# A heat exchanger design for the separated window target of the EADF 

C. Aragonese, S. Buono, G. Fotia, L. Maciocco, V. Moreau, L. Sorrentino


#### Abstract

The spallation target of the Energy Amplifier Demonstration Facility (EADF) [1] is cooled by a liquid lead-bismuth eutectic (LBE), while the secondary coolant is a diathermic oil. The reasons for these choices have been extensively discussed in [2] and [3]. Here we present the design and the optimisation of a heat exchanger using these fluids, whose additional requirements are the need of fitting into the top of the annular downcomer section of the target and the minimisation of the pressure losses on the LBE side, allowing the use of natural convection for the circulation of the primary fluid. Heat exchanger working temperatures are between 250 and $180^{\circ} \mathrm{C}$ in the LBE side, and between 150 and $190^{\circ} \mathrm{C}$ in the oil side (cold fluid), while the power to be removed is up to 3 MW . We selected a bayonet-type heat exchanger, as suggested in [4] for the primary loop of the EADF vessel, which seems to be the most appropriate choice to satisfy all the requirements.


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## Heat Exchanger description

The target heat exchanger is a bayonet-type with a triangular tube arrangement. It is located at the top of the downcomer channel, occupying all the available space in the annular section. A simple element of the bayonet heat exchanger consists of a couple of concentric pipes where the cold fluid (oil) flows downwards inside the inner tube and turns upwards in the annular section between the two tubes (figure 1).
The radius of the external pipes and the ratio between the radius of the two pipes are the most important parameters for the optimisation. These two parameters affects the pressure drops at both LBE and oil side, as well as the length of the tubes needed for the prescribed heat load of the exchanger.

## Assumptions and Requirements

The following input data were used in the heat exchanger design.

- The input and output LBE temperatures were set respectively to 250 and $180^{\circ} \mathrm{C}$, that is the operating range of the target at a LBE flow rate of $250 \mathrm{~kg} / \mathrm{s}$ [5].
- The heat load was estimated from a FLUKA [6] simulation and set to 2.6 MW .
- The inlet and outlet oil temperatures were set to 150 and $190^{\circ} \mathrm{C}$ respectively.

The requirements of the heat exchanger, which are peculiar of the EADF design, are:

- Very small pressure drops at the LBE side.
- A maximum length of 2 meters.

Both requirements go in the direction of enhancing natural circulation, so reducing the need of using additional pumping devices.

## Model development

The heat exchangers overall modelling is usually performed using the Fourier Equation:

$$
\begin{equation*}
\mathrm{Q}=\mathrm{UA} \Delta \mathrm{~T} \tag{1}
\end{equation*}
$$

where $Q$ is the heat removal load, $U$ is the overall heat transfer coefficient, $A$ is the exchange area and $\Delta \mathrm{T}$ is the effective temperature difference which, for countercurrent heat exchangers, is given by the Logarithmic Mean Temperature Difference (LMTD):

$$
\begin{equation*}
\mathrm{LMTD}=\frac{\left(\mathrm{T}_{2}-\mathrm{t}_{1}\right)-\left(\mathrm{T}_{1}-\mathrm{t}_{2}\right)}{\ln \left[\frac{T_{2}-t_{1}}{T_{1}-t_{2}}\right]} \tag{2}
\end{equation*}
$$

The above expression is usually extended to not countercurrent-flow heat exchangers through empirical correction factors [7]. However, since bayonet exchangers are quite peculiar, we preferred to obtain the effective difference temperature by the integration of the differential heat balances along the length of the heat exchanger.

The scheme of a single tube of the heat exchanger is reported in figure 1. Using the procedure reported in appendix B, the following expression was obtained [9] (for symbols meaning see the nomenclature in appendix A):

$$
\begin{equation*}
\Delta T=\left(t_{2}-t_{1}\right) \frac{2 E}{\ln \left[\frac{V+E}{V-E}\right]} \tag{3}
\end{equation*}
$$

where:

$$
\begin{aligned}
& E=\frac{1}{2} \sqrt{ }(R-1)^{2}+4 F \\
& V=\frac{1}{2} \frac{\left(T_{1}-t_{2}\right)+\left(T_{2}-t_{1}\right)}{t_{2}-t_{1}} \\
& F=\frac{u p}{U P} \quad \text { and } \quad R=\frac{\dot{m} c_{p}}{\dot{M} C_{p}}=\frac{T_{1}-T_{2}}{t_{2}-t_{1}}
\end{aligned}
$$

It can be seen from the previous equations that the effective temperature difference depends not only on the fluid temperature, as usually happens for common heat exchangers, but also on the ratio of the overall heat transfer coefficients of internal and external tubes.
In order to keep the efficiency of a bayonet heat exchanger high enough, the F ratio has to be kept as small as possible: this implies that the heat exchanges between the two oil columns should be minimised.


