Generalized Analytical Approach of the Calculation of the Harmonic Effects of Single Phase Multilevel PWM Inverters

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Abstract **This paper introduces a generalized analytical approach for calculating the total harmonic distortion THD and its weighted value WTHD for multilevel PWM inverters. The calculation considers one single phase and it can apply to any number of levels of the inverter in general. Although the analysis is based on the assumption of a high number of pulses, the developed equations can also be applied for lower frequency ratios fs/f1. The analytical formulas require the characteristic parameters of the PWM only, which are the modulation factor m, the switching frequency fs, the fundamental frequency f1, the effective inductance L and the DC link voltage. Voltage inverters with a number of levels N are considered. It must be noticed that some differences appear between the case of N odd and N even. Several parametric curves are calculated to define the specifications of an inverter with N levels in order to fulfill the harmonic voltage recommendations trying to reduce the output signal filtering.**

I. INTRODUCTION

Multilevel PWM inverters find increasing interest for high power DC to AC conversion [1-4]. The calculation of THD and WTHD of multilevel inverters is the main subject of some authors. In [5] this calculation was presented but was not generalized. Only the results of several levels of the inverter were presented. Besides, the presented results were completely individuals and the formulas were not generalized for a number N of levels. Using the way of calculation presented in this paper, THD and WTHD can be calculated by generalizing the formulas and studiying all the cases.

Figure 1 shows one possible single-phase topology of a 6- level inverter. The analysis in this paper is based on the assumption of constant DC voltages. One possibility of generating the control signals for a multilevel inverter is the carrier based pulse width modulation.

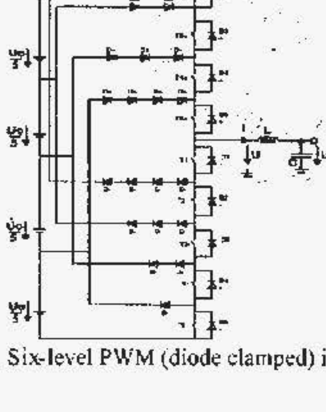

Fig l. Six-level PWM (diode clamped) inverter

II THD AND WTHD

The performance of different PWM techniques and the influence of parameter variations can be best compared by the total harmonic distortion THD and the weighted total harmonic distortion WTHD. The THD is defined by the root of the sum of all squared harmonics of the pulse width modulated voltage U(t).

$$THD = \sqrt{\sum_{i=2}^{\infty}\left[\frac{\hat{U}_i}{\hat{U}_1}\right]^2} = \sqrt{\sum_{i=2}^{\infty}\left[\frac{\tilde{U}_i\sqrt{2}}{mU_0}\right]^2} \qquad (1)$$

It is normalized to the fundamental amplitude $\hat{U}_1 = mUo$ where m is the modulation factor. The goal of this paper is to calculate some important parameters of a multilevel inverter depending on the number of levels N of the inverter. These parameters are the duty cycle, the averaged ripple of the current in the single phase leg and the factor $(U/Uo)^2$. Calculating these parameters, THD and WTHD can be found easily. The weighted total harmonic distortion WTHD is also based on the sum of all squared harmonics but it considers the order of the harmonics in addition. The higher order (i) of the harmonics, lower their influence to the WTHD factor.

$$WTHD = \sqrt{\sum_{i=2}^{\infty}\left[\frac{\tilde{U}_i\sqrt{2}}{imU_0}\right]^2} \qquad (2)$$

It must be noticed that floor(N/2) will be denoted as N_{ce} in future. The floor(x) operator determines the greatest integer less than or equal to the number x. In order to carry out these calculations, we must discriminate between the cases of N even and N odd. Positive output voltages are only considered due to the fact that the system is completely symmetrical. In [5], this way of calculation is presented but it is generalized in this paper studying all the possible cases. It is defined u as the average output voltage over a period.

