Vibrational Mechanics in an Optical Lattice: Controlling Transport via Potential Renormalization

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We demonstrate theoretically and experimentally the phenomenon of vibrational resonance in a periodic potential, using cold atoms in an optical lattice as a model system. A high-frequency (HF) drive, with a frequency much larger than any characteristic frequency of the system, is applied by phase modulating one of the lattice beams. We show that the HF drive leads to the renormalization of the potential. We used transport measurements as a probe of the potential renormalization. The very same experiments also demonstrate that transport can be controlled by the HF drive via potential renormalization.

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The control of transport is a recurrent topic in physics, chemistry, and biology. The typical scenario corresponds to particles diffusing on a periodic substrate, with transport controlled by the application of dc and ac external fields [1,2]. The ultimate limit for the control of transport is often the impossibility of tuning the periodic potential, as it is usually the case in solid state.

In this work we provide a proof-of-principle of how this limitation can be overcome, and demonstrate theoretically and experimentally the control of a periodic potential amplitude via a strong high-frequency (HF) oscillating field. The potential is renormalized, with its amplitude controlled by the strength and frequency of the HF field. The mechanism underlying the potential renormalization is the so-called vibrational resonance, intially introduced [3] and observed [4–7] in bistable systems. Our experiment uses cold atoms in a dissipative optical lattice as a model system. However, the phenomenon demonstrated here is very general, and is relevant to any classical system of particles in a periodic potential. This may also offer a possibility of tuning the potential in solid state systems, where this is usually considered impossible. Combined with previous work which showed how ac fields can be used to control transport via dynamical symmetry breaking [1,2] and tunnel coupling renormalization [8,9], the present work demonstrates that a complete control of transport can be achieved via ac fields.

Our experimental work relies on the study of the transport properties of atoms in an optical lattice for different strengths of the applied HF field. We will demonstrate that by tuning the HF field it is possible to control the amplitude of the potential, and to make it vanish. In this respect, the use of cold atoms in dissipative optical lattices is very convenient as the transport properties in these systems have been studied in detail [10–12], and this allows us to use the transport measurements to characterize the potential. We will provide two different sets of measurements, as supporting evidence of the potential renormalization. First, we will demonstrate that the diffusion properties, which are known to strongly depend on the potential depth [10,11], can be controlled by the HF field in a way which corresponds to the potential renormalization. Second, we will show that also directed transport, as induced by harmonic-mixing (HM) [13] of a biharmonic drive, can be controlled by the HF field. In fact, anharmonicity, together with the breaking of a dynamical symmetry, leads to the creation of directed currents in harmonic-mixing. Therefore, whenever the potential, renormalized by the HF field, vanishes, directed transport should cease [14–16].

Before discussing the experimental results, we introduce a model useful for the understanding of the potential renormalization of a dissipative optical lattice as produced by a HF oscillating field. We consider the simplest model of a dissipative optical lattice: a $J_g = 1/2 \rightarrow J_e = 3/2$ atom, of mass m, illuminated by two counter-propagating laser fields with orthogonal linear polarizations. This configuration generates a 1D optical lattice [12]. The atom in the $\pm$ ground state experiences the potential $U_{\pm}(z) = U_0[-2 \pm \cos(2kz)]/2$, where z is the laser beam propagation axis, k the laser field wave vector and U_0 the optical lattice depth [12,17].

