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Metropolitan Educational Research Consortium
The Community Foundation
Serving Richmond and Central Virginia
Bridging Richmond
Collective Action — Cradle to Career

Middle Grades Math: Assessing the Debate Over the When and How of Algebra

Fall 2014

by

Jesse Senechal, Ph.D.

METROPOLITAN EDUCATIONAL RESEARCH CONSORTIUM

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Background

Virginia Commonwealth University and the school divisions of Chesterfield, Colonial Heights, Goochland, Hanover, Henrico, Powhatan, and Richmond established the Metropolitan Educational Research Consortium (MERC) in 1991. The founding members created MERC to provide timely information to help resolve education problems identified by practicing professional educators. MERC currently provides services to over 12,000 teachers in eight school divisions. MERC has base funding from its membership. Its study teams are composed of university investigators and practitioners from the membership.

MERC is organized to serve the interests of its members by conducting and disseminating research to enhance teaching and learning in metropolitan educational settings. MERC's research and development agenda is built around five goals:

- To improve educational decision-making through the joint development of practice-driven research.
- To anticipate significant educational issues and needs that can be researched.
- To identify proven strategies for improving instruction, leadership, policy and planning.
- To enhance the effective dissemination of research to practitioners.
- To provide research oriented professional development opportunities for school practitioners.

In addition to conducting research, MERC conducts technical and educational seminars, program evaluations, and an annual conference, and publishes reports and research briefs.

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Introduction

Within discussions of K-12 math, the middle grades are framed as a critical period of transition between the foundational concepts presented in elementary math classes and the more abstract upper-level math classes that are traditionally associated with the high school level (i.e. Algebra, Geometry, Trigonometry, and Calculus). Over the past two decades important debates have occurred among educational researchers, math educators, and education policy makers as to the proper approach to middle grades math policy. At the center of this debate is the question of how and when to integrate algebra into the math course sequence. This focus on algebra has to do with its position as the first of the higher-level math classes, leading many to frame it as a "gatekeeper" course to future academic progress and opportunity (Adelman, 1999; Horn & Nunez, 2000; National Mathematics Advisory Panel, 2008; Spielhagen, 2011).

On one side the algebra debate are the advocates of Algebra-for-All (AfA) or universal algebra policies. They argue that access to algebra has been too selective, and that there are significant numbers of students that are ready to take algebra in middle grades that are not given the opportunity. What is more alarming, they argue, is that these excluded students are more likely to be Black and Latino, from lower socio-economic classes, and from homes where the parents do not have high levels of education (Cogan, Schmidt, & Wiley, 2001; Filer & Chang, 2008; Gamoran & Hannigan, 2000; Horn & Bobbit, 2000; McCoy 2005; Walston and McCarroll, 2010), making access to early algebra a basic civil rights issue (Kaput, 1998; Moses, 1993; Moses & Cobb, 2001; Spielhagen, 2011). The solution, from their perspective, is to dismantle the selective barriers that inhibit access to early algebra. It is also worth noting that this push for early algebra is closely connected to the broader push for academic intensification (Allensworth, Nomi, & Montgomery, 2009; Domina, McEachin, Penner, et al., 2014) that has driven the argument for the standards-based accountability movement.

On the other side of the debate are those who argue that blanket policy mandates that push more students into early algebra are having a variety of unintended negative consequences on curriculum, instruction and academic achievement in elementary, middle and high school (Loveless, 2008; Allensworth et al., 2009). This counter argument is often supported by the claim that many middle grades students are not developmentally prepared for the abstract thought necessary to succeed in algebra, and that blanket mandates for early algebra are leaving less time in the curriculum for building the important pre-algebraic foundations that are typically assigned to middle grades math (e.g. proportional reasoning, ratios, fractions) (National Mathematics Advisory Panel, 2008). Far from being a solution to the inequities of the system, opponents suggest that AfA policies have in certain cases become a barrier to student mathematical understanding, and for some populations, have had profoundly negative effects.

Considering this context, the purpose of this paper is (1) to provide and overview of the rationale of AfA policies, (2) to explore the arguments both for and against AfA policies, and (3) to present a set of policy recommendations around middle grades math that may be used to inform a regional action network. Although the national debate around AfA policies tends to be presented in polarizing arguments, the premise of this paper is that there are legitimate concerns and ideas on both sides that need to be considered when formulating a sound middle grades math policy. It is important to tackle the disparities in access to algebra and advanced level math and science classes, however this problem cannot be solved by pushing unprepared students through a math curriculum that leaves them with only perfunctory knowledge of math concepts. The hope is that this paper will synthesize what has been learned from the prior implementation of these policies in various contexts and the research that has tracked the policy outcomes in order to present a sound middle ground of policy recommendations.

