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Three Quarter Plackett-Burman Designs for Estimating All Main Effects and Two-Factor Interactions

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Three Quarter Plackett-Burman Designs for Estimating All Main Effects and Two-Factor Interactions

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by

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Abstract

Plackett-Burman designs and three quarter fractional factorial designs are both well established in the statistical literature yet have never been combined and studied. Plackett-Burman designs are often non-regular and are thus subject to complex aliasing. However, Plackett-Burman designs have the advantage of run-size efficiency (over the usual 2^k factorials) and taking three quarters of a Plackett-Burman design further improves this benefit. By considering projections of these designs, we constructed a catalog of designs of resolution V and ranked by D-efficiency.

This thesis is dedicated to my friends and family, new and old.

For my parents, Bolt, Beau, Roy a.k.a. Re a.k.a. Tha Royal Bandit

and everyone in QA who changed my entire perspective for the better.

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CHAPTER 1

BACKGROUND/OVERVIEW

The field of statistical analysis has completely transformed with the advent of modern computers. The newly available computational power has opened the door to exponential growth within every area of statistics. Experimental design has especially seen great advancement in productivity and the expansion of its knowledge base that continues to develop. Not only are completely new concepts being established every year, more and more researchers are finding novel and exciting applications for existing areas of study. This thesis aims to establish an innovative application for three quarter fractional factorial designs [John(1969)] and Plackett-Burman designs [Plackett and Burman(1946)], two concepts that have yet to be combined and studied.

Three quarter FFDs and Plackett-Burman designs are two areas of statistics that have been thoroughly studied on their own. Both concepts have been in existence for a number of decades and have undergone different levels of scrutiny through the years. However, it seems that there has not been any formal research to date on the outcome of combining these two types of designs. The purpose of this thesis is to establish a catalog of designs that are projections of three quarter Plackett-Burman designs with resolution V, i.e. all main effects and two-factor interactions are estimable.

Chapter 2 consists of a literature review of pertinent material. This includes two-level full and fractional factorial designs, Plackett-Burman designs and three quarter fractional factorial designs. Chapter 3 is entitled Designs, Research Method & Results and contains detailed information about the research method. It begins by outlining the method of design construction, from creating a Plackett-Burman design, subsetting it to construct a three quarter design to taking projections of it. Next, construction of a catalog of resolution V designs is explained. The catalog is then tested for efficacy with the use of estimation efficiency and simulation studies. The final chapter is a conclusion to this thesis including recommendations and possibilities of future research.

CHAPTER 2

LITERATURE REVIEW

2.1 Two-Level Full and Fractional Factorial Designs

Two-level factorial designs are some of the most widely used designs. They are simple to build and can be efficient when properly constructed. A "two-level" design is one that only deals with independent variables consisting of two predetermined levels, e.g., factor A will only be at 100°F or 200°F and factor B will only be 10 minutes or 20 minutes. These designs are represented as 2^k designs where k is the number of factors included in the model. For example, if $k = 4$ then the design would be a 2^4 full factorial consisting of 16 runs (because $2 * 2 * 2 * 2 = 16$). What this tells the researcher is that an experiment that contains 4 two-level independent variables can have all possible factor combinations included in the design using a minimum of 16 runs. When treatment combinations are exhaustively included in a design, it is called a *full factorial design* [Fisher(1935)]. The regression model contains k main effects, $\binom{k}{2}$ two-factor interactions, $\binom{k}{3}$ three-factor interactions, etc., depending on how many effects are desired in the model. The designs are presented in a *design matrix* consisting of 1s (signifying a factor's high level) and -1s (signifying a factor's low level). The design matrices also show all treatment combinations that are to be tested for the experiment [Montgomery(2009)]. An example of a 2^3 full factorial model is shown in Table 2.1.

Table 2.1: A 2^3 Full Factorial Design

A	B	C
1	1	1
1	1	-1
1	-1	1
1	-1	-1
-1	1	1
-1	1	-1
-1	-1	1
-1	-1	-1

It can be seen that every possible high-level and low-level factor combination of A, B and C is observed.

In practice, a full factorial model is often not desirable because of its lack of run size economy as the number of factors increases. As more factors are added to a design, the number of runs needed for a full factorial increases exponentially. For example, a design with 5, 6, 7 and 8 factors needs 32, 64, 128 and 256 runs, respectively, in order to be a full factorial and run all possible factor combinations. This can become cumbersome, time consuming and costly, or altogether impossible depending on the circumstance. The best and most widely used solution to this problem is the usage of *fractional factorial designs* (FFD) [Box and Hunter(1961)]. FFDs are set up specifically to use only a fraction of the runs needed while at the same time maximizing the amount of information able to be collected from the data. FFDs are denoted by 2^{k-p} where p is the fraction of the design to be constructed. 2^{k-p} can also be written as $\frac{2^k}{2^p}$ or $\frac{1}{2^p}2^k$; when p is 4, the design would be $\frac{1}{2^4}2^k$ which equals $\frac{1}{16}2^k$ or a 16th fraction of the 2^k design [Wu and Hamada(2009)]. The number of runs are calculated the same way as before by solving 2^{k-p}, e.g., if $k = 3$ and $p = 1$ then the design is 2^{3-1}, which equals 2^2, so the FFD would have 4 runs. Four runs is one half of the 8 runs necessary for the full 2^3 factorial design.

The trade-off with using FFDs is that factor aliasing becomes a necessity. Since fewer degrees of freedom are available, some factors are aliased (biased) with each other. Consider a one-half fraction of the 2^3 design in Table 2.1, written as 2^{3-1}. Beginning with a full factorial in A and B, we use C = AB to generate the C main effect column. *Generators* are used to construct fractional factorial designs and in the case where I = ABC, C = AB is the generator. In 2^{k-p} notation, p also represents the number of generators that are needed in the defining relation. Table 2.2 shows the 2^{3-1} design constructed with the generator C = AB.

Table 2.2: A 2^{3-1} Fractional Factorial Design

A	B	C
-1	-1	1
-1	1	-1
1	-1	-1
1	1	1

It can be seen that each point in the factor column C was generated by multiplying the design points for A and B together. Factor generators determine which runs will be left out of the design in order to utilize only a fraction of the runs. The *defining relation*, determined by the generators, states which factorial effects will be aliased. For instance, in the 2^{3-1} example, the defining relation is written as I = ABC where I is the intercept. Thus, A = BC, B = AC and C = AB makes up the alias structure, indicating which effects are aliased together. A *word* is any factor group from the defining relation. A defining relation outlines all of the aliasing between effects and the generators serve as the foundation for how the aliasing is established. For this reason, a defining relation may consist of a total of more words than there are generators [Montgomery(2009)]. Additionally, a defining relation for a particular design will always contain $2^p - 1$ words. For example, a possible defining relation for a 2^{6-2} design is I = ABE = ACDF = BCDEF. Only E = AB and F = ACD are generators but there are a total of three words in the defining relation once the generators are combined, creating the last word BCDEF [Wu and Hamada(2009)].

The size of the words in the defining relation determines the *resolution* of the design. Resolution is a very important tool in design because it is a quick way of determining effects in the model that can be clearly estimated. The number of letters in the smallest word of a defining relation decides the resolution. For example, the defining relation from above, I = ABE = ACDF = BCDEF, has a resolution of III because the smallest word, ABE, has three letters in it (resolutions are usually denoted in Roman numeral form). Wu and Hamada [2009] give a very concise table of rules for Resolution IV and V designs:

1. In any resolution IV design, the main effects are clear.

2. In any resolution V design, the main effects are strongly clear and the two-factor interactions are clear.

3. Among the resolution IV designs with given k and p, those with the largest number of clear two-factor interactions are the best.

Effects that are *clear* are not aliased with main effects or two-factor interaction and effects that are *strongly clear* are not aliased with main effects, two-factor interactions or three-factor interactions [Wu and Chen(1992)].

Resolution V designs are widely considered to be the most desirable because three-factor interactions or greater are usually treated as negligible during analysis. For the purpose of this thesis, this was also held true throughout research and the acquisition of resolution V designs was considered to be the main objective.

2.2 Plackett-Burman Designs

Non-regular designs are a variety of designs that are widely used because they allow for a little more flexibility in run size. Two-level FFDs are severely limited in choices of run size because they must be powers of 2. Non-regular designs help alleviate this problem by allowing for better management of run size economy. Some of the most widely used non-regular designs are Plackett-Burman designs. These designs were originated by Plackett and Burman [1946], with many being cyclic in nature, meaning that the second column is created by shifting the first column, the third column is created by shifting the second column, etc. Additionally, these designs are available in run sizes that are multiples of four (but not powers of two as these are already normal, fractional factorial designs). Table 2.3 below shows an example of a small Plackett-Burman design in 12 runs.

Table 2.3: 12-run Plackett-Burman Design

1	1	1	1	1	1	1	1	1	1	1
-1	1	-1	1	1	1	-1	-1	-1	1	-1
-1	-1	1	-1	1	1	1	-1	-1	-1	1
1	-1	-1	1	-1	1	1	1	-1	-1	-1
-1	1	-1	-1	1	-1	1	1	1	-1	-1
-1	-1	1	-1	-1	1	-1	1	1	1	-1
-1	-1	-1	1	-1	-1	1	-1	1	1	1
1	-1	-1	-1	1	-1	-1	1	-1	1	1
1	1	-1	-1	-1	1	-1	-1	1	-1	1
1	1	1	-1	-1	-1	1	-1	-1	1	-1
-1	1	1	1	-1	-1	-1	1	-1	-1	1
1	-1	1	1	1	-1	-1	-1	1	-1	-1

It can be seen that every column is the same as the one prior, only every point is shifted one place down until the cycle is complete (this is with the exception of the first row which consists of all +1s). With the additional options that these designs grant in terms of run size, they prove to be very useful and Plackett-Burman designs continue to be some of the most widely used non-regular designs. The drawback to non-regular designs such as these is that they introduce *complex aliasing* into the model. Complex aliasing, or partial aliasing, creates an alias structure that no longer deals with fully aliased effects and thus a defining relation can not be constructed. The aliasing structure now has fractions of effects aliased with each other and can cause problems with analysis if not properly handled [Wu and Hamada(2009)]. An example of complex aliasing is shown on the next page. This is the alias structure that would result from taking the first five factors of the 12-run Plackett-Burman design in Table 2.3.

$$A = \tfrac{1}{3}BC - \tfrac{1}{3}BD - \tfrac{1}{3}BE + \tfrac{1}{3}CD + \tfrac{1}{3}CE + \tfrac{1}{3}DE$$
$$B = \tfrac{1}{3}AC - \tfrac{1}{3}AD - \tfrac{1}{3}AE + \tfrac{1}{3}CD - \tfrac{1}{3}CE + \tfrac{1}{3}DE$$
$$C = \tfrac{1}{3}AB + \tfrac{1}{3}AD + \tfrac{1}{3}AE + \tfrac{1}{3}BD - \tfrac{1}{3}BE + \tfrac{1}{3}DE$$
$$D = -\tfrac{1}{3}AB + \tfrac{1}{3}AC + \tfrac{1}{3}AE + \tfrac{1}{3}BC + \tfrac{1}{3}BE + \tfrac{1}{3}CE$$
$$E = -\tfrac{1}{3}AB + \tfrac{1}{3}AC + \tfrac{1}{3}AD - \tfrac{1}{3}BC + \tfrac{1}{3}BD + \tfrac{1}{3}CD$$

Notice that the main effects are no longer aliased with other whole effects. Now, many of the aliased effects are not even positive and all of them are fractions. This is where the name “partial aliasing” comes from since effects are partially aliased with each other, causing the alias structure to look even more intricate and confusing. It is for this reason that there is no such thing as a defining relation for non-regular designs. In many cases, however, the disadvantage of complex aliasing can prove to be acceptable given the benefits of Plackett-Burman designs.

One of these benefits is the fact that Plackett-Burman designs hold *hidden projection properties*. Hidden projection properties means that taking projections of a design can result in a desirable outcome that is not immediately evident. In the case of Plackett-Burman designs, one such hidden projection property is that resolution V designs can be constructed out of projections of their factors. For example, if any 4-factor projection of a 12-run Plackett-Burman design (like the one shown in Table 2.3) is taken, then it will be able to estimate all main effects and two-factor interactions of those four factors. This is especially important because of how it relates to the comparable fractional factorial design. The closest fractional factorial design to a 12-run Plackett-Burman is the 2^{4-1} design in 8 runs (a full factorial design with 4 factors would require 16 runs). This design has a defining relation of I = ABCD and is only able to estimate main effects and three two-factor interactions. This hidden projection property allows the 12-run Plackett-Burman design to estimate all main effects and two-factor interactions as a 2^4 full factorial design but uses four fewer runs [Wang and Wu(1995)].

