# A dynamic-network slacks-based measure with an application to Japanese Prefectures

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**Abstract:** This paper develops a dynamic-network DEA (data envelopment analysis) model where total output is jointly produced from two sectors: a human capital sector and a physical capital sector. While human capital is treated as an exogenous input, physical capital production is an intermediate output of one period that becomes an input to a subsequent period. The method is applied using pooled data on 47 Japanese prefectures during the period 2007-2009.
**Keyword:** Dynamic DEA, network DEA, dynamic-network model

## 1. INTRODUCTION

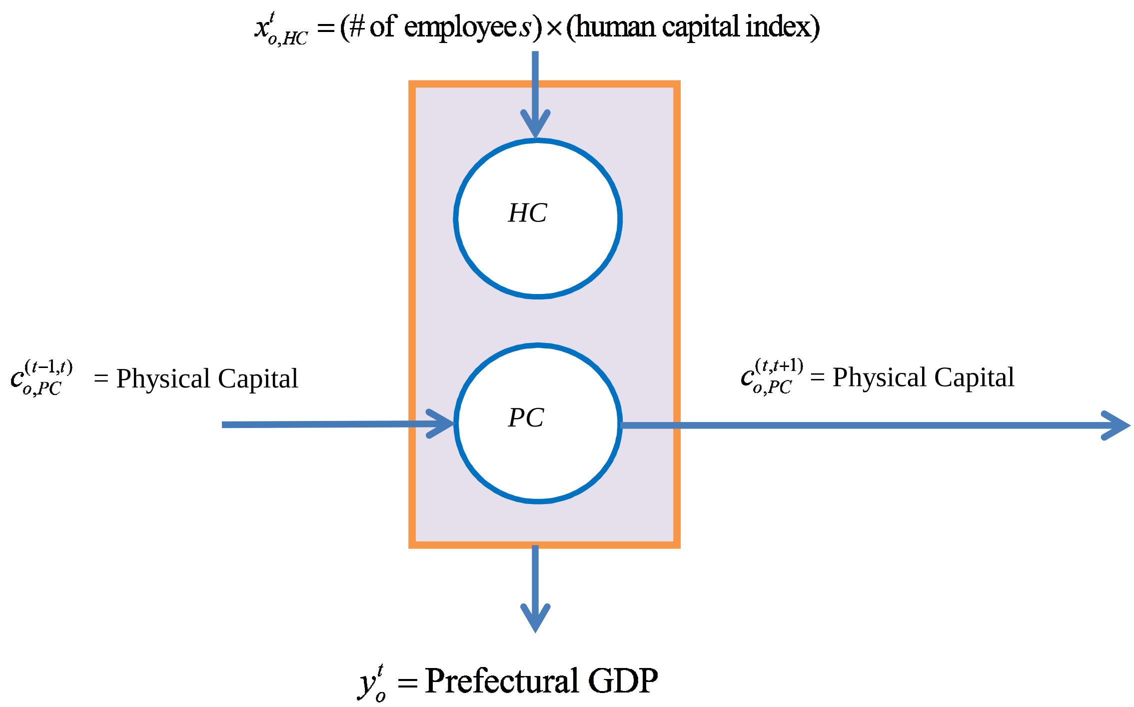
In this paper, we develop a measure of prefectural productive efficiency (PPE) to evaluate the performance of Japanese prefectures. Our method builds on Färe and Grosskopf's (1996) dynamic-network framework and Tone and Tsutsui's (2010, 2014) slacks-based network model. In addition, human capital has been shown to be an important driver of productivity growth (Henderson and Russell 2005, Badunenko, Henderson, and Russell 2013) and we control for human capital differences in labor use between prefectures. The model used to estimate PPE has the following characteristics. First, the prefectural technology consists of a human capital (HC) sector and a physical capital (PC) sector. The two sectors differ in that the HC input is an augmented labor input and the PC input to the current period was produced in the preceding period. Research incorporating lagged intermediate outputs (or carryover outputs) from a preceding period to a current period include Nemoto and Goto (2003), Bogetoft et al. (2009), Färe, Fukuyama, and Weber (2010) and Akther, Fukuyama, and Weber (2013). The PC sector uses physical capital from a previous period to produce final output in the current period and to reproduce itself as an intermediate output or carryover output for use in a subsequent period. Second, we employ an output oriented form of PPE that accounts for the slacks in final output and the PC carryover output.[1] Third, final output is jointly produced by the two sectors. Fourth, the two sector network system allows resources to be reallocated between periods so that larger final outputs can be achieved through inter-temporal optimization. We estimate the model of dynamic-network performance for 47 Japanese prefectures during 2007-2009.

