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Poverty Measures and Anti-Poverty Policy

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Poverty Measures and Anti-Poverty Policy

Abstract

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This paper is concerned with one of these areas, namely, the measurement of poverty and the implications for anti-poverty policy. In the 1960's and 1970's those who were working in the poverty field held a number of somewhat incompletely articulated views as to the extent of poverty in an economy. One was the judgment that a country is poorer the larger is the number or fraction of its people below an agreed-upon poverty standard. Second, the severity of poverty depends on how poor the poor are. As formulated then, the larger is the average income shortfall among the poor, the more severe is poverty. Thirdly, it was recognized that some of the poor are poorer than others, and the extent of poverty should also depend on the distribution of income among the poor.

Keywords

poverty, famine, development, public policy

Disciplines

Economics | Income Distribution | Labor Economics | Labor Relations | Public Policy | Social Policy | Social Welfare

Comments

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Poverty Measures and Anti-Poverty Policy

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1 Introduction

Amartya Sen has made fundamental contributions to the study of distributional aspects of economic growth and decline. Among his pathbreaking works are his lectures on the economics of inequality (Sen, 1973), his article on the axiomatics of poverty measurement (Sen, 1976), and his book on anti-poverty policy in the context of famines (Sen, 1981)¹.

This paper is concerned with one of these areas, namely, the measurement of poverty and the implications for anti-poverty policy. In the 1960's and 1970's those who were working in the poverty field held a number of somewhat incompletely articulated views as to the extent of poverty in an economy. One was the judgment that a country is poorer the larger is the number or fraction of its people below an agreed-upon poverty standard. Second, the severity of poverty depends on how poor the poor are. As formulated then, the larger is the average income shortfall among the poor, the more severe is poverty. Thirdly, it was recognized that some of the poor are poorer than others, and the extent of poverty should also depend on the distribution of income among the poor.

In a justly-celebrated paper, Sen (1976) made two enduring contributions to poverty analysis. The first was to show that these three considerations could be combined into a single poverty measure

$$S = H[I + (1 - I)G] \quad (1)$$

where S is the Sen poverty measure, H is the headcount ratio (*i.e.*, the fraction of the population with incomes below an agreed-upon poverty

* We thank an anonymous referee for useful comments.

¹ Some of his astonishingly prolific output is collected in two volumes (Sen, 1982 and 1984).

line z), I is the average income shortfall among the poor (*i.e.*, the average amount by which their incomes fall below the poverty line, expressed as a percentage of the poverty line itself), and G is the Gini coefficient of income inequality among the poor. Sen's other contribution in that same paper was to show that the particular components of the poverty measure (H , I , and G) and the particular functional form used to combine them (eq. 1) are not arbitrary but may instead be derived from fundamental axiomatic judgments.

Sen's approach to the measurement of poverty spawned a large literature in the 1970's and the 1980's. One group of researchers has generalized the Sen index and analyzed the properties of the extended class (Anand, 1977; Thon, 1979; Kakwani, 1980 and Blackorby and Donaldson, 1980). Another group of researchers has modified Sen's ideas and constructed and developed a related but distinct class of measures, known as the P_α class (Foster, Greer, and Thorbecke, 1984).

For the policy-maker, the availability of several poverty measures poses a practical problem. Supposing that he were inclined to use an anti-poverty budget to minimize poverty, how should he spend it? Would the optimal allocation of the anti-poverty budget depend upon whether the poverty measure to be minimized is H , I , S , or P_α ? If the preferred allocation is sensitive to the choice of poverty measure, and we show in this paper that it is, then policy-makers (and those who advise them) must first decide what particular poverty measure they are seeking to minimize. Then and only then will they be in a position to evaluate the effects on poverty of alternative resource allocations.

In this paper, we derive two fundamental results regarding the Sen poverty index and its variants². First, we show that although the Sen index is decisive in any particular set of circumstances (meaning that it can always be used to decide how best to allocate a given anti-poverty budget) the main feature of the poverty-minimizing allocation varies from one circumstance to another. Sometimes the optimal allocation is to spend all the money to raise the incomes of the poorest of the poor, other times it is best to use all the money to lift as many people out of poverty as possible, and other times a mixture of these two policies is best. Second, we show that although the Sen index is decisive, the answer it gives on how best to spend an anti-poverty budget does not always accord with the answer given by other poverty measures. From this point of view, the choice of a poverty measure,

²From the point of view taken in this paper, the variants of the Sen index available in the literature have the same properties as the Sen index itself. Even when not explicitly mentioned, all the results given for the latter also apply to the former.

or, equivalently, of the social judgement axioms behind it, is indeed consequential. The last section of the paper shows that the apparently innocuous 'normalization axiom' imposed by Sen is responsible for the divergence between Sen's measure —and its variants— and the P_α class, and implicitly departs from the conventional utilitarian approach to income inequality and poverty measurement.