Figure 1: Bayonet Exchanger. Sketch of double pipe arrangement

## Model parameters and correlation

The two overall heat transfer coefficients are composed of a series of local contributions:

$$
\begin{align*}
& U=\left(R_{f, L B E}+1 h_{\text {LBE }}+R_{\text {steel }}+1 h_{h_{\text {oil,ext_ann }}}+R_{\text {oil }}\right)^{-1}  \tag{4}\\
& u=\left(R_{f, \text { iil }}+1 h_{h_{\text {oil, int_ann }}}+R_{\text {steel }}+1 / h_{\text {oil,inn_tube }}+R_{\text {oil }}\right)^{-1} \tag{5}
\end{align*}
$$

The single contribution can be calculated using the appropriate correlation for the dimensionless Nusselt number $\left(N u=\frac{h_{i} \mathrm{~L}}{\mathrm{k}}\right)$, namely:

- The Martinelli equation for liquid metals:
- The Sieder-Tade equation for tubes:
- The Monrad and Pelton equation for annuli:

$$
\begin{aligned}
& \mathrm{Nu}=7.0+0.025\left(\mathrm{Pr}^{0.8} \operatorname{Re}^{0.8}\right) \\
& \mathrm{Nu}=0.023 \operatorname{Pr}^{1 / 3} \operatorname{Re}^{0.8} \\
& \mathrm{Nu}=0.020 \operatorname{Pr}^{1 / 3} \operatorname{Re}^{0.8}\left(\frac{\mathrm{rexxt}}{\mathrm{r}_{\text {int }}}\right)^{0.53}
\end{aligned}
$$

The Prandtl and Reynolds numbers are calculated at average fluid conditions. It is worth to point out that the equivalent diameters to be used in Reynolds numbers may differ depending on the considered phenomena. In particular, in annular section three equivalent diameters have to be considered: one for the pressure drops, one for heat exchange with the inner wall of the annulus and one for heat exchange with the outer wall of the annulus.
This can be explained considering the definition of the equivalent diameter:

$$
\begin{equation*}
D_{\text {eq }}=4 \frac{S_{w}}{P_{w}} \tag{6}
\end{equation*}
$$

were the wetted perimeter $P_{w}$ relative to the considered phenomena should be used. For pressure drops, both internal and external circumferences must be considered, while for heat transfer only the relevant one must be used.
The pressure drops can be computed from the Fanning equation:

$$
\begin{equation*}
\Delta P=\frac{2 \rho f L v^{2}}{D_{\text {eq }}} \tag{7}
\end{equation*}
$$

using the appropriate value of the friction factor

$$
\begin{equation*}
f=a R^{-0.25} \tag{8}
\end{equation*}
$$

where a is equal to 0.087 in the annular section and 0.079 elsewhere. The above expression is valid for Reynolds number values between 2100 and 100000.
The number of tubes in the heat exchanger is calculated from geometrical considerations. Empirical correlation relating bundle diameter, tube diameter and tube arrangement with the number of tubes are available in literature [7,8].
The following correlation was used considering circular bundle geometry and a triangular pipe arrangement with a pitch of 1.25 D :

$$
\begin{equation*}
N_{\text {tubes }}=0.319\left(\frac{D_{\text {bundle }, \text { ext }}}{D_{\text {pipe }}}\right)^{2.142}-0.319\left(\frac{D_{\text {bundle }, \text { int }}}{D_{\text {pipe }}}\right)^{2.142} \tag{9}
\end{equation*}
$$

## Computational algorithm

The final output of the computational algorithm is the tube length starting from the following input data:

- Tubes geometry (diameters, thickness, pitch).
- Bundle geometry.
- Inlet and outlet LBE temperatures.
- Inlet and output oil temperatures.
- Total heat load.
- Fouling factors.

Since the effective temperature difference depends on the overall heat transfer coefficients an iterative procedure is needed, which is described by the following steps:

1. Assume first guess values for $U$ and $u$.
2. Calculate $\Delta \mathrm{T}$ from eq. (3).
3. Calculate the length of the HE from eq. (1).
4. Calculate $U$ from the data of the previous step.
5. Calculate $Q_{\text {est }}$ from the last value of $U$.
6. Compare $Q_{\text {est }}$ with $Q_{\text {set }}$. If convergence is not reached go to point 2 .