## 2. NOTATION AND MODEL

In traditional growth theory, it is common to assume that a single product is produced by labor and physical capital. Extending this framework, we assume that the output is prefectural GDP (gross domestic product), the HC input is labor, and the PC input consists of public infrastructure and private physical capital. We regard the prefectural production technology as that carried out by two sub-processes or distinct sectors: the HC sector and the PC sector. Note that network systems of the two-sectors are parallel but non-symmetric. Figure 1 shows the prefectural production structure.

To implement the dynamic structure depicted in Figure 1, we first introduce relevant notations and define production technologies. In period $t$ $(t = 1,...,T)$, consider a set of two sectors (HC and PC) of prefectures, each of which converts its physical capital stock produced at the preceding periods as well as the exogenous human capital input in the current period, to produce the final output of GDP and physical capital stock to the subsequent period. The quantity $x^t_{j,HC}$ is the observed input of the HC sector of the $j^{th}$ prefecture (denoted $\mathrm{PREF}_j$) in period t. The quantity $c^{(t-1,t)}_{j,PC}$ is the amount of physical capital produced (as an intermediate output) in period $t$-1 that is used as an input in period $t$. The quantity $c^{(t,t+1)}_{j,PC}$ is the amount of physical capital produced in period $t$ to be used as an input in period $t$+1. The final output $y^t_j$ is produced jointly by the two sectors.

[1] Fukuyama and Mirdehghan (2012) discuss the identification of divisional efficiency from a Pareto-Koopmans efficiency perspective.




Legend:
HC: Human Capital
PC: Physical Capital

**Figure 1: Prefectural Production for Prefecture "*o*"**

We define a dynamic-network prefectural productive efficiency ( $\mathrm{PPE}_o$ ) model for prefecture "*o*" by

$$\frac{1}{PPE_o} = \max\ 1 + \frac{1}{2} \times \left( \frac{s^+}{y_o^t} + \frac{s_{PC}^{(t,t+1)+}}{c_{o,PC}^{(t,t+1)}} \right) \tag{1}$$

subject to:

(HC= *Human capital*)

$$\sum_{j=1}^{J} x_{j,HC}^{t} \lambda_{j,HC} \le x_{o,HC}^{t} \tag{2}$$

$$\sum_{j=1}^{J} y_j^t \lambda_{j,HC} = y_o^t + s^+ \tag{3}$$

$$\sum_{j=1}^{J} \lambda_{j,HC} = 1, \tag{4}$$

( $PC$ =*physical capital*)

$$\sum_{j=1}^{J} y_j^t \lambda_{j,PC} = y_o^t + s^+ \tag{5}$$

$$\sum_{j=1}^{J} c_{j,PC}^{(t-1,t)} \lambda_{j,PC} \le c_{o,PC}^{(t-1,t)} \tag{6}$$

$$\sum_{j=1}^{J} c_{j,PC}^{(t,t+1)} \lambda_{j,PC} = c_{o,PC}^{(t,t+1)} + s_{PC}^{(t,t+1)+} \tag{7}$$

$$\sum_{j=1}^{J} \lambda_{j,PC} = 1 \tag{8}$$

$$s_{PC}^{+} \ge 0 \tag{9}$$

$$s_{PC}^{(t,t+1)+} \geq 0 \quad (10)$$

$$\lambda_{j,HC} \geq 0, \quad \lambda_{j,PC} \geq 0 \qquad (\forall j). \quad (11)$$

We note that the intensity variables for the HC sector and PC sector need not be the same implying different reference technologies for the two sectors. However, both sectors are subject to variable returns to scale which is implied by the intensity variables summing to one: $\sum_{j=1}^{J} \lambda_{j,HC} = 1$ and $\sum_{j=1}^{J} \lambda_{j,PC} = 1$.

