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Abstract. Quantum nonlocality is typically assigned to systems of two or more well-separated particles, but nonlocality can also exist in systems consisting of just a single particle when one considers the subsystems to be distant spatial field modes. Single particle nonlocality has been confirmed experimentally via a bipartite Bell inequality. In this paper, we introduce an N-party Hardy-like proof of the impossibility of local elements of reality and a Bell inequality for local realistic theories in the case of a single particle superposed symmetrically over N spatial field modes (i.e. N qubit W state). We show that, in the limit of large N, the Hardy-like proof effectively becomes an all-versus-nothing (or Greenberger–Horne–Zeilinger (GHZ)-like) proof, and the quantum-classical gap of the Bell inequality tends to be the same as that in a three-particle GHZ experiment. We describe how to test the nonlocality in realistic systems.

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Contents

1. Introduction

Bell [1] and others [2] constructed a series of inequalities that are satisfied by pairs of particles submitted to measurements x (on particle A) and y (on particle B) with outcomes a and b, respectively, under the assumption that the joint probability distributions can be written as $P(x = a, y = b) = \sum_{\lambda} P(\lambda) P_A(x = a, \lambda) P_B(y = b, \lambda)$, where λ are pre-established classical correlations. However, experiments [3–8] testing these inequalities have consistently violated them, hence proving that it is impossible to fully describe the world with theories satisfying this assumption, called local realism.

Local realism is not the same as 'local elements of reality', defined as physical quantities whose outcomes can be predicted with certainty from outcomes of space-like separated measurements [9]. Both concepts are related, but the former has the advantage of being independent of predictions with certainty only existing in ideal experiments. Bell's original target was to prove the incompatibility between local elements of reality and quantum mechanics, but the violation of Bell's inequalities actually proves much more: it proves that the world is incompatible with local realism.

The Greenberger–Horne–Zeilinger (GHZ) proof [10] is a proof of no local elements of reality, which can be converted into a proof of no local realism through the violation of a Bell inequality [11]. This violation has been observed in the laboratory [12, 13]. In the case of the GHZ proof, the corresponding Bell inequality is maximally violated by the GHZ class of many-qubit entangled states. A striking fact is that the degree of violation increases exponentially with the number of particles.

Until relatively recently, nonlocality (i.e. the impossibility of local realism) was presumed to be a property of two (or more) well-separated particles. However, Tan *et al* [14] pointed out that nonlocality could, in principle, be determined via a Bell inequality test with a single photon in a superposition of two distinct spatial field modes. The fact that nonlocality could be considered an intrinsic property of a single excitation of a quantum field caused a flurry of discussion [15–18], the upshot of which was the experimental verification of entanglement of a single photon in two separate sites [20].

Here, we are concerned with the nonlocality generated by a single particle as it is symmetrically superposed over an increasing number of distant field modes to form the single particle implementation of an N-qubit W state [19]. First we prove the impossibility of local elements of reality and then we derive a Bell inequality to detect nonlocality experimentally.

In the proof of no local elements of reality, we see that for a small number of sites (around $N \leqslant 10$) the conflict with local elements of reality only occurs for a fraction of events. However, as the number of sites tends to infinity (or effectively $N \gg 10$), the conflict tends to occur for all events, in a similar way to that in a GHZ proof [12]. This is particularly surprising as the GHZ class is usually thought to be the only one capable of exhibiting this type of nonlocality. More interestingly, while it is impossible to create a GHZ state using less than three *particles*, here we show that a similar proof of no local elements of reality can be obtained for a state *with just one particle*.

We start in section 2 with a theoretical proof of no local elements of reality and then in section 3 we derive a Bell inequality to test nonlocality experimentally and explain how one may check for the nonlocality of a single photon over N sites in practice. In order to give full weight to our results, in section 4 we suggest how to implement the required measurements in realistic conditions. After the writing of this paper, we noted that two other papers have appeared that consider the nonlocality of a W state using our results as a starting point [21, 22]; all their results are consistent with our original findings.

