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DISCRETE MODEL DATA IN STATISTICAL PROCESS CONTROL

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1. Introduction

Control charts are widely used in industry as a tool to monitor process characteristics. Deviations from process targets can be detected based on evidence statistical significance. It can be said that the birth of modern statistical process control (SPC) took place when Walter A. Shewart developed the concept of a control chart en 1920's.Traditional attribute chart such as *p* and *c* charts are not suitable in automated high yield manufacturing and continuos production processes. Failure in the selection of the underlying distribution can result incorrect conclusions regarding the statistical control of a process. Traditionally, standard statistical control charts for a discrete random are based on Poisson or binomial distributions. This paper presents the latest development of statistical control charts for a shifted geometric distribution.

2. CCC charts

The cumulative count of conforming chart (CCC) is a powerful technique for process control when a large number of consecutive conforming items are observed between two nonconforming ones. A control chart can be set up to monitor this number and decisions made based whether this number is too large or small. The CCC chart is very useful for one at a time inspections or tests which are common in automated manufacturing process. It is a technique for high quality processes when nonconforming items are rarely observed. High quality process are usually associated with low counts of nonconforming items. Calvin(1983) firstly studied the CCC charts to monitor zero defects processes. The use of CCC type control chart has been further studied by Lucas(1989), Bourke(1991), Kaminsky et al(1992), Xie and Goh(1995), Ermer(1995), Glushkovsky(1994), Xie et al.(1995) and Joyner-Motley(1998), Wu et al(1999, 2000), Kuralmani et al(2002), among others.

3. Average Run Length

The run length of a chart is the number of samples a prior to observing an out of limits point on the chart. Since it is assumed that the data are generated by underlying probabilistic model, the run length is a random variable. Depending on the actual state of the system, we can suppose the process is in-control run length (RLIN) or out of control run length (RLOUT). If we have enough historical process data, we can consider μ_0 and σ known, we can suppose the run length in a in-control process is distributed as a geometric distribution, RLIN~G(α), and the run length in a out.process is distributed, ROUT~G(1-β),where α is the probability a false alarm and β is the probability of not detecting a shift at a sampling time.

The Average Run Length (ARL) is a property of a control chart. The ARL is the expected number of runs to an alarm and is expressed depending of the case

$$\text{ARL(IN)}=\frac{1}{\alpha}$$

$$\text{ARL(OUT)}=\frac{1}{1-\beta}=\frac{1}{1-(1-p)^{LCL}+(1-p)^{UCL}}$$

The standard deviation of the run length SRL(IN) is defined as

$$\text{SRL(IN)}=\frac{\sqrt{1-\alpha}}{\alpha}$$

Therefore, the standard deviation and the average of the run length (a number of a consecutive units between two successive alarm signals) distribution will be very close if α is small.

And SRL(OUT) as

$$\text{SRL(OUT)}=\frac{\sqrt{\beta}}{1-\beta}$$

So, SRL(OUT)≈ARL(OUT) for a large ARL(OUT).

The lower and upper control limits of the CCC charts are given by

$$UCL=\frac{\ln(\alpha/2)}{\ln(1-p)}$$

$$LCL=\frac{\ln\left[1-\alpha/2\right]}{\ln(1-p)}$$

The control limts are highly asymmetric. The log-scale can be used for the plotting. The CCC chart is a powerful charting technique when the process is near ZD and samples are mosttly conforming, and also is useful for detecting process improvement, which conventional techniques would fail even with the common rules.The resolution of the CCC chart on a log-scale is higher when the process quality is higher.

Xie and Goh(1992) introduced the concept of certainty level *s*, which is the probability that the process is actually out-of-control. The certainty level is related to false alarm probability when interpreting CCC chart signal. The relation between the proportion of nonconforming items *p* and number of items inspected *n*, which is given as, $(1-p)^n=s$, The number of conforming items inspected before a nonconforming one is allowed for the process to still be considered in control can be expressed $n=\ln s/\ln(1-p)$.