At the end of the simulation, it must be verified that the pressure losses and the heat exchanger length are acceptable, otherwise the geometry has to be changed.

## Results and Discussion

A diameter of about 15 mm was chosen in order to fit the requirements of having a tube length of the order of 2 meters and not having too thin tubes (due to structural resistance requirements). In fact, the first parameter investigated was the outer tube diameter. All the other input data were kept constant, except for the inner tube diameter and the pitch, which were constrained to a fixed ratio with the outer tube diameter. The following diameter ratios were considered:
$R=0.714 ; 0.75 ; 0.80 ; 0.81$ and 0.85 .
As shown in figure 2 to 6 the number of tubes decreases with the increase of the outer tube diameter, while the needed tube length increases. It can also be seen that the tube diameters ratio strongly affects the heat transfer coefficient and, therefore, the required tube length, being the higher the ratio the shorter the tubes. However, due to the increased oil velocity in the annular section, the pressure drops increases too much, reaching unacceptable values.


Figure 2: EADF target heat exchanger: number of tubes and tube length (left), fluids velocity and pressure drops (right). Diameter ratio $R=0.714$


Figure 3: EADF target heat exchanger: number of tubes and tube length (left), fluids velocity and pressure drops (right). Diameter ratio $\mathrm{R}=0.75$


Figure 4: EADF target heat exchanger: number of tubes and tube length (left), fluids velocity and pressure drops (right). Diameter ratio $\mathrm{R}=0.80$


Figure 5: EADF target heat exchanger: number of tubes and tube length (left), fluids velocity and pressure drops (right). Diameter ratio $R=0.81$


Figure 6: EADF target heat exchanger: number of tubes and tube length (left), fluids velocity and pressure drops (right). Diameter ratio $\mathrm{R}=0.85$

We selected a diameter ratio of about 0.8 in order to have a velocity in the annulus high enough to enhance oil-LBE heat transfer preventing too high pressure drops. This is more evident in figures 7 and 8 .
After this optimisation, the outer and inner tube diameters were finally set to a commercial available dimension [7] as close as possible to the desired values.
In table 1 the main parameters of the heat exchanger are reported.
It can be seen that the resulting pressure drops in the bayonet heat exchanger are acceptable both in the oil ( 1.5 bar ) and in the LBE side ( 0,025 bar). This is particularly important since it may allow basing our target design on natural convection cooling.


Figure 7: Tubes length for different ratios R. Figure 8: Oil pressure drops for different diameter ratios R

|  | Value | Dimensions | Note |
| :---: | :---: | :---: | :---: |
| Outer pipe |  |  |  |
| External diameter | 15.88 | mm | 5/8 in BGW 20 <br> (Standards of Tubular Exchanger Manufacturers Association)) |
| Internal diameter | 14.10 | mm |  |
| Thickness | 0.89 | mm |  |
| Inner pipe |  |  |  |
| External diameter | 12.7 | mm | 1/2 in BGW 22 |
| Internal diameter | 11.28 | mm |  |
| Thickness | 0.71 | mm |  |
| Bundle |  |  |  |
| Internal bundle diameter | 342 | mm |  |
| External bundle diameter | 591.5 | mm |  |
| Number of tubes | 512 |  |  |
| Tubes length | 1.81 | m |  |
| Tube pitch | 19.84 | mm | $1.25 \mathrm{D}_{\text {out }}$ |
| Flow areas |  |  |  |
| Total bundle area | 0.1829 | $\mathrm{m}^{2}$ |  |
| LBE flow area | 0.08158 | $\mathrm{m}^{2}$ |  |
| Inner pipes flow area | 0.05114 | $\mathrm{m}^{2}$ |  |
| Annulus flow area | 0.01505 | $\mathrm{m}^{2}$ |  |
| Cold fluid data (oil) |  |  |  |
| Oil inlet temperature | 150 | ${ }^{\circ} \mathrm{C}$ |  |
| Oil outlet temperature | 190 | ${ }^{\circ} \mathrm{C}$ |  |
| Oil mass flow rate | 31.21 | Kg/s |  |
| Oil velocity inner pipes | 0.66 | $\mathrm{m} / \mathrm{s}$ |  |
| Oil velocity in annulus | 2.25 | $\mathrm{m} / \mathrm{s}$ |  |
| Average oil temperature | 170 | ${ }^{\circ} \mathrm{C}$ |  |
| Thermal conductivity | 0.1119 | W/m K |  |
| Density | 920.50 | $\mathrm{kg} / \mathrm{m}^{3}$ |  |
| Specific heat | 2066.83 | $\mathrm{J} / \mathrm{kg} \mathrm{K}$ |  |
| Dynamic viscosity | $1.07 \mathrm{E}-3$ | kg/m s |  |
| Cold fluid dimensionless number |  |  |  |
| Pr number | 20 |  |  |
| Re number inner pipe | 6400 |  |  |
| Equivalent Diameter (pressure drops) | 1.397 | mm |  |
| Equivalent Diameter (internal side heat transfer) | 2.948 | mm |  |
| Equivalent Diameter (external side heat transfer) | 2.656 | mm |  |
| Re number annulus (pressure drops) | 2700 |  |  |
| Re number annulus (internal side heat transfer) | 5700 |  |  |
| Re number annulus (external side heat transfer) | 5150 |  |  |
| Nu number inner pipe | 69 |  |  |
| Nu number annulus (internal side) | 52 |  |  |
| Nu number annulus (external side) | 48 |  |  |
| Heat transfer coefficient inner pipe | 686 | W/ m ${ }^{2} \mathrm{~K}$ |  |