2. Proof of no elements of reality for a single photon W state

We consider a system containing a single excitation, in this case a photon, which is symmetrically superposed over N sites. Each site represents a spatial field mode and we count the number of photons in each mode [23]. The state of the system is then

$$|\psi_{\text{W}}\rangle_N = \frac{1}{\sqrt{N}}(|100\cdots 0\rangle + |010\cdots 0\rangle + \ldots + |000\cdots 1\rangle), \tag{1}$$

where $|100\cdots 0\rangle$ indicates that the photon occupies the first site, while the rest are empty. This is just the single photon implementation of the N-qubit W state—we have chosen this implementation as it gives the most striking violation of local realism that is known in a system containing only one quanta.

Since only zero or one photon occupies each site at any instance, the outcomes $z_i = \pm 1$ of a Pauli, $\hat{Z}_i$, measurement applied to the ith site indicate whether the photon is present in that site or not, i.e. $\hat{Z}_i|0\rangle_i = |0\rangle_i$ and $\hat{Z}_i|1\rangle_i = -|1\rangle_i$. We will also consider local measurements in the Pauli $\hat{X}$ basis, whose outcomes $x_i = \pm 1$ correspond to finding the ith site in $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$ after the measurement.

2.1. Local elements of reality of a W state

In what follows, we will use some properties of the W state (points (i) and (ii) below) to derive a further measurement setting (point (iii)) whose outcome is fixed for models satisfying the criterion for local elements of reality proposed by Einstein *et al* [9]—*If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity*—but is incompatible with some predictions of quantum mechanics.

(i) According to quantum mechanics, state (1) has the following property:

$$P_{\psi_{\text{W}}}(\underbrace{z_i = \cdots = z_j}_{N-1\ \text{qubits}} = +1) = 1, \tag{2}$$

which is the probability of finding zero photons in $N-1$ sites (although we cannot tell which ones) when we measure $\hat{Z}$ on all sites. It does not matter that we do not know which $N-1$ sites will be empty until after the measurement (we will discuss this point further shortly). Here the photon is found in the kth site (which could be any of the N sites: $k \in 1, \ldots, N$), such that $P_W(z_k = -1) = 1$: it is thus impossible for the photon to be found on two or more sites simultaneously.

(ii) We will define a further $N-1$ properties of state (1), but since all such properties are similar, we will define first only one. The argument proceeds in a counterfactual manner. In the measurement setting defined by point (i) every site was measured in the local $\hat{Z}$ basis. However, we could have instead measured two of the sites in the Pauli $\hat{X}$ basis. We shall call these two sites the $k(\in 1, \ldots, N)$th site, which is the site the particle would have been found in upon the particle number measurement, and the ith site, which can be any other site. With this new measurement setting, the outcomes, x_k and x_i, are always correlated, such that

$$P_{\psi_W}(x_k = x_i | \underbrace{z_m = \cdots = z_r}_{N-2\,\text{qubits}} = +1) = 1. \tag{3}$$

That is, the conditional probability of the local $\hat{X}_k$ and $\hat{X}_i$ measurements resulting in the same outcome, given that the photon is not found in the remaining $N-2$ sites, is unity. Since we only have a single photon in the state, there are always $N-2$ sites containing no photons, so that we are conditioning on a property that is certain. For a model satisfying the criterion of local elements of reality to reproduce the W state $x_i = x_k$ must therefore hold.

The remaining $N-2$ properties have the same form as (3), always with one $\hat{X}$ measurement on the kth site, but with the other $\hat{X}$ measurement on a different site each time:

$$P_{\psi_W}(x_j = x_k | z_m = \cdots = z_r = +1) = 1, \quad \forall\, j \neq i,\, k. \tag{4}$$

The fact that (4) also holds follows, since we could have instead measured the $j(\neq i)$th site along with the kth site in the $\hat{X}$ basis to obtain $x_j = x_k$ and so on. Using the properties (2), (3) and (4) of the W state, we can conclude that the local outcomes, z and x, should have predefined values (corresponding to local elements of reality) for all sites before any measurement, since all of these statements occur with certainty.