The factor $U(u)^2$ can be easily calculated using the next formula where U(t) is the output voltage of the single phase in a period.

$$\tilde{U}(\overline{u})^2 = \int_0^T U(t)^2\,dt \qquad (3)$$

The RMS value of the fundamental voltage $\hat{U}_1(t)$ is simply given by $\hat{U}_1 = m\ U_0$ while the RMS value of the PWM voltage U can be determined. U^2 is identical to $U(u)^2$ averaged over a period. For symmetrical reasons it is sufficient to consider one quarter of a period only. It is denoted a as the duty cycle. The harmonic content of current $I_-(t)$, the current ripple peak to peak (ΔI) and the RMS value of $I_-(t)$ ($\tilde{I}_-(\bar{u})$) can be determined as

$$I_-(t)=\frac{1}{L}\int_0^t (U(t)-\bar{u}U_0)dt$$

$$\Delta I=\frac{1}{L}\int_{\frac{1-a}{2}T}^{\frac{1+a}{2}T}(U(t)-\bar{u}U_0)dt \quad (4)$$

$$\tilde{I}_-(\bar{u})=\frac{1}{2\sqrt{3}}\Delta I(\bar{u})$$

The function u can be considered in general u = m[sin(a) $-k_3$sin(3a+f_3)+$k_5$sin(5a+f_5)]. Therefore, this study can include reference voltages with third and fifth harmonic content. So, several cases are studied.

A. Calculation of the parameters

1. N odd

The possible output voltages of a multilevel inverter with N odd are 0, U_0/N_{ce}, $2U_0/N_{ce}$, ... , $(N_{ce}-1)U_0/N_{ce}$, U_0. Therefore, N_{ce} possible intervals can be defined as

Interval 1 ? {0, U_0/N_{ce}}
Interval 2 ? {U_0/N_{ce}, $2U_0/N_{ce}$}
....
Interval Nce-1 ? {$(N_{ce}-2)U_0/N_{ce}$, $(N_{ce}-1)U_0/N_{ce}$}
Interval Nce ? {$(N_{ce}-1)U_0/N_{ce}$, U_0}

It must be noticed that it can be denoted a_k as the duty cycle of interval k, ΔI_k as the ripple of the current in the interval k averaged over a period and $(U/U_0)_k^2$ with k = 1, 2, ... , N_{ce}. These parameters have been calculated by increasing iterative operations with N levels using the formulas commented before.

$$a_k=\frac{(N-1)\bar{u}}{2}-k+1$$
$$\frac{\Delta I_k f_s L}{U_0}=\frac{2k(1-k)}{N-1}+\bar{u}(2k-1)-\frac{(N-1)\bar{u}^2}{2} \quad (5)$$
$$\left(\frac{\tilde{U}}{U_0}\right)_k^2=\frac{4\left(\frac{\bar{u}(N-1)(2k-1)}{2}+k(1-k)\right)}{(N-1)^2}$$

1.
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2. N even

The possible output voltages of a multilevel inverter with N even are $-U_0/(N-1)$, $U_0/(N-1)\}$, $2U_0/(N-1)\}$, ..., $(N_{ce}-1)U_0/(N-1)$, Uo. Therefore, N_{ce} intervals can be defined as

Interval 0 ? {$-U_0/(N-1)$, $U_0/(N-1)$} (central interval)
Interval 1 ? {$U_0/(N-1)$, $2U_0/(N-1)$}
Interval 2 ? {$2U_0/(N-1)$, $3U_0/(N-1)$}
....
Interval Nce-2 ? {$(N_{ce}-2)U_0/(N-1)$, $(N_{ce}-1)U_0/(N-1)$}
Interval Nce-1 ? {$(N_{ce}-1)U_0/(N-1)$, U_0}

It must be noticed that a central interval appears. This special interval has an output voltage negative (its value is $-U_0/(N-1)$) and the other is positive (its value is $U_0/(N-1)$).