| Heat transfer coefficient annulus internal side | 1965 | W/ m ${ }^{2} \mathrm{~K}$ |  |
| :---: | :---: | :---: | :---: |
| Heat transfer coefficient annulus external side | 2006 | $\mathrm{W} / \mathrm{m}^{2} \mathrm{~K}$ |  |
| Hot fluid data (Lead Bismuth Eutectic) |  |  |  |
| LBE inlet temperature | 250 | ${ }^{\circ} \mathrm{C}$ |  |
| LBE outlet temperature | 180 | ${ }^{\circ} \mathrm{C}$ |  |
| LBE mass flow rate | 250 | kg/s |  |
| LBE velocity | 0.30 | m/s |  |
| Average LBE temperature | 215 | ${ }^{\circ} \mathrm{C}$ |  |
| Thermal conductivity | 9.756 | W/m K |  |
| Density | 10441 | $\mathrm{kg} / \mathrm{m}^{3}$ |  |
| Specific heat | 146.54 | J/kg K |  |
| Dynamic viscosity | $2.14 \mathrm{E}-3$ | kg/m s |  |
| Hot fluid dimensionless number |  |  |  |
| Pr number | 3.21E-2 |  |  |
| Equivalent diameter | 13.05 | mm |  |
| Re number | 18400 |  |  |
| Nu number | 11.96 |  |  |
| Heat transfer coeff. | 8500 | W/ m ${ }^{2} \mathrm{~K}$ |  |
| Heat exchanger data |  |  |  |
| Tube conductivity | 26 |  |  |
| Heat exchanged | 2.6 | MW |  |
| Effective bayonet $\Delta T$ | 39 | ${ }^{\circ} \mathrm{C}$ |  |
| Overall heat transfer coefficient | 1452 | W/ m ${ }^{2} \mathrm{~K}$ | without fouling factor |
| Heat exchanger efficiency | 0.39 |  |  |
| Pressure drops LBE side | 0.025 | bar |  |
| Pressure drops oil side | 1.5 | bar |  |

Table 1:Heat exchanger design data and parameters.

## Conclusions

A bayonet-type heat exchanger of annular section was designed and optimised for the EADF spallation target. An optimisation procedure was performed to obtain an heat exchanger with a length of about 2 meters while keeping pressure losses as low as possible, in order to allow the use of natural circulation for the regime operation of the spallation target.
An optimised heat exchanger has been obtained with the prescribed length and pressure losses of about 2500 Pa for a LBE mass flow rate of $250 \mathrm{~kg} / \mathrm{s}$.

## References

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[4] Calculation Report - Intermediate Heat Exchanger Functional Sizing, Ansaldo Nucleare, ADS 5 TMLX 0123, June 1999.
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[7] Perry, R. H. and Don W Green (Eds.), Perry's Chemical Engineers' Handbook, sixth edition, Mc Graw Hill, 1984.
[8] Sinnot, R.K., An Introduction to Chemical Engineering Design, Chemical Engineering Series, Pergamon Press, 1986.
[9] Kern, D.Q., Process Heat Transfer, Mc Graw Hill, 1965.