(iii) We can now use the statements ((3) and (4)) about local elements of reality from points (i) and (ii) and the rules of classical probability theory to construct a logical argument that any theory satisfying the criterion of local elements of reality must satisfy, namely

$$P(\underbrace{x_1 = \cdots = x_N}_{N\,\text{qubits}}) = 1, \tag{5}$$

i.e. that an $\hat{X}$ measurement on all of the sites must result in outcomes that are equal. This follows, since from point (3) we can calculate

$$P_{HV}(x_k = x_i) = P_{\psi_W}(x_k = x_i | \underbrace{z_m = \cdots = z_r}_{N-2\,\text{qubits}} = +1) P(z_m = \cdots = z_r = +1)$$

$$+ P_{\psi_W}(x_k = x_i | z_m = -1\, z_n = \cdots = z_r = +1) P(z_m = -1\, z_n = \cdots = z_r = +1)$$

$$= 1 \times 1 + 0 = 1, \tag{6}$$

where $m \neq i \neq k$ and likewise for other pairs of sites. The term $P_{\psi_W}(x_k = x_i | z_m = -1\, z_n = \cdots = z_r = +1) P(z_m = -1\, z_n = \cdots = z_r = +1)$ is zero because the photon can be found in only one
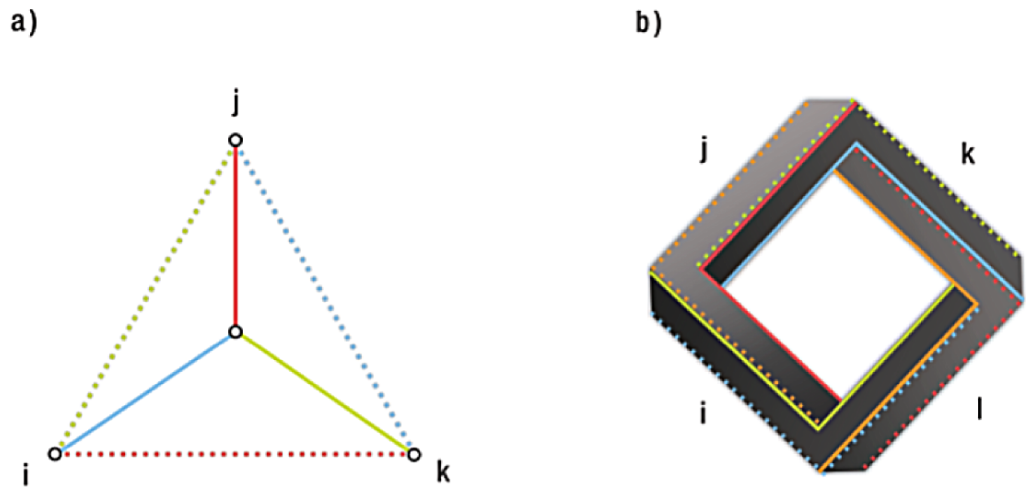
a)

b)

Figure 1. (a) Measurement outcomes inducing local elements of reality. The sites containing no photon upon a number measurement are labeled i, j and k and situated at the edges of the triangle and the site that would contain the photon is labeled l and sits in the middle of the triangle. A dotted line between two sites indicates that they both contain no photon. Given this, it is guaranteed that the remaining two sites are perfectly correlated in the $\hat{X}$ basis, which is represented by a solid line of the same color linking these sites. Models with local elements of reality always satisfy (5), since all of the sites are connected by solid lines. However, for four sites quantum mechanics violates this prediction half of the time. (b) Conflict with quantum mechanics. Here each edge represents a different site and the colors indicate different sets of photon number (dotted lines) and superposition basis (solid lines) measurements. Each measurement setting is itself consistent with local elements of reality. However, since the different observables do not commute, one cannot simply add the settings and expect to obtain perfect correlations for X measurements on all of the sites. The conflict with local elements of reality here is as transparent as the impossibility of this Penrose square.

site, which we have labeled k, and thus cannot also be in site $m \neq k$. For a pictorial depiction with four sites, see figure 1(a).