2.3 Three Quarter Fractional Factorial Designs

Much like the fractional factorial designs covered in Section 2.1, three quarter fractional factorial designs seek to lower the number of runs used in a design while maximizing effect estimability. Three quarter designs do this by eliminating 25% of the runs of a design and thus only utilize the remaining three quarters of the runs.

A common notation for three quarters of regular FFDs is $3(2^{k-p})$. There are several ways to construct three quarters of regular designs. The simplest way to build a three quarter design is to start with a full factorial design, partition the design into four blocks, then omit the runs contained in one of the blocks. Mee [2009] uses the example of taking a 2^4 full factorial design and blocking on ABC and ABD. The result is the design in Table 2.4.

Table 2.4: A 2^4 Full Factorial Design in Four Blocks

A	B	C	D
-1	-1	-1	-1
1	-1	1	1
-1	1	1	1
1	1	-1	-1
-1	-1	1	-1
1	-1	-1	1
-1	1	-1	1
1	1	1	-1
-1	-1	-1	1
1	-1	1	-1
-1	1	1	-1
1	1	-1	1
-1	-1	1	1
1	-1	-1	-1
-1	1	-1	-1
1	1	1	1

If any one block in Table 2.4 is omitted, this leaves three quarters of the design that is able to estimate all main effects and two-factor interactions. A design that only uses three quarters of the original run size and is still able to estimate the effects of a resolution V design is very helpful in situations where run size economy is key, e.g. when runs are very expensive, time-consuming, etc.

John [1962] uses the same example of a 2^4 full factorial design, but instead utilizes the defining relations of the blocks to illustrate the method of taking three quarters of a design. The four blocks above can be expressed as 2^{4-2} fractional factorial designs with defining relations as shown on the next page.

i)	I = CD = -ABC = -ABD	or (1), *acd*, *bcd*, *ab*
ii)	I = -CD = ABC = -ABD	or *c*, *ad*, *bd*, *abc*
iii)	I = -CD = -ABC = ABD	or *d*, *ac*, *bc*, *abd*
iv)	I = CD = ABC = ABD	or *cd*, *a*, *b*, *abcd*

Now, any set of treatment combinations constructed from its defining relation can be omitted to create a three quarter design. Something that is very important to note is that the above examples for constructing three quarter designs have all been for regular designs. These methods can not be used for non-regular designs since they have complex aliasing and no defining relations. For this reason, the method of subsetting must be used to construct three quarters of non-regular designs.

Subsetting is performed by using the columns of a design matrix to determine which runs to omit. Operating on any two columns, the signs ($\pm$) of the design points are used to separate the runs into four groups. These groups are $(+,+)$, $(+,-)$, $(-,+)$ and $(-,-)$, e.g. $(+,+)$ subsetted on columns 1 and 2 indicates that for all the runs selected, all the entries for columns 1 and 2 are $+1$. Table 2.5 shows a 2^3 design subsetted on columns 1 and 2 for $(+,+)$.

Table 2.5: A 2^3 Full Factorial Design with $(+,+)$ Subsetting

	1	2	3
	-1	-1	-1
	-1	-1	1
	-1	1	-1
	-1	1	1
	1	-1	-1
	1	-1	1
$\Rightarrow$	1	1	-1
$\Rightarrow$	1	1	1

The two runs with positive entries in columns 1 and 2 can now be omitted, creating three quarters of the original 2^3 design. If instead it was preferable to subset on columns 1 and 2 with $(-,-)$, the first two runs in Table 2.5 would be omitted. This method of subsetting for three quarter designs is applied to Plackett-Burman designs in Chapter 3, Section 3.2.

CHAPTER 3

DESIGNS, RESEARCH METHOD & RESULTS

3.1 Plackett-Burman Design Construction

Each design that was analyzed first started as a full Plackett-Burman design. Designs of run-size 24 through 100, in multiples of four, were examined with the exception of designs of sizes 32 and 64 because they were already regular designs. The Plackett-Burman designs of run-size n were generated using JMP by adding n-1 factors to the Screening Design option. The only exceptions to this method were designs of run-size 92, 96 and 100. Designs of size 92 and 100 were constructed by manipulating appropriate-sized Hadamard matrices [Sloane(2011)]. The first column of each Hadamard matrix was multiplied by all other columns of the matrix, thus creating a column of 1's in the first column that was deleted. The 96-run design was constructed using the 48-run Plackett-Burman design as shown below where D_n signifies the Plackett-Burman design of run-size n and $\underline{\pm 1}$ indicates a column of 1's of appropriate size.

$$D_{96} = \begin{bmatrix} \underline{1} & D_{48} & D_{48} \\ \underline{-1} & -D_{48} & D_{48} \end{bmatrix}$$

3.2 Subsetting for Three Quarter Designs

Each Plackett-Burman design had to be pared down to three quarters of its original size. The method to do this was fairly simple and involved subsetting. To subset, two columns of the Plackett-Burman design are selected along with a treatment combination of $(+,+)$, $(+,-)$, $(-,+)$ or $(-,-)$. If $(+,+)$ is selected along with columns 1 and 2, then every row of the Plackett-Burman design where both columns 1 and 2 are $+1$ is deleted, thus leaving three quarters of the original runs. Because of the cyclic nature of the Plackett-Burman designs used and the fact that every column has a balanced number of even and odd entries, this method always results in one quarter of the runs being deleted. The progression of designs in Figure 3.1 illustrates this method with the use of an 8-run Plackett-Burman design.

Figure 3.1: Subsetting Progression

8-run Plackett-Burman

A	B	C	D	E	F	G
1	-1	-1	1	-1	1	1
1	1	-1	-1	1	-1	1
1	1	1	-1	-1	1	-1
-1	1	1	1	-1	-1	1
1	-1	1	1	1	-1	-1
-1	1	-1	1	1	1	-1
-1	-1	1	-1	1	1	1
-1	-1	-1	-1	-1	-1	-1

→

Subsetted on (+,+) of A and B

A	B	C	D	E	F	G
1	-1	-1	1	-1	1	1
~~1~~	~~1~~	~~-1~~	~~-1~~	~~1~~	~~-1~~	~~1~~
~~1~~	~~1~~	~~1~~	~~-1~~	~~-1~~	~~1~~	~~-1~~
-1	1	1	1	-1	-1	1
1	-1	1	1	1	-1	-1
-1	1	-1	1	1	1	-1
-1	-1	1	-1	1	1	1
-1	-1	-1	-1	-1	-1	-1

→

¾th of an 8-run Plackett-Burman

A	B	C	D	E	F	G
1	-1	-1	1	-1	1	1
-1	1	1	1	-1	-1	1
1	-1	1	1	1	-1	-1
-1	1	-1	1	1	1	-1
-1	-1	1	-1	1	1	1
-1	-1	-1	-1	-1	-1	-1

→

A 2-factor projection of Res. V

A	B	C	D	E	F	G
1	-1	-1	1	-1	1	1
-1	1	1	1	-1	-1	1
1	-1	1	1	1	-1	-1
-1	1	-1	1	1	1	-1
-1	-1	1	-1	1	1	1
-1	-1	-1	-1	-1	-1	-1

It quickly became evident that the number of searchable design combinations rapidly increased to the millions. Considering all $(\pm,\pm)$ combinations over all possible subsets was clearly an impossibility. Through the use of generalized word-length patterns [Xu and Wu(2001)] it became clear that while there were differences between the combinations of $(\pm,\pm)$, overall they were interchangeable and it was not necessary to search over all permutations. For consistency, $(+,+)$ was always subsetted upon for the remainder of this study.

3.3 Projections and Building the Catalog

The purpose of this thesis was to find three quarter Plackett-Burman designs capable of estimating all main effects and two-factor interactions. In order to do this, projections of the subsetted Plackett-Burman designs needed to be taken. A *projection* is when a subset of factors are taken from a design, thus creating a new, smaller design. Table 3.1 shows all the Plackett-Burman designs considered, their three quarter run size and possible projection sizes.

Table 3.1: Possible Projections for each Plackett-Burman Design

PB size	3/4 size	Possible projections
24	18	5
28	21	5
36	27	5,6
40	30	5-7
44	33	5-7
48	36	5-7
52	39	5-8
56	42	5-8
60	45	5-8
68	51	5-9
72	54	5-9
76	57	5-10
80	60	5-10
84	63	5-10
88	66	5-10
92	69	5-11
96	72	5-11
100	75	5-11

The task of considering all possible combinations of projections through an exhaustive search was computationally prohibitive. This raised the question: What is the "best" k-factor projection and how do we go about finding it? First, in order to begin answering this question, a method of ranking designs had to be chosen.

For regular, orthogonal designs, ranking is often easily achieved through either resolution (determined by alias structure) or minimum aberration (determined by the defining relation). Unfortunately, neither of these methods are available for designs that employ complex aliasing because of the lack of a defining relation. D-efficiency was decided upon as the ranking method because of its wide usage in the statistical community and relative ease to understand and compute [Box and Draper(1971)]. Additionally, D-efficiency was one of the primary methods of ranking that Mee [2004] used in his paper titled *Efficient Two-Level Designs for Estimating All Main Effects and Two-Factor Interactions*. By using the same method of ranking as in Mee's paper, a direct comparison could be made later on between his designs and the ones found in this thesis (these comparisons are made in Chapter 3).

The equation for calculating the D-efficiency for each design is shown below:

$$\text{D-efficiency} = \left(\frac{|X'X|}{|X^{*\prime}X^{*}|} \right)^{1/p},$$

where $\mathbf{X}$ is the model matrix for the three quarter design and $\mathbf{X}^*$ is the model matrix for the equivalent D-optimal design; p is equal to the number of parameters in the model. Model matrices, often denoted as $\mathbf{X}$, contain the levels of all of the independent variables included in the regression model, e.g. the intercept, main effects and two-factor interactions would be in the model matrix if those are the effects to be estimated [Montgomery(2009)].

Using this method of ranking by D-efficiency, the "best" projections were acquired with the use of Matlab. Code was written in Matlab to build an exhaustive matrix containing all possible projection combinations (this code can be found in Appendix A). For example, if all possible two-factor projections of a four-column design were desired, Matlab would build a matrix that looks like the one as follows:

$$\begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 2 & 3 \\ 2 & 4 \\ 3 & 4 \end{bmatrix}$$

The projection matrix is then used in iterations to pare down the previously created $(+,+)$ subsets of the Plackett-Burman designs. Now, all that was left for Matlab to do was test for estimability of all main effects and two-factor interactions. Testing all possible five-factor projections of a three quarter, 24-run Plackett-Burman design calls for $\binom{23}{5}$ = 33,649 iterations (all 5-factor combinations of the 23 factors in the 24-run Plackett-Burman design). When this is multiplied by $\binom{23}{2}$ = 253 subset iterations (all 2-factor combinations of the 23 factors in the 24-run Plackett-Burman design), we get 8,513,197 total possible combinations. In our case, this is the smallest possible number of permutations to search over and is a clear problem. For comparison sake, if the same example was taken with a Plackett-Burman design of size 40 (only two designs larger), we now have $\binom{39}{5}$ = 575,757 projection possibilities times $\binom{39}{2}$ = 741 subsets, which equals 426,635,937 total combinations. With a Plackett-Burman design of size 48, we cross the one billion threshold. This being the case, along with the fact that our research extended all the way to Plackett-Burman designs of run-size

100, only a small portion of the projections were able to be searched.

Beginning at the first entry of the projection matrix, Matlab sequentially searched through the possible $(+,+)$ subsets of the Plackett-Burman design until it found a projection/subset combination that was able to estimate all main effects and two-factor interactions. The projection, subset and D-efficiency for this design was recorded. Matlab then immediately moved on to the next entry in the projection matrix and began the sequential subset search over again. One hundred resolution V designs were found during each of these searches and the projection with the highest D-efficiency was noted. The process was repeated by skipping through the projection matrix by millions of entries at a time in even intervals, always taking note of the designs with the highest D-efficiency. Once the single "best" design was found from the groups of 100 designs, an expanded search was centered around this location in the projection matrix and 1,000 resolution V designs were examined. The projection with the highest D-efficiency from this expanded search was entered into a catalog. This method proved inadequate once 44-run Plackett-Burman designs with six or more projections were reached; Matlab no longer had the memory capabilities needed to build projection matrices. To alleviate this situation, the use of *for loops* was employed. Instead of building the entire projection matrix and then proceeding, code was written to build each projection, sequentially search through subsets, record it if it was of full rank, then simply move on to the next projection in the sequence (this code can be found in Appendix B). Different starting points were selected and the same method of searching over 100 then 1,000 resolution V designs was used. The design with the highest overall D-efficiency was cataloged. This catalog is shown in its entirety in Appendix C and a small section of the catalog is shown in Table 3.2.

