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Mario Rabinowitz
Electric Power Research Institute

T. McMullen
Virginia Commonwealth University

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Mario Rabinowitz
Electric Power Research Institute, Palo Alto, California 94303

T. McMullen
Department of Physics, Commonwealth University, Richmond, Virginia 23284

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Reasonably good agreement with the superconducting transition temperatures of the cuprate high-T_c superconductors can be obtained on the basis of an approximate phenomenological theory. In this theory, two criteria are used to calculate the superconducting transition temperature. One is that the quantum wavelength is of the order of the electron-pair spacing. The other is that a fraction of the normal carriers exist as Cooper pairs at T_c. The resulting simple equation for T_c contains only two parameters: the normal carrier density and effective mass. We calculate specific transition temperatures for 12 cuprate superconductors.

As the temperature of an ideal Bose gas is lowered, the particles undergo a Bose–Einstein (BE) condensation when the thermal wavelength

$$\lambda_T = h/(2\pi m_B k_\beta T_c)^{1/2} \quad (1)$$

is comparable to the interparticle spacing.[1,2] Here, h is Planck's constant, k_β is the Boltzmann constant, m_B is the boson mass, and T_c is the transition temperature. λ_T is called the thermal wavelength, because $\lambda_T \approx \lambda$, the de Broglie wavelength.

For a three-dimensional ideal Bose gas, the BE condensation temperature[1,2] is

$$T_c^{\mathrm{BE}} = \frac{h^2 n^{2/3}}{2\pi m_B k_\beta [\zeta(\frac{3}{2})]^{2/3}} = \frac{h^2 n^{2/3}}{11.92 m_B k_\beta}, \quad (2)$$

where n is the three-dimensional number density, $\zeta(x)$ is the Riemann zeta function of x, and $\zeta(3/2) = 2.612...$. There are two shortcomings related to the ordinary BE condensation. One is that T_c^{BE} is orders of magnitude higher than the experimental T_c of three-dimensional superconductors like the metallics if a sizable fraction of the normal carriers are bosons. The other is that the BE gas does not condense in two dimensions so that unmodified BE statistics are not a sufficient condition for BE condensation. More general considerations argue against a one-dimensional condensation. However, with an energy gap (binding energy between particles), a BE type condensation in lower dimensions is described by Blatt,[3] although this appears to be contrary to Hohenberg's theorem.[4]

Two conditions are met by known superconductors. One is the existence of bosons. The other is a condensation in momentum space. This led to a consideration that there may be two temperatures: T_p, the pairing temperature to form bosons out of fermion pairs; and the condensation temperature T_C. If $T_p < T_C$, then T_p is the limiting temperature for superconductivity which we call the transition temperature T_c. However, it is difficult to calculate T_p because this requires knowledge of the pairing interaction and its strength, although this is the usual strategy. Three possibilities were considered first by Rabinowitz[5–9] in 1987:

(1) That T_C might be less than or equal to T_p, in which case $T_c = T_C$.

(2) That a fraction of fermions $\sim k_\beta T_c / E_F$ are incipient Cooper pairs at T_c.

(3) Reduced dimensionality effects properly enter into λ via the equipartition principle.

His paradigm led to good agreement with experiment for broad classes of superconducting materials from 1 to 10^9 K.[5–9] More recently this has been extended to include 10^{-3} K for the superfluid state of ^{3}He for a range of 12 orders of magnitude.[10,11]

Even if $T_C > T_p$, the T_C is found to be a much better upper limit to T_c than T_c^{BE}. We are not aware of any T_c calculations for specific cuprates, nor of any other attempts to calculate a reduced condensation temperature which does not include interactions. Here, we present a more detailed test of this approach by comparing its results with the observed T_c's of specific cuprates. Harshman and Mills[12] have compiled a comprehensive table of the relevant parameters, extracted from experimental data, which makes this possible. BE condensation approaches to superconductivity were first presented by Schafroth, Butler, and Blatt.[13,14] These approaches differ from ours, and appear not to have been successful in predicting T_c's. Standard approaches have been unsuccessful with the cuprates.

The point of view adopted here is that as the temperature is decreased in a gas similar to a Bose–Einstein (BE) gas, particles should start a BE type condensation into the superconducting state when the de Broglie wavelength is of the order of the interparticle spacing for the fraction of carriers that have become Cooper pairs.[5–11] Thus we can avoid the difficult calculations encountered when the nature of the interaction is known, and which are impossible to do when the pairing interaction is not known.

We assume that a BE type condensation in three dimensions of the free Bose gas involves electron pairs within a shell of energy $\sim k_\beta T_c$ around the Fermi energy, E_F. The number of electrons involved is the number of k values within a shell Δk of energy width $\sim k_B T_c$ within the Fermi surface, so that Δk is given by

$$k_\beta T_c \approx \frac{\hbar^2 k_F^2}{2m} - \frac{\hbar^2 (k_F - \Delta k)^2}{2m}, \quad (3)$$

TABLE I. Comparison of experimental and theoretical cuprate transition temperatures.