We can also calculate the parameters a_k, ΔI_k and and $(U/U_0)_k^2$. These parameters have been calculated where k = 0,1, 2, ... , N_{ce}-1.

$$a_k=\frac{(N-1)\bar{u}}{2}-k+\frac{1}{2}$$
$$\frac{\Delta I_k f_s L}{U_0}=\frac{1-4k^2}{2(N-1)}+2k\bar{u}-\frac{(N-1)\bar{u}^2}{2} \quad (6)$$
$$\left(\frac{\tilde{U}}{U_0}\right)_k^2=\left(\frac{1}{2}-\frac{(N-1)\bar{u}}{2}+k\right)u_n^2+\left(\frac{1}{2}+\frac{(N-1)\bar{u}}{2}-k\right)u_d^2$$

These expressions are completely valid for the central interval taking into account that in this case k is equal to zero. Therefore, the expressions for the central interval are the following.

$$a_0=\frac{(N-1)\bar{u}}{2}+\frac{1}{2}$$
$$\frac{\Delta I_0 f_s L}{U_0}=\frac{1}{2(N-1)}-\frac{(n-1)\bar{u}^2}{2} \quad (7)$$
$$\left(\frac{\tilde{U}}{U_0}\right)_0^2=\frac{1}{(N-1)^2}$$

B. Calculation of KU factor

The KU factor is defined as

$$KU=\frac{1}{U_0^2}\sum_{i=2}^{\infty}\tilde{U}_i^2=\frac{1}{U_0^2}\sum_{i=1}^{\infty}\tilde{U}_i^2-\frac{\tilde{U}_1^2}{U_0^2}=\frac{U^2-\tilde{U}_1^2}{U_0^2} \quad (8)$$

and U and U_1 are determined as

$$\tilde{U}_1=\frac{mU_0}{\sqrt{2}} \qquad \tilde{U}^2=\frac{2}{\pi}\int_0^{\frac{\pi}{2}}\tilde{U}(\bar{u})^2 d\alpha \quad (9)$$

It must be noticed that firstly it will be considered u = m[sin(a)]. Therefore, third and fifth harmonics will be considered in the nextsection of this work.

1. N odd

For the first interval, applying the formula described before, it can be used the following expression

$$U_{C_1}=\frac{2}{\pi}\int_0^{\frac{\pi}{2}}\frac{2\bar{u}}{N-1}d\alpha \quad (10)$$

If the number of levels of the inverter is greater or equal than 5, a second interval appears and an angle β must be calculated. β is the angle where the modulation changes the low level to the up level. Therefore, for example if the number of levels is equal to 5, β is the angle where m changes between m=0.5 and m>0.5. In general, Nce angles β_k must be calculated with N=5. These angles follow the next expression.

$$\beta_0 = 0$$
$$\beta_k = \arcsin\left(\frac{2(k-1)}{m(N-1)}\right) \quad (11)$$
$$\beta_{N_{ce}} = \frac{\pi}{2} \qquad \text{with } k = 1 \ldots N_{ce}-1$$

In general, Uc_j can be determined as

$$Uc_j = \frac{2}{\pi}\sum_{k=1}^{j}\int_{\beta_{k-1}}^{\beta_k}\left(\frac{\tilde{U}}{U_o}\right)_k^2 d\alpha$$

$$Uc_j = \frac{2}{\pi}\sum_{k=1}^{j}\int_{\beta_{k-1}}^{\beta_k}\frac{4\left(\frac{\overline{u}(N-1)(2k-1)}{2}+k(1-k)\right)}{(N-1)^2}d\alpha \qquad (12)$$

with $j = 1 \ldots N_{ce}$

The parameter Uc_j is associated with the interval $j/N_{ce} > m = (j-1)/N_{ce}$. The function U is defined as the sum of terms Uc_j with j=1, 2, ..., N_{ce}. Therefore, U is a function where m can change between 0 and 1. Finally, factor KU and factor THD can be calculated as

$$KU = U - \frac{m^2}{2} \qquad THD = \frac{\sqrt{2KU}}{m} \quad (13)$$

The evolution of KU factor and THD factor with the number of levels of the inverter can be calculated. It is shown in figures 8 and 9.