We now briefly discuss the use of counterfactuals in the derivation of (5). For simplicity, here we consider only three sites, but our reasoning is straightforward to generalize to any number of sites. In point (i), we illustrated the trivial property that upon a local particle number measurement of all sites, the photon would be found in only one site; we labeled that site k, which could be site 1, 2 or 3, and there is a probability of one third to be found in each site. We will now show why our argument holds no matter which site the photon is found in.

In the model with local elements of reality of three sites, we consider three rows of $\hat{Z}$ outcomes, namely

$$
\begin{aligned}
z_1 &= -1, & z_2 &= +1, & z_3 &= +1, \\
z_1 &= +1, & z_2 &= -1, & z_3 &= +1, \\
z_1 &= +1, & z_2 &= +1, & z_3 &= -1,
\end{aligned}
\tag{7}
$$

which are three different, mutually exclusive 'worlds' that we can work within to derive our prediction for these models. No matter which world we find ourselves in when we make a $\hat{Z}$ measurement on all of the sites, the same prediction (equation (5)) always follows. For instance, if the photon had been found in the second site, $P(z_2 = -1) = 1$, the properties $P(x_1 = x_2 | z_3 = +1) = 1$ and $P(x_2 = x_3 | z_1 = +1) = 1$ lead us to conclude that $P_{\mathrm{LER}}(x_1 = x_2 = x_3) = 1$. On the other hand, if the photon is found in the first site, $P(z_1 = -1) = 1$ (i.e. the first row of values above), we would obtain the same local realist prediction $P_{\mathrm{LER}}(x_1 = x_2 = x_3) = 1$ from the properties $P(x_1 = x_2 | z_3 = +1) = 1$ and $P(x_1 = x_3 | z_2 = +1) = 1$. That is why we remarked in point (i) that it does not matter which site the photon is found in after a $\hat{Z}$ measurement, as all the three 'worlds' lead to the same prediction.

2.2. Quantum mechanical result

Quantum mechanics can, however, contradict the prediction given by equation (5). The conflict for a three-qubit W state was studied earlier [24], and the outcomes of local $\hat{X}$ measurements on each of the sites were shown to disagree with the outcome for models with local elements of reality one quarter of the time. In this paper, we are concerned with how the probability of violating the local realist prediction equation (5) scales with the number of sites. Remarkably, as the single photon is superposed over an increasing number of sites, even though the average number of photons per site goes to zero, the probability of having a conflict exponentially approaches unity:

$$
\begin{aligned}
P_v^{(N)} &= 1 - P_{\psi_{\mathrm{W}}}(x_1 = \cdots = x_N) \\
&= 1 - \frac{N}{2^{N-1}}
\end{aligned}
\tag{8}
$$

(for instance, in the case of 20 sites the probability of having a conflict with local elements of reality is $P_v^{(N=20)} = 0.999962\ldots$). Thus, the outcomes of local $\hat{X}$ measurements on the state, $|\psi_W\rangle_N$, can *never* be completed by elements of reality for sufficiently large N.

For few sites (less than 10), ours is a Hardy-type proof [25]. However, in the limit of *many* sites, the W state created from a single photon behaves similarly to a GHZ state and surprisingly demonstrates, to our knowledge for the first time, an always–always–...–always–never contradiction. The 'always' clauses refer to the fact that for any zero photon measurement of $N - 2$ sites, the remaining two sites will always be correlated in the $\hat{X}$ basis. The 'never' clause, on the other hand, implies that the measurement of all sites in the $\hat{X}$ basis will never result in all outcomes being the same, in the large N limit (see [24] for a more detailed explanation of this notation). For all practical purposes our test is as non-statistical as the GHZ test, since no matter how good the measurement system is in the GHZ case, it will always have probabilities of success that are below e.g. $P_v^{(N=20)} = 0.999962$.