2 Poverty Measures and the Optimal Anti-Poverty Budget Allocation Problem

Suppose a policy-maker has available a budget of T for use in reducing poverty. We assume that the finances for this budget have already been raised by a process which need not concern those who decide how to spend the available money.

A dollar amount z defines how many of the n persons in the economy are poor. Let q be the number of poor and let (y_1, y_2, \dots, y_q) be the income distribution among them. Without loss of generality, we index them

$$y_1 \leq y_2 \leq \dots \leq y_q$$

With the poverty line $z (> y_q)$, a poverty measure is defined as a function

$$P(q; y_1, y_2, \dots, y_q)$$

The optimal allocation policy corresponding to a poverty measure P is then given by the following program:

$$(P) \quad \begin{cases} \min_{(t)} P(Q; y_1 + t_1, y_2 + t_2, \dots, y_Q + t_Q) \\ \text{subject to} \\ t_i \geq 0 \quad \forall i = 1, 2, \dots, q \\ \sum_i t_i = T \\ y_i + t_i \geq z \quad \forall i = Q + 1, \dots, q \end{cases}$$

That formulation makes it clear that any anti-poverty budget allocation involves a drop in the number of poor (Q is necessarily below or equal to q) and/or an increase in the income of those who remain poor. This feature is of central concern in the present paper.

Note that all transfers in program (P) are required to be non-negative. In other words, we assume that the anti-poverty budget is financed either by foreign aid or by taxation and that the only concern

of the policy-maker is about how to spend it. In the case where it was raised through taxation, such negative transfers may be implicitly accounted for by considering that (y_1, y_2, \dots, y_q) are post-tax incomes.

Note also that program (P) is based on the assumption that *poverty targeting* is perfect in the sense that policy-makers may transfer any amount of income to any specific individual in the population of poor. From that point of view, the present analysis departs from the related literature on *imperfect targeting* (see Kanbur, 1987)³.

The simplest measures of poverty are the headcount ratio

$$H = \frac{q}{n}$$

and the average relative income shortfall:

$$I = \frac{z - \bar{y}_q}{z} \equiv \frac{1}{q} \sum_{i=1}^q \frac{z - y_i}{z} \equiv \frac{1}{q} \sum_{i=1}^q \frac{g_i}{z}$$

where g_i is the absolute shortfall of the i^{th} individual. Both measures H and I may be combined into the poverty gap, HI . This measure represents the fraction of the poverty line, z , that would have to be spent per head of the (whole) population to eliminate poverty.

The Sen measure combines H , I , and G (the Gini coefficient associated with the (y_1, y_2, \dots, y_q) distribution) to obtain:

$$S = H[I + (1 - I)G].$$

Another set of measures is that proposed by Foster, Greer and Thorbecke (1984). In a discrete setting it simply writes:

$$P_\alpha = \sum_{i=1}^q \frac{(z - y_i)^\alpha}{z^\alpha}$$

where α is a positive coefficient.

For α larger than one the FGT measure may be interpreted as the sum of the individual relative poverty shortfalls, g_i/z weighted by a function describing normatively the intensity of poverty $((z - y_i)/z)^{\alpha-1}$.

The measures H and HI are special cases of the Sen and P_α measures. It may easily be checked that $P_0 = H$ and $P_1 = HI$. One also sees that HI corresponds to the Sen index in the case of zero income inequality among the poor (i.e., $y_1 = y_2 = \dots = y_q$).

³ Although the issues dealt with here are of obvious relevance for the *imperfect targeting* case.

3 Optimal Anti-Poverty Allocation with H , HI and P_α

Interestingly enough, the solution of program (P) yields radically different results depending on what poverty measure is used. At one extreme, stands the headcount ratio H , and the P_α measures with $\alpha < 1$. For these measures, the optimal allocation policy consists of transferring T to the richest persons among the poor, so as to have as few people as possible remaining in poverty. Accordingly the optimal allocation policy is given by:

$$\begin{aligned}
 t_i &= z - y_i & i &= Q + 1, \dots, q \\
 t_Q &= T - \sum_{i=Q+1}^q (z - y_i) \\
 t_i &= 0 & i &= 1, 2, \dots, Q - 1
 \end{aligned}$$

with Q the *optimal* number of poor being given by:

$$\sum_{i=Q}^q (z - y_i) \leq T \leq \sum_{i=Q+1}^q (z - y_i) \tag{2}$$

Note, moreover, that the positive transfer requirement prevents some additional redistribution from the poorest among the poor to the remaining richest ones. Such a transfer, if it took place, would reduce the headcount ratio, H , even more. As for the P_α measure, with $\alpha < 1$, it is a concave decreasing function of individual incomes. It follows that total poverty is more sensitive to a dollar given to the least poor than to somebody lower in the poverty scale.