The reciprocal of $\mathrm{PPE}_o$ gives the average proportional expansion in final output and capital carried forward to the next period that is feasible given the human capital augmented labor and physical capital from the previous period. A prefecture is efficient for the dynamic network model if $\mathrm{PPE}_o$ =1. Values of $\mathrm{PPE}_o$ less than 1 indicate inefficiency with smaller values indicating greater inefficiency.

The PPE model adopts the following assumptions:

Assumption 1:

The objective function of our framework includes slacks of the GDP output and physical capital output (carryovers to the next period) but does not include slacks associated with the human capital input (exogenous input).

Assumption 2:

PC in period t-1 plays a role as a “quasi-fixed” input to period *t*, whereas HC used in period *t* is formulated as a standard input.

Assumption 3:

Prefectural output is produced jointly by the two sectors.

Assumption 1 is employed because each prefectural government’s primal interest will be whether or not the prefecture attains the potential output (not input reductions). Assumption 2 is adopted using growth theory which often considers quasi-fixed inputs. Hence, the dynamic production system is subject to lagged effects. In a bank efficiency context, nonperforming loans are used as a nondiscretionary input by Akther et al. (2013) and Fukuyama and Weber (2013), where this input negatively affects production in later periods.

Regarding Assumption 3, it is of great importance to note why we use (3) and (5), for which we originally used

$$\sum_{j=1}^{J} y_j^t \lambda_{j,HC} = y_o^t + s_{HC}^{+} \quad \text{and}$$

$$\sum_{j=1}^{J} y_j^t \lambda_{j,PC} = y_o^t + s_{PC}^{+}, \quad \text{respectively.}$$

However, in order to obtain the common GDP output target for both sectors, the right hand side of the two equations must be the same, i.e.,

$$\sum_{j=1}^{J} y_j^t \lambda_{j,HC} = y_o^t + s_{HC}^{+} = y_o^t + s_{PC}^{+} = \sum_{j=1}^{J} y_j^t \lambda_{j,PC}$$

at the optimum. Setting $s^{+} = s_{HC}^{+} = s_{PC}^{+}$, we combine (3) and (5) to obtain

$$\sum_{j=1}^{J} y_j^t \lambda_{j,HC} = \sum_{j=1}^{J} y_j^t \lambda_{j,PC} = y_o^t + s^{+}.$$

## 3. DATA

Human capital consists of general education and training investments and is an important factor of economic growth. In our study, the data on human capital is constructed according to the procedure of Fukao and Yue[2] (2000), who developed a 47 prefectural human resource index for the years 1955-1995. For a complete account of their calculation procedure, see http://www.ier.hit-u.ac.jp/~fukao/japanese/data/fuken2000/datamaking.pdf.

Multiplying the human capital index by the number of workers yields the human capital augmented measure of labor. A similar approach has been used by Henderson and Russell (2005) and Badunenko, Henderson, and Russell (2013) in their examination of labor productivity across countries.

Physical capital includes the basic infrastructure and producer goods needed to support various economic activities. As a proxy of this input, we use prefectural capital formation (Cabinet Office, Government of Japan). The PC sector employs the capital stock that is carried over from the previous period to maintain or enhance the capital stock for use in a subsequent period. The HC sector and PC sector jointly produce the final prefectural product of GDP.

Formally, the input-output data are defined as follows:

$x_{j,HC}^{t}$ = # of workers times the human capital index for Prefecture *j*

$c_{PC}^{(t-1,t)}, c_{PC}^{(t,t+1)}$ = capital formation carry-over within the capital formation sector of Prefecture *j*

$y_{j}^{t}$ = gross domestic product (GDP) of Prefecture *j*.