## Appendix A

## Nomenclature

| A | Heat exchange area | [ $\mathrm{m}^{2}$ ] |
| :---: | :---: | :---: |
| $\mathrm{C}_{\mathrm{p}}$ | Lead bismuth specific heat | [J/kg K] |
| $\mathrm{c}_{\mathrm{p}}$ | Oil specific heat | [J/kg K] |
| $\mathrm{D}_{\text {bundle }}$ | Bundle diameter | [m] |
| $\mathrm{D}_{\text {eq }}$ | Equivalent diameter | [m] |
| $\mathrm{D}_{\text {pipe }}$ | Outer pipe outside diameter | [m] |
| f | Friction factor |  |
| h | Heat transfer coefficient | [W/ m ${ }^{2} \mathrm{~K}$ ] |
| L | Heat exchanger length | [m] |
| $\mathrm{N}_{\text {tubes }}$ | Number of tubes |  |
| Nu | Nusselt number |  |
| P | Outer tube heat transfer wetted perimeter | [m] |
| p | Inner tube heat transfer wetted perimeter | [m] |
| Pr | Prandtl number |  |
| $\mathrm{P}_{\mathrm{w}}$ | Wetted perimeter | [m] |
| Q | Heat removal load | [W] |
| Re | Reynolds number |  |
| $\mathrm{R}_{\mathrm{f}}$ | Fouling resistance | $\mathrm{m}^{2} \mathrm{~K} / \mathrm{W}$ |
| $\mathbf{R}_{\text {steel }}$ | Steel heat resistance | $\mathrm{m}^{2} \mathrm{~K} / \mathrm{W}$ |
| $\mathrm{S}_{\mathrm{w}}$ | Cross flow area | [m²] |
| T | Lead bismuth temperature | [ ${ }^{\text {C }}$ ] |
| t | Oil temperature | [ ${ }^{\text {C] }}$ |
| U | Overall heat transfer coefficient | [W/ m ${ }^{2} \mathrm{~K}$ ] |
| u | Inner tube overall heat transfer coefficient | [W/ m $\left.{ }^{2} \mathrm{~K}\right]$ |
| v | Fluid velocity | [m/s] |
| $\dot{M}$ | Lead bismuth mass flow rate | [kg/s] |
| $\dot{\mathrm{m}}$ | Oil mass flow rate | [kg/s] |
| $\rho$ | Density | [ $\left.\mathrm{kg} / \mathrm{m}^{3}\right]$ |
| - ${ }^{\text {T }}$ | Effective temperature difference | [ ${ }^{\text {C] }}$ |
| subscripts |  |  |
| 1 | Heat exchanger inlet |  |
| 2 | Heat exchanger outlet |  |
| LBE | Lead Bismuth Eutectic |  |
| oil | Diathermic oil |  |
| inn_tube | Inner tube |  |
| int_ann | Annulus internal side wall |  |
| ext_ann | Annulus external side wall |  |

## Appendix B

## Effective temperature difference calculation for bayonets heat exchangers.

As reported in [9], different flow arrangements for bayonet exchanger are available. We refer to the arrangement shown in Figure 1.
Imposing the differential balance on each stream:
$U\left(T-t^{9}\right) P d x-u\left(t^{\oplus}-t^{9}\right) p d x=-\dot{m} c_{p} d t^{\Phi}$
$u\left(t^{\oplus}-t^{9}\right) p d x=\dot{m} c_{p} d t^{\circledR}$
Adding A1 and A2:
$U P\left(T-t^{9}\right) d x=\dot{m} c_{p}\left(d t^{\oplus}-d t \Phi\right)$
$\dot{M} C_{p}\left(T-T_{2}\right)=\dot{m} c_{p}\left(t^{\Phi_{-}} t 9\right.$
Differentiating
$\dot{M} C_{p} \frac{d T}{d x}=\dot{m} c_{p}\left(\frac{d t^{\oplus}}{d x}-\frac{d t^{\ominus}}{d x}\right)$
(A4bis)