No.	Compound	T_{exp} (K)	T_{c2} (K)	$n(10^{21}/cm^3)$	m/m_0
1	$La_{1.9}Sr_{0.1}CuO_4$	33	46	~0.5	~2
2	$La_{1.875}Sr_{0.125}CuO_4$	36	55	~1.0	~2.6
3	$La_{1.85}Sr_{0.15}CuO_4$	39	51	5.2	8.6
4	$YBa_2Cu_3O_{6.67}$	60	60	1.1	2.6
5	$YBa_2Cu_3O_7$	92	80	16.9	12
6	$YBa_2Cu_4O_8$	80	76	2.8	3.8
7	$HoBa_2Cu_4O_8$	80	145	~1.3	~1.2
8	$Bi_2Sr_2CaCu_2O_8$	89	43	3.5	7.8
9	$(Bi_{1.6}Pb_{0.4})Sr_2Ca_2Cu_3O_{10}$	107	42	3.5	7.8
10	$Tl_2Ba_2CaCu_2O_8$	99	50	4.9	8.4
11	$Tl_2Ca_2Ba_2Cu_3O_{10}$	125	67	4.2	5.7
12	$(Tl_{0.5}Pb_{0.5})Sr_2CaCu_2O_7$	80	90	2.8	3.2

where k is the wave vector and m is the effective mass of the electron. Thus

$$\frac{\Delta k}{k_F}=\frac{k_\beta T_c}{2[(\hbar^2/2m)k_F^2]}=\frac{k_\beta T_c}{2E_F}. \tag{4}$$

The number density of Cooper pairs for a roughly spherical Fermi surface is then

$$n_B=\frac{1}{2}\frac{\Delta k(4\pi k_F^2)}{\frac{4}{3}\pi k_F^3}n=\frac{3\Delta k}{2k_F}n. \tag{5}$$

Combining Eqs. (4) and (5), we have

$$n_B=\frac{3k_\beta T_{c3}}{4E_F}n, \tag{6}$$

where we introduce T_{c3}, a "three-dimensional" transition temperature that may apply to some less anisotropic materials, and which can be thought of here as providing the energy cutoff that selects the effective bosons. The procedure will be to use T_{c3} to determine the interboson spacing in order to calculate T_{c2}, which will be our transition temperature estimate for the cuprates. The choice of "two-dimensional" motion for these materials is based on their layered structure and highly anisotropic normal state properties.

Two electron pairs are encompassed by $(1/2)\lambda$ when

$$\lambda\approx 2(n_B)^{-1/3}. \tag{7}$$

For a pair of electrons of effective mass $2m$, momentum p, and kinetic energy $(f/2)k_\beta T$, the de Broglie wavelength is

$$\lambda=\frac{h}{p}=\frac{h}{\{2(2m)[(f/2)k_\beta T_c]\}^{1/2}}, \tag{8}$$

where f is the number of degrees of freedom per particle pair. For three-dimensional motion of the bosons we take $f=3$. For two-dimensional motion we will take $f=2$. Rabinowitz gives a more general discussion of f in Ref. 7.

Combining Eqs. (6)–(8) we obtain

$$T_{c3}=0.218\left(\frac{\hbar^2 n^{2/3}}{2mk_\beta}\right). \tag{9}$$

To obtain the "two-dimensional" transition temperature, λ as given by Eq. (7) is equated to λ as given by Eq. (8) with $f=2$, as we restrict the kinetic energy to two dimensions by the equipartition of energy principle. This yields the two-dimensional transition temperature which we will compare with experiment:

$$T_{c2}=0.328\left(\frac{\hbar^2 n^{2/3}}{2mk_\beta}\right). \tag{10}$$

Table I shows a comparison of the experimentally observed transition temperatures with T_{c2}, together with the relevant input data as obtained from Harshman and Mills.[12] In Eq. (10) we use the three-dimensional (3D) carrier density n_{3D} for n, and the effective mass for m, where m_0 is the free-electron mass.

It is remarkable how well our calculated transition temperatures agree with the experimental values without explicitly introducing a pairing mechanism. Perhaps this should not be entirely surprising, as T_c itself is a measure of the interaction strength. In this model, T_c enters into the equations in two different ways so that it is possible to solve for T_c. It is a self-consistent condition that the number of pairs that BE condense is $\propto T_c$ itself. This may be why we can obtain the transition temperature without prior knowledge of the interaction mechanism. This works best for short coherence length pairs, and not so well for metallics with the greatest pair-pair interactions because of their long coherence lengths with a large overlap of pairs.

[1] D. S. Betts, Contemp. Phys. **10**, 241 (1969).
[2] K. Huang, *Statistical Mechanics* (Wiley, New York, 1963).
[3] J. M. Blatt, *Theory of Superconductivity* (Academic, New York, 1964).
[4] P. C. Hohenberg, Phys. Rev. **158**, 383 (1967).
[5] M. Rabinowitz, in Proceedings of the EPRI Workshop on High-Temperature Superconductivity, 1987, pp. 2-3 to 2-33, EL/ER-5894.
[6] M. Rabinowitz, in Proceedings of the EPRI Conference on Electrical Applications of Superconductivity, 1988, pp. 1-1 to 1-18, EL-6325-D.
[7] M. Rabinowitz, Int. J. Theor. Phys. **28**, 137 (1989).
[8] M. Rabinowitz, Physica C **162-164**, 249 (1989).
[9] M. Rabinowitz, *Advances in Cryogenic Engineering* (Plenum, New York, 1990), Vol. 36A, p. 21.
[10] M. Rabinowitz, Int. J. Theor. Phys. **32**, 565 (1993).
[11] M. Rabinowitz, Appl. Phys. Commun. **12**, 55 (1993).
[12] D. R. Harshman and A. P. Mills, Phys. Rev. B **45**, 10684 (1992).
[13] M. R. Schafroth, Phys. Rev. **100**, 463 (1955).
[14] M. R. Schafroth, S. T. Butler, and J. M. Blatt, Helv. Phys. Acta **30**, 93 (1957).