Table 3.2: Sample of Design Catalog

PB size	Best D-eff. %	Proj. size	Projection	Subset
24	0.8831	5	9,10,16,17,21	1,2
28	0.9985	5	6,7,9,12,16	1,2
36	0.9853	5	14,16,22,24,27	1,2
36	0.8892	6	12,15,20,22,23,34	1,2

PB size indicates the size of the original Plackett-Burman design, not the three quarter design; Best D-eff. % shows the best D-efficiency found during the search; Proj. size is the number of columns in the projection; Projection is the "best" design specified by column number of the three quarter Plackett-Burman design; Subset is the two columns used to subset on by $(+,+)$ in order to build the three quarter design. Each of these designs can estimate all main effects and two-factor interactions with relatively good D-efficiencies. The next task was to affirm these results through comparison, estimation efficiency and simulation studies.

3.4 Estimation Efficiency and Degree of Freedom Efficiency

The first method that was used to test the efficacy of these designs was *estimation efficiency.* For estimation efficiency, the standard error for each of the factorial effects in a design are compared to the "ideal" standard error, i.e. the smallest possible. The smallest standard error for a factorial effect is $\frac{\sigma^2}{N}$ where σ^2 is the error variance and N is the total number of observations. Estimation efficiency is the ratio between the ideal standard error and that for the effect being tested [Mee(2004)]. For example, if the standard error of the effect being tested is $\frac{\sigma^2}{32}$ and the smallest possible standard error is $\frac{\sigma^2}{48}$, then the estimation efficiency is calculated as such:

$$\text{Estimation efficiency: } \frac{\sigma^2}{48} / \frac{\sigma^2}{32} = 32/48 = .6666667$$

Thus the estimation efficiency for this example is about 67%, indicating that some regression coefficient is estimated with 67% efficiency. For regular, orthogonal designs, the estimation efficiencies do not fluctuate between effects because the error variances are the same. However, for non-regular designs such as the ones examined in this paper, complex alising causes the estimation efficiencies to appear rather eratic since the error variances for individual effects are not the same. Table 3.3 shows an example of the estimation efficiencies calculated for five-factor projections of Plackett-Burman designs of sizes 24, 28, 36, 40 and 44. The estimation efficiencies were calculated using Matlab, code for which can be found in Appendix D.

Table 3.3: Example of Estimation Efficiencies for Some Five-Factor Projections

PB size (3/4 size)	24(18)	28(21)	36(27)	40(30)	44(33)
interaction	0.7588	0.9337	0.9068	0.8906	0.9668
main effects	0.7827	0.8964	0.9129	0.9300	0.8710
	0.6382	0.9033	0.9129	0.8051	0.9258
	0.6349	0.8964	0.9343	0.8051	0.8986
	0.5556	0.9033	0.9343	0.8661	0.8986
	0.6382	0.8964	0.8674	0.8661	0.8850
two-factor int.	0.7111	0.9054	0.8088	0.8906	0.9146
	0.6253	0.8964	0.8227	0.8906	0.8963
	0.7277	0.9054	0.8400	0.9232	0.8963
	0.7111	0.8964	0.8604	0.9232	0.8884
	0.6191	0.9033	0.8400	0.8373	0.8954
	0.6191	0.8818	0.8227	0.8373	0.8954
	0.6253	0.9033	0.8604	0.8373	0.9453
	0.5657	0.9033	0.8088	0.8373	0.9083
	0.6191	0.8964	0.8364	0.8373	0.8822
	0.6191	0.9033	0.8364	0.8373	0.8822

As was mentioned earlier, the estimation efficiencies for each effect are seldom the same as a result of the complex aliasing. It can be seen that as the design size gets larger, the estimation efficiencies get better.

The estimation efficiencies were calculated for every design in the catalog and compared to the efficiencies listed by Mee[2004]. The results of this comparison are shown in Table 3.4. It should be noted that since there was a wide variety of efficiencies found across effects for each design, the ranges of these efficiencies are broken up by general similarity and shown in the table.

Table 3.4: Comparison of Estimation Efficiencies Between Catalog and Mee's Designs

	Catalog		Mee	
	# runs	est. eff. range	#runs	est. eff
5 factor proj.	18 21 27-75	56%-78% 88%-90% 81%-99%	16	100%
6 factor proj.	27-30 33-39 42-75*	31%-87% 50%-93% 77%-99%	22	87%
7 factor proj.	30-42 45-75	15%-94% 53%-95%	29	69%-76%
8 factor proj.	39-57 60-75	6%-83% 43%-90%	48	67%-89%
9 factor proj.	51-63 66-75	14%-76% 34%-85%	64	100% for ME and 14 2fi's
10 factor proj.	57-72 75	4%-75% 23%-61%	64	100% for ME and 13 2fi's
11 factor proj.	69-75	4%-47%	96	67%-100%

*Excluding run-size 54 that had an est. eff. range of 52%-87%

The estimation efficiencies of the cataloged designs are high for the low factor projections, but generally decrease as the number of factor progressions increases. However, it is important to note that the upper bounds of the ranges for the cataloged designs have just as good, and in some cases better, estimation efficiencies as the Mee designs. This ceases to be the case at 9-factor projections although starting at 8-factor projections, the cataloged designs have better run-size economy.

Mee [2004] also used degree of freedom efficiency (df-efficiency) as a method of comparing designs. Df-efficiency is expressed as:

$$\text{df-efficiency} = (\text{no. of effects estimated})/(\text{total df}) = .5k(k+1)/(n-1)$$

where k is the number of factors in the design and n is the run size. This gives a sense of how economical a design is in using degrees of freedom to estimate main effects and two-factor interactions. Mee provides a table of the smallest regular designs that are resolution V along with their df-efficiencies. The catalog designs that beat the regular designs' run sizes and df-efficiencies have been added and are shown in Table 3.5. Also included are the df-efficiencies of the designs that Mee recommends (the same designs that are used for estimation efficiency comparison). The full list of catalog df-efficiencies is shown in Appendix F.

Table 3.5: Comparison of Catalog and Mee's Df-efficiencies

k	6	7	8	9	10	11
Regular 2_V^{k-p}	32	64	64	128	128	128
df-efficiency	68%	44%	57%	35%	43%	52%
Recommended design size	22	29	48	64	64	96
df-efficiency	100%	100%	76.6%	71.4%	87.3%	69.5%
Catalog design size range	27 & 30	30-63	39-63	51-75	57-75	69-75
df-efficiency range	77.8 & 70%	93.3-44.4%	92.3-57.1%	88.2-60%	96.5-73.3%	95.7-88%

The vast majority of the catalog designs have better df-efficiencies than the smallest comparable regular designs. The smaller catalog designs in 8, 9 and 10 factors and all of the designs in 11 factors have better df-efficiencies than Mee's recommended designs. This indicates that for the most part, the three quarter Plackett-Burman designs make better use of degrees of freedom than the regular and recommended designs.

3.5 Simulation Study and Results

Next, a simulation study was performed in order to address whether or not these designs could really identify active factors in a real-world setting.

The simulation study began by establishing active effects in a model. These active effects consisted of three, four or five main effects and their associated two-factor interactions. The simulations were performed by way of a "power" equation that calculated the percentage of the time that the designs were able to identify the active effects. Below is the power equation where the active effects are A and B.

$$\text{Power} = \frac{(\#\text{ times A's } p\text{-val} < .05) + (\#\text{ times B's } p\text{-val} < .05) + (\#\text{ times AB's } p\text{-val} < .05)}{(\#\text{ active effects})(1000\text{ iterations})}$$

If there were three active effects established, then there would be six entries in the numerator of the equation (for A, B, C, AB, AC and BC). 1,000 simulation iterations were performed by Matlab to test the cataloged designs (code for this can be found in Appendix G. The number of design iterations that accurately identified the active effects was divided by (# of active effects)*(1,000) thus giving a percentage that was the power of the design. This study was repeated looking at columns (1,2,3), (1,2,3,4) and (1,2,3,4,5) as active factors with magnitudes of 1, 2 and 3 (nine total combinations). The study was also performed with the non-sequential factors (1,3,5), (1,4,5,7) and (2,4,6,9), again repeated with magnitudes 1, 2 and 3 of all effects. It follows that only some of the designs were able to be used for the simulations with non-sequential factors because the rest simply did not have enough columns in the projection to accommodate the simulation; e.g. the 68-run Plackett-Burman design was the first design able to estimate all main effects and two-factor interactions in a projection of nine columns and so it was able to run the simulation for (2,4,6,9) because it actually contained a ninth column in its projection.

All the designs were able to identify active effects at least 92% of the time except the ones shown in Table 3.6. The full list of results for the simulation study can be found in Appendix H. For comparison, the same simulation study was run on the D-optimal designs for each run-size and projection combination. The corresponding D-optimal simulation results are shown in the Table 3.7. Note: non-sequential active effects, i.e. (1,3,5), (1,4,5,7) and (2,4,6,9), were not included in the table because the difference between the sequential and non-sequential effects was deemed negligible.

Table 3.6: Simulation Study for Catalog Designs

PB size	Proj. size	act: 1,2,3 mag: 1	act: 1,2,3 mag: 2	act: 1,2,3 mag: 3	act: 1,2,3,4 mag: 1	act: 1,2,3,4 mag: 2	act: 1,2,3,4 mag: 3	act: 1,2,3,4,5 mag: 1	act: 1,2,3,4,5 mag: 2	act: 1,2,3,4,5 mag: 3
24	5	49.05%	91.10%	99.68%	47.53%	90.96%	99.57%	47.56%	90.95%	99.38%
36	6	91.30%	99.98%	100%	87.85%	99.93%	100%	85.94%	99.97%	100%
40	7	22.03%	40.48%	56.73%	22.15%	40.70%	55.35%	19.88%	40.17%	56.45%
44	7	69.58%	99.32%	100%	73.86%	99.34%	100%	75.68%	99.67%	100%
52	8	32.58%	73.15%	93.02%	31.65%	75.42%	91.63%	33.28%	73.20%	91.49%
56	8	96.93%	100%	100%	97.50%	100%	100%	88.75%	99.71%	99.99%
68	9	79.72%	99.88%	100%	85.13%	99.95%	100%	88.59%	100%	100%
76	10	14.15%	26.87%	39.42%	16.47%	27.11%	42.98%	13.81%	29.35%	41.53%
80	10	76.72%	99.82%	100%	71.57%	99.71%	100%	73.77%	99.53%	100%
92	11	37.53%	79.68%	95.52%	35.18%	75.36%	93.56%	35.49%	77.45%	94.17%
96	11	83.15%	99.93%	100%	79.42%	99.07%	100%	82.86%	99.49%	99.98%

Table 3.7: Simulation Study for D-Optimal Designs

PB size	Proj. size	act: 1,2,3 mag: 1	act: 1,2,3 mag: 2	act: 1,2,3 mag: 3	act: 1,2,3,4 mag: 1	act: 1,2,3,4 mag: 2	act: 1,2,3,4 mag: 3	act: 1,2,3,4,5 mag: 1	act: 1,2,3,4,5 mag: 2	act: 1,2,3,4,5 mag: 3
24	5	57.38%	96.18%	99.97%	60.67%	96.62%	100%	57.19%	96.11%	99.90%
36	6	95.67%	100%	100%	96.56%	100%	100%	96.33%	100%	100%
40	7	30.10%	53.05%	75.18%	29.68%	54.34%	74.52%	31.09%	54.23%	73.89%
44	7	96.05%	100%	100%	96.08%	100%	100%	95.70%	100%	100%
52	8	76.08%	99.47%	100%	75.15%	99.50%	100%	73.92%	99.51%	100%
56	8	99.35%	100%	100%	99.32%	100%	100%	99.38%	100%	100%
68	9	99.65%	100%	100%	99.43%	100%	100%	99.63%	100%	100%
76	10	32.20%	56.25%	76.82%	35.30%	58.72%	78.43%	34.23%	62.18%	78.57%
80	10	99.30%	100%	100%	98.98%	100%	100%	98.86%	100%	100%
92	11	84.57%	100%	100%	82.86%	99.77%	100%	83.88%	99.95%	100%
96	11	99.88%	100%	100%	99.75%	100%	100%	99.74%	100%	100%

For the most part, the power of effect estimation is comparable between the catalog designs and the D-optimal designs. This is further evidence to support the fact that the catalog designs are beneficial. It is clear to see that the designs that had the most issues detecting the active effects were the smallest projections possible. These designs are not recommended because of their overall lack of success. Even though the estimation efficiencies were not stellar, the outcome of these simulations gave credence to the efficacy of these designs.