We shall term the type of anti-poverty policy that transfers all of the available budget to the richest of the poor a “Type-r policy”. Plotting post-transfer against pre-transfer incomes as in Figure 1 yields a very characteristic schedule: the 45° line up to some income level and then a horizontal segment. Note, on the other hand, that this shape does not depend on the initial distribution of incomes or the budget to be allocated. Both these variables only determine the critical level of income above which transfers are positive — condition (2) above.

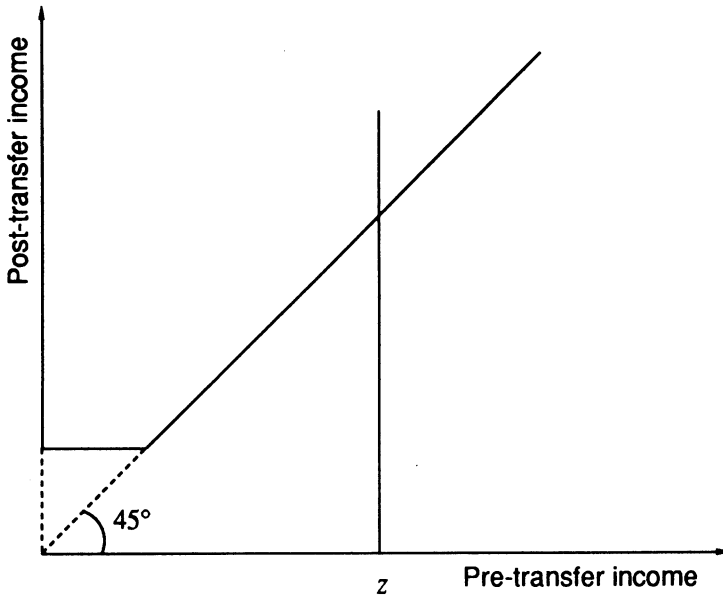


Figure 1
Optimal transfers with H or P_α ($\alpha < 1$)

We have thus shown :

Proposition 1. *For the headcount index H or for the P_α class with $\alpha < 1$, the optimal allocation of an anti-poverty budget is a Type-r allocation.*

The “Type-p policy” does the opposite. Only the poorest of the poor receive a transfer, which brings them all up to the same income level, still below the poverty line (see Figure 2).

It may be shown that a Type-p policy is optimal for the P_α measure for all α strictly larger than unity. The reason is that P_α now is a decreasing convex function of individual incomes more sensitive to transfers at the bottom than at the top of the distribution. With budget T , the optimal allocation will then be given by:

$$\begin{aligned}
 t_i &= y^* - y_i & i &= 1, 2, \dots, p \\
 t_i &= 0 & i &= p + 1, \dots, q
 \end{aligned}$$

with y^* and p given by:

$$\sum_{i=1}^{p-1} (y_p - y_i) \leq \sum_{i=1}^p (y^* - y_i) = T \leq \sum_{i=1}^p (y_{p+1} - y_i). \tag{3}$$

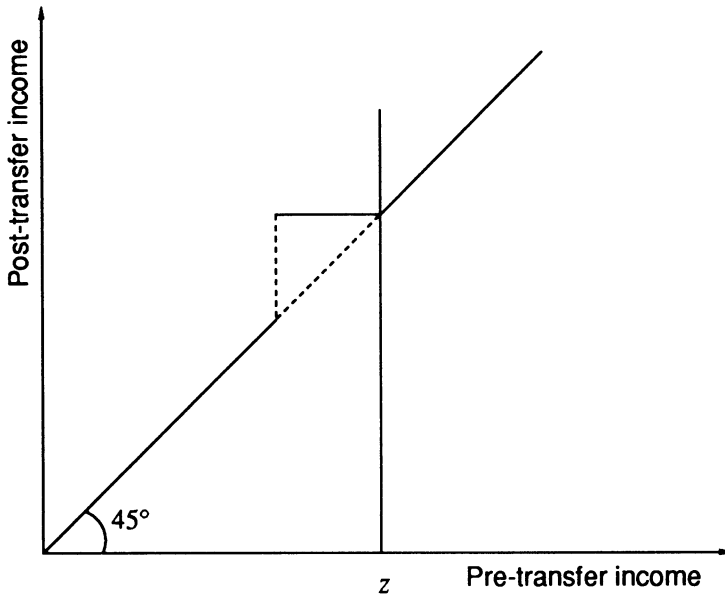


Figure 2
Optimal transfers with P_α ($\alpha > 1$)