We now introduce a HF oscillating force with frequency ω_{HF} and amplitude A_{HF}:

$$F_{\mathrm{HF}}(t) = A_{\mathrm{HF}} \sin(\omega_{\mathrm{HF}} t + \phi_0), \tag{1}$$

with ϕ_0 a (mainly irrelevant) phase which describes the state of the oscillating force at $t = 0$. Of interest here is the high-frequency case, where the frequency of the HF drive is much larger than any characteristic frequency of the system, in the present case the vibrational frequency ω_v of the atoms at the bottom of the well. In the asymptotic limit of infinite amplitude and frequency of the drive ($\omega_{\rm HF} \to \infty$, $A_{\rm HF} \to \infty$), it is possible to show [18] that, consistently with Refs. [3,14–16], the atomic dynamics corresponds to the motion in a static (i.e., without HF field) dissipative optical lattice, with renormalized amplitude $\tilde{U}$:

$$\tilde{U}_{\pm}(\hat{z}) = U_0[-2 \pm J_0(2kr)\cos(2k\hat{z})]/2, \qquad (2)$$

where J_0 is the Bessel function of the first kind, and $r = A_{\rm HF}/(m\omega_{\rm HF}^2)$ is the parameter—here and thereafter termed the HF ratio—which controls the renormalization of the optical lattice.

The above analysis shows that, in the asymptotic limit of infinite frequency and strength, a HF field leads to an effective renormalization of the potential. We now consider finite values of driving away from infinity that are experimentally accessible. The atomic transport in the optical lattice in the presence of a HF field is numerically studied for two different setups, which correspond to the ones used to provide the experimental evidence.

In the first setup, a HF force of finite amplitude and frequency is applied to atoms in a dissipative optical lattice. For this scheme, the effective renormalization of the optical potential can be detected by studying the diffusion properties of the atoms through the lattice. In fact, for a dissipative optical lattice of the type considered here, it is well established [10,11] that there is a critical potential depth located at about $U_{\rm cr} \sim 100E_r$, with $E_r = \hbar^2k^2/(2m)$ the recoil energy, which separates two very different regimes. For potential depths larger than the critical one, the diffusion is normal. Instead, for potential depth lower than the critical one, the diffusion becomes anomalous, with the exponent of the diffusion dependent on the potential amplitude. To be quantitative, we define the diffusion exponent α as $\langle x^2(t)\rangle - \langle x(t)\rangle^2 \sim t^\alpha$ in the limit $t \to \infty$. According to this definition, $\alpha = 1$ corresponds to normal diffusion, while $\alpha > 1$ characterizes superdiffusion. In an undriven optical lattice, superdiffusion is encountered at shallow optical potentials (below the critical depth), with the exponent α increasing for decreasing potential depth. Thus we will take the exponent of the diffusion as a measure of the potential depth. To assess the effective renormalization of the potential by a HF field, we numerically simulated the dynamics of the atoms in a deep optical lattice with a HF drive. We determined the diffusion exponent as a function of the HF ratio r which for infinite frequency and amplitude of the drive determines the lattice renormalization. Our results, reported in Fig. 1, show that the diffusion exponent α can be controlled by the HF field, with a dependence consistent with the potential renormalization derived in the infinite limit, Eq. (2): α increases whenever the potential depth is decreased, with the largest values of α produced by the r values corresponding to the zeros of the Bessel function, i.e., to vanishing potentials. The upper bound of $\alpha = 3$ for $U_0 = 0$ corresponds to the vanishing of the friction mechanism (Sisyphus cooling