CHAPTER 4

CONCLUSION

4.1 Recommendations

More often than not, a statistician will attempt to use the simplest possible design while still being able to accurately and efficiently estimate all the effects necessary. Given this fact, non-regular designs like Plackett-Burman designs are often overlooked for similarly-performing fractional factorial designs. The designs cataloged in this paper were researched with run-size efficiency in mind. However leery one may be of non-regular designs, it is important to note the ease of taking a Plackett-Burman design and simply subsetting it down to three quarters of its original size. For this reason, these designs can be effectively implemented by a wide variety of users.

The smallest projections possible, i.e. 5 factors of a three quarter Plackett-Burman design in 24 runs, 6 factors in 36 runs, 7 factors in 40 runs, 8 factors in 52 runs, 9 factors in 68 runs, 10 factors in 76 runs and 11 factors in 92 runs, are not recommended due to issues with estimation efficiency. These designs prove to be too limited in ability to accurately identify accurate effects. However, depending on the situation, most of the other design and projection combinations are powerful enough to be used with just as good, if not better, run-size efficiency. Additionally, all of these designs are extremely easy to construct and do not require the assistance of software (as does the construction of D-optimal designs).

4.2 Future Research

An exhaustive search across all design combinations was impossible for the purpose of this paper. There are certainly "better" D-efficiencies that could be found for many of the designs in the catalog and this may be a worthwhile area to follow-up on, given the ability to perform an exhaustive search. Along the same lines would be a search for isomorphic designs. Two designs are isomorphic if manipulating the run order and/or switching the levels of factors of one design produces the other. Knowledge of isomorphism in designs could help cut down on the number of designs to be searched across. In this case, there is no one good method of searching for isomorphic designs other than implementing an exhaustive search.

Ranking by D-efficiency was one of the simplest and most widely used methods of ranking. It was for this reason, and the fact that easy comparisons could be made, that D-efficiency was chosen to rank the designs established for the catalog. If a different method of ranking were to be implemented, then the entire catalog would have been built using dissimilar choices for the entries.

4.3 Conclusion

One of the benefits of three quarter Plackett-Burman designs is that they are relatively easy to construct without the assistance of software. Additionally, they allow for greater run-size economy. Performing an experiment in fewer runs while still having the ability to estimate all main effects and two-factor interactions is extremely important - especially if runs are expensive or time-consuming. The designs constructed in this thesis are, for the most part, able to efficiently and accurately determine significant factors in a model in fewer runs than comparable regular designs and Plackett-Burman designs. This fact can prove to be beneficial to experimenters who require prudence in their selection of designs when resources are limited. Constructing a resolution V design can also mean eliminating the need for design augmentation down the line, which costs even more runs, money and resources. The potential benefit of these designs outweighs the cautiousness that traditionally surrounds their use.

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APPENDIX A

MATLAB CODE FOR FINDING DESIGNS AND THEIR D-EFFICIENCIES USING PROJECTION MATRICES

```
PP=[];
Proj=[];
Sub=[];
Deff=[];
f=input('What size projection? ');
a=1.0576e+028;                  %Enter the D-optimal design det(X'*X) here
cols=size(PB,2);
Y=nchoosek(1:cols,f);           %Y is the projection matrix; Y can be cut down
for i=1:size(Y,1)               %to allow searching in different locations
    subset=nchoosek(1:cols,2);
    for j=1:size(subset,1)
        A=find(PB(:,subset(j,1))==1 & PB(:,subset(j,2))==1); %subsets on (+,+)
        A=setdiff(1:size(PB,1),A);
        PP=PB(A,:);
        projPP=PP(:,Y(i,:));            %makes the projection
        X=x2fx(projPP,'i');
        rX=rank(X);
        if rX==size(X,2)        %checks if the design can est. all ME's and 2fi's
            b=((det(X'*X))/a)^(1/size(X,2)); %if so, its D-eff is calculated
            Deff=[Deff;b];
            Proj=[Proj;Y(i,:)];
            Sub=[Sub;subset(j,:)];
            break
        end
        PP=[];
    end
    if size(Deff,1)==100
        break
    end
end
DeffProj=[Deff,Proj,Sub]; %DeffProj builds the list of full-rank designs
clear a b f PP projPP Proj Sub subset X rX i j cols A;
```

This code finds the first 100 designs capable of estimating all ME's and 2fi's. If more/less than 100 designs are desired, simply replace "100" everywhere that says `if size(Deff,1)==100` with the new number.

APPENDIX B

MATLAB CODE FOR FINDING DESIGNS AND THEIR D-EFFICIENCIES USING FOR LOOPS

```
PP=[];
Proj=[];
Sub=[];
Deff=[];
a=8.4625e+072;                      %Enter the D-optimal design det(X'*X) here
cols=size(PB,2);
% for p=1:(size(PB,2)-10);          %11 column projection
%     if size(Deff,1)==100
%         break
%     end
% for q=(p+1):(size(PB,2)-9);       %10 column projection
%     if size(Deff,1)==100
%         break
%     end
% for r=(q+1):(size(PB,2)-8);       %9 column projection
%     if size(Deff,1)==100
%         break
%     end
for s=(r+1):(size(PB,2)-7);         %8 column projection
    if size(Deff,1)==100
        break
    end
    for t=(s+1):(size(PB,2)-6);     %7 column projection
        if size(Deff,1)==100
            break
        end
        for u=(t+1):(size(PB,2)-5); %6 column projection
            if size(Deff,1)==100
                break
            end
            for v=(u+1):(size(PB,2)-4); %5 factor projection
                if size(Deff,1)==100
                    break
                end
                for w=(v+1):(size(PB,2)-3);
                    if size(Deff,1)==100
                        break
                    end
                    for x=(w+1):(size(PB,2)-2);
                        if size(Deff,1)==100
                            break
                        end
                        for y=(x+1):(size(PB,2)-1);
                            if size(Deff,1)==100
                                break
                            end
                            for z=(y+1):size(PB,2);
    Y=[r,s,t,u,v,w,x,y,z];  %ADD correct amount of vars here for #projections
    subset=nchoosek(1:cols,2);
    for j=1:size(subset,1)
        A=find(PB(:,subset(j,1))==1 & PB(:,subset(j,2))==1); %subsets on (+,+)
```

```
        A=setdiff(1:size(PB,1),A);
        PP=PB(A,:);
        projPP=PP(:,Y);      %makes the projection
        X=x2fx(projPP,'i');
        rX=rank(X);
        if rX==size(X,2)     %checks if the design can est. all ME's and 2fi's
            b=((det(X'*X))/a)^(1/size(X,2)); %if so, its D-eff is calculated
            Deff=[Deff;b];
            Proj=[Proj;Y];
            Sub=[Sub;subset(j,:)];
            break
        end
        PP=[];
    end
    if size(Deff,1)==100
        break
    end
                        end
                    end
                end
            end
        end
       end
    end
end
% end
% end
% end
DeffProj=[Deff,Proj,Sub]; %DeffProj builds the list of full-rank designs
clear a b f PP projPP Proj Sub subset X rX i j cols A Y p q r s t u v w x y z;
```

As is, this code finds 8-factor projections of full rank. More or less code in the beginning can be commented out or in depending on how large or small of projections are desired. Note that `Y=[r,s,t,u,v,w,x,y,z];` needs to be changed according to the projection size.

This code finds the first 100 designs capable of estimating all ME's and 2fi's. If more/less than 100 designs are desired, simply replace "100" everywhere that says `if size(Deff,1)==100` with the new number.

APPENDIX C

FULL CATALOG OF RESOLUTION V THREE QUARTER PLACKETT-BURMAN DESIGNS

PB size	Best D-eff. %	Proj. size	Projection	Subset
24	0.8831	5	9,10,16,17,21	1,2
28	0.9985	5	6,7,9,12,16	1,2
36	0.9853	5	14,16,22,24,27	1,2
	0.8892	6	12,15,20,22,23,34	1,2
40	0.9598	5	13,16,18,20,24	1,2
	0.8862	6	4,10,11,28,31,34	1,2
	0.7802	7	4,13,14,24,28,32,33	1,2
44	0.9591	5	11,19,33,36,38	1,2
	0.851	6	1,2,3,4,18,30	1,5
	0.7894	7	3,8,10,12,16,18,19	1,2
48	0.9679	5	5,8,26,32,42	1,2
	0.9094	6	21,30,33,38,43,46	1,2
	0.8054	7	9,14,16,22,27,30,35	1,2
52	0.992	5	11,12,13,26,43	1,2
	0.9226	6	10,11,16,18,22,36	1,2
	0.8407	7	6,11,13,18,26,31,34	1,2
	0.7035	8	4,5,6,11,15,24,25,27	1,2
56	1	5	6,18,21,33,34	1,2
	0.9821	6	8,10,15,20,28,44	1,2
	0.8928	7	9,14,16,23,26,30,33	1,2
	0.8056	8	20,25,26,27,28,30,34,47	1,4
60	0.9802	5	13,16,17,30,58	1,2
	0.9436	6	11,13,23,28,44,47	1,2
	0.8741	7	30,31,32,33,34,46,51	1,2
	0.7905	8	30,31,32,33,34,35,46,51	1,2
68	0.9768	5	15,32,34,39,48	1,2
	0.9414	6	10,11,12,13,14,22	1,2
	0.8921	7	30,31,32,33,34,35,61	1,2
	0.8386	8	30,31,32,33,34,35,60,61	1,2
	0.7799	9	30,31,32,33,34,35,36,61,63	1,2
72	0.9837	5	12,46,54,56,65	1,2
	0.9461	6	61,62,63,66,69,71	1,2
	0.8924	7	61,62,63,66,69,70,71	1,2
	0.8277	8	10,11,12,13,14,15,33,63	1,2
	0.7819	9	30,31,32,33,34,35,36,39,40	1,2
76	0.989	5	60,64,66,69,70	1,2
	0.9615	6	5,6,7,8,11,44	1,2
	0.8935	7	57,58,59,60,65,67,74	1,2
	0.8402	8	61,65,66,68,69,70,72,74	1,2
	0.802	9	30,35,36,37,38,40,54,73,74	1,2
	0.6778	10	35,45,46,47,48,50,57,61,73,74	1,2

PB size	Best D-eff. %	Proj. size	Projection	Subset
80	0.9821	5	60,61,62,66,73	1,2
	0.9622	6	60,61,62,67,73,74	1,2
	0.9007	7	30,31,32,33,34,46,49	1,2
	0.8488	8	30,31,32,33,34,35,46,63	1,2
	0.8063	9	30,31,32,33,34,35,36,60,77	1,2
	0.7367	10	30,31,32,33,34,35,36,37,61,73	1,2
84	0.9764	5	30,31,32,35,69	1,2
	0.9437	6	50,51,52,53,55,63	1,2
	0.8945	7	40,41,42,43,44,47,63	1,2
	0.8398	8	30,31,32,33,34,35,37,81	1,2
	0.8123	9	30,31,32,33,34,35,36,70,75	1,2
	0.7661	10	70,72,73,74,75,76,79,80,82,83	1,2
88	0.9877	5	78,80,82,85,86	1,2
	0.9639	6	15,20,25,26,32,80	1,2
	0.9104	7	65,70,76,78,81,82,86	1,2
	0.8563	8	50,55,56,57,58,63,72,74	1,2
	0.8343	9	15,20,25,26,27,28,32,78,80	1,2
	0.7631	10	45,55,65,66,67,68,71,72,74,76	1,2
92	0.9874	5	30,31,32,44,71	1,2
	0.9555	6	50,51,52,53,57,70	1,2
	0.9236	7	30,31,32,33,34,46,52	1,2
	0.8646	8	20,21,22,23,24,25,30,91	1,2
	0.8271	9	30,31,32,33,34,35,36,46,67	1,2
	0.771	10	20,21,22,23,24,25,26,27,29,48	1,2
	0.6764	11	50,51,52,53,54,55,56,57,58,64,70	1,2
96	0.9927	5	10,11,12,13,39	1,2
	0.9726	6	10,11,12,13,18,39	1,2
	0.9331	7	30,31,32,33,34,41,91	1,2
	0.8771	8	30,31,32,33,34,35,41,74	1,2
	0.8117	9	1,2,3,4,5,6,7,9,61	1,8
	0.7645	10	46,47,48,49,50,51,52,53,63,88	1,2
	0.7005	11	40,47,48,49,50,51,52,53,55,58,95	1,7
100	0.9869	5	20,21,22,23,24	1,2
	0.9673	6	20,21,22,23,24,35	1,2
	0.94	7	20,21,22,23,24,25,82	1,2
	0.8883	8	80,81,82,83,84,85,87,96	1,2
	0.8426	9	80,81,82,83,84,85,86,87,96	1,2
	0.7895	10	80,81,82,83,84,85,86,87,88,96	1,2
	0.7204	11	80,81,82,83,84,85,86,88,89,90,96	1,2