This shows :

Proposition 2. *For the P_α class with $\alpha > 1$, the optimal allocation of an anti-poverty budget is a Type-p allocation⁴.*

For the 'poverty-gap' index HI , the optimal policy is indeterminate. Define a "poverty-efficient allocation" of an anti-poverty budget to be one where all transfers are made to persons who ex ante were poor and, for all persons i , the amount of the transfer $t_i \leq z - y_i$. In other words, when a poverty-efficient allocation is made, no money is "wasted" on persons with incomes above the poverty line. From the definition of H and I , it may be checked that the poverty gap depends on the total income of those below the poverty lines and not on their number as such. It then follows :

Proposition 3. *HI is reduced by the same amount for all poverty-efficient allocations.*

⁴ Clearly, a Type-p allocation is also optimal for the average relative shortfall measure (I). However, it results in the same reduction in poverty as any other allocation that maintains the number of poor constant.

Proposition 3 has two consequences. First, it suggests that the aggregate poverty gap measure HI offers no guidance on how to allocate an anti-poverty budget: as long as no poor person receives a transfer that would put him more than above the poverty line, any allocation of the budget — whether Type-r, Type-p, or mixed — will lead to exactly the same amount of reduction in poverty by the HI measure. Second, HI is a component of the Sen index and of other possible poverty measures. The fact that this component changes by equal amounts for all poverty-efficient allocations implies that in determining the optimal anti-poverty allocation, we need only direct our attention at the remaining components — in the case of the Sen index, for example, only at $H(1 - I)G$.

4 Optimal Anti-Poverty Policy with the Sen Index

We have distinguished three possible allocations of an anti-poverty budget:

- **Type-r:** Allocate all the money to the richest of the poor, thereby reducing the headcount ratio the most.
- **Type-p:** Allocate all the money to the poorest of the poor, thereby reducing the average income shortfall and the Gini coefficient among the poor the most.
- **Mixed allocation:** Allocate a (strictly positive) fraction of the money to the richest of the poor and another (strictly positive) fraction of the money to the poorest of the poor.

In this section, we show that all three types of allocation may be optimal when the poverty measure being used is the Sen index (or known generalizations of it).

The Sen index is given by

$$S = H[I + (1 - I)G].$$

The presence of G in this formula makes it difficult to determine analytically how the extent of poverty varies with the allocation of the anti-poverty budget. The various possibilities are more easily demonstrated using special cases

Example One

Consider first the case in which q_1 of the persons below the poverty line (z) have incomes y_1 and the remaining q_2 of the poor have incomes y_2 . Assume $y_1 < y_2 < y_z$ and let $q_1 + q_2 = q$. Normalize the total population n so that $n = 1$. We shall now show:

Proposition 4. *In the two-income case, the optimal allocation of the anti-poverty budget using the Sen index is a pure allocation, either Type-r or Type-p. A mixed allocation is never optimal.*

Proof: For a population with two incomes y_1 and y_2 with corresponding frequencies f_1 and f_2 , the Gini coefficient is given by

$$G = \frac{f_1}{f_1 + f_2} - \frac{f_1 y_1}{f_1 y_1 + f_2 y_2}$$

Applying this to the case of two-incomes within the poverty group and substituting into the formula for the Sen index, we obtain

$$S = HI + \frac{q_1 q_2}{q_1 + q_2} \frac{y_2 - y_1}{z}$$

Suppose the anti-poverty budget is $\$T$. This budget may be expressed in units of p , *i.e.*, the number of people (in group 2) who *could* be lifted out of poverty. Thus, $T = (z - y_2)p$. It is assumed that $p < q_2$ ⁵. Let the number who actually *are* lifted out of poverty by a particular budgetary allocation be denoted by i . Then, the value of the Sen index is $S = HI + Y(i)$, where

$$Y(i) = q_1 \left(\frac{q_2 - i}{q_1 + q_2 - i} \right) \frac{1}{z} \left[y_2 - \left(y_1 + \frac{(z - y_2)(p - i)}{q_1} \right) \right]$$

We have shown that HI is reduced by the same amount for any efficient allocation (See Proposition 3). The problem therefore comes down to reducing $Y(i)$ the most by choice of i :

$$\min_{(i)} Y_i = \left[\frac{1}{z} \frac{q_2 - i}{q_1 + q_2 - i} \right] [q_1(y_2 - y_1) - (z - y_2)(p - i)]. \tag{4}$$

A plot of the components of Y_i is shown in Figure 3 for alternative choices of i . The first bracketed term yields a downward-sloping curve (C) which begins positive and turns negative at $i = q_2$. By assumption, $q_2 > p$, and since $i \leq p$, it follows that only part of the positive portion is relevant. The second bracketed term is an upward-sloping straight line (L), the intercept of which may be positive, zero, or negative as $\{q_1(y_2 - y_1) - (z - y_2)p\} \geq 0$.