The data set consists of 47 prefectures over the period 2007-2009. The yen values are deflated by 2005 GDP deflator. Table 1 reports descriptive statistics. The average prefectural contribution to real GDP shrank from 11.59 trillion yen in 2007 to 11.28 trillion yen in 2008 to 10.8 trillion yen in 2009. Physical capital stock carried over to the next period declined from 32 trillion yen in 2007 to 31.9 trillion yen in 2009. Offsetting the decline in physical capital, the human capital augmented labor input grew by 8.3% from 2007 to 2009.

[2] Fukao and Yue (2000) estimated this index for the period 1955-1995.

**Table 1: Descriptive Statistics**

| Period | | Human capital | Previous period physical capital= $c_{j,PC}^{(t-1,t)}$ (Mill. Yen) | Current period physical capital= $c_{j,PC}^{(t,t+1)}$ (Mill. Yen) | GDP (Mill. Yen) |
|---|---|---|---|---|---|
| 2007-2009 | mean | 1,659,470 | 31,829,810 | 31,876,883 | 11,227,607 |
| | std dev | 2,062,459 | 24,730,713 | 24,891,723 | 15,285,715 |
| | max | 13,628,005 | 159,045,537 | 159,618,978 | 100,061,637 |
| | min | 344,775 | 13,879,239 | 13,871,527 | 2,027,794 |
| 2007 | mean | 1,594,230 | 31,721,117 | 31,968,306 | 11,588,971 |
| | std dev | 1,920,465 | 24,472,426 | 24,926,152 | 15,774,189 |
| | max | 11,818,594 | 155,783,075 | 159,045,537 | 100,061,637 |
| | min | 344,775 | 13,905,438 | 13,934,699 | 2,160,115 |
| 2008 | mean | 1,657,262 | 31,968,306 | 31,800,007 | 11,280,426 |
| | std dev | 2,064,125 | 24,926,152 | 24,790,722 | 15,364,942 |
| | max | 12,797,412 | 159,045,537 | 158,373,044 | 97,840,393 |
| | min | 351,277 | 13,934,699 | 13,879,239 | 2,092,722 |
| 2009 | mean | 1,726,917 | 31,800,007 | 31,862,335 | 10,813,422 |
| | std dev | 2,191,712 | 24,790,722 | 24,957,689 | 14,688,356 |
| | max | 13,628,005 | 158,373,044 | 159,618,978 | 93,842,542 |
| | min | 365,369 | 13,879,239 | 13,871,527 | 2,027,794 |

## 4. Dynamic-network prefectural productive efficiency estimates and their determinants

Table 2 reports the estimates of overall efficiency, and Figure 2 compares the prefectural productive efficiency estimates calculated under output orientation and variable returns to scale. Average efficiency is approximately 0.92 in 2007 and 0.93 in 2009. Nine different prefectures are efficient ($PPE = 1$) in at least one year. The prefectures of Tokyo, Kanagawa, Fukui, Tottori, and Okinawa are efficient in all three sample years. The prefectures of Saga, Aichi, Toyama, and Tochigi are efficient in at least one year. Three of Japan's four biggest cities (Tokyo, Yokohama, and Nagoya) belong to these prefectures. Productive efficiency is higher in these urbanized and industrialized prefectures than it is in rural agricultural prefectures. Yamagata prefecture has the ninth largest geographic area but the $35^{th}$ largest population–is the least efficient in all three years with efficiency averaging 0.82. This evidence suggests that there might be important relations among productive efficiency and agglomeration economies (benefits that firms obtain by the clustering of activities external to the firms).