The figure 1 facilities decisions on the state of control of a process whenever a non conforming item is observed

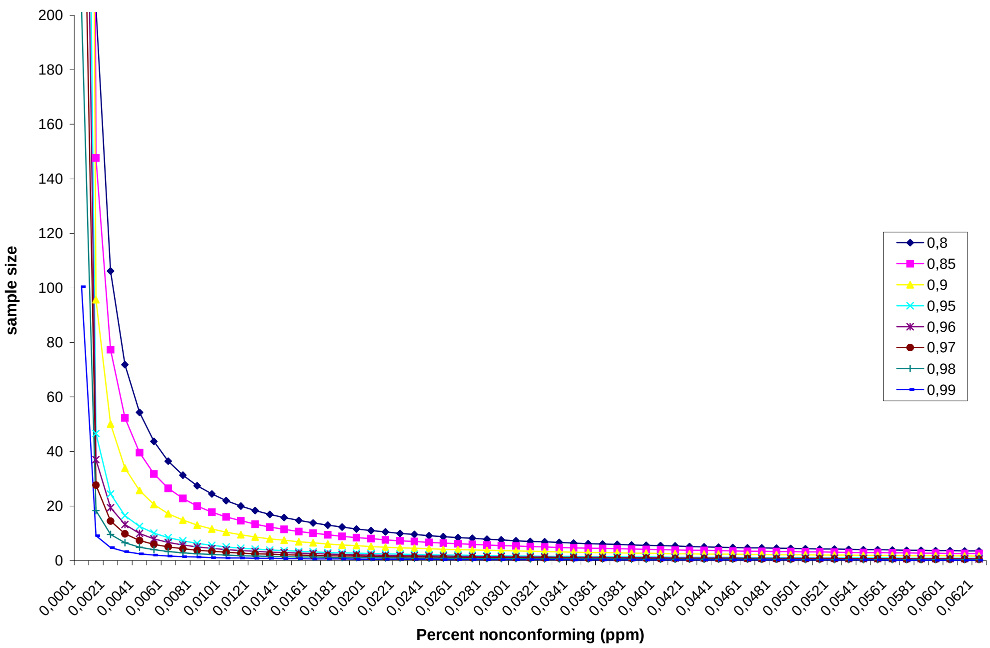

Figure 1. States of control of a process.

For a high quality products, a good approximation of certainty level is, $s=e^{-np}$

In the case grouped data are considered, three sigma limits can be used to count data. Assuming the underlying distribution to be geometric, if we have a subgroup of size *n*, then the total number of counts in the subgroup, $X=\sum_{i=1}^{n} X_i$ ~Negbin(*n*,*p*). In general, based on the conventional idea of k-sigma, control limits for $Z=X-n$, is given as

$$ULC=\frac{n(1-p)}{p}+k\sqrt{\frac{n(1-p)}{p^2}}$$

$$LCL=\frac{n(1-p)}{p}-k\sqrt{\frac{n(1-p)}{p^2}}$$

where $p<1-\frac{k^2}{n}$.

Although the geometric charts are shown to be suitable for the monitoring of high quality process, it has also problems since the standard assumptions are similar to those of the traditional Shewart chart. One of the problems is relationated with the average run length. It is due to the skewness of geometric distribution, the ARL curve does not have its maximum at the process level used to compute the control limits, so the process is shifted slightly and it will take a longer time to raise an alarm. Xie et al. (2000) proposed an optimization method to obtain the maximum ARL at the desired value of *p*. The control limits obtained are:

$$LCL = \gamma_\alpha \frac{\ln(1-\frac{\alpha}{2})}{\ln(1-p_0)}; \qquad UCL = \gamma_\alpha \frac{\ln(\frac{\alpha}{2})}{\ln(1-p_0)}$$

where γ_α is a adjustment factor, a function of false alarm probability, and its expression is $\gamma_\alpha = \dfrac{\ln\left[\dfrac{\ln(1-\alpha/2)}{\ln(\alpha/2)}\right]}{\ln\left[\dfrac{\alpha/2}{1-(\alpha/2)}\right]}$

Kuralmani et al (2002) proposed a conditional procedure making use of the information earlier counts. This procedure operates as follows. We counts the conforming units until a conforming unit is found. Then it is concluded that the process is in control is.