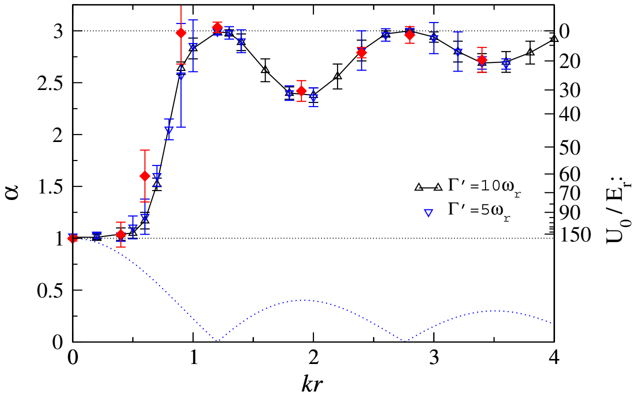

FIG. 1 (color online). Left axis: numerical results, as obtained by Monte Carlo simulations, for the spatial diffusion exponent as a function of the HF ratio r for an optical lattice with a depth $U_0 = 200E_r$. Right axis: value of U_0 which corresponds to the exponent α for an undriven lattice. Triangles correspond to results obtained in the infinite limit, for a photon scattering rate $\Gamma' = 5\omega_r$ and $\Gamma' = 10\omega_r$, were ω_r is the recoil frequency. The filled diamonds refer to simulations with a HF field of finite amplitude, with frequency $\omega_{\rm HF}/\omega_v = 20$ and $\Gamma' = 10\omega_r$. The solid line is a guide for the eye for the results corresponding to the infinite limit with $\Gamma' = 10\omega_r$. The dotted line is $|J_0(2kr)|$. The error bars on the numerical results correspond to the finite statistics of the Monte Carlo simulations.

[12]) associated with the optical lattice. Figure 1 also reports the value of U_0 which corresponds to the exponent α for an undriven lattice, so to make explicit the correspondence between a driving with HF ratio r and the depth of the renormalized potential. Finally, we notice that our results for the diffusion exponent α essentially coincides with the values derived in the infinite limit. We can thus conclude that the potential is effectively renormalized according to the dependence obtained in the infinite limit [see Eq. (2)].

In the second setup, besides the HF drive, a biharmonic force of the form

$$F(t) = F_0[A_1\cos(\omega t) + A_2\cos(2\omega t + \phi)] \qquad (3)$$

is also applied to the atoms in the lattice. Here ω is the frequency of the drive, of the same order of magnitude or smaller than the vibrational frequency, and ϕ the relative phase between harmonics. The amplitude of the lattice, and its renormalization by the HF field, can be determined by observing the directed motion of the atoms through the lattice. In fact, the two harmonics of the drive are mixed by the nonharmonic potential, thus producing directed motion of the atoms through the lattice [19]. The average current being proportional to the nonharmonicity of the potential, directed transport measurements give access to the potential amplitude. More precisely, for weak driving the average atomic velocity is expected to be of the form $v = v_{\max}\sin(\phi - \phi_d)$, with ϕ_d a dissipation-induced phase lag [19]. In our simulations we determined the velocity v for different values of the phase ϕ, so to derive the maximum velocity $v_{\max}$. Then by varying the strength of the HF drive, we were able to determine $v_{\max}$ as a function of the r parameter, with results as in Fig. 2. Once again, the results produced with a field of large, but finite, frequency and amplitude essentially coincide with those obtained in the infinite limit. These results also show that current measurements can be used to probe the potential renormalization. Whenever the HF field leads to a shallower potential, as from Eq. (2), the current is reduced, with zero current observed for those values of r leading to a vanishing potential.

Our experimental demonstration of potential renormalization via HF field relies on the two detection schemes outlined above. In both setups, ^{87}Rb atoms are cooled and trapped in a magneto-optical trap (MOT). After a compression phase of 50 ms, and 8 ms of optical molasses, the atoms are loaded into a 1D dissipative optical lattice. The lattice is created by the interference of two linearly polarized and counterpropagating laser beams, red detuned from resonance with the D_2-line $F_g = 2 \rightarrow F_g = 3$ atomic transition. One of the lattice beams is sent through a double pass electro-optical modulator (EOM), so to be able to apply a HF phase modulation. In the reference frame of the lattice, such a phase modulation translates into a rocking force of the form of Eq. (1). Quantitatively, a phase modulation $\alpha(t)$ leads to a force $F(t) = m\ddot{\alpha}(t)/(2k)$ in the reference frame of the lattice [19]. In the experiments, the modulation is progressively turned on starting after 1 ms equilibration time in the optical lattice, with a turn-on ramp of 1 ms. Thereafter the procedure differed for the two setups.