APPENDIX D

MATLAB CODE FOR CALCULATING ESTIMATION EFFICIENCIES

```
p=input('Projection column 1: ');
q=input('Projection column 2: ');
r=input('Projection column 3: ');
s=input('Projection column 4: ');
t=input('Projection column 5: ');
u=input('Projection column 6: ');
v=input('Projection column 7: ');
w=input('Projection column 8: ');    %only include as many input lines as the
x=input('Projection column 9: ');    %size of the projection to be analyzed
y=input('Projection column 10: ');
z=input('Projection column 11: ');
A=find(PB(:,1)==1 & PB(:,2)==1);     %subsets on (+,+) on (1,2)
A=setdiff(1:size(PB,1),A);
PP=PB(A,:);
D=PP(:,[p,q,r,s,t,u,v,w,x,y,z]);     %this should match the number of


X=x2fx(D,'i');
V=inv(X'*X);
denom=1./diag(V);
eff=denom/size(X,1);                 %eff gives the estimation efficiency

clear p q r s t u v w x y z A PP D X V denom
```

APPENDIX E

FULL TABLES OF ESTIMATION EFFICIENCIES

5-Factor Projections

PB size (3/4 size)	24(18)	28(21)	36(27)	40(30)	44(33)	48(36)	52(39)	56(42)	60(45)	68(51)	72(54)	76(57)	80(60)	84(63)	88(66)	92(69)	96(72)	100(75)
interaction	0.7588	0.9337	0.9068	0.8906	0.9668	0.9222	0.9702	0.9624	0.9883	0.9641	0.9657	0.9669	0.9558	0.9701	0.9836	0.9730	0.9879	0.9776
main effects	0.7827	0.8964	0.9129	0.9300	0.8710	0.9424	0.9199	0.9624	0.9343	0.9064	0.9385	0.9645	0.9262	0.9432	0.9636	0.9722	0.9826	0.9562
	0.6382	0.9033	0.9129	0.8051	0.9258	0.9343	0.9451	0.9624	0.9338	0.8944	0.9525	0.9633	0.9598	0.9710	0.9571	0.9499	0.9867	0.9461
	0.6349	0.8964	0.9343	0.8051	0.8986	0.8803	0.9408	0.9624	0.9520	0.9563	0.9638	0.9607	0.9366	0.9576	0.9564	0.9570	0.9794	0.9777
	0.5556	0.9033	0.9343	0.8661	0.8986	0.9372	0.9451	0.9624	0.9479	0.9046	0.9431	0.9794	0.9551	0.9818	0.9833	0.9566	0.9840	0.9569
	0.6382	0.8964	0.8674	0.8661	0.8850	0.9080	0.9255	0.9624	0.9334	0.9369	0.9374	0.9788	0.9612	0.9598	0.9635	0.9786	0.9797	0.9762
two-factor int.	0.7111	0.9054	0.8088	0.8906	0.9146	0.8765	0.9548	0.9624	0.9213	0.9141	0.9440	0.9611	0.9486	0.9311	0.9666	0.9615	0.9610	0.9465
	0.6253	0.8964	0.8227	0.8906	0.8963	0.8844	0.9447	0.9624	0.9124	0.9734	0.9584	0.9269	0.9333	0.9103	0.9615	0.9495	0.9558	0.9715
	0.7277	0.9054	0.8400	0.9232	0.8963	0.8877	0.9353	0.9624	0.9547	0.9019	0.9411	0.9485	0.9453	0.9323	0.9784	0.9683	0.9866	0.9607
	0.7111	0.8964	0.8604	0.9232	0.8884	0.9134	0.9199	0.9624	0.8945	0.9483	0.9312	0.9479	0.9358	0.9539	0.9746	0.9786	0.9590	0.9754
	0.6191	0.9033	0.8400	0.8373	0.8954	0.8510	0.9364	0.9624	0.9240	0.9670	0.9461	0.9597	0.9570	0.9135	0.9587	0.9615	0.9548	0.9611
	0.6191	0.8818	0.8227	0.8373	0.8954	0.9150	0.9717	0.9624	0.9679	0.9196	0.9542	0.9609	0.9627	0.9335	0.9815	0.9528	0.9826	0.9482
	0.6253	0.9033	0.8604	0.8373	0.9453	0.9293	0.9451	0.9624	0.9062	0.9537	0.9531	0.9656	0.9647	0.9566	0.9613	0.9621	0.9592	0.9594
	0.5657	0.9033	0.8088	0.8373	0.9083	0.8568	0.9596	0.9624	0.9462	0.9645	0.9500	0.9417	0.9471	0.9387	0.9807	0.9549	0.9832	0.9742
	0.6191	0.8964	0.8364	0.8373	0.8822	0.8558	0.9408	0.9624	0.9211	0.9577	0.9570	0.9387	0.9408	0.9507	0.9656	0.9641	0.9496	0.9699
	0.6191	0.9033	0.8364	0.8373	0.8822	0.9204	0.9451	0.9624	0.9590	0.9595	0.9357	0.9579	0.9670	0.9557	0.9803	0.9713	0.9751	0.9727

6-Factor Projections

PB size (3/4 size)	36(27)	40(30)	44(33)	48(36)	52(39)	56(42)	60(45)	68(51)	72(54)	76(57)	80(60)	84(63)	88(66)	92(69)	96(72)	100(75)
interaction	0.4878	0.7034	0.8344	0.8985	0.9317	0.9442	0.9020	0.9169	0.8640	0.8881	0.9182	0.9423	0.9853	0.9467	0.9809	0.9626
main effects	0.7557	0.7239	0.7080	0.7388	0.7617	0.9579	0.8549	0.8857	0.5545	0.9441	0.9072	0.8963	0.9079	0.9058	0.9595	0.9420
	0.7862	0.6747	0.6461	0.7546	0.7553	0.9298	0.8187	0.8863	0.7047	0.8658	0.9233	0.9072	0.9226	0.8527	0.9502	0.9073
	0.6505	0.7382	0.7109	0.7266	0.7292	0.8759	0.8608	0.8922	0.5467	0.8742	0.9072	0.8860	0.9098	0.9463	0.9560	0.9568
	0.6336	0.7650	0.7090	0.7566	0.7437	0.9240	0.8155	0.9126	0.5185	0.9603	0.9156	0.8109	0.8918	0.8952	0.9679	0.9050
	0.5959	0.7983	0.6148	0.5029	0.8084	0.9230	0.7763	0.8643	0.5592	0.9569	0.8713	0.8744	0.9090	0.8543	0.9447	0.9603
	0.7206	0.7526	0.5371	0.8549	0.7175	0.9230	0.9051	0.8344	0.6467	0.9021	0.9110	0.8919	0.8987	0.9532	0.9633	0.9196
two-factor int.	0.7322	0.4664	0.6559	0.8160	0.8157	0.9147	0.8918	0.9049	0.6462	0.8192	0.8967	0.8879	0.9772	0.8724	0.8971	0.9018
	0.7594	0.5795	0.7510	0.7383	0.7363	0.9215	0.8940	0.8597	0.6413	0.8600	0.9155	0.8527	0.8880	0.9227	0.8884	0.9289
	0.6038	0.5908	0.5950	0.6867	0.8311	0.8794	0.8456	0.8672	0.5663	0.9220	0.9097	0.8780	0.9341	0.8847	0.9246	0.9307
	0.5569	0.8380	0.6888	0.7870	0.8139	0.9099	0.8458	0.8824	0.5932	0.9153	0.8890	0.8838	0.9373	0.8677	0.8706	0.9546
	0.6762	0.7768	0.7155	0.9043	0.7339	0.9099	0.9067	0.8323	0.6377	0.9118	0.9136	0.9051	0.9444	0.9166	0.9295	0.9165
	0.4880	0.7034	0.7595	0.5763	0.7198	0.9038	0.8013	0.8341	0.8663	0.8750	0.9168	0.8460	0.9691	0.9128	0.9131	0.9157
	0.4467	0.4743	0.6264	0.8037	0.8389	0.9091	0.8255	0.8447	0.6649	0.8788	0.9105	0.9110	0.8824	0.8538	0.9231	0.8895
	0.3678	0.5964	0.6350	0.7106	0.8575	0.9498	0.7863	0.8709	0.5743	0.8590	0.8942	0.8718	0.9056	0.8557	0.8828	0.9427
	0.8682	0.4825	0.6533	0.8533	0.7813	0.9498	0.8580	0.7945	0.5576	0.8491	0.8834	0.8917	0.8950	0.9024	0.9128	0.8744
	0.3135	0.5693	0.7221	0.8050	0.7631	0.8866	0.8764	0.8595	0.6819	0.8717	0.9082	0.8738	0.9375	0.8970	0.9321	0.9236
	0.3920	0.4746	0.6646	0.7453	0.8349	0.8808	0.8276	0.9026	0.6398	0.8421	0.8486	0.8529	0.9304	0.9152	0.8917	0.9348
	0.7739	0.5673	0.6549	0.7263	0.7896	0.8808	0.8347	0.7853	0.7657	0.8497	0.8725	0.8902	0.9381	0.9266	0.9299	0.8887
	0.3865	0.7899	0.6739	0.7896	0.8128	0.8688	0.8052	0.9286	0.5389	0.9246	0.8949	0.8656	0.8953	0.8703	0.8898	0.9098
	0.6277	0.8112	0.6504	0.6871	0.7681	0.8688	0.8664	0.8208	0.5786	0.9511	0.9137	0.9068	0.8630	0.8967	0.9502	0.8770
	0.6540	0.6648	0.6896	0.8530	0.7878	0.8258	0.8909	0.8597	0.5273	0.9220	0.8830	0.7995	0.9162	0.9090	0.9199	0.9111