The Structure of this Paper

Following this introduction, this paper is divided into three main sections

- **Algebra-for-All**. This section will provide a general overview of the emergence and current state of the "Algebra-for-All" policy push.

- **Summary of Arguments For and Against Algebra-for-All.** This section will outline arguments on the different sides of the debate. Through this section, research on the effects of AfA and related policies will be cited. Local examples will be used in certain cases to illustrate the points.

- **Policy Recommendations**. This section will provide a series of policy recommendations related to middle grades math. These recommendations will include ideas related to the reform of curriculum and instruction, assessment, professional development, and teacher preparation.

Bridging Richmond's Middle Level Focus

This white paper is an initiative supported by Bridging Richmond (BR), a regional partnership modeled after *StriveTogether*, a national network designed to promote regional, cross-sector collaborations around the cradle-to-career pipeline. BR's vision is that '*every person in our region will have the education and talent necessary to sustain productive lifestyles.'* To realize this vision, Bridging Richmond engages its regional partners from the education, business, government, civic, and philanthropic communities to (1) facilitate community vision and agenda for college- and career-readiness, (2) establish shared measurement and advance evidence-based decision making, (3) align and coordinate strategic action, and (4) mobilize resources and community commitment for sustainable change. BR's region includes eight school divisions (Richmond City, Chesterfield County, Henrico County, Hanover County, Goochland County, Powhatan County, New Kent County, and Charles City County) serving over 160,000 students including over 36,000 in the middle grades (6-8).

One focus of BR's work is the middle level learning space. The work in this area has included (1) support for the administration and use of the Gallup Student Poll for middle grade students in surrounding school divisions and communities, (2) planning and hosting a series of Middle Level Learning Summits that bring regional stakeholders together to discuss the challenges and opportunities of middle level learning, (3) support for the organization and facilitation of a Middle Level Learning Interest Group comprised of higher education faculty and K-12 researchers to help inform the regional conversations around middle level school reform, and (4) support for the MSR2020 out-of-school time system within Richmond Public Schools.

This paper is part of a series of white papers on research and best practices in middle level education. Other papers in this series include:

- **Best Practice in Out-of-School Time Systems (February 2013)** - Out-of-School Time (OST) programming is defined as both after school and summer learning opportunities for youth designed to offer alternative learning experiences or supplement and support traditional school-based education. This paper presents a review of current research and best practices in the design and implementation of citywide Out-of-School Time Systems as well as an overview of possible performance measures and community indicators for OST systems. The report also includes the perspectives gained from semi-structured phone interviews with five program leaders from four established OST citywide systems.

- **Middle Level Learning: Compendium of Research and Best Practice (July 2014)** – This paper examines the core principles underlying the “middle school” model, and reviews the research on its core components as well as models of comprehensive middle level school reform. This paper is designed to serve as a resource for practitioners, administrators, policy makers, and community members from the Richmond-area who are interested in developing a better understanding of the history and core themes of the middle level learning space and grounding their work and decision-making in the national research and literature on best practice for middle level learning.

Middle School, Middle Grades, or Middle Level?

Through this paper several different terms are used to represent the educational spaces that serve young adolescents. This includes *middle school*, *middle grades* and *middle level*. Before going on, it is worth clarifying the use of these terms.

- The term “middle school” is used to represent a school reform movement and a particular school model that emerged in the late 1960s and persists today. The middle school model is the dominant model across the country (60% of all middle grade schools are designated middle schools), across the state (64% are middle schools), and in the eight regional BR school divisions surrounding Richmond (89% of the schools serving the middle grades are six through eight middle schools).

- The term “middle grades,” as used in this paper, includes any school space that serves students in the period of young adolescence – generally grades five through nine. Middle grades schools include middle schools as well as junior highs, intermediate schools, and the later grades of K-8 schools.

- The term “middle level” is used in the title and throughout this paper to be inclusive of all of the middle grades school models as well as out-of-school learning spaces for this age group (e.g., afterschool, summer school, youth development programs).