In particular, it is surprising that such a contradiction is obtained using the properties of a nonstabilizing state (i.e. a state without perfect correlations). Moreover, we emphasize that our result is true for any N-qubit W state, no matter how it is represented physically. Note that it has been shown in [26] that W states comprised of $N > 10$ qubits lead to more robust (against noise admixture) violations of local realism than do the GHZ states, indicating further that large W states have properties very different from those of small W states. Further investigation of the properties of large versus small W states would be fruitful.

3. Bell inequality for a single photon W state

To test this kind of nonlocality in actual experiments where the *perfect* correlations required to define local elements of reality are never achieved, we have to derive the N-party Bell inequality corresponding to the previous proof (of no elements of reality). This Bell inequality holds for any local theory (as defined in the introduction) and does not require the notion of local elements of reality.

For $N = 3$, the method in [27] shows that for any local realistic theory,

$$
\begin{aligned}
\beta = {} & P(z_1 = +1, z_2 = +1, z_3 = -1) + P(z_1 = +1, z_2 = -1, z_3 = +1) \\
& + P(z_1 = -1, z_2 = +1, z_3 = +1) \\
& - P(z_1 = +1, x_2 = +1, x_3 = -1) - P(z_1 = +1, x_2 = -1, x_3 = +1) \\
& - P(x_1 = +1, z_2 = +1, x_3 = -1) - P(x_1 = -1, z_2 = +1, x_3 = +1) \\
& - P(x_1 = +1, x_2 = -1, z_3 = +1) - P(x_1 = -1, x_2 = +1, z_3 = +1) \\
& - P(x_1 = +1, x_2 = +1, x_3 = +1) - P(x_1 = -1, x_2 = -1, x_3 = -1) \leqslant 0.
\end{aligned} \tag{9}
$$

The probabilities appearing in inequality (9) are exactly those involved in the argument of impossibility of local elements of reality. The Bell inequality is tight: there are local models that saturate the bound; for example, the one in which $z_1 = z_2 = x_1 = x_2 = x_3 = +1$ and $z_3 = -1$.

The single-photon W state (4) violates inequality (9). Specifically, it gives

$$
\beta_{\psi_W} = \tfrac{1}{4}, \tag{10}
$$

since

$$
\begin{aligned}
P_{\psi_W}(z_1 = +1, z_2 = +1, z_3 = -1) & = P_{\psi_W}(z_1 = +1, z_2 = -1, z_3 = +1) \\
& = P_{\psi_W}(z_1 = -1, z_2 = +1, z_3 = +1) = \tfrac{1}{3}, \\
P_{\psi_W}(z_1 = +1, x_2 = +1, x_3 = -1) & = P_{\psi_W}(z_1 = +1, x_2 = -1, x_3 = +1) = 0, \\
P_{\psi_W}(x_1 = +1, z_2 = +1, x_3 = -1) & = P_{\psi_W}(x_1 = -1, z_2 = +1, x_3 = +1) = 0, \\
P_{\psi_W}(x_1 = +1, x_2 = -1, z_3 = +1) & = P_{\psi_W}(x_1 = -1, x_2 = +1, z_3 = +1) = 0, \\
P_{\psi_W}(x_1 = +1, x_2 = +1, x_3 = +1) & = P_{\psi_W}(x_1 = -1, x_2 = -1, x_3 = -1) = \tfrac{3}{8}.
\end{aligned}
$$

This inequality can be generalized to any $N > 3$ as follows:

$$
\begin{aligned}
\Omega = {} & P(z_1 = +1, \ldots, z_{N-1} = +1, z_N = -1) + \cdots + P(z_1 = -1, z_2 = +1, \ldots, z_N = +1) \\
& - P(z_1 = +1, \ldots, z_{N-2} = +1, x_{N-1} = +1, x_N = -1) \\
& - P(z_1 = +1, \ldots, z_{N-2} = +1, x_{N-1} = -1, x_N = +1) - \cdots \\
& - P(x_1 = +1, x_2 = -1, z_3 = +1, \ldots, z_N = +1) \\
& - P(x_1 = -1, x_2 = +1, z_3 = +1, \ldots, z_N = +1) \\
& - P(x_1 = +1, \ldots, x_N = +1) - P(x_1 = -1, \ldots, x_N = -1) \leqslant 0.
\end{aligned} \tag{11}
$$