⁵ If p were larger than q_2 , the only issue would be whether to allocate the first q_2 unit of the budget to the richest (group 2) or the poorest of the poor (group 1), the allocation of the remaining $(p - q_2)$ units being trivial.

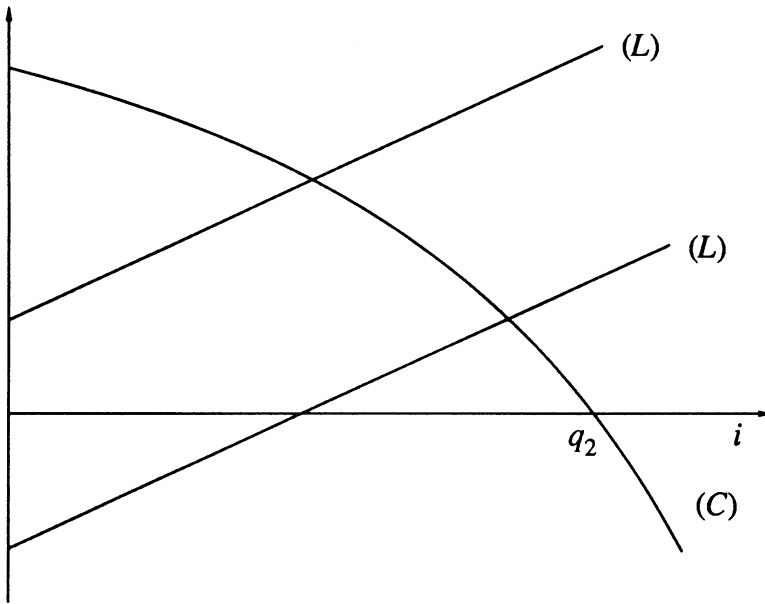


Figure 3

Analysis of the graph yields the following results :

- i) If $\{q_1(y_2 - y_1) - (z - y_2)p\} < 0$ the optimal number to lift out of poverty, i^* , is zero. In this case, the largest possible poverty reduction is found by choosing i so that the first bracketed term in (4) is the most negative and the second the most positive. This occurs when $i^* = 0$.
- ii) In the case where $\{q_1(y_2 - y_1) - (z - y_2)p\} \geq 0$, it is shown in the appendix that $Y(i)$ is a convex function of i so that the optimal policy is always $i^* = 0$ or $i^* = p$.

Type-r and type-p allocations are thus the only optimal allocations⁶. □

As could be expected, the preceding conditions — and those reported in the appendix — show that the optimal allocation will be of

⁶ Note that this is true only if the anti-poverty budget happens to be an integer multiple of $(z - y_2)$. If it is not the case, the optimal policy may consist of lifting at the poverty line as many individuals from group 2 as possible and allocating the remaining budget to group 1. We ignore here this *infra-marginal* allocation problem.

Type-p or Type-r depending on how close are the richest poor to the poverty line — *i.e.*, $(z - y_2)$ —, the initial inequality among the poor (which depends on $(y_2 - y_1)$ and q_1) and the size of the budget (p).

In view of this result, we might ask whether a mixed allocation is *ever* optimal using the Sen index. The following example shows that it is.

Example Two

Consider a population of 100 people, 10 of whom are poor ($q = 10$) using a poverty line z of \$20. Let the initial distribution of income among the poor be

$$Y_0 = \left[0.1, \underbrace{10, 10, 10, 10, 10, 10, 10, 10, 10}_8, 19.9 \right],$$

for which the corresponding Sen index is $S = .0589$, and suppose that the anti-poverty budget T is \$10.1 — an amount just sufficient to lift two people out of poverty if used in that way. The question is whether such an allocation is optimal.

The resultant income distributions for these ten people under the three possible allocations — Type-p, Type-r, and mixed — are:

Policy	New Income Distribution	Sen Index
Type-p	$\left[\underbrace{10.02, \dots, 10.02}_9, 19.9 \right]$.0494
Type-r	$\left[0.1, \underbrace{10, \dots, 10}_7, 20, 20 \right]$.0493
Mixed	$\left[\underbrace{10.01, \dots, 10.01}_9, 20 \right]$.0450

The mixed allocation is the poverty-minimizing one.