In the first experiment, we study the diffusion of the atoms in the optical lattice in the presence of the HF drive. The width of the atomic cloud is measured by fluorescence imaging after diffusive expansion inside the driven lattice. The width is measured at a fixed set of expansion times within the interval between 1 and 16 ms from the lattice turn-on. The time range over which images are taken is limited by the atom loss, particularly important for the values of the HF ratio r leading to a vanishing renormalized potential. Measurements of the spatial width of the atomic cloud on such a short temporal range do not allow us to derive an accurate value of the exponent of the diffusion α [20]. Instead, we characterize the diffusion by an effective diffusion coefficient D, as obtained by fitting the data with $\langle x^2(t)\rangle - \langle x(t)\rangle^2 = 2Dt$. Clearly, superdiffusion leads to a large enhancement of the derived effective diffusion coefficient. Thus a large increase in the effective diffusion coefficient can be taken as signature of the reduction of the potential, as produced by the renormalization by the HF field.

Our experimental results for the effective diffusion coefficient as a function of the HF ratio r are reported in Fig. 3. The data clearly show that the atomic diffusion is significantly modified by the HF drive, with a dependence of the effective diffusion coefficient on the HF ratio r consistent with the potential renormalization [see, e.g., Eq. (2) for the analytic expression in the limit of infinite frequency and amplitude]. Indeed, the effective diffusion coefficient increases whenever the HF ratio r corresponds to decreasing depth of the optical lattice, with the largest values of the diffusion constant observed in correspondence of the

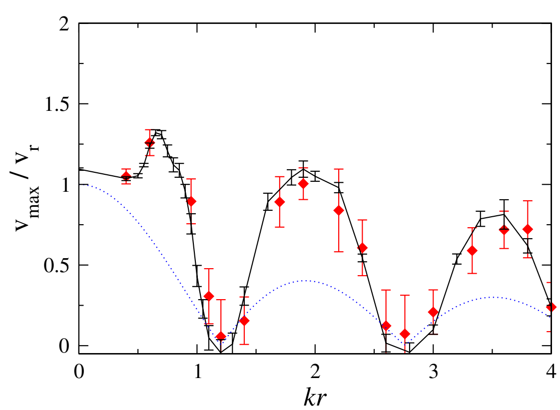

FIG. 2 (color online). Numerical results, as obtained by Monte Carlo simulations, for the amplitude of the current $v_{\max}$, rescaled by the recoil velocity v_r, as a function of the HF ratio r under a biharmonic driving force of the form of Eq. (3) with $A_1 = A_2 = 1$, $F_0 = 140\hbar k\omega_r$, and $\omega = \omega_v$. The solid line corresponds to the results obtained in the infinite limit and the diamonds to the simulation results with $\omega_{\mathrm{HF}}/\omega_v = 20$, both cases with $\Gamma' = 10\omega_r$. The dotted line is $|J_0(2kr)|$. The error bars on the numerical results correspond to the finite statistics of the Monte Carlo simulations.

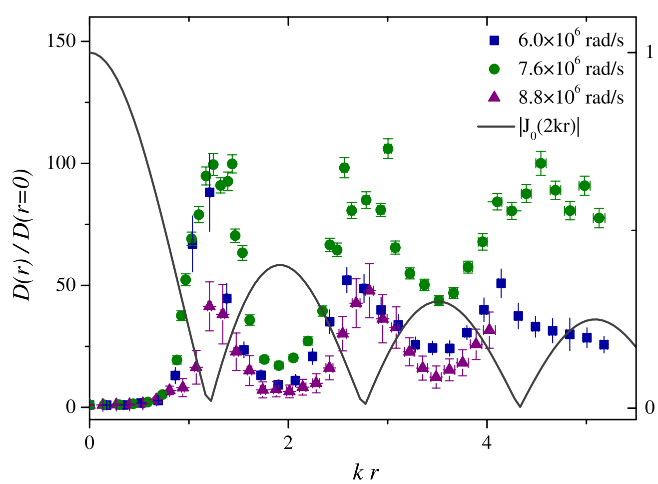

FIG. 3 (color online). Experimental results for the effective diffusion coefficient D as a function of the HF ratio r for different values of ω_{HF}, as indicated in the figure. The data are rescaled by the value of the diffusion constant for an undriven lattice. The vibrational frequency of the atoms at the bottom of the well, as determined by measuring the lattice beam power and waist, is $\omega_v = (9 \pm 1) \times 10^5$ rad/s. The solid line, with values on the right axis, is $|J_0(2kr)|$.