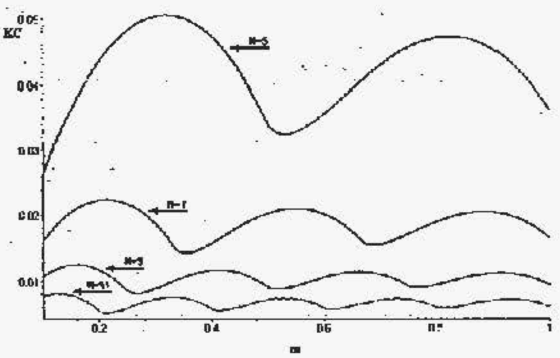

Fig 8. KU factor evolution for N odd

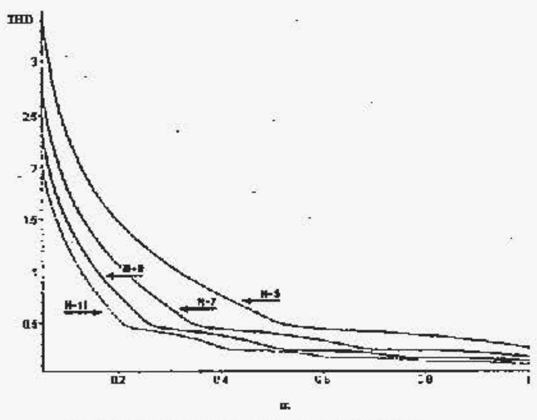

Fig 9. THD factor evolution for N odd

3. N even

For the central interval the factor Uc_0 can be easily calculated.

$$Uc_0 = \frac{2}{\pi}\int_0^{\frac{\pi}{2}}\frac{1}{(N-1)^2}d\alpha \quad (14)$$

For the first interval (this interval only exists if N=4) the angle β where the modulation changes the low level for the high level must be calculated. So, for example, in the case N=4, this angle marks the change between m=1/3 and m>1/3. In general, the number of angles β_k that must be determined is Nce-1 where N=4. The analytical expression of β_k is

$$\beta_0 = 0$$
$$\beta_k = \arcsin\left(\frac{2k-3}{m(N-1)}\right) \quad (15)$$
$$\beta_{N_{ce}} = \frac{\pi}{2} \qquad k = 1 \ldots N_{ce}-1$$

It is defined u_{il} as the initial output voltage of interval k, u_f as the final output voltage of interval k.. So, in general it can be calculated the expression:

$$Uc_j = \frac{2}{\pi}\sum_{k=0}^{j}\int_{\beta_k}^{\beta_{k+1}}\left(\frac{\tilde{U}}{U_o}\right)_k^2 d\alpha \quad (16)$$

$$Uc_j = \frac{2}{\pi}\sum_{k=0}^{j}\int_{\beta_k}^{\beta_{k+1}}\left(\left[\frac{1}{2}-\frac{\overline{u}(N-1)}{2}+k\right]u_{il}^2+\left[\frac{\overline{u}(N-1)}{2}-k+\frac{1}{2}\right]u_{f}^2\right)d\alpha$$

with $j = 0 \ldots N_{ce}-1$.

It must be taken into account that the factor Uc_0 only exists in the interval 1/(N-1)>m=0. For the other intervals, Uc_j exists in the interval (2j+1)/(N-1)>m=(2j-1)/(N-1). So, the function U can be built as the sum of this Uc_j factors with j=0, 1, ..., N_{ce}-1. Therefore, U is a function where m changes between 0 and 1. Finally, factor KU and factor THD can calculated using (13).

So, the evolution of KU factor and THD factor with the number of levels of the inverter can be calculated. It is shown in figures 10 and 11.