The state (4) violates inequality (11). Specifically,

$$
\Omega_{\psi_W} = 1 - \frac{N}{2^{N-1}}, \tag{12}
$$

which tends to one when N tends to infinity. A unity quantum-classical gap is characteristic of the violation of the three-party Bell inequality [11] (in terms of probabilities of elementary propositions [27]) by a GHZ state: there is a local model that mimics all but one of the quantum predictions, but the price for the existence of such a model is that it gives a prediction, $P_L(x_1 = +1, \ldots, x_N = +1) + P_L(x_1 = -1, \ldots, x_N = -1) = 1$, which is the opposite of the quantum one, $P_{\psi_W}(x_1 = +1, \ldots, x_N = +1) + P_{\psi_W}(x_1 = -1, \ldots, x_N = -1) \to 0$ as N increases. This shows that, in the limit of large N, the violation of the Bell inequality for the single-photon W state resembles the violation of the Bell inequality [11] by a three-qubit GHZ state.

To test the nonlocality of a single photon W state in an experiment, one needs to observe the violation of inequality (11). The test goes as follows. One first tests whether one has a single photon in the system by checking the probabilities $P(z_1 = +1, \ldots, z_{N-1} = +1, z_N = -1), \ldots, P(z_1 = -1, z_2 = +1, \ldots, z_N = +1)$. Then, one tests the probabilities $P(z_1 = +1, \ldots, z_{N-2} = +1, x_{N-1} = +1, x_N = -1), \ldots, P(x_1 = -1, x_2 = +1, z_3 = +1, \ldots, z_N = +1)$. Finally, one tests the probabilities $P(x_1 = +1, \ldots, x_N = +1)$ and $P(x_1 = -1, \ldots, x_N = -1)$. To ensure locality, these measurements should ideally be performed at a speed higher than that of any communication between the sites.

4. Experimental implementations

To implement our test, we need to first prepare a photon in a symmetric superposition of many distant sites and then make $\hat{X}$ and $\hat{Z}$ measurements on each of the N sites. We will describe below how to prepare this state with specific implementations, but here we note that they are, in general, considerably easier to create than the GHZ states. Measuring $\hat{Z}_i$ means detecting whether there is one photon in site i. A measurement in the $\hat{X}$ basis can be achieved by applying a Hadamard gate on the site and then measuring in the $\hat{Z}$ basis. Note that by performing these gates on each site we end up adding photons to the system. However, this addition is local and does not change the nonlocality of the system as a whole, which is solely due to the spread of the original photon.

The basic element of any experimental test of our work, therefore, has to be a unit that is capable of deterministically performing a Hadamard gate, i.e. creating a superposition of Fock states out of the vacuum. A scheme that achieves this is described in [28]. It relies on the selective manipulation of a three-level atom with an external classical field and the cavity mode, which is briefly pictured in figure 2. For particular choices of detuning Δ and coupling constants g and Ω, one can adiabatically eliminate the third level $|h\rangle$, obtaining an effective resonant coupling between the atom and the cavity mode that depends on the number of photons of the latter. In fact, this coupling can be tuned in such a way that only one chosen subspace of the entire Fock state basis, in our case $|0\rangle, |1\rangle$, is resonant with the $|g\rangle \to |e\rangle$ atomic transition. Then, the quantum circuit described in [28] can be used to deterministically rotate states $|0\rangle$ and $|1\rangle$, respectively, into states $(|0\rangle + |1\rangle)/\sqrt{2}$ and $(-|0\rangle + |1\rangle)/\sqrt{2}$.