values of r leading to a vanishing (in the infinite limit) optical lattice. This shows that the HF drive renormalizes the optical potential, in agreement with the general theory [3] and with our numerical analysis for the specific system.

In the second experiment, we probe the amplitude of the renormalized potential by studying directed transport following harmonic mixing of two harmonics, as outlined in the numerical analysis. With respect to the previous experiment devoted to the study of the atomic diffusion, an additional biharmonic drive, with frequencies ω, 2ω and phase difference ϕ is introduced. This is done using additional acousto-optical modulators (AOMs). In the reference frame of the lattice, the biharmonic phase modulation corresponds to a driving force of the form of Eq. (3). In the experiment, the HF driving is first ramped up, as in the previous experiment. Then the biharmonic drive is progressively turned on with a ramp-up time of 4 ms. The velocity of the center-of-mass of the atomic cloud is derived by position measurements obtained via fluorescence imaging. The measurements are repeated for 10 different values of the phase difference ϕ between harmonics. The data are then fitted by the expected dependence $v = v_{\max} \sin(\phi - \phi_d)$, thus deriving a value for $v_{\max}$ which can be taken as a measure of the renormalized potential depth. In fact, the mixing of harmonics requires an anharmonic potential, with the current generated proportional to the anharmonicity. Our results for $v_{\max}$ as a function of the HF ratio r are presented in Fig. 4. These data for the directed transport amplitude are consistent with the renormalization of the potential depth by the HF drive. In fact, whenever the value of the HF ratio r corresponds to a reduced potential depth, the current decreases, with zero current observed for the r values corresponding to the zeros of the Bessel function, a signature of the vanishing optical lattice. These results also demonstrate a new scheme for the control of the transport via ac fields: the amplitude of the current can be controlled by a variation in the HF field and the direction reversed via a π shift in the relative phase between harmonics. Finally, we notice that there is a small deviation, both in the experiment and in the numerical simulations (see Fig. 2) from the behavior expected from the Bessel function at small values of r, with the data showing an extra peak at $kr \sim 0.75$. This peak could be explained by a superimposed resonance corresponding to the matching of the frequency of the biharmonic force with the oscillation frequency of the atoms at the bottom of the renormalized well.

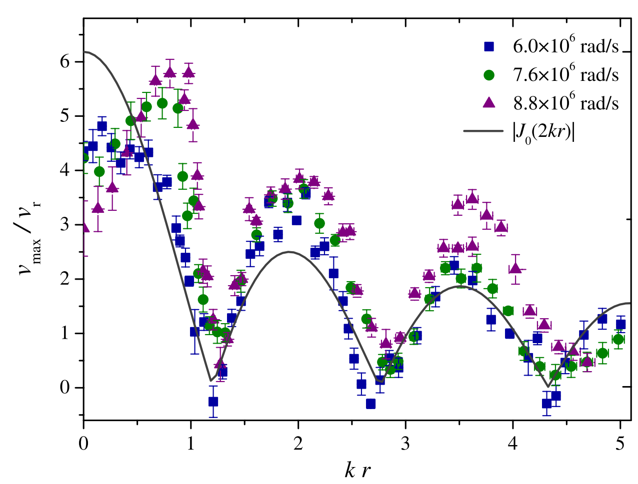

FIG. 4 (color online). Experimental results for the amplitude of the current $v_{\max}$, rescaled by the recoil velocity v_r, of directed transport through the optical lattice as a function of the HF ratio r. In addition to the HF drive, a biharmonic force of the form of Eq. (3) is applied to the atoms, with parameters $A_1 = 1$, $A_2 = 2$, $\omega = 9.42 \times 10^4$ rad/s, $F_0 = 112\hbar k \omega_r$. The vibrational frequency of the atoms at the bottom of the well is $\omega_v = (9 \pm 1) \times 10^5$ rad/s. The solid line, with values on the right axis, is $|J_0(2kr)|$.