We have thus proved:

Proposition 5. *Using the Sen index, the optimal allocation may be a mixed allocation.*

The intuition of Example Two is instructive. Because the richest poor person was just \$0.1 away from the poverty line, and the Sen index gives credit to reducing the headcount ratio H , it would seem that some

part of the anti-poverty budget should go to getting that person out of poverty. But getting the next person out of poverty would be more expensive — in this example, it would use up the entire remaining budget. If the rest of the budget were instead to be used to raise the income of the poorest of the poor, the average income shortfall would be reduced, as would the Gini coefficient among the poor. This latter use of resources might therefore reduce poverty more than would further reduction in the headcount.

The preceding considerations suggest that a mixed allocation is apt to be optimal when some of the poor are close to the poverty line, when the budgetary cost of reducing the headcount increases sufficiently rapidly, and when the poorest of the poor are very poor. We have not been able to prove this conjecture in general (indeed, it may not be provable in general) but we have been able to show its validity for an example using the uniform distribution.

Example Three

In this example, we assume that the distribution of poor incomes on the interval $[0, z]$ is uniform, with density $1/z$. A combination of Type-r/Type-p policy leads to the poorest incomes being raised to the income level a , and all individuals with initial income larger than b being lifted out of poverty. T_r and T_p , being the corresponding transfers (per head of the original population of poor), the income limits a and b are given by:

$$a = \sqrt{2zT_p}, \quad b = z - \sqrt{2zT_r}.$$

Using the uniform distribution, it is easily shown that the Gini coefficient among the remaining poor is given by:

$$G = \frac{b^3 - 3a^2b + 2a^3}{3b(a^2 + b^2)}$$

and the Sen index by:

$$S(a, b) = \frac{b}{z^2} \left(z - \frac{a^2 + b^2}{2b} \right) + \frac{b}{z} \cdot \frac{a^2 + b^2}{2bz} \cdot G$$

where $(a^2 + b^2)/2b$ is the mean income of the remaining poor.

Minimizing $S(a, b)$ under the constraint $T_r + T_p = T$, yields the optimal allocation T_r/T depicted in Figure 4 as a function of the total transfer T . It may be checked that the optimal policy is of Type-r only if the amount of the transfer is small enough. As in the preceding example, lifting those people infinitely close to the poverty line out of

poverty costs little and is the best to be done when the total budget is limited. For larger budgets, however, it becomes more and more costly to lift more people out of poverty, and this policy is less advantageous than raising the income of the remaining poor. Accordingly, the share of Type-r transfer as a fraction of the total transfer decreases continuously until all remaining poor have the same income ($a = b$). At that stage, the marginal budgetary amounts are equally distributed between the two types of transfer.

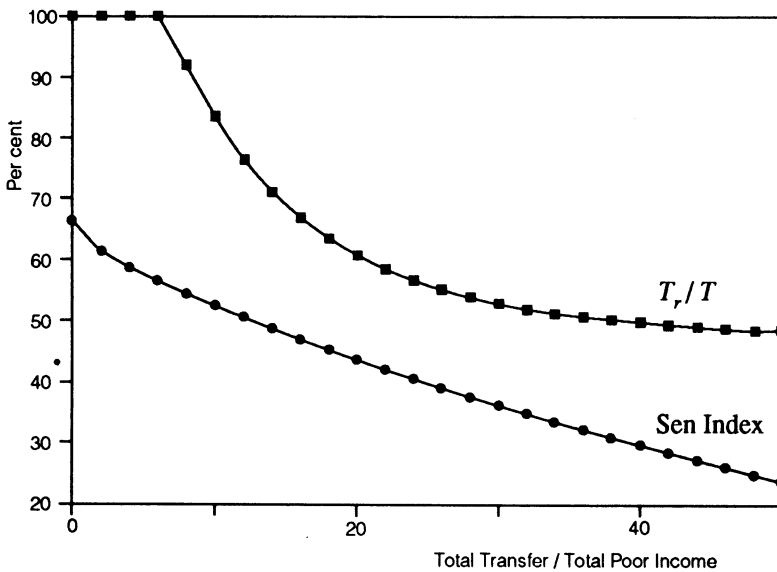


Figure 4
Optimal transfer allocation (T_r/T) (Uniform distribution)

In sum, we have shown that the Sen index can give different answers on how to allocate an anti-poverty budget. A Type-r allocation is optimal in some circumstances, a Type-p allocation in others, and a mixed allocation in others. In this respect, the Sen index is fundamentally unlike H or P_α with $\alpha < 1$ or P_α with $\alpha > 1$ which necessarily lead to *corner* solutions where the optimal policy is Type-p or Type-r only, whatever the initial distribution of poor incomes. Only for the Sen index (and related poverty measures) are both kinds of pure allocations ever optimal, and only for the Sen index is a mixed allocation ever optimal, depending on the actual distribution of incomes among the poor, and the size of the anti-poverty budget.