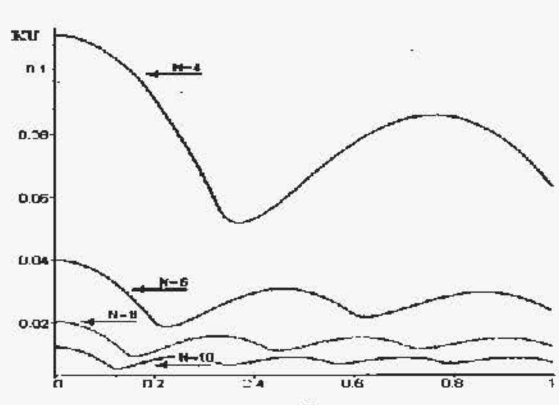

Fig 10. KU factor evolution for N even

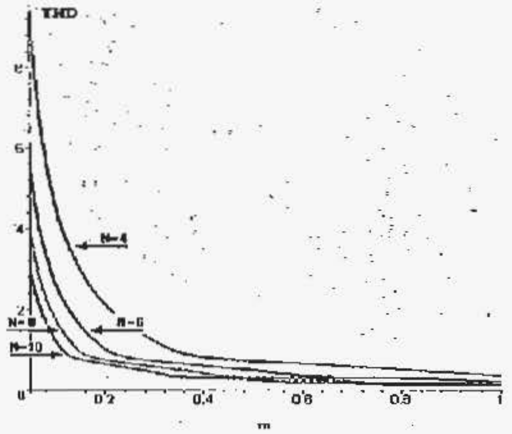

Fig 11. THD factor evolution for N even

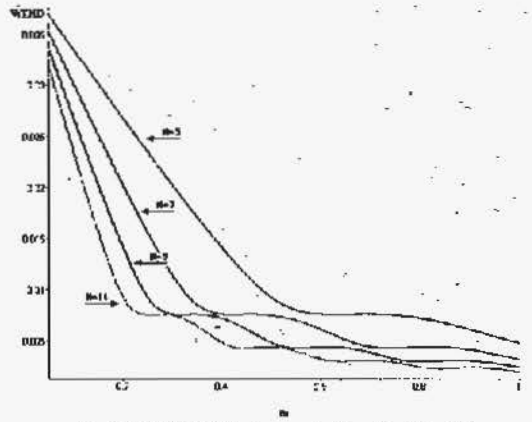

Fig 13. WTHD factor evolution for N odd

C. Calculation of KI factor

1. N odd

In general, the factors Ic_j can be calculated as

$$Ic_j = \frac{2}{\pi}\sum_{k=1}^{j}\int_{\beta_{k-1}}^{\beta_k}\left(\frac{\Delta I_k f_c L}{U_0}\right)^2 d\alpha$$

$$Ic_j = \frac{2}{\pi}\sum_{k=1}^{j}\int_{\beta_{k-1}}^{\beta_k}\left(\frac{2k(1-k)}{N-1}+\bar{u}(2k-1)-\frac{(N-1)\bar{u}^2}{2}\right)^2 d\alpha \quad (17)$$

with $j = 1 \ldots N_{ce}$.

These factors Ic_j exist in the interval $j/N_{ce}>m=(j-1)/N_{ce}$. So, the function I can be built as the sum of these parameters Ic_j with j=1, ..., Nce. Therefore, I is a function where m changes between 0 and 1. Finally, factor KI and factor can be calculated WTHD as

$$KI = \frac{2\pi I f_1^2}{3 fs^2} \qquad WTHD = \frac{\sqrt{2KI}}{m} \quad (18)$$

Using these formulas, the calculation of the parameters is very fast and easy. So, the evolution of KI factor and WTHD factor with the number of levels of the inverter can be calculated. It is shown in figures 12 and 13.

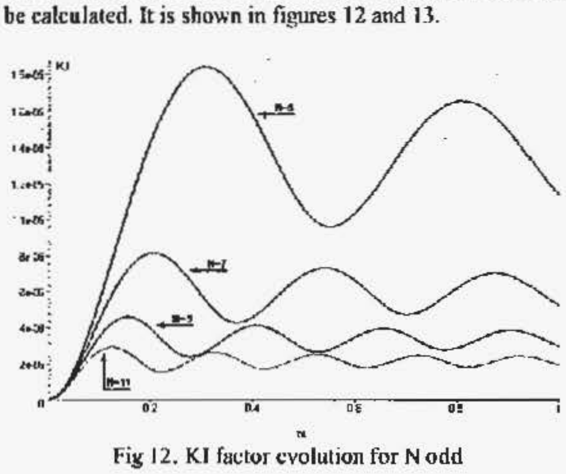

Fig 12. KI factor evolution for N odd

2. N even

In general, it can be used

$$Ic_j = \frac{2}{\pi}\sum_{k=0}^{j}\int_{\beta_k}^{\beta_{k+1}}\left(\frac{\Delta I_k f_c L}{U_0}\right)^2 d\alpha$$