Method

The process for developing this paper involved both a review of national literature on middle level math policy as well as an ongoing process of engaging local researchers, practitioners, and policy makers. The review included scholarly literature, professional literature, and the policy positions and resources provided by national organizations. Sources for the literature review were identified through (1) searches of scholarly databases and general web searches on a variety of topics related to middle level math, (2) the review of bibliographies of key studies, and (3) a review of websites of national organizations that are focused on math and middle level education. The review of literature and the organization and writing of the paper were also supported by engaging local stakeholders, primarily through a Middle Level Math Study Team.

<u>Middle Level Math Study Team</u>

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- Aimee Ellington, VCU Department of Math
- Ingrid Grant, Henrico County Public Schools
- Patricia Fox, Chesterfield County Public Schools
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Algebra-for-All

Generally the origins of early algebra efforts are traced back to the broader policy push that started in the 1980s for increased rigor in K-12 Schools. The seminal document of this movement, *A Nation At Risk* (Gardner, Larsen, Baker & Campbell,1983), suggested that our nation's K-12 schools were succumbing to a "rising tide of mediocrity" that threatened not only individual opportunity but also national economic and political security. A particular focus of *A Nation at Risk* – and the subsequent rigor debates over the past three decades – is the status of math and science education in our country. Many arguments in this tradition use international comparisons of math achievement to establish the need for math policy reform. For example, on the Program for International Student Assessment (PISA), the United States typically ranks among the bottom of developed countries in math achievement (Spielhagen, 2011). These arguments also draw on stories of students entering post-secondary institutions unprepared for basic math classes (Steen, 1999).

Early in this debate, reforming the approach to teaching algebra was identified as one of the keys to improving national math performance. Not only was the content of algebra seen as an important foundation for the types of abstract thinking critical for advanced math literacy (Carraher & Schiemann, 2007; Howe, 2005; Vogel, 2008), but the course was also viewed as a gatekeeper that provided access to upper level math and science classes in high school, to certain district-level specialty programs, and ultimately to expanded post-secondary opportunities (Adelman, 1999; Attwell & Domina, 2008; Barger & McCoy, 2010; Moses, 2001). This focus on early algebra is evident in President Bill Clinton's 1998 address to an Education Roundtable where he raised concern that "around the world, middle school students are learning algebra and geometry. Here at home just a quarter of all students take algebra before high school" (quoted in Loveless, 2008).

The focus on algebra through the 1990s and 2000s, led many states and school districts to develop policies that opened access to algebra for middle grades students. These Algebra-for-All policies (also known as Universal Algebra) increase the number of students taking algebra in eighth grade. In certain cases AfA policies mandate particular course sequencing or use incentive structures to promote early algebra policies at the district and school level. As a result, the number of eighth grade students enrolled in algebra nationally almost doubled from 16% in 1990 to 31% in 2007 (Loveless, 2008). It is important to note that this rise in algebra taking varied dramatically by state and district. The most prominent example of state level initiative is in California where in the mid-1990s the California Department of Education and the state legislature developed policies that emphasized early algebra. As a result of this initiative, California's 8th grade algebra taking rate is twice that of the national average (60%) (Domina, McEachin, Penner, & Penner, 2014).

However, the California example is the exception. In most states, including Virginia, it is individual school districts that develop and implement math policies. While the Virginia Department of Education tracks 8th grade algebra taking as a state level performance measure, it has no policies in place that encourage or discourage AfA policies. As a result, in Virginia, there is a wide range of approaches to middle grades math policy and early algebra. For example, within the eight school divisions that comprise the BR region, 8th grade algebra (or higher) course taking ranges from a low of 25% in one division to a high of 95% in another.

Key Cases in the Algebra-for-All Debate

In the literature on Algebra-for-All efforts there are several key cases that have helped build our understanding of the effects of these policies. This includes policies that have been implemented at the state level as well district level. Much of the research cited in this paper was conducted in these settings. Below are brief discussions of how the policy was developed, and the key findings from research and evaluation efforts into these cases.

California

California has long been on the forefront of early algebra efforts. In 1997, a revision of the California State Content standards recommended all students take algebra by 8th grade. This curricular push from the California Department of Education was supported by a 1999 state law that instituted penalties for districts that did not move toward the early algebra goal. While the penalties created an incentive to implement these policies, it was not a mandate. The policy also left the specifics of how to promote early algebra up to the individual districts creating a wide range of strategies used across the state. As a result of this initiative, California's 8th grade algebra taking rate has grown steadily over the past 15 years to a point where it is now twice that of the national average (60%). Looking at district-level data from all California public schools Domina, McEachin, Penner and Penner (2014) estimate the effect of early algebra initiatives on 10th grade math achievement. The researchers found that enrolling more students in advanced courses has negative average effects on students' achievement, driven by negative effects in large districts. They suggest that the overall negative results from the policy should lead us to consider the level and quality of implementation of early algebra courses (Domina, McEachin, Penner, & Penner, 2014). Similar negative effects were found in a 2012 study of California's policy by Liang, Heckman, and Abedi.