1. the count of conforming units are within the lower and upper control limits or
2. *h* previous runs were in control even if the count of conforming units are not within the control limits.

For a fixed value of *h* and a prescribed false alarm probability α the conditional limits are given as

$$LCL_{cond} = \frac{\ln[1-(1-\delta)/2]}{\ln[1-p]}; \qquad UCL_{cond} = \frac{\ln[(1-\delta)/2]}{\ln[1-p]}$$

where δ is the solution of the following equation

$$1-\alpha = \delta + (1-\delta)\delta^h$$

When *h* approaches infinitely, δ will aproach 1-α, which means that the CCC chart is an asymptotic case of the conditional procedure.

To compare the ARL's between the conditional, optimun conditional and traditional charts, graphs are drawn in figure 2.

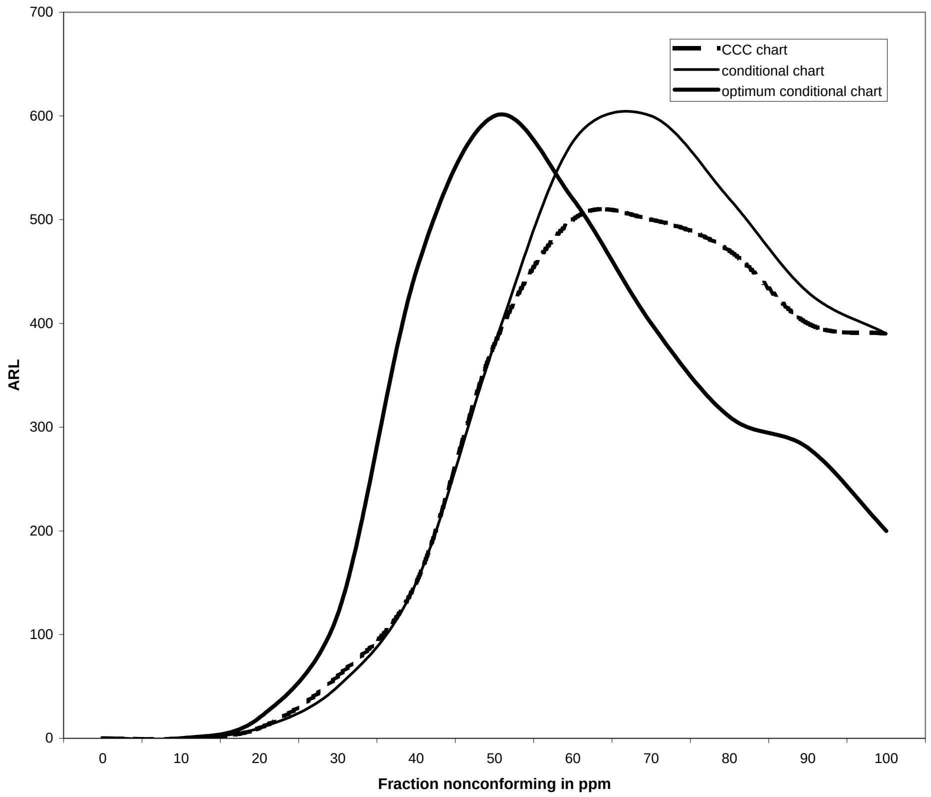

Figure 2. ARL curves of conditional and CCC charts for p=50 ppm.

The ARL curve is almost shifted towards to the left in order to achieve maximun ARL at the desired process average. The maximun of the average run lengh peaks at just p=50 ppm An alarm will be raised quickly no matter the direction the process shifts towards from this value.

4.Conclusions

In this paper, the importance of underlying distribution in SPC data is studied. Geometric distributions is useful in manufacturing industries. It has been exposed traditional methods based on a single count, it is relatively insensitive to process shifts. In addition optimal limts are defined in such a way that the average run lengh becomes maximun when the process average is at the nominal level. These methods could be easily implemented in manufacturing industry. This method is highly suited to automatic production environments where the products are inspected one after another and the count of conforming is accumulated automatically.

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