7-Factor Projections

PB size (3/4 size)	40(30)	44(33)	48(36)	52(39)	56(42)	60(45)	68(51)	72(54)	76(57)	80(60)	84(63)	88(66)	92(69)	96(72)	100(75)
interaction	0.1469	0.6454	0.7808	0.8869	0.8180	0.7926	0.8511	0.8768	0.7656	0.9236	0.9294	0.9243	0.9468	0.9499	0.9514
main effects	0.4651	0.4730	0.5402	0.5707	0.5462	0.6963	0.7050	0.6767	0.8812	0.7472	0.7943	0.8933	0.8000	0.9087	0.8353
	0.7251	0.2162	0.3278	0.8036	0.5353	0.8111	0.7249	0.7475	0.7108	0.8875	0.7078	0.8011	0.8200	0.8432	0.8276
	0.3917	0.5311	0.5222	0.6314	0.8089	0.7346	0.8194	0.6786	0.7069	0.6586	0.7247	0.8231	0.8231	0.8880	0.8953
	0.3295	0.6139	0.5848	0.4814	0.5605	0.6596	0.6803	0.7653	0.7658	0.8071	0.7222	0.7557	0.8597	0.8513	0.9139
	0.2596	0.3788	0.5451	0.5499	0.3534	0.6620	0.7356	0.8322	0.7928	0.8061	0.7562	0.9058	0.7897	0.8283	0.8470
	0.4336	0.4366	0.3833	0.6926	0.8461	0.6403	0.7221	0.7595	0.7890	0.8166	0.7668	0.8850	0.9247	0.8638	0.7810
	0.6513	0.4165	0.5325	0.6825	0.8461	0.7057	0.6394	0.8245	0.7951	0.7842	0.7233	0.8379	0.8526	0.8307	0.8576
two-factor int.	0.3196	0.3168	0.4475	0.5006	0.2931	0.6325	0.6971	0.6612	0.7719	0.8145	0.8432	0.7508	0.8218	0.8967	0.8301
	0.2709	0.3091	0.4865	0.4984	0.5307	0.7250	0.7859	0.7517	0.7030	0.6876	0.7899	0.7870	0.7924	0.7368	0.8580
	0.4541	0.2685	0.2708	0.4121	0.4144	0.6374	0.7133	0.7002	0.7235	0.7854	0.7530	0.8001	0.8124	0.8380	0.8630
	0.2536	0.3967	0.6289	0.6769	0.5431	0.6524	0.7169	0.7537	0.7988	0.7131	0.7452	0.6685	0.7433	0.7722	0.7823
	0.3305	0.4237	0.5585	0.4365	0.8156	0.5965	0.6888	0.5269	0.7845	0.7341	0.8058	0.8707	0.8820	0.8076	0.8047
	0.3199	0.3292	0.5948	0.5473	0.8156	0.6257	0.7723	0.7036	0.8252	0.6954	0.8628	0.8236	0.8485	0.8784	0.8041
	0.3113	0.2668	0.5613	0.4531	0.7057	0.6418	0.6534	0.7141	0.7793	0.7571	0.7389	0.7428	0.8186	0.8622	0.8234
	0.4863	0.5219	0.6161	0.5497	0.5316	0.5889	0.7810	0.6910	0.6858	0.6994	0.7865	0.8154	0.8046	0.8433	0.9116
	0.4681	0.3322	0.3363	0.5297	0.4219	0.7210	0.6813	0.8261	0.6969	0.7780	0.7689	0.7281	0.7678	0.8427	0.8496
	0.2310	0.3291	0.5096	0.5858	0.7856	0.7114	0.6385	0.5942	0.6473	0.6696	0.8071	0.8271	0.8256	0.8558	0.8541
	0.1494	0.6264	0.2604	0.6307	0.7856	0.6141	0.5848	0.6508	0.7078	0.7864	0.7924	0.7515	0.8709	0.8433	0.7874
	0.2751	0.4281	0.4459	0.3852	0.6653	0.6190	0.8107	0.8272	0.6139	0.7797	0.7180	0.7534	0.8058	0.8333	0.9024
	0.4068	0.4447	0.3397	0.4922	0.4192	0.6515	0.7187	0.7890	0.6583	0.7788	0.6801	0.7488	0.8116	0.7236	0.8558
	0.1550	0.2233	0.3732	0.5135	0.9357	0.5697	0.7293	0.7257	0.6589	0.7068	0.8321	0.8281	0.8538	0.8227	0.8178
	0.1572	0.3459	0.5187	0.5404	0.9357	0.7385	0.6472	0.7511	0.5910	0.7141	0.8558	0.7805	0.7637	0.7923	0.8771
	0.3715	0.5102	0.7039	0.5460	0.2714	0.7501	0.6853	0.8432	0.7985	0.7171	0.7793	0.7618	0.7588	0.8330	0.8745
	0.2812	0.5162	0.6181	0.4727	0.7677	0.5639	0.7797	0.6538	0.7193	0.7721	0.6940	0.8132	0.8196	0.8548	0.8181
	0.4629	0.5177	0.6865	0.5371	0.7677	0.5841	0.7829	0.8078	0.7221	0.7332	0.7139	0.8120	0.8138	0.8783	0.9226
	0.2466	0.3822	0.3935	0.4488	0.8461	0.6022	0.7288	0.7512	0.7414	0.7519	0.7988	0.8650	0.8092	0.7369	0.8214
	0.3058	0.4953	0.5099	0.6891	0.8461	0.5822	0.7039	0.7401	0.8075	0.7572	0.8197	0.7829	0.8184	0.7943	0.8518
	0.1469	0.4416	0.2322	0.5970	0.7427	0.6466	0.6695	0.6531	0.7162	0.8438	0.7950	0.8557	0.8651	0.9179	0.9060

8-Factor Projections

PB size (3/4 size)	52(39)	56(42)	60(45)	68(51)	72(54)	76(57)	80(60)	84(63)	88(66)	92(69)	96(72)	100(75)
interaction	0.2072	0.8093	0.7288	0.7759	0.7958	0.6717	0.8526	0.8782	0.7204	0.8999	0.8927	0.8515
main effects	0.3095	0.8330	0.5679	0.4840	0.6089	0.4647	0.7475	0.7858	0.7497	0.6687	0.8049	0.8285
	0.1211	0.7586	0.5734	0.4710	0.6687	0.7005	0.6864	0.7451	0.7212	0.7360	0.8479	0.7320
	0.2951	0.6613	0.5554	0.6141	0.5741	0.5576	0.6592	0.6924	0.6800	0.6278	0.7342	0.7673
	0.2951	0.6241	0.3771	0.5405	0.5190	0.7301	0.7705	0.7298	0.6915	0.6834	0.7143	0.6458
	0.3701	0.4883	0.4286	0.5114	0.5312	0.5411	0.7124	0.7828	0.7928	0.7380	0.7966	0.6426
	0.4639	0.2203	0.2492	0.6157	0.5132	0.7117	0.6836	0.7718	0.6960	0.6852	0.7438	0.8026
	0.2432	0.4482	0.4426	0.3863	0.5735	0.7401	0.6494	0.6968	0.6458	0.6271	0.6884	0.7536
	0.3188	0.3891	0.2662	0.4957	0.6541	0.5318	0.6264	0.7134	0.7020	0.7226	0.6642	0.7167
two-factor int.	0.1302	0.6159	0.2766	0.5193	0.6109	0.5372	0.6765	0.7223	0.7954	0.6435	0.7683	0.7981
	0.1080	0.4523	0.4499	0.7259	0.5415	0.7142	0.5945	0.6865	0.4262	0.6029	0.5802	0.7664
	0.1671	0.5190	0.3305	0.6148	0.5525	0.4952	0.5689	0.6636	0.4962	0.6818	0.7515	0.7470
	0.0554	0.2588	0.4736	0.5652	0.6288	0.4215	0.6304	0.5704	0.7710	0.6543	0.6033	0.7779
	0.0913	0.2720	0.4773	0.5783	0.5922	0.5908	0.6136	0.6239	0.7363	0.6215	0.6710	0.8047
	0.1319	0.3820	0.4764	0.3905	0.5689	0.7340	0.5987	0.7475	0.6957	0.6814	0.7149	0.8152
	0.1596	0.2129	0.3822	0.5647	0.5111	0.4290	0.6026	0.7901	0.7099	0.7810	0.7711	0.7743
	0.1923	0.4801	0.3589	0.5507	0.5290	0.6935	0.6267	0.5014	0.6270	0.6593	0.7485	0.7868
	0.1943	0.6940	0.3552	0.6667	0.4854	0.4517	0.5647	0.5980	0.5185	0.6390	0.6217	0.6778
	0.2060	0.1252	0.6346	0.4913	0.5601	0.5273	0.6304	0.6995	0.8407	0.7296	0.6454	0.6719
	0.1600	0.1291	0.3623	0.4632	0.5585	0.5060	0.6388	0.5257	0.7004	0.6476	0.6802	0.7413
	0.1178	0.7004	0.5827	0.4788	0.7580	0.5027	0.5932	0.6329	0.7254	0.7067	0.7017	0.6751
	0.2150	0.3079	0.3221	0.3851	0.5187	0.6186	0.5272	0.5397	0.6974	0.7213	0.7973	0.6924
	0.0821	0.4924	0.4728	0.5643	0.4458	0.6075	0.6408	0.4785	0.7204	0.7530	0.6755	0.7182
	0.1814	0.2490	0.4163	0.5772	0.5195	0.5791	0.7025	0.6777	0.6748	0.6667	0.7369	0.7536
	0.1885	0.1394	0.2733	0.5227	0.4624	0.7515	0.5492	0.5843	0.5796	0.6062	0.7315	0.7088
	0.2565	0.2541	0.3453	0.6349	0.6535	0.6603	0.5798	0.6482	0.5587	0.7067	0.6771	0.7494
	0.1322	0.2078	0.2547	0.5422	0.5265	0.6461	0.6515	0.6235	0.6591	0.6878	0.7795	0.7333
	0.1498	0.2576	0.5791	0.4989	0.5857	0.6703	0.6643	0.6534	0.6947	0.7148	0.6057	0.6301
	0.3215	0.2001	0.3187	0.5876	0.5332	0.5564	0.5417	0.6569	0.5412	0.6895	0.5638	0.7823
	0.0863	0.1698	0.3305	0.5546	0.5749	0.4053	0.6570	0.6514	0.6104	0.7627	0.6606	0.6413
	0.2542	0.1607	0.4547	0.6622	0.5465	0.5970	0.5761	0.6976	0.6380	0.8590	0.7600	0.7201
	0.0829	0.3995	0.2543	0.5993	0.4892	0.6085	0.6825	0.5797	0.7786	0.7075	0.5519	0.7279
	0.1948	0.4591	0.3454	0.4349	0.5901	0.5090	0.6296	0.6338	0.7638	0.8046	0.6706	0.6811
	0.3099	0.5487	0.2170	0.4243	0.5550	0.3998	0.7955	0.7310	0.7062	0.7103	0.7465	0.7028
	0.3508	0.4421	0.1782	0.5015	0.5951	0.6220	0.6033	0.6339	0.6686	0.7639	0.6877	0.6782
	0.1654	0.5013	0.3830	0.5295	0.5539	0.7958	0.6456	0.6157	0.8295	0.7215	0.7415	0.7229
	0.1187	0.6656	0.3607	0.5547	0.5955	0.6381	0.6647	0.5836	0.7526	0.7666	0.7620	0.8361

9-Factor Projections

PB size (3/4 size)	68(51)	72(54)	76(57)	80(60)	84(63)	88(66)	92(69)	96(72)	100(75)
interaction	0.5955	0.5244	0.4347	0.7579	0.7300	0.6958	0.8501	0.7268	0.7486
main effects	0.2226	0.3040	0.1665	0.4243	0.5459	0.6777	0.4550	0.7268	0.6371
	0.2639	0.2478	0.4310	0.4980	0.5065	0.5914	0.5107	0.5320	0.6537
	0.3051	0.2623	0.4906	0.4789	0.6665	0.5457	0.3744	0.5026	0.6687
	0.3287	0.3885	0.3208	0.6096	0.3576	0.5330	0.5808	0.5524	0.5968
	0.4581	0.2329	0.4130	0.5042	0.5898	0.6167	0.6413	0.5088	0.5167
	0.3864	0.2846	0.3708	0.4868	0.5571	0.5121	0.4795	0.6158	0.6710
	0.2488	0.2141	0.3519	0.6727	0.5246	0.6094	0.4962	0.6395	0.5486
	0.2956	0.3289	0.3304	0.4605	0.6113	0.4753	0.7492	0.6266	0.5750
	0.3051	0.3281	0.6307	0.4476	0.4566	0.6972	0.5851	0.4274	0.6305
two-factor int.	0.3661	0.3679	0.5609	0.5412	0.4168	0.7614	0.6748	0.4889	0.6427
	0.1892	0.4881	0.4573	0.3687	0.5073	0.6312	0.5206	0.4194	0.5656
	0.3384	0.4164	0.3538	0.4034	0.4384	0.6149	0.6023	0.3821	0.6725
	0.3768	0.3597	0.4981	0.5019	0.6181	0.4529	0.5673	0.3715	0.6389
	0.2838	0.4768	0.3739	0.4074	0.3567	0.5680	0.5627	0.4759	0.6956
	0.2056	0.2701	0.3405	0.5334	0.6084	0.6600	0.5783	0.4243	0.4568
	0.3187	0.2945	0.4460	0.4848	0.3542	0.4884	0.5738	0.4656	0.6735
	0.1648	0.3541	0.2449	0.4462	0.4176	0.6281	0.5962	0.6391	0.6464
	0.1763	0.2971	0.4174	0.3465	0.5206	0.6542	0.5067	0.4834	0.5659
	0.5737	0.3552	0.3167	0.3731	0.3736	0.4720	0.5024	0.4598	0.6224
	0.4519	0.3570	0.3447	0.4291	0.4329	0.5022	0.5758	0.5470	0.5537
	0.2734	0.2922	0.3493	0.3159	0.4785	0.4850	0.4733	0.5040	0.5016
	0.2362	0.2427	0.4029	0.4164	0.4876	0.4735	0.6340	0.3399	0.4917
	0.2311	0.5235	0.2241	0.5776	0.5417	0.5431	0.4073	0.5674	0.5249
	0.1353	0.4076	0.4414	0.3793	0.4173	0.6747	0.6040	0.5894	0.5804
	0.3366	0.2954	0.5188	0.4843	0.3273	0.6230	0.5188	0.6162	0.6220
	0.4481	0.2099	0.1943	0.3218	0.5553	0.4097	0.5662	0.5496	0.7140
	0.4418	0.1878	0.4696	0.4322	0.4111	0.6305	0.4101	0.5264	0.6378
	0.3230	0.2667	0.1978	0.4776	0.5108	0.6166	0.5872	0.4823	0.5061
	0.2809	0.3272	0.4514	0.3984	0.6075	0.6571	0.5629	0.4370	0.6663
	0.3107	0.4038	0.2834	0.2536	0.5296	0.4908	0.4723	0.7351	0.5820
	0.3989	0.2366	0.3733	0.4468	0.4615	0.6958	0.5071	0.3546	0.4890
	0.3260	0.2582	0.4112	0.5758	0.3752	0.3678	0.4470	0.4250	0.7314
	0.3996	0.3086	0.4022	0.3034	0.4488	0.4867	0.4919	0.5075	0.6198
	0.4738	0.3969	0.2506	0.4731	0.3711	0.3828	0.5409	0.5305	0.5945
	0.3777	0.3882	0.3634	0.4445	0.4151	0.4823	0.4463	0.6461	0.6430
	0.3785	0.5222	0.3060	0.4637	0.3922	0.3900	0.4919	0.6017	0.6170
	0.4697	0.3253	0.2716	0.3950	0.3661	0.4312	0.4380	0.5438	0.6163
	0.3568	0.5191	0.3437	0.4091	0.4228	0.6108	0.5985	0.5247	0.5458
	0.3866	0.3527	0.3280	0.4212	0.5675	0.4754	0.6488	0.6962	0.5521
	0.3354	0.1810	0.3543	0.4270	0.5076	0.4899	0.4090	0.5259	0.6632
	0.2289	0.3330	0.4434	0.4128	0.4138	0.4637	0.5915	0.5379	0.6197
	0.3004	0.3851	0.3431	0.4470	0.5361	0.4441	0.5192	0.7293	0.6490
	0.2237	0.3995	0.5254	0.5087	0.5496	0.4590	0.5915	0.5222	0.5544
	0.2428	0.3278	0.3293	0.3880	0.4775	0.4225	0.5367	0.7920	0.5726
	0.2425	0.3054	0.3254	0.4240	0.5728	0.6462	0.4433	0.7703	0.6442