Chicago

In 1997, as part of a push for increased academic rigor, Chicago implemented a policy that required algebra for all ninth grade students and eliminated all high school remedial math courses. This policy led to an immediate shift in enrollment. By 1999, algebra enrollment numbers in 9th grade went up to almost 100%. Although this effort does not involve pushing algebra to the middle grades, it is based on similar premises of universal access and increased rigor. For this reason, the research on the Chicago policy is often sited in discussions of early algebra. Despite the dramatic change in algebra course taking, researchers from the Chicago Consortium of School Research (CCSR) found that the academic outcomes for target students – low-ability students previously excluded from algebra – did not change in meaningful ways (Allensworth et al., 2009). The research showed that test scores did not increase and that failure rates increased for low-performing students. In a follow up study by the CCSR, Nomi (2012) found that the policy also had negative effects on high-ability students, presumably due to the heterogeneous grouping of students in algebra classes. However, it is important to note that the CCSR researchers did not conclude that universal algebra polices should not be pursued. Rather as Allensworth, et al. (2009) states "curricular policies need to be accompanied by other profound changes in the educational system with greater attention to instruction and with concomitant efforts to improve the academic behaviors that have been shown to be associated with better school performance" (p. 385).

Chesterfield County, Virginia

In the early 2000s, Chesterfield school leaders responded to disparities they found in course quality and access to algebra across schools by revising their eighth grade algebra placement policies, and implementing a comprehensive plan for getting schools, teachers and students prepared for this major shift in math instruction. As a result of this policy shift, the number of eighth grade algebra takers grew quickly from 30% in 2004-2005 to over 90% by the 2008-2009 school year. Spielhagen (2011) who acted as the evaluator for the districts policy through its implementation, found that despite the fact that algebra in Chesterfield was now open to a broader ability-level group of students, the rates of passing the standardized algebra assessment remained constant at both the pass and pass advanced rate. Essentially, this meant that not only were more students taking algebra, but more students were also successful in algebra. Chesterfield had taken strides to closing the opportunity gap for students by opening up the algebra program to the majority of students in eighth grade, regardless of demographic indicators and school placement.

Summary of Arguments For and Against AfA Policies

The shift to early algebra requires attention and possible adjustments to a number of curriculum and school organizational issues. For example, making algebra a standard eighth grade class means rethinking the sequencing of math curriculum up to eighth grade, rethinking placement policies for advanced level math classes, and assessing the middle grades math teacher preparation for teaching algebra. In addition, successful change related to this type of policy must involve shifting the attitudes and beliefs about teaching and learning math that are held by the students, teachers, parents and administrators who exist under these new policies. Due to the fact that the AfA policies require such dramatics shift in approach, attitudes and resources, they have been controversial from the start. In the following section the core arguments around AfA policies are reviewed. These arguments have been organized into two main categories: (1) the effect of AfA policies on access and equity in math enrollment, and (2) the effect of AfA policies on student math achievement.

AfA policy impact on access and equity in math enrollment

As mentioned above, perhaps the primary concern driving the early algebra push – as well as increased academic rigor arguments generally – is the issue of equity and access. The claim that access to algebra is a key to closing achievement gaps between student sub-groups is supported by several lines of research. However, there are also critiques of this research that challenge the efficacy of AfA policies in addressing equity issues.

Disparities in algebra access

Much of the argument for AfA policies is based on research that shows eighth grade algebra enrollment is lower among Black and Hispanic students (Cogan et al. 2001; Gamoran & Hannigan, 2000; McCoy 2005; Watson and McCarroll, 2010), students with lower SES (Filer & Chang, 2008; Gamoran & Hannigan, 2000; Waltson and McCarroll, 2010), and those whose parents have fewer years of education (Horn & Bobbit, 2000; Watson and McCarroll, 2010). Stein, Kaufman, Sherman and Hillen (2011) suggest that this imbalance in algebra enrollment has to do with different levels of math preparedness among these populations, but is also a result of subjective placement factors such as peer and parent encouragement (Filer & Chang, 2008), urban and rural schools that do not offer algebra (Cogan et al. 2001), and less encouragement from teachers and academic counselors (Singh and Granville, 1999; Spielhagen, 2011). The effect of early ability-level tracking in math classes has also been identified as a significant factor in inequitable algebra course taking outcomes (Speilhagen, 2011).