$$Ic_j = \frac{2}{\pi}\sum_{k=0}^{j}\int_{\beta_k}^{\beta_{k+1}}\left(\frac{1-4k^2}{2(N-1)}+2k\bar{u}-\frac{(N-1)\bar{u}^2}{2}\right)^2 d\alpha \quad (19)$$

with $j = 0, \ldots, N_{ce}-1$.

The factor Ic_0 only exists in the interval $1/(N-1)>m=0$. For the other intervals, Ic_j exists in the interval $(2j+1)/(N-1)>m=(2j-1)/(N-1)$. The function I can be built as the sum of this Ic_j factors with j=0, 1, ..., $N_{ce}-1$. Therefore, I is a function where m changes between 0 and 1. Finally, factor KI and factor WTHD can be calculated with (18).

So, the evolution of KI factor and WTHD factor with the number of levels of the inverter can be calculated. It is shown in figures 14 and 15.

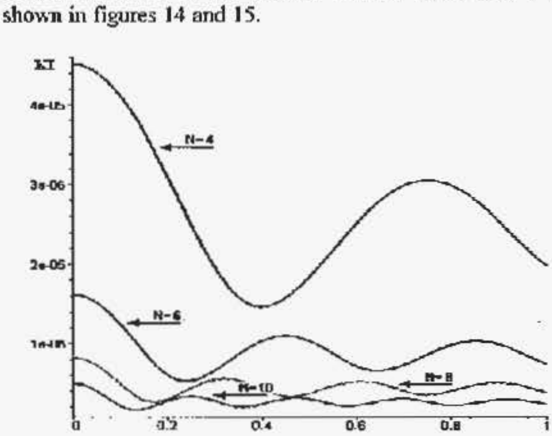

Fig 14. KI factor evolution for N even

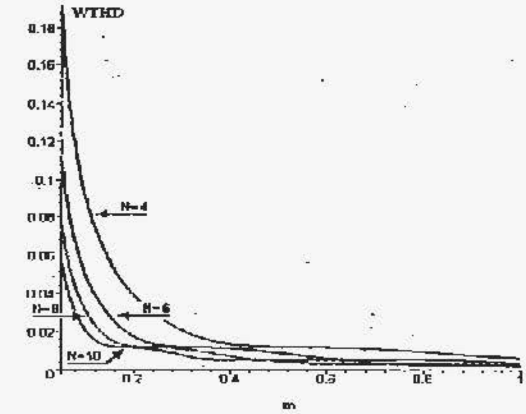

Fig 15. WTHD factor evolution for N even

III. COMPARISION BETWEEN N ODD/EVEN

In order to compare the THD and WTHD of the inverter with N odd and N even, several figures can be shown. THD factor is shown in figures 16 and 17.