10-Factor Projections

PB size (3/4 size)	76(57)	80(60)	84(63)	88(66)	92(69)	96(72)	100(75)
interaction	0.1464	0.4771	0.5275	0.3827	0.4926	0.7510	0.6079
main effects	0.0692	0.1450	0.3105	0.3457	0.3393	0.3204	0.5142
	0.0481	0.2022	0.2744	0.3179	0.4655	0.1739	0.5759
	0.1356	0.2756	0.2671	0.2864	0.4404	0.4128	0.5421
	0.1357	0.1466	0.3492	0.3768	0.4206	0.5442	0.4247
	0.1277	0.2561	0.2378	0.4425	0.2722	0.3293	0.3494
	0.1052	0.3683	0.3463	0.2216	0.5241	0.3975	0.4444
	0.0493	0.3240	0.2068	0.4216	0.3271	0.3207	0.4065
	0.0998	0.1628	0.2579	0.3075	0.5806	0.5057	0.4288
	0.1026	0.2647	0.3455	0.5342	0.3919	0.3447	0.4243
	0.1227	0.2172	0.2665	0.3893	0.3808	0.5639	0.4509
two-factor int.	0.1571	0.2453	0.2425	0.2919	0.3938	0.2546	0.4481
	0.0823	0.2458	0.4347	0.3011	0.3851	0.1485	0.4521
	0.0750	0.1920	0.2815	0.1691	0.3127	0.3451	0.5204
	0.0644	0.3387	0.3365	0.3690	0.2279	0.5138	0.5749
	0.0369	0.2882	0.3696	0.3011	0.2101	0.2127	0.5344
	0.1089	0.1415	0.2704	0.3025	0.1821	0.3296	0.3990
	0.1328	0.2784	0.2441	0.3406	0.2192	0.4369	0.4838
	0.1313	0.2147	0.4402	0.4948	0.3145	0.4271	0.5411
	0.0780	0.1732	0.3301	0.3626	0.3363	0.5109	0.5429
	0.1053	0.1870	0.1910	0.1906	0.3706	0.4227	0.4299
	0.1285	0.1628	0.2089	0.2184	0.5307	0.1799	0.4727
	0.0680	0.1815	0.3879	0.2835	0.2237	0.3108	0.4066
	0.0820	0.1773	0.3021	0.2249	0.2309	0.3510	0.3556
	0.3324	0.1299	0.2505	0.2341	0.3185	0.4381	0.4426
	0.0620	0.1801	0.2951	0.2458	0.3203	0.3747	0.4400
	0.1624	0.1935	0.2188	0.3649	0.3175	0.4249	0.3993
	0.0765	0.2818	0.3524	0.2865	0.4040	0.5122	0.3626
	0.1551	0.1655	0.1938	0.2158	0.4523	0.3555	0.4153
	0.1230	0.1132	0.2275	0.3058	0.4096	0.4402	0.6141
	0.1790	0.1040	0.4506	0.2937	0.3901	0.3471	0.4209
	0.2407	0.1409	0.2410	0.3528	0.4716	0.3532	0.3538
	0.1753	0.0917	0.3272	0.2800	0.4386	0.5288	0.3719
	0.2234	0.2285	0.3456	0.3660	0.5164	0.4942	0.5012
	0.0745	0.2261	0.3732	0.2293	0.4612	0.4000	0.4460
	0.0495	0.2791	0.2717	0.3827	0.4905	0.3818	0.3827
	0.0808	0.1677	0.2964	0.1997	0.2342	0.2490	0.5076
	0.1441	0.1436	0.1837	0.1958	0.3543	0.2304	0.4645
	0.1864	0.0900	0.4221	0.3400	0.2933	0.3693	0.3828
	0.1522	0.2671	0.2789	0.3422	0.4078	0.2182	0.2293
	0.1389	0.1233	0.1411	0.2142	0.4524	0.2936	0.4736
	0.0927	0.2241	0.2986	0.2360	0.2976	0.3156	0.5394
	0.1290	0.1511	0.2619	0.3075	0.2015	0.4213	0.4615
	0.1298	0.2083	0.3710	0.2845	0.2647	0.2672	0.3955
	0.1502	0.2251	0.3637	0.2830	0.3931	0.2765	0.5256
	0.0603	0.3456	0.3684	0.2576	0.3225	0.3963	0.4379
	0.1354	0.0887	0.2492	0.2822	0.3767	0.4562	0.4853
	0.0847	0.2340	0.3745	0.4666	0.3953	0.2810	0.4774
	0.1137	0.2414	0.2518	0.4110	0.3599	0.4269	0.4060
	0.1643	0.1197	0.2253	0.3681	0.3631	0.3350	0.4395
	0.1612	0.1761	0.4067	0.3353	0.1756	0.4678	0.3565
	0.0797	0.2371	0.3552	0.2839	0.3434	0.2759	0.3850
	0.1777	0.3436	0.3781	0.3073	0.4164	0.6366	0.4893
	0.2092	0.3050	0.3625	0.4189	0.4439	0.6250	0.4240
	0.1188	0.1175	0.2042	0.3666	0.4238	0.2744	0.4199
	0.1321	0.1420	0.4228	0.3291	0.2946	0.3770	0.2689

11-Factor Projections

PB size (3/4 size)	92(69)	96(72)	100(75)
interaction	0.4531	0.2110	0.2443
main effects	0.0597	0.2132	0.4064
	0.1372	0.2167	0.2662
	0.1248	0.4015	0.1880
	0.0645	0.0611	0.1980
	0.0476	0.1264	0.1507
	0.1362	0.1604	0.2665
	0.1254	0.1710	0.1679
	0.1518	0.1025	0.2143
	0.1299	0.2588	0.1993
	0.1872	0.1218	0.2160
	0.1371	0.0911	0.1671
two-factor int.	0.1386	0.1389	0.2187
	0.1571	0.1428	0.3131
	0.2076	0.2422	0.3984
	0.1733	0.2283	0.2701
	0.1936	0.1562	0.2364
	0.1795	0.1477	0.3552
	0.2729	0.2064	0.2677
	0.1514	0.2380	0.3192
	0.1011	0.2694	0.2768
	0.2353	0.2645	0.3127
	0.1053	0.1881	0.3134
	0.0669	0.2460	0.3093
	0.1370	0.2420	0.3170
	0.0416	0.1581	0.2098
	0.1973	0.2026	0.2214
	0.1242	0.2810	0.1847
	0.0666	0.4701	0.1765
	0.1044	0.2490	0.2532
	0.1156	0.1693	0.2820
	0.0783	0.1535	0.2211
	0.1422	0.2924	0.2994
	0.0624	0.1407	0.1726
	0.2033	0.1360	0.3364
	0.1849	0.2111	0.4398
	0.1775	0.1390	0.2471
	0.0881	0.0987	0.1913
	0.1622	0.2110	0.2831
	0.1341	0.4495	0.3529
	0.1188	0.2206	0.2142
	0.1805	0.1748	0.3218
	0.0870	0.1504	0.2146
	0.0838	0.1515	0.3030
	0.1836	0.1854	0.2223
	0.1125	0.0837	0.2282
	0.0570	0.2114	0.3502
	0.0437	0.0870	0.3145
	0.1112	0.1521	0.2274
	0.1086	0.1998	0.1279
	0.2829	0.1099	0.3067
	0.1074	0.1703	0.3122
	0.0557	0.2062	0.1836
	0.0936	0.2101	0.1797
	0.1121	0.1532	0.1881
	0.0906	0.2542	0.1031
	0.1331	0.1469	0.2916
	0.1696	0.1835	0.2438
	0.0975	0.1424	0.1815
	0.1948	0.0998	0.2064
	0.0904	0.1313	0.3463
	0.2270	0.1878	0.1027
	0.2072	0.1152	0.1626
	0.1182	0.0701	0.2283
	0.0599	0.1387	0.2049
	0.0655	0.1039	0.2596
	0.1621	0.0401	0.3477

APPENDIX F

FULL TABLE OF DF-EFFICIENCIES FOR CATALOG DESIGNS

3/4 PB size	Proj. size	Df-eff.
18	5	83.3%
21	5	71.4%
27	5	55.6%
	6	77.8%
30	5	50%
	6	70%
	7	93.3%
33	5	45.5%
	6	63.6%
	7	84.8%
36	5	41.7%
	6	58.3%
	7	77.8%
39	5	38.5%
	6	53.8%
	7	71.8%
	8	92.3%
42	5	35.7%
	6	50%
	7	66.7%
	8	85.7%
45	5	33.3%
	6	46.7%
	7	62.2%
	8	80%
51	5	29.4%
	6	41.2%
	7	54.9%
	8	70.6%
	9	88.2%
54	5	27.8%
	6	38.9%
	7	51.9%
	8	66.7%
	9	83.3%
57	5	26.3%
	6	36.8%
	7	49.1%
	8	63.2%
	9	78.9%
	10	96.5%

3/4 PB size	Proj. size	Df-eff.
60	5	25%
	6	35%
	7	46.7%
	8	60%
	9	75%
	10	91.7%
63	5	23.8%
	6	33.3%
	7	44.4%
	8	57.1%
	9	71.4%
	10	87.3%
66	5	22.7%
	6	31.8%
	7	42.4%
	8	54.5%
	9	68.2%
	10	83.3%
69	5	21.7%
	6	30.4%
	7	40.6%
	8	52.2%
	9	65.2%
	10	79.7%
	11	95.7%
72	5	20.8%
	6	29.2%
	7	38.9%
	8	50%
	9	62.5%
	10	76.4%
	11	91.7%
75	5	20%
	6	28%
	7	37.3%
	8	48%
	9	60%
	10	73.3%
	11	88%