Access to advanced math and science courses

Beyond increasing access to early algebra, some research also suggests that taking early algebra is a strong positive predictor of more advanced math course taking in high school (Allensworth, et al. 2009; Atanda, 1999; Burris, Hubert, & Levin, 2006; Horn & Bobbit, 2000; Paul, 2005; Spielhagen, 2006). Other studies have also correlated early algebra with increased enrollment in advanced science classes (Paul, 2005). What's more, Edmunds et al. (2012) found that the access to higher level math and science that results from AfA policies has led to higher percentages of non-white, first generation college going, and low-SES students taking higher level math.

Critique of the access and equity research

While the critics of AfA policies do not dispute inequity in access to algebra and other higher-level math and science classes, they suggest that these types of universal policies – especially when poorly implemented – may only serve to shift or mask the inequities in the system. One example of this is the practice within some systems of creating different types of algebra classes that are given to different types of students, seemingly a form of covert tracking (Cogan et al. 2001; Schmidt, 2009). There is also the concern that pushing what might be considered "un-prepared" students into algebra, could lead to higher rates of failure within those groups. For example, Loveless (2008) found that students who were low performing coming into early algebra, fell behind similarly low performing students who did not have early algebra.

Another important critique of the research on early algebra points to the issue of selection bias in the study design. That is to say, while many research studies have shown correlation between early algebra and advanced course taking, these studies were looking at non-AfA systems where the students in early algebra were generally higher achieving anyway (Loveless, 2008; Chang, 2008; Domina, et al. 2014).

Critics have also suggested that even with AfA policies in place, many students are not taking higher-level classes. For example, some studies have found that between one half and two thirds of students who completed algebra in 8th grade do not go on to advanced math classes beyond Algebra II (Atanda, 1999; Ma, 2000; Spielhagen, 2006). Others have noted that taking algebra early and not pursuing higher level math classes leaves many students with no math classes during their junior and senior year, putting them at a disadvantage when entering post-secondary.

AfA policy impact on student math achievement

The other main topic of debate surrounding algebra is the effect of AfA policies on student math achievement. Within this debate math achievement is defined in several ways including (1) math course passing rates; (2) standardized achievement tests (e.g., mandated state tests or National Assessment of Educational Progress (NAEP); and, in some cases longer term outcomes such as (3) graduation rates and post secondary math achievement. Overall, the research findings related to academic achievement are very mixed. In some cases this seems to be an effect of different policies and populations being studied (e.g., California versus Chesterfield), while in other cases, it may be a result of researchers coming to different conclusions about the same data.

Course Passing

The studies of math course passing are a good example of researchers coming to alternative interpretations of similar data. For example, while some research has shown AfA polices leading to higher numbers of students passing algebra (Speilhagen 2011), others have highlighted the fact that the same policies have led to higher failure rates (Stein et al., 2011; Williams, Haertel, Kirst, Rosin, & Perry, 2011). More students take algebra, which leads to more students passing and more students failing.

Standardized Test Achievement

The research on the impact of early algebra on student achievement is also mixed when the outcome measure is standardized test achievement. Research that is used by proponents of AfA policies can be categorized into two types of studies. The first are studies that look at the relationship between early algebra and math standardized test achievement *in the absence of AfA policies*. For example, there is a large body of research showing that early algebra is a predictor of higher achievement, even when controlling for race, SES, and prior math achievement (Filer & Chang, 2008; Gamoran & Hannigan, 2000; Kurlaender & Reardon, 2008; Shakrani, 1996; Smith 1996). However, these studies – like the advanced course enrollment research above – have been critiqued on the grounds of selection bias (Loveless, 2008; Chang, 2008; Domina, et al. 2014). The other category of research looks specifically at settings with AfA policies in place. Here too there are a number of studies that have established relationships between early algebra and higher test achievement (Balfanz, Legters, & Jordan, 2004; Burris et al., 2006; Kemple et al., 2005; Nomi & Allensworth, 2009; Williams, et al., 2011).