Combining with A3:
$\frac{d T}{d x}=\frac{U P}{\dot{M} C_{p}}\left(t^{\oplus}-T\right)$
Differentiating again:
$\frac{d^{2} T}{d x^{2}}=\frac{U P}{\dot{M} C_{p}}\left(\frac{d t^{\oplus}}{d x}-\frac{d T}{d x}\right)$
from A4bis:
$\frac{d t^{\oplus}}{d x}=\frac{d t^{\oplus}}{d x}+\frac{\dot{M} C_{p}}{\dot{m} c_{p}} \frac{d T}{d x}$
and from A2 and A4:
$\frac{d t^{\ominus}}{d \mathrm{x}}=\frac{\text { up }}{\dot{\mathrm{m}} \mathrm{c}_{\mathrm{p}}}\left(\mathrm{t}^{\oplus}-\mathrm{t}^{\oplus}\right)=\frac{\mathrm{up}}{\dot{\mathrm{m}} \mathrm{c}_{\mathrm{p}}} \frac{\dot{\mathrm{m}} \mathrm{C}_{\mathrm{p}}}{\dot{\mathrm{m}}}\left(\mathrm{T}-\mathrm{T}_{\mathrm{p}}\right)$
Using A7 and A8 in the A6:
$\frac{d^{2}\left(T-T_{2}\right)}{d x^{2}}+\left(\frac{U P}{\dot{M} C_{p}}-\frac{U P}{\dot{m} c_{p}}\right) \frac{d\left(T-T_{2}\right)}{d x}-\frac{U P}{\dot{m} c_{p}} \underset{\mathrm{~m} c_{p}}{\dot{u p}}=0$
T can be obtained by solving the second order differential equation A9. Applying the boundary conditions ( $T_{1}-T_{2}$ ) is given by:

$$
\begin{equation*}
\mathrm{T}_{1}-\mathrm{T}_{2}=2 \frac{\mathrm{t}_{2}-\mathrm{T}_{1}}{\left[\frac{\dot{M} C_{p}}{\dot{m} c_{p}}-1-\Gamma\left(\frac{e^{\mathrm{x}_{1} \mathrm{~L}}+\mathrm{e}^{\mathrm{x}_{2} L}}{\mathrm{e}^{\mathrm{x}_{1} \mathrm{~L}}-\mathrm{e}^{\mathrm{x}_{2} \mathrm{~L}}}\right)\right]} \tag{A10}
\end{equation*}
$$

where:
$\left.\Gamma=\sqrt{\left(1-\frac{\dot{M} C_{p}}{\dot{m} c_{p}}\right.}\right)^{2}+4 \frac{u p}{U P}\left(\frac{\dot{M} C_{p}}{\dot{m} c_{p}}\right)^{2}$
and

$$
\begin{equation*}
X_{1-2}=1 / 2 \frac{U P}{\dot{M} C_{p}}\left(\frac{\dot{M} C_{p}}{\dot{m C_{p}}}-1 \pm \Gamma\right) \tag{A12}
\end{equation*}
$$

Comparing eq. A10 with the Fourier equation:
$\mathrm{Q}=\dot{\mathrm{M}} \mathrm{C}_{\mathrm{p}}\left(\mathrm{T}_{1}-\mathrm{T}_{2}\right)=\mathrm{UPL} \Delta \mathrm{T}$
it is possible to eliminate the $L$ variable and obtain the bayonet heat exchanger effective temperature difference:

$$
\begin{equation*}
\Delta t=\frac{\sqrt{\left(T_{1}-T_{2}-t_{2}+t_{1}\right)^{2}+\frac{4 u p}{U P}\left(t_{2}-t_{1}\right)^{2}}}{\ln \left[\frac{T_{2}-t_{2}+T_{1}-t_{1}+\sqrt{\left(T_{1}-T_{2}-t_{2}+t_{1}\right)^{2}+\frac{4 u p}{U P}}\left(t_{2}-t_{1}\right)^{2}}{T_{2}-t_{2}+T_{1}-t_{1}-\sqrt{\left(T_{1}-T_{2}-t_{2}+t_{1}\right)^{2}+\frac{4 u p}{U P}}\left(t_{2}-t_{1}\right)^{2}}\right]} \tag{A14}
\end{equation*}
$$

Defining:

$$
\begin{array}{ll}
R=\frac{\dot{m} c_{p}}{\dot{M} C_{p}}=\frac{T_{1}-T_{2}}{t_{2}-t_{1}} & F=\frac{u p}{U P} \\
E=1 / 2 \sqrt{ }(R-1)^{2}+4 F & V=1 / 2 \frac{\left(T_{1}-t_{2}\right)+\left(T_{2}-t_{1}\right)}{\left(t_{2}-t_{1}\right)}
\end{array}
$$

Equation A14 can be written in shorter form:

$$
\begin{equation*}
\Delta t=\left(t_{2}-t_{1}\right) \frac{2 E}{\ln \left[\frac{V+E}{V-E}\right]} \tag{3}
\end{equation*}
$$

