of cost a to another vector b if $u(a) \leqslant u(b)$.

In order to find a condition that permits to divide among the players a total cost $z^N \in V(N)$ accordingly with a given strictly increasing linear utility function, u, we will define the following scalar game (N, v_u):

$$v_u(\emptyset) = 0, \quad v_u(S) = \min_{z^S \in V(S)} u(z^S), \; \forall S \subseteq N, S \neq \emptyset. \tag{1}$$

Using Bird's rule in the scalar game (N, v_u), we can construct dominance core allocations for some $z^N \in V(N)$.

Let $x = (x^1, \dots, x^n)$ be the Bird's allocation of the game (N, v_u). This vector allows us to give a proportional allocation of $z^N \in V(N)$ defined by

$$X = (X^1, \dots, X^n), \quad \text{where } X^i = \frac{x^i}{u(z^N)} z^N \ \forall i \in N. \tag{2}$$

The following theorem states a condition ensuring that these allocations belong to the dominance core.

Theorem 3.3. *If* $v_u(N) = u(z^N)$, $z^N \in V(N)$ *then the proportional allocation* X *defined in* (2) *belongs to the dominance core of* (N, V).

Proof. As u is a linear function, (N, v_u) is the mcst-game associated to the graph G with scalar cost on his edges, $u(e^{ij})$. Let x be the allocation of $u(z^N)$ obtained by Bird's rule applied in (N, v_u). Then x is a core allocation for the scalar game. Let $X \in \mathbb{R}^{k \times n}$ be the proportional allocation defined in (2). It is straightforward that $X^N = \sum_{i=1}^n \frac{x^i}{u(z^N)} z^N = z^N$ and then $X \in I^*(N, V)$. Moreover, if we assume that $X \notin C(N, V; \not\geqslant)$ then, there exists a coalition $S \subset N$ and a vector $w^S \in V(S)$ such that $X^S \geqslant w^S$, $X^S \neq w^S$. Then, as u is linear strictly increasing utility function and as x belongs to the core of the game (N, v_u),

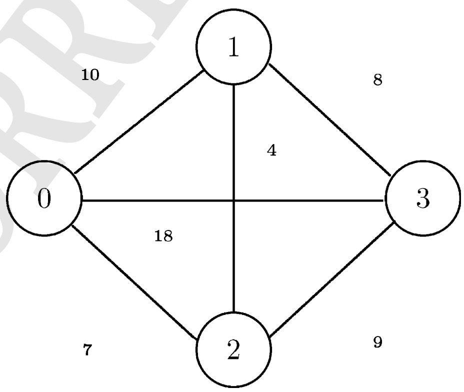
$$\min_{z^S \in V(S)} u(z^S) \leqslant u(w^S) < u(X^S) = \sum_{i \in S} u(X^i) = \frac{\sum_{i \in S} x^i}{u(z^N)} u(z^N) = x^S \leqslant v_u(S) = \min_{z^S \in V(S)} u(z^S).$$

This is a contradiction. □

Example 2.1 (*Continued*). Suppose that the strictly increasing linear utility function, u, used to compare the worth of the coalitions consist of giving triple importance to the second criterion, that is, the utility of vector a is $u(a) = a_1 + 3a_2$. Then, the scalar game (N, v_u) is:

S	{1}	{2}	{3}	{1,2}	{1,3}	{2,3}	N
$v_u(S)$	10	7	6	11	18	16	19

In this case, $v_u(N) = u((4,5)^t)$, the mcst for the weighted graph is the Pareto-optimal tree associated to $z^N = (4,5)^t$ and (N, v_u) is the mcst-game associated to the weighted graph.

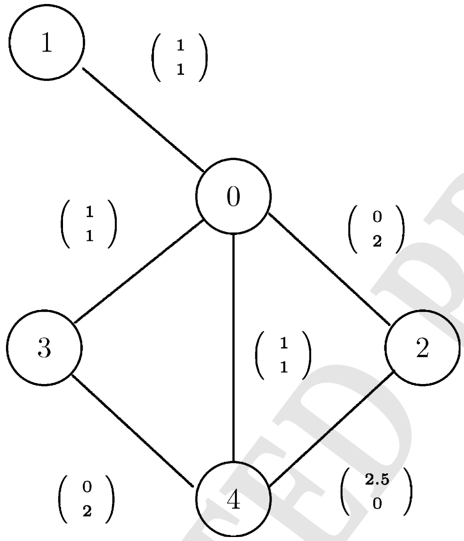


Therefore Bird's cost allocation $x = (4, 7, 8)$ is in the core of (N, v_u). Then the proportional allocation $X = \begin{pmatrix} \frac{16}{19} & \frac{28}{19} & \frac{32}{19} \\ \frac{20}{19} & \frac{35}{19} & \frac{40}{19} \end{pmatrix} \in C(N, V, \not\geqslant)$.

Notice that the allocations obtained with this method are different from those obtained applying the vectorial Bird's rule to the corresponding Pareto> mcst-game.

Unfortunately is not always possible to find a topological order, defined through an increasing linear utility function, for every $z^N \in V(N)$.

Example 3.2. Consider the complete graph where all the links $\{i,j\}$ not drawn below have costs $(5,5)^t$:



Three Pareto-minimum cost spanning trees for this graph are

$$T_1 = \{\{0,1\},\{0,2\},\{0,3\},\{0,4\}, \quad \text{with associated cost } (3,5)^t,$$

$$T_2 = \{\{0,1\},\{2,4\},\{0,3\},\{5.5,3\}\}, \quad \text{with associated cost } (5.5,3)^t,$$

$$T_3 = \{\{0,1\},\{2,4\},\{0,3\},\{3,4\}\}, \quad \text{with associated cost } (4.5,4)^t.$$

Then $z^N = (4.5,4)^t$ is a solution of $\min_{z^N \in V(N)} u(z^N)$ for no increasing linear utility function u.

4. Final remarks

This paper contains the methodological developments of the multi-criteria mcst-games through the analysis of their dominance and preference cores. However, there is enough room for further research. An interesting avenue of research should aim to construct new core concepts being consistent with the preference structure of the decision-maker. In particular, this scheme would lead to construct interactive procedures, which could be used by the decision-maker in decision support systems when the original core sets do not reduce to a singleton.

Another issue that deserves future research is the application of this model to real problems. There are challenging economical models in the new Europe that clearly fall into this category: design and operation of a common oil pipeline network, operation of existing inter-Europe electrical distribution network, … These topics are currently under research and will be considered in a follow up paper.

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