Research that is used to critique AfA policies has pointed to negative or non-significant increases in test achievement (Allensworth et al., 2009; Burris et al., 2006). This research also often points out that the benefits of early algebra are not for all students. For example, studies have found that early algebra has a negative effect on the standardized test achievement of low and moderately performing students (Clotfelter et al., 2012; Domina, McEachin, Penner & Penner, 2014; Gamoran & Hannigan, 2000; Loveless, 2008), while Nomi (2010) found that the AfA policy in Chicago led to lower achievement for high-achieving students, presumably due to lowered expectations in mixed ability classes (Nomi, 2010). However, it is also worth noting that some of the researchers that found negative or non-significant increases in test scores, ultimately recommended broad reforms to district math curriculum and teacher professional development policies rather than an elimination of AfA policies (Allensworth, et al. 2009; Nomi, 2010). Essentially, they argue if AfA polices are implemented, they must be implemented well.

Graduation Rates

One of the concerns voiced by the critics of AfA policies is that pushing “un-prepared” students into early algebra might lead to increased course failure and negative attitudes toward math which might then in turn lead to increased likelihood of dropping out. While not much research has been conducted on the long-term effects of AfA policies on student academic outcomes, Allensworth et al. (2009) found that Chicago’s universal policy did not have positive or negative effect on graduation rates, a finding that counters the suggestion of the critics that early algebra might lead to increased drop outs. In a similar vein Spielhagen (2006) found that students who participated in early algebra took more advanced math classes and were more likely to attend college than those who did not.

Policy Recommendations

Although there are significant disagreements that surround Algebra-for-All policies, there are also some important points of common ground. For all involved there is an agreement that (1) a middle grades math policy should promote advanced mathematical understanding that includes a strong pre-algebra foundation for all students and that (2) the policy should lead to equitable outcomes across gender, racial/ethnic, and socioeconomic groups. The question is not whether these are desirable goals, but rather how to get there. With this in mind, below are a series of policy recommendations that are grounded in research and best practice. These recommendations are organized into the following five categories:

1. Curriculum and Assessment Policies
2. Course Placement Policies
3. Teacher Preparation and Professional Development Policies
4. Out-of-School Support Services
5. Research and Evaluation

Curriculum and Assessment Policies

- ***Ensure that students master the content of middle level math.*** It is critical that policies that re-order the math course sequence do not weaken the pre-algebra foundation that is provided by the middle-level math curriculum. Along these lines it is important that schools and teachers define content mastery as more than course passing or benchmark scores on standardized assessments.

- ***Elementary and early middle curriculum review***. Review elementary and middle level curriculum to ensure proper development of pre-requisite algebra skills and conceptual understanding. Focus should be on providing students with deep understanding of pre-algebraic concepts such as proportional reasoning, fractions, and ratios. Also school leaders should review math assessment practices (state, district and school level) to determine alignment with standards and curriculum frameworks.

- ***Develop and allow for pedagogical approaches that increase student engagement in pre-algebra math content.*** One problem identified by many math educators is lack of innovative pedagogical approaches to math instruction in the middle grades. If students are to succeed at this level, teaching strategies need to be developed and implemented that engage students with the content.

- ***Early intervention in elementary and middle to mitigate achievement gap.*** Research on issues of access to early algebra suggests that part of the problem relates to insufficient access to support services in elementary and early middle school. Providing more early interventions when students fall behind in math may improve algebra readiness at the middle grades level.

- ***Increasing instructional time.*** When algebra is moved earlier, it is important that content in the elementary and middle grades is not eliminated. Increasing math instructional time, for example to double math periods, might be a strategy that avoids having to eliminate curriculum. However, it should be considered that when math time is increased, instructional time in other subjects will need to be cut.

- ***Reviewing approach to higher level math classes***. As policies increase enrollment in upper level classes, curriculum materials and pedagogical strategies will have to be adjusted to address heterogeneous student groupings.

- ***Expanding high school course options and coordinating with post-secondary institutions***. While early algebra gives students earlier access to advanced high school math classes, there are many students who finish their high school math requirements early and do not choose to go beyond the required courses. In these cases there are one or two year gaps between the final high school math course and post-secondary math requirements. Creating alternative elective math courses to fill these gaps may lead to higher levels of math achievement at the post-secondary level. Courses might include applied mathematics such as CTE classes, STEM classes, and computer science.

- ***Curriculum articulation across levels.*** It is recommended that division leaders align curriculum across levels, with special attention to the transitions between elementary, middle and high school. For example, divisions should look at differences in instructional materials (Schielack, 2010), pedagogical approaches and assessment strategies to ensure that students are not getting lost as approaches to math instruction change. Convening teachers across the levels to review the curricular approaches may lead to better alignment. Articulation with post-secondary math departments – at the college and community college level – might also be worth considering.