5 Axiomatics of Alternative Poverty Measures

It is clear from the preceding arguments that the main difference between the Sen Index (and its variants) and the P_α class (for $\alpha > 1$) is that the former are essentially based on the *poverty gap* ratio HI where both the headcount and the average income shortfall enter symmetrically, whereas the latter give more weight to the income shortfall. For $\alpha = 2$, for instance, Foster et al. (1984) mention that the P_α measure is

$$P_2 = H (I^2 + (1 - I)^2 C_p^2)$$

where C_p is the coefficient of variation of the incomes of the poor. Conceptually, that definition is formally equivalent to that of Sen, except for the fact that the shortfall component has been squared and the coefficient of variation rather than the Gini coefficient is used to measure inequality among the poor.

Some of the axioms leading to the Sen index are substantive while some others may be considered as *ad hoc*. This seems to be the case for the *Normalization Axiom* (N-axiom) which postulates that, in the absence of inequality among the poor, poverty should be measured by the poverty gap HI . It is precisely this axiom which leads to an optimal anti-poverty policy that combines Type-r and Type-p transfers under certain conditions.

The HI formulation has a number of desirable features: it is increasing in both H and I ; a given change in H contributes more to a change in poverty the larger is I ; a given change in I contributes more to a change in poverty the larger is H ; and when no one is in poverty, the poverty index has the value zero. But these same features would be satisfied by a wide range of other functions — for instance, HI^2 , which is the P_α index in the case $\alpha = 2$ when all the poor have the same income.

A possible justification of the N-axiom, not given by Sen, relies on social welfare dominance considerations. Considering the whole income distribution, rather than only that part below the poverty line, Atkinson (1970) has shown that the (utilitarian) social welfare associated with distribution $f(\cdot)$ — $f(\cdot)$ being the density function — is larger than that associated with $f^*(\cdot)$ for all increasing and concave individual utility functions if and only if:

$$\int_0^u (u - x) f(x) dx \leq \int_0^u (u - x) f^*(x) dx \quad (D)$$

for all values of u . As noted by Foster and Shorrocks (1988a, 1988b), this is equivalent to saying that the poverty gap (HI) corresponding to

the first distribution is smaller than that corresponding to the second for *all* poverty lines u .

Now consider two distributions such that all individuals below a well-defined poverty line, z , have identical incomes. So, all poor have the income y in distribution $f(\cdot)$ and their share in the population is $F(z)$, whereas the common income is y^* with distribution $f^*(\cdot)$ and the headcount ratio is $F^*(z)$. If $y < y^*$, distribution $f(\cdot)$ cannot dominate distribution $f^*(\cdot)$ according to the dominance criterion (D). On the other hand, if $y > y^*$ and $F(z) < F^*(z)$, then (D) is necessarily satisfied for all $u \leq z$. It thus remains to analyze the ambiguous case $y > y^*$ and $F(z) > F^*(z)$, *i.e.* there are more poor with $f(\cdot)$ than with $f^*(\cdot)$ but they are less poor. Applying the dominance criterion (D) on the interval $[0, z]$ shows that it is equivalent to:

$$HI \leq H^*I^*$$

So, measuring poverty with the poverty gap HI is equivalent to using a dominance criterion for that part of the distributions below the poverty line⁷, provided that there is no inequality among the poor.

Allowing now for inequality among the poor, we might follow various possible routes. The first one is simply to add a corrective term to the poverty gap ratio. Generalizing Sen's approach, poverty may then be defined by:

$$P = HI(1 + D_p)$$

where D_p is some inequality measure of incomes among the poor⁸.

A second route is to ignore the N-axiom, and, possibly, to stick to the dominance criterion. Although they did not state it explicitly, this is precisely what Foster *et al.* (1984) have done. Indeed Type-p allocations, which are optimal for P_α measures with $\alpha > 1$, dominate all other allocations in the sense of the dominance criterion (D).

Different normalization rules give a quite different tradeoff between a reduction in H and a reduction in I than does the Sen index. In this sense, the N-axiom is more than just a normalization axiom: it is consequential for a number of purposes, among which is the optimal allocation of an anti-poverty budget, the problem we have addressed here.

⁷This argument is in the line of the approach to poverty taken in Atkinson (1987).

⁸This is the generalization of Sen index proposed by Blackorby and Donaldson (1980).