In conclusion, in this work we demonstrated experimentally the phenomenon of vibrational resonance in a dissipative optical lattice. The application of a HF drive, with frequency much larger than any characteristic frequency of the system, leads to the renormalization of the potential. The renormalized amplitude can be controlled by the HF drive parameters. We used transport measurements as a probe of the potential renormalization. The very same experiments also demonstrated that transport can be controlled by the HF drive via potential renormalization.

The possibility to renormalize a potential via ac fields, as demonstrated here, is very general, and it is applicable to any system of particles in a periodic potential. As such, it paves the way to the control of potentials in systems in which they are not directly accessible, and it may also be applicable to solid state systems where ac drives can be introduced by the application of electric fields.

Finally, our setup can also be taken as the demonstration of a sensor able to detect signals with frequency exceeding any internal frequency of the sensor [14–16]. Here, the signal detected is the HF drive whose presence, although not coupling to any internal mode of the system, can be precisely detected due to its effect via the potential renormalization.

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[1] P. Reimann, Phys. Rep. **361**, 57 (2002).
[2] P. Hänggi and F. Marchesoni, Rev. Mod. Phys. **81**, 387 (2009).
[3] P. Landa and P. V. E. McClintock, J. Phys. A **33**, L433 (2000).
[4] V. N. Chizhevsky, E. Smeu, and G. Giacomelli, Phys. Rev. Lett. **91**, 220602 (2003).
[5] J. Casado-Pascual and J. P. Baltanás, Phys. Rev. E **69**, 046108 (2004).

[6] J. Casado-Pascual, D. Cubero, and J. P. Baltanás, Europhys. Lett. **77**, 50 004 (2007).

[7] D. Cubero, J. P. Baltanás, and J. Casado-Pascual, Phys. Rev. E **73**, 061102 (2006).

[8] A. Eckardt, C. Weiss, and M. Holthaus, Phys. Rev. Lett. **95**, 260404 (2005).

[9] H. Lignier, C. Sias, D. Ciampini, Y. Singh, A. Zenesini, O. Morsch, and E. Arimondo, Phys. Rev. Lett. **99**, 220403 (2007).

[10] S. Marksteiner, K. Ellinger, and P. Zoller, Phys. Rev. A **53**, 3409 (1996).

[11] H. Katori, S. Schlipf, and H. Walther, Phys. Rev. Lett. **79**, 2221 (1997).

[12] G. Grynberg and C. Mennerat-Robilliard, Phys. Rep. **355**, 335 (2001).

[13] K. Seeger and W. Maurer, Solid State Commun. **27**, 603 (1978); W. Wonneberger and H. J. Breymayer, Z. Phys. B **43**, 329 (1981); F. Marchesoni, Phys. Lett. A **119**, 221 (1986).

[14] M. Borromeo and F. Marchesoni, Europhys. Lett. **72**, 362 (2005).

[15] M. Borromeo and F. Marchesoni, Phys. Rev. E **73**, 016142 (2006).

[16] M. Borromeo and F. Marchesoni, Phys. Rev. Lett. **99**, 150605 (2007).

[17] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.108.020603 for a complete definition of the model.

[18] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.108.020603 for the complete derivation of the system renormalization by the HF field.

[19] F. Renzoni, Adv. At. Mol. Opt. Phys. **57**, 1 (2009).

[20] Atom losses lead to an underestimate of the diffusion exponent, as many superdiffusing atoms are lost.