- ***Review accountability policies***. Divisions should review how accountability systems tied to test performance may impact teaching and learning in the math curriculum. This is especially important in cases where there are gaps between the content of the course and the assessments. If assessments drive the instruction, key content that is not tested might not be covered.

- ***Explore the use of technology and digital curriculum to enhance math instruction*****.** Divisions should consider ways that technology and digital curriculum could supplement algebra and other math instruction. This may be especially effective in heterogeneously grouped classes where students need appropriately leveled material. This might also be a valuable approach in smaller divisions that have fewer resources available for tiered levels of instruction.

The complicated range of middle grades math sequencing

The table below presents a comparison of the middle grade mathematics course sequencing and test taking patterns for three school divisions. For each division it shows courses offered to students in grades 6, 7 and 8; the percentage of students taking those courses; and the Standards of Learning test that the student takes. While these divisions are based on information collected from local school division leaders, the information has been re-presented in ways that mask the identities of the divisions. The point of this comparison is to illustrate the range of division-level approaches, and, more importantly, to raise questions about the implications of various course sequencing approaches.

Division 1			Division 2			Division 3		
% taking	Course(s) Offered	SOL Test taken	% taking	Course(s) Offered	SOL Test taken	% taking	Course(s) Offered	SOL Test taken
GRADE 6								
9%	* Math 6 Self-contained	Math 6				4%	6 General Mathematics	Math 6
49%	* Math 6 Standard	Math 6	73%	* Math 6 Regular	Math 6	72%	6 Mathematics	Math 6
			23%	* Math 6 Honors	Math 7	10%	6 Mathematics Honors	Math 6
42%	Pre-algebra 6 Honors	Math 8	4%	* Math 7 Honors	Math 8	14%	7 Mathematics Advanced	Math 7
GRADE 7								
5%	* Pre-Algebra 7 Self-Contained	Math 7				5%	7 General Mathematics	Math 7
27%	* Pre-Algebra 7	Math 8	64%	* Math 7 Regular	Math 7	69%	7 Mathematics	Math 7
26%	Pre-Algebra 7 Accelerated	Math 8	29%	* Math 7 Honors	Math 8	11%	7 Mathematics Honors	Math 7
41%	Algebra I 7 Honors	Algebra I	7%	* Algebra I Honors	Algebra I	3%	7 Mathematics Advanced	Math 8
						12%	7 Mathematics Accelerated	Math 8
GRADE 8								
3%	* Math MS Self-Contained	Math 8				5%	8 General Mathematics	Math 8
53%	* Algebra I Standard	Algebra I	7%	* Algebra Prep	Math 8	69%	8 Pre-Algebra	Math 8
26%	* Algebra I Accelerated	Algebra I	86%	* Algebra I (3 levels: Regular, Intermediate and Honors)	Algebra I	6%	8 Pre-Algebra Honors	Math 8
						20%	8 Algebra I Advanced	Algebra I
						<1%	Geometry	Geometry
18%	Geometry Honors	Geometry	7%	* Geometry	Geometry	1%	Geometry Advanced	Geometry

* Double period classes

Observations

- Each division has a unique system for labeling and assigning levels to their classes. For example, what seem to be equivalent tracks across divisions use a variety of labels including “general”, “standard” and “regular.”

- Divisions accelerate their middle grades math courses at much different rates. For example, Division 1 accelerates the curriculum the quickest, with 41% of the students taking Algebra by the 7th grade. This allows a significant number of grade 8 students in Division 1 (18%) to take Geometry in the 8th grade year. In contrast, Division 3 accelerates the slowest with only 20% of grade 8 students taking algebra and less than 2% in geometry.

- In the three example divisions there seems to be a significant degree of ability level tracking within algebra courses. For example, in Division 2 there are five levels of algebra.

- To support students as they accelerate the delivery of math content, each division has taken a different approach to accelerating math instruction. In Division 1, double period math is used primarily with the standard/regular level students, while Division 2 has made all levels of math double period. Division 3 does not use double period math classes at all.

- The patterns of SOL taking are very different across the three divisions as well. The three middle level math SOL tests (grade 6, 7, and 8) are not necessarily given to all students. One example of this is in Division 1 where only a small fraction (5%) of students take the grade 7 SOL test. In Division 3 by contrast, essentially all students take the grade 7 SOL test either during their grade 6 year (14%) or grade 7 year (85%).