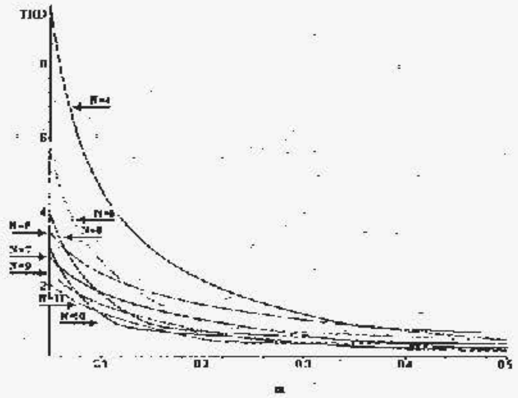

Fig 16. THD factor evolution with 0.05<m<0.5

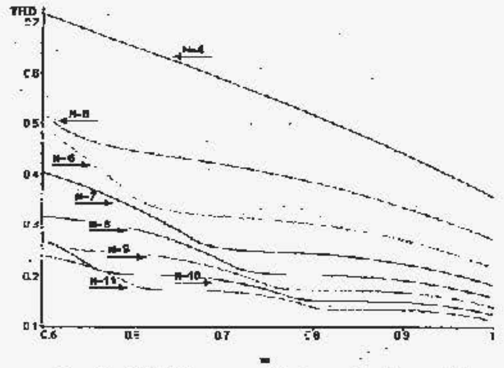

Fig 17. THD factor evolution with 1>m>0.5

It can be observed clearly that inverters with an even number of levels present THD factors greater than inverters with an odd number of levels when m is small. This phenomenon occurs due to the fact that inverters with N odd present zero vectors whereas inverters with N even do not present that kind of vectors. When m is small, these vectors make easy to follow the reference vector and the error is low.

When m grows this phenomenon loses importance and the evolution of THD factor is completely logical. Therefore, for example, THD factor with N = 6is greater than the THD factor with N= 7 and lower than THD factor with N= 5.

In the same way, it can be shown the evolution of WTHD factor with the number of levels. It is shown in figures 18 and 19.

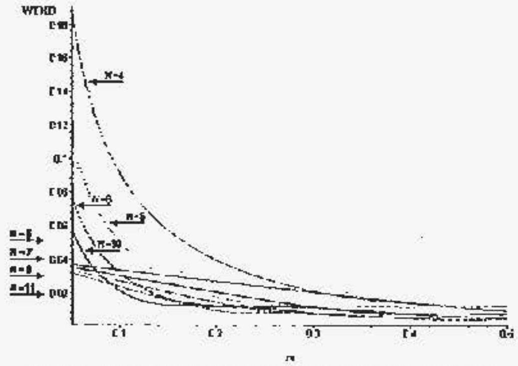

Fig 18. WTHD factor evolution with 0.5>m>0.05

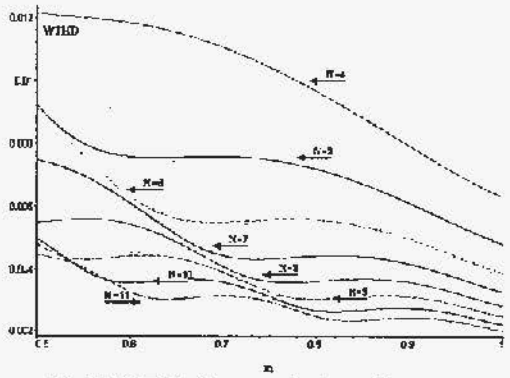

Fig 19. WTHD factor evolution with 1>m>0.5

In the same way on that it was commented previously, it can be observed clearly that inverters with an even number of levels present WTHD factors greater than inverters with an odd number of levels when m is small. Equally, when m grows this phenomenon loses importance and the evolution of WTHD factor is completely logical.

IV. THIRD AND FIFTH HARMONIC CONTENT

Now, it will be considered that the function u including harmonics. So, in general, $u = m[\sin(a) + k_3 \sin(3a+f_3) + k_5 \sin(5a+f_5)]$. Therefore, this study includes reference voltages with third and fifth harmonic content.

The evolution of the factors with harmonic content can be easily calculated using the same formulas commented before. As an example, third harmonic content will be considered. The evolution is represented in the plane m-k3. The results are shown in figures 20-23. It must be noticed that these curves include the figures 16-19 because the 2-D presented figures of THD and WTHD are the figures 20-23 with k3equal to zero. So, these 3-D parametric curves are the summary of the calculation.