APPENDIX G

MATLAB CODE FOR SIMULATION STUDY

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p=input('Projection column 1: ');
q=input('Projection column 2: ');
r=input('Projection column 3: ');
s=input('Projection column 4: ');
t=input('Projection column 5: ');
u=input('Projection column 6: ');
v=input('Projection column 7: ');
w=input('Projection column 8: ');     %only include as many input lines as the
x=input('Projection column 9: ');     %size of the projection to be analyzed
y=input('Projection column 10: ');
z=input('Projection column 11: ');
A=find(PB(:,1)==1 & PB(:,2)==1);      %subset on (+,+) on (1,2)
A=setdiff(1:size(PB,1),A);
PP=PB(A,:);
D=PP(:,[p,q,r,s,t,u,v,w,x,y,z]);      %this should match the number of input lines above

power=[];
iters=1000;                           %number of simulations to be run
alpha=0.05;
X=x2fx(D,'i');
beta=zeros(size(X,2),1);

for mag=1:3       %For 3 active factors
    active=[2 4 6 14 16 33];          % Which factors to be active (A,C,E,AC,AE,CE)
    beta(active)=mag;                 % magnitude of coefficient
    sigma=1;                          % error value
    for i=1:iters
        Y=X*beta+randn(size(X,1),1)*sigma;
        stats=regstats(Y,D,'i',{'rsquare' 'tstat'}); % finds p-values
        pvals(:,i)=stats.tstat.pval;
    end
    pwr=(length(find(pvals(2,:)<=alpha))+length(find(pvals(4,:)<=alpha))+
    length(find(pvals(6,:)<=alpha))+length(find(pvals(14,:)<=alpha))+
    length(find(pvals(16,:)<=alpha))+length(find(pvals(33,:)<=alpha)))/(6*iters);
    power=[power,pwr];
end

for mag=1:3       %For 4 active factors
    active=[2 5 6 8 15 16 18 40 42 48]; % active factors (A,D,E,G,AD,AE,AG,DE,DG,EG)
    beta(active)=mag;                   % magnitude of coefficient
    sigma=1;                            % error value
    for i=1:iters
        Y=X*beta+randn(size(X,1),1)*sigma;
        stats=regstats(Y,D,'i',{'rsquare' 'tstat'}); % finds p-values
        pvals(:,i)=stats.tstat.pval;
    end
    pwr=(length(find(pvals(2,:)<=alpha))+length(find(pvals(5,:)<=alpha))+
    length(find(pvals(6,:)<=alpha))+length(find(pvals(8,:)<=alpha))+
    length(find(pvals(15,:)<=alpha))+length(find(pvals(16,:)<=alpha))+
    length(find(pvals(18,:)<=alpha))+length(find(pvals(40,:)<=alpha))+
    length(find(pvals(42,:)<=alpha))+length(find(pvals(48,:)<=alpha)))/(10*iters);
```

```
    power=[power,pwr];
end

for mag=1:3     %For 4 active factors
    active=[3 5 7 10 24 26 29 41 44 55]; % active factors (B,D,F,I,BD,BF,BI,DF,DI,FI)
    beta(active)=mag;                    % magnitude of coefficient
    sigma=1;                             % error value
    for i=1:iters
        Y=X*beta+randn(size(X,1),1)*sigma;
        stats=regstats(Y,D,'i',{'rsquare' 'tstat'}); % finds p-values
        pvals(:,i)=stats.tstat.pval;
    end
    pwr=(length(find(pvals(3,:)<=alpha))+length(find(pvals(5,:)<=alpha))+
    length(find(pvals(7,:)<=alpha))+length(find(pvals(10,:)<=alpha))+
    length(find(pvals(24,:)<=alpha))+length(find(pvals(26,:)<=alpha))+
    length(find(pvals(29,:)<=alpha))+length(find(pvals(41,:)<=alpha))+
    length(find(pvals(44,:)<=alpha))+length(find(pvals(55,:)<=alpha)))/(10*iters);
    power=[power,pwr];
end

clear X Y active alpha beta i iters sigma stats pvals D
clear A PP p q r s t u v w x y z mag pwr
```

The factors that are "active" are entered into the array according to their corresponding factor number in the X model matrix. The output variable "power" gives the simulation percentages for the active effects.

APPENDIX H

FULL TABLES OF SIMULATION STUDY RESULTS

PB size	Proj. size	act: 1,2,3 mag: 1	1,2,3 2	1,2,3 3	1,2,3,4 1	1,2,3,4 2	1,2,3,4 3	1,2,3,4,5 1	1,2,3,4,5 2	1,2,3,4,5 3	1,3,5 1	1,3,5 2	1,3,5 3	1,4,5,7 1	1,4,5,7 2	1,4,5,7 3	2,4,6,9 1	2,4,6,9 2	2,4,6,9 3
24	5	49.05%	91.10%	99.68%	47.53%	90.96%	99.57%	47.56%	90.95%	99.38%	48.75%	90.08%	99.35%	-	-	-	-	-	-
28	5	92.80%	100%	100%	92.72%	100%	100%	92.72%	100%	100%	94.03%	100%	100%	-	-	-	-	-	-
36	5	98.85%	100%	100%	99.11%	100%	100%	99.17%	100%	100%	99.28%	100%	100%	-	-	-	-	-	-
	6	91.30%	99.98%	100%	87.85%	99.93%	100%	85.94%	99.97%	100%	88.27%	99.98%	100%	-	-	-	-	-	-
40	5	99.75%	100%	100%	99.70%	100%	100%	99.73%	100%	100%	99.73%	100%	100%	-	-	-	-	-	-
	6	99.75%	100%	100%	99.70%	100%	100%	99.73%	100%	100%	97.08%	100%	100%	-	-	-	-	-	-
	7	22.03%	40.48%	56.73%	22.15%	40.70%	55.35%	19.88%	40.17%	56.45%	21.28%	40.10%	52.33%	19.86%	41.10%	59.43%	-	-	-
44	5	99.87%	100%	100%	99.92%	100%	100%	99.92%	100%	100%	99.97%	100%	100%	-	-	-	-	-	-
	6	99.22%	100%	100%	99.04%	100%	100%	98.77%	100%	100%	99.25%	100%	100%	-	-	-	-	-	-
	7	69.58%	99.32%	100%	73.86%	99.34%	100%	75.68%	99.67%	100%	77.55%	99.90%	100%	79.31%	99.80%	100%	-	-	-
48	5	100%	100%	100%	99.95%	100%	100%	99.95%	100%	100%	100%	100%	100%	-	-	-	-	-	-
	6	99.52%	100%	100%	99.65%	100%	100%	99.68%	100%	100%	99.42%	100%	100%	-	-	-	-	-	-
	7	92.60%	100%	100%	92.41%	99.97%	100%	92.15%	100%	100%	94.08%	100%	100%	94.86%	99.99%	100%	-	-	-
52	5	99.98%	100%	100%	99.99%	100%	100%	99.99%	100%	100%	99.98%	100%	100%	-	-	-	-	-	-
	6	99.93%	100%	100%	99.89%	100%	100%	99.94%	100%	100%	99.88%	100%	100%	-	-	-	-	-	-
	7	98.35%	100%	100%	97.63%	100%	100%	97.60%	100%	100%	98.50%	100%	100%	98.29%	100%	100%	-	-	-
	8	32.58%	73.15%	93.02%	31.65%	75.42%	91.63%	33.28%	73.20%	91.49%	35.25%	75.13%	90.67%	32.82%	73.12%	91.14%	-	-	-
56	5	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-	-	-	-
	6	100%	100%	100%	99.97%	100%	100%	100%	100%	100%	99.98%	100%	100%	-	-	-	-	-	-
	7	97.97%	100%	100%	98.33%	100%	100%	97.38%	100%	100%	98.27%	100%	100%	97.73%	100%	100%	-	-	-
	8	96.93%	100%	100%	97.50%	100%	100%	88.75%	99.71%	99.99%	89.40%	100%	100%	87.03%	99.86%	100%	-	-	-
60	5	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-	-	-	-
	6	100%	100%	100%	100%	100%	100%	99.99%	100%	100%	100%	100%	100%	-	-	-	-	-	-
	7	99.85%	100%	100%	99.89%	100%	100%	99.95%	100%	100%	99.95%	100%	100%	99.86%	100%	100%	-	-	-
	8	96.38%	100%	100%	95.62%	100%	100%	96.39%	100%	100%	97.70%	100%	100%	96.14%	100%	100%	-	-	-
68	5	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-	-	-	-
	6	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-	-	-	-
	7	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-
	8	99.70%	100%	100%	99.80%	100%	100%	99.81%	100%	100%	99.83%	100%	100%	99.40%	100%	100%	-	-	-
	9	79.72%	99.88%	100%	85.13%	99.95%	100%	88.59%	100%	100%	87.03%	99.93%	100%	87.70%	99.99%	100%	85.97%	99.80%	100%
72	5	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-	-	-	-
	6	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-	-	-	-
	7	100%	100%	100%	100%	100%	100%	99.99%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-
	8	99.95%	100%	100%	99.94%	100%	100%	99.86%	100%	100%	99.92%	100%	100%	99.95%	100%	100%	-	-	-
	9	94.30%	100%	100%	95.19%	100%	100%	93.71%	100%	100%	92.48%	100%	100%	92.70%	100.00%	100%	95.21%	100%	100%
76	5	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-	-	-	-
	6	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-	-	-	-
	7	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-
	8	99.98%	100%	100%	99.98%	100%	100%	99.92%	100%	100%	99.87%	100%	100%	99.94%	100%	100%	-	-	-
	9	95.98%	100%	100%	97.06%	100%	100%	96.83%	100%	100%	93.93%	100%	100%	96.44%	100%	100%	98.72%	100%	100%
	10	14.15%	26.87%	39.42%	16.47%	27.11%	42.98%	13.81%	29.35%	41.53%	15.57%	27.52%	39.20%	13.57%	28.47%	41.95%	16.40%	29.21%	44.18%

PB size	Proj. size	act: 1,2,3 mag: 1	1,2,3 2	1,2,3 3	1,2,3,4 1	1,2,3,4 2	1,2,3,4 3	1,2,3,4,5 1	1,2,3,4,5 2	1,2,3,4,5 3	1,3,5 1	1,3,5 2	1,3,5 3	1,4,5,7 1	1,4,5,7 2	1,4,5,7 3	2,4,6,9 1	2,4,6,9 2	2,4,6,9 3
80	5	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-	-	-	-
	6	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-	-	-	-
	7	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-
	8	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-
	9	99.48%	100%	100%	99.62%	100%	100%	99.64%	100%	100%	99.50%	100%	100%	99.55%	100%	100%	99.55%	100%	100%
	10	76.72%	99.82%	100%	71.57%	99.71%	100%	73.77%	99.53%	100%	76.33%	99.25%	99.98%	72.21%	99.44%	100%	74.92%	99.62%	100%
84	5	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-	-	-	-
	6	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-	-	-	-
	7	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-
	8	100%	100%	100%	100%	100%	100%	99.99%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-
	9	100%	100%	100%	99.75%	100%	100%	99.80%	100%	100%	99.98%	100%	100%	99.79%	100%	100%	99.81%	100%	100%
	10	93.62%	100%	100%	92.46%	100%	100%	93.29%	100%	100%	94.67%	100%	100%	92.40%	100%	100%	93.86%	100%	100%
88	5	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-	-	-	-
	6	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-	-	-	-
	7	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-
	8	99.98%	100%	100%	99.99%	100%	100%	99.98%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-
	9	100%	100%	100%	99.99%	100%	100%	99.94%	100%	100%	99.97%	100%	100%	99.99%	100%	100%	99.96%	100%	100%
	10	96.50%	100%	100%	94.96%	100%	100%	96.22%	100%	100%	98.70%	100%	100%	96.87%	100%	100%	96.52%	100%	100%
92	5	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-	-	-	-
	6	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-	-	-	-
	7	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-
	8	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-
	9	99.93%	100%	100%	99.99%	100%	100%	99.97%	100%	100%	99.97%	100%	100%	99.97%	100%	100%	99.97%	100%	100%
	10	99.63%	100%	100%	99.72%	100%	100%	98.99%	100%	100%	98.63%	100%	100%	97.55%	100%	100%	99.08%	100%	100%
	11	37.53%	79.68%	95.52%	35.18%	75.36%	93.56%	35.49%	77.45%	94.17%	37.50%	74.98%	92.90%	35.60%	75.55%	92.12%	28.89%	71.20%	91.62%
96	5	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-	-	-	-
	6	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-	-	-	-
	7	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-
	8	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-
	9	100%	100%	100%	99.99%	100%	100%	99.97%	100%	100%	99.98%	100%	100%	99.94%	100%	100%	99.99%	100%	100%
	10	95.52%	100%	100%	96.19%	100%	100%	97.74%	100%	100%	97.30%	100%	100%	99.52%	100%	100%	97.65%	100%	100%
	11	83.15%	99.93%	100%	79.42%	99.07%	100%	82.86%	99.49%	99.98%	85.42%	99.90%	100%	77.25%	98.70%	99.99%	80.48%	99.14%	99.99%
100	5	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-	-	-	-
	6	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-	-	-	-
	7	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-
	8	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	-	-	-
	9	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
	10	100%	100%	100%	100%	100%	100%	99.97%	100%	100%	99.97%	100%	100%	99.96%	100%	100%	99.79%	100%	100%
	11	96.88%	100%	100%	96.47%	100%	100.00%	96.32%	100%	100%	94.43%	100%	100%	95.74%	100%	100%	93.77%	100%	100%