Course Placement Policies

- ***Review the idea of algebra-readiness***. At the heart of the Algebra-for-All debate is the issue of algebra readiness. However, this term has come to take on a range of meanings depending on who is using it in what school policy context (Spielhagen, 2011). Divisions should engage in open discussions of the idea of algebra-readiness. Discussions should occur about the criteria of readiness especially when they involve less objective indicators such as teacher assessments (e.g., grades, ability to apply content, ability to communicate mathematically, problem attack and solving skills, persistence, organization skills, and maturity).

- ***Review placement policies and practices***. Divisions should work to eliminate placement practices associated with disparities in math access and achievement. For example, math course placements, in some cases, are subject to pressure from involved parents (Useem, 1992). In other cases math placements have been connected to particular student behaviors such as teacher pleasing, not necessarily ability. For example, in the pre-AfA research on Chesterfield, Spielhagen (2011) identified an “overlap group” of students who had the same entrance credentials for entrance into 8th grade algebra but some got into the course while others did not. This research found that placement ultimately was due to human factors, i.e. parent or teacher intervention. The Chesterfield AfA reform sought to eliminate this disparity.

- ***Attend to issue of possible tracking.*** Ensure that opening access of algebra does not mean creating levels of algebra that carry different levels of rigor and expectations. De-tracking often means making significant adjustments in the organization of schools as well addressing resistance to de-tracking from parents, teachers and students. It is important that schools actively tackle these issues as they work to de-track classes.

- ***Examine barriers to access and pathways to success in algebra prior to grade 9.*** Barriers could include under- or over-identification for elementary acceleration programs and late entry for students who transfer in from other school divisions or from out-of-state.

Teacher Preparation and Professional Development Policies

- ***Work with teacher preparation programs to recruit and retain math teachers***. Many schools, especially in the middle grades, do not have enough qualified math teachers to support the increased enrollment brought about by AfA policies. This is especially true for schools that have shifted to double period math classes. Improving math outcomes will require coordination with teacher preparation programs to recruit and retain qualified teachers.

- ***Provide sustained professional development to teachers.*** Target professional development math instructional strategies and models that address the learning needs of heterogeneous groups of learners. The development of content mastery is especially important among elementary school teachers, who are likely not to have math content expertise, while secondary math teachers may need additional support with innovative pedagogical approaches to math instruction.

- ***Review and potentially increase the use of math specialists to support teachers.*** An effective strategy for sustained professional development is the use of division-level and building-level math specialists.

- ***Provide professional development to address teacher attitudes and dispositions that may inhibit student math achievement.*** Shifting middle school level algebra from an honors track class to a standard class for all students may also mean shifting teacher attitudes towards students of varying ability levels. There are multiple ways that teachers attitudes toward math and toward student ability can be a barrier to achievement (Spielhagen, 2011). In certain cases teachers may need training in cultural competency to overcome personal biases and practices that inhibit equal treatment of students.

Out-of-School Support

- ***Connecting with parents.*** As algebra classes open up to a broader range of students, it is important for schools and teachers to connect with parents about expectations of the curriculum and the support service available. For example, parents need to understand the importance of middle grades math curriculum, so they don't push schools to promote their children to higher-level math classes without proper preparation.

- ***Out-of-School time (OST) support.*** Schools should coordinate with out-of-school time providers whose programs support K-12 math goals. This could include after-school programs or summer programs that are run through non-profits or are university-based. School leaders and OST leaders could work to align OST activities with the school day curriculum.

Research and Evaluation

- ***Conduct research and evaluation to understand the impact of AfA policies.*** As new middle grades math policies are developed and implemented it is critical that school divisions work independently and collaboratively to understand the impact of the policies on school and student success. This work could include a range of research and evaluation activities that use formative designs to support continuous improvement efforts as well as summative studies that examine the long-term impact of these policies on student achievement in high school and post secondary settings. Below are a list of questions that might guide research and evaluation efforts:

    - To what extent does grade algebra placement (8^{th} or 9^{th} grade) affect student success (course passing, test achievement, post-secondary access and success)?

    - What is the mathematics curriculum path of students who took and passed the Algebra SOL in 8^{th} grade?

    - Under AfA policies, what are the characteristics of the students who struggle with eighth grade algebra versus those who do not struggle?

    - What is the impact of professional development programs designed to prepare teachers for pre-algebra, algebra and advanced math courses?

    - What constitutes appropriate post-algebra mathematics curriculum for students who struggle with algebra, regardless of when they study it?

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