6 Conclusion

Here, then, is the essential difference. The Sen index gives a higher weight to a reduction in the poverty headcount ratio than does the P_α class. When a person ε below the poverty line is removed from poverty, the Sen index values that reduction as the average income shortfall adjusted for the extent of inequality among the poor. The P_α class, $\alpha > 1$, gives that same change a much smaller weight: the P_α index changes by ε^α , which is necessarily smaller than the average income shortfall. In the limit, as $\varepsilon \rightarrow 0$, the change in P_α goes to zero, whereas the change in the Sen index remains finite.

These differences among the various classes of measures must themselves be axiomatized, precisely in the spirit of Sen's approach to poverty measurement. It is clear, in particular, that P_α measures are very much like standard (utilitarian) social welfare functions limited to the poor segment of a given population. As such they do not give any particular weight to a transfer which permits somebody to be lifted out of poverty. In contrast, there is an implicit non-zero social gain in moving an individual out of poverty in Sen's measure, however close he may be to the poverty line. That social gain and how it compares with an ordinary increase in income need further axiomatic elaboration, possibly in some of the directions mentioned by Sen himself in his discussion of the concept of welfare.

Following the argument in this paper, an alternative approach to the axiomatics of poverty would be to start from assumptions about the optimal allocation of an anti-poverty budget. Usual measures all lead to optimal allocations of type-p, type-r or mixed. But one may consider that another type of allocation should be optimal in some circumstances. For instance, the budget T could be optimally transferred to everyone proportionally to his/her poverty shortfall ($z - y_i$). The problem would then be to characterize all poverty measures consistent with such optimal allocations and some other basic axioms.

APPENDIX

This appendix derives the optimal allocation with the Sen index where there are two incomes among the poor and

$$q_1(y_2 - y_1) - (z - y_2)p > 0$$

Take the logarithmic derivative of $Y(i)$ in (4) with respect to i :

$$\hat{Y}_i = \frac{dY}{di} \frac{1}{Y}$$

Ignoring $\frac{1}{z}$, since the analysis does not depend on it, we derive

$$\hat{Y}_i = -\frac{q_1}{(q_2 - i)(q_1 + q_2 - i)} + \frac{z - y_2}{q_1(y_2 - y_1) - (z - y_2)p + (z - y_2)i}$$

The absolute value of the first term is an upward-sloping function (F_1) of i , and the second term is a downward-sloping function (F_2) of i .

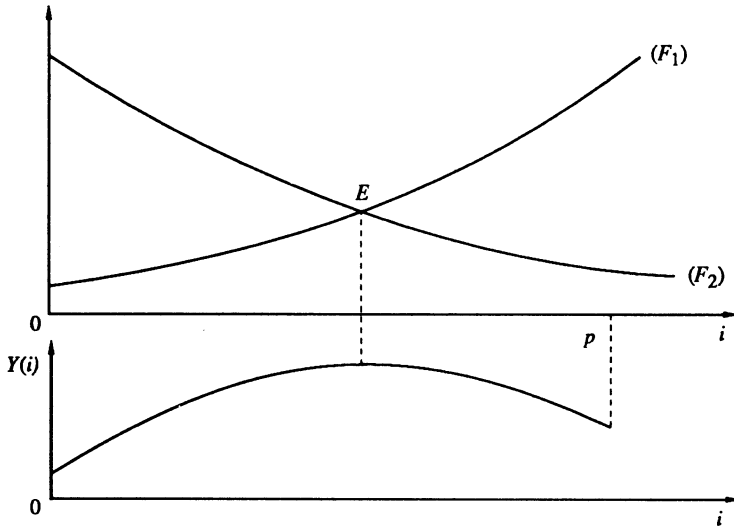


Figure A

Suppose the first term (F_1) is smaller than the second (F_2) at $i = 0$. Then, the graph is as in Figure A. \hat{Y}_i is positive to the left of the intersection point E and negative to the right of it, producing an interior *maximum* for

$Y(i)$. Given that the goal is to *minimize* poverty, this means to go to the corners. To know whether $i^* = 0$ or $i^* = p$, compare $Y(0)$ with $Y(p)$ and choose the smaller. In the case illustrated, $i^* = 0$

If the first term (F_1) in \hat{Y}_i is larger in absolute value than the second (F_2) at $i = 0$, then Y is a decreasing function of i over the interval $[0, p]$. So Y is minimized at $i = p$.

Putting together the previous conditions, it is easily shown that the optimal allocation is $i^* = p$ iff

$$\frac{q_i(y_2 - y_1) - (z - y_2)p}{q_1 + q_2} > \frac{(q_2 - p)q_1(y_2 - y_1)}{q_2(q_1 + q_2 - p)}$$

and $i^* = 0$ otherwise.

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