Once one is able to implement a Hadamard rotation, this basic element can be used in a number of different ways to perform tests of nonlocality suggested in this paper. Here we briefly describe an implementation, which is well within the experimental state of the art.

It involves using an unbiased multiport beam splitter [31] to create the W state (here the aforementioned difficulty of preparing GHZ states is apparent; a GHZ state would require no photons in any port to be coherently superposed with one photon in every port!). Note
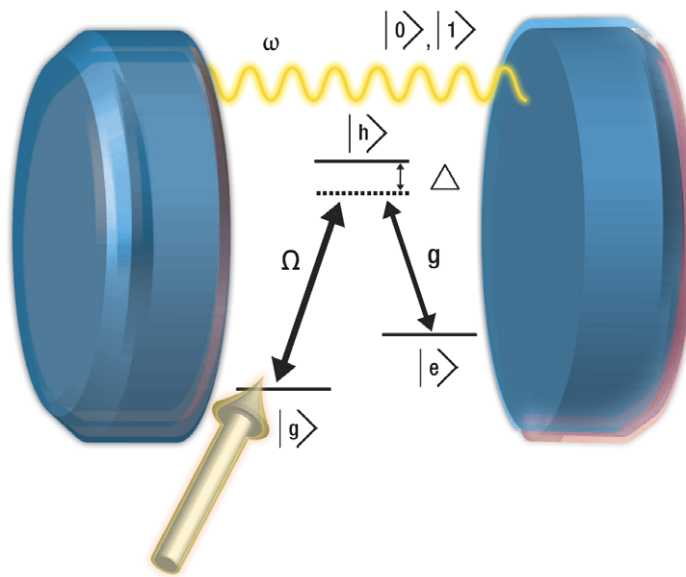
Figure 2. Hadamard implementing unit: the combination of a three-level atom and an external classical field allows for the Hadamard rotation in the Fock basis $|0\rangle, |1\rangle$ of the state of the cavity mode. The same atom can then be used to measure the state of the cavity field. When $\Delta \sim 10g \sim 10\,\Omega$, level $|h\rangle$ can be adiabatically eliminated and the off-resonant Raman transition between $|g\rangle$ and $|e\rangle$ will strongly depend on the number of photons of the cavity mode [29]. The transition can then be tuned to resonance for a given Fock subspace, in our case $|0\rangle, |1\rangle$, and the circuit described in [28] can be applied in order to implement deterministic rotations in this subspace as required for the measurement in the $\hat{X}$ basis.

that a heralded four-mode single-photon W state has recently been created via a sequence of beamsplitters [30], which would allow us to test the Hardy-type regime of our Bell theorem equation (8). To each port we couple an optical fiber, which guides the photon to a cavity [32, 33] containing the above-described unit Hadamard element. In this way, we are able to carry out both the $\hat{X}$ and $\hat{Z}$ measurements in each cavity and therefore test our violations of nonlocality. In order to guarantee the existence of the photon entering the multiport beamsplitter, one can generate a twin beam in parametric down-conversion and use one of the photons as the trigger for the experiment.

    Another method, utilizing unbalanced homodyne detection, has recently been put forward to test our proposal [34]. Within this scheme, a quantum efficiency of 69% is required for a single-photon, three-mode W state to exhibit a detectable violation of local realism. It would be interesting to investigate further how this efficiency scales with the number of sites.

    A much more challenging experiment would be to test the W nonlocality with one massive (instead of massless) particle. There is a longstanding lively debate [35–37] about the question of whether mode entanglement of massive particles can be used in tests of nonlocality, which is due to the presence of superselection rules that are not present in the photonic case. We hope that this paper will stimulate further research in this important direction.

5. Conclusions

In this paper, we have derived a Hardy-like proof of the impossibility of local elements of reality for an N-site single photon W state. We have shown that in the limit of a large number of sites this proof effectively becomes an all-versus-nothing proof, similar to the GHZ test of nonlocality. We have derived a Bell inequality that allows one to check experimentally for local realistic theories, and we indicate how this test could be implemented in realistic systems.

Acknowledgment

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