V. CONCLUSIONS

In this work, a fast and easy method to calculate the THD and WTHD factors has been developed. This method is completely generalized and any number of levels can be studied. This calculation can be carry out in order to know the inverter specifications to fulfill the harmonics recommendation. Besides, the filtering reduction of the output signals can be done thanks to decreasing THD and WTHD harmonics . In this paper, it is shown that inverters with a number even of levels present THD and WTHD factors higher than inverters with a number odd of levels when m is small. It must be noticed that inverters with N>11 achieve harmonic parameters very same and it has not sense the use of inverters with more levels.

There are several practical uses for the method. Firstly, it can be determined the number of levels of a prototype in order to achieve the specifications of distortion knowing the switching frequency *fs*. Secondly, it can be determined the maximum switching frequency *fs* of a real prototype with N levels to fulfill the distortion specifications. Thirdly, the maximum modulation index *m* can be calculated knowing the specifications of the prototype (*fs, N*).

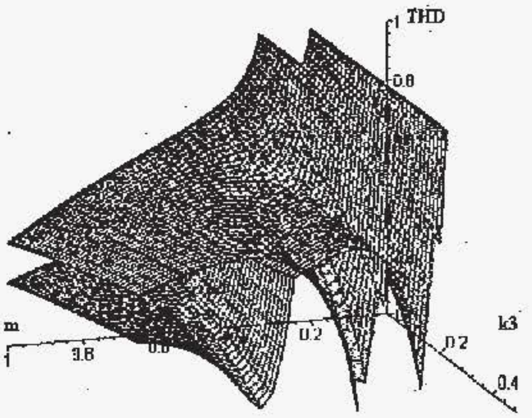

Fig 20. THD factor evolution with third harmonic content with N=4 and N=6

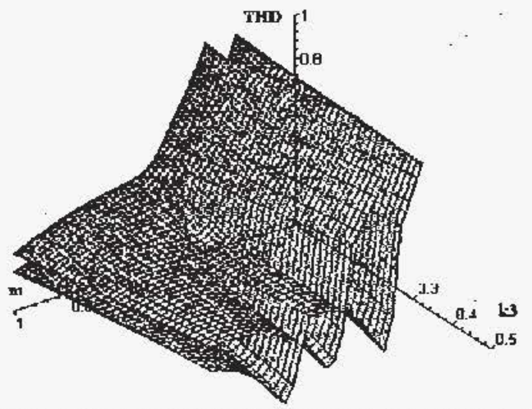

Fig 21. THD factor evolution with third harmonic content with N=5 and N=7

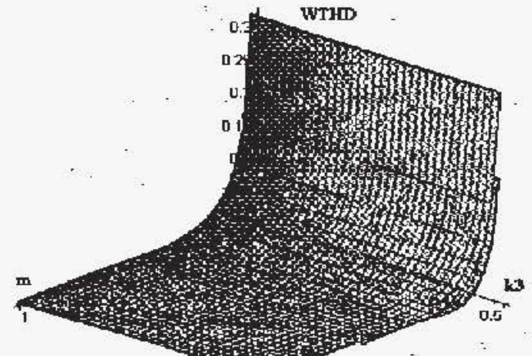

Fig 22. WTHD factor evolution with third harmonic content with N=4 and N=6

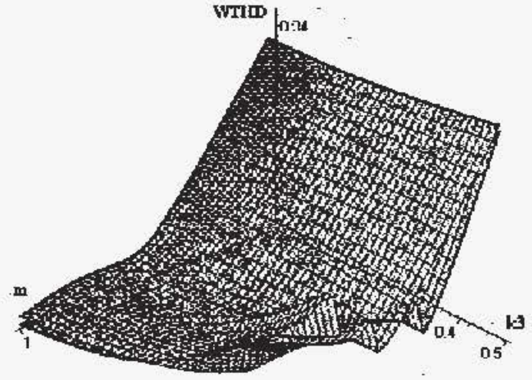

Fig 23. WTHD factor evolution with third harmonic content with N=5 and N=7

So, this method is a very useful tool to know a real prototype or to determine a possible prototype that fulfills the distortion specifications. Besides, the method includes the study of any possible harmonic content. The evolution of THD and WTHD factor can be easily shown.

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