**Table 2.** Prefectural Productivity Estimates

| | 2007 | 2008 | 2009 | All years |
|---|---|---|---|---|
| Mean | 0.924 | 0.930 | 0.931 | 0.929 |
| Std. dev. | 0.052 | 0.054 | 0.047 | 0.052 |
| Min. | 0.822 | 0.818 | 0.816 | 0.816 |
| Max | 1 | 1 | 1 | 1 |
| # on frontier | 7 | 8 | 6 | 5 |

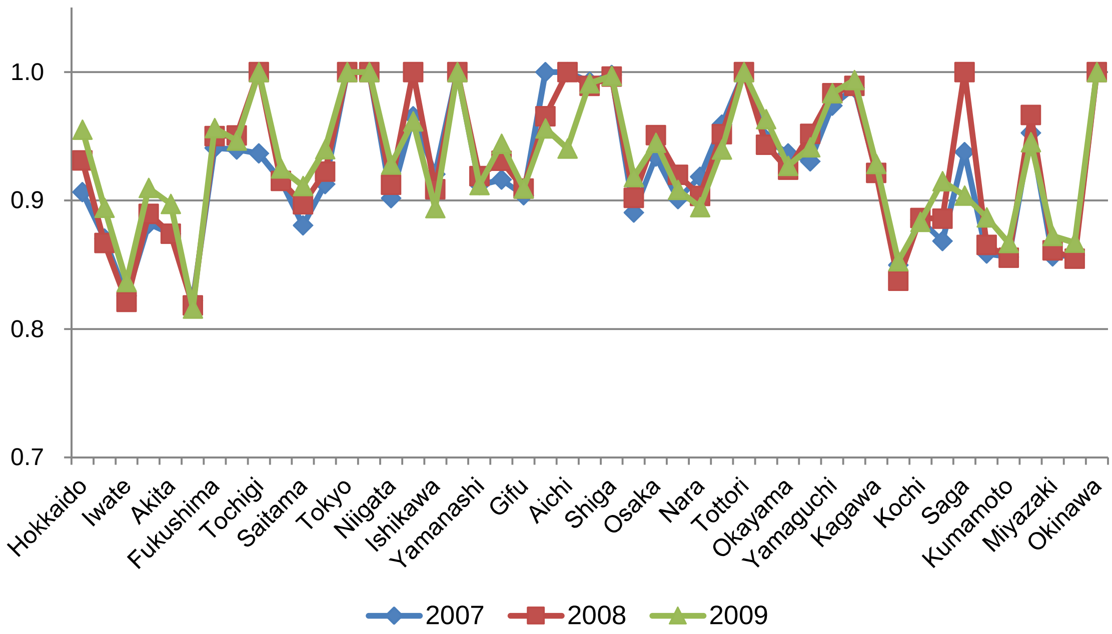



**Figure 2: Prefectural Efficiencies**

Therefore, we further examine the effects of prefectural location and characteristics on prefectural productive efficiency. Otsuka et al. (2010) examined whether or not market access (MA), population density (DEN), and public fiscal transfer (FT) have impact on the efficiency of Japanese regional industries. Otsuka et al.'s (2010) estimated equation took the form

$$Overall\ Efficiency = f(\text{MA, DEN, FT, REG}) \quad (12)$$

where REG are regional dummy variables. Market access and population density are considered to be proxy variables of agglomeration economies. Public fiscal transfer (FT) is defined as the national tax revenue allocated to local governments (prefectures). In Otsuka et al. (2010), the following market accessibility index for market $j$ is used:

$$MA_j = \sum_{k \neq j} \left[ \left( \frac{d_{jk}^{-1}}{\sum_{k \neq j} d_{jk}^{-1}} \right) \times GRP_k \right] \quad (13)$$

where $d_{jk}$ is the automobile travelling time between prefectures $j$ and $k$ and the gross production $GRP_k$ is the market size at prefecture *k*.

Similar to Otsuka et al.'s (2010) model we estimate

$$\hat{PPE} = \alpha + \alpha_1 DEN + \alpha_2 MA + \alpha_3 FT + \delta_1 MAN + \delta_2 SER + \sum_i \beta_i \, dummy_i + \varepsilon \quad (14)$$

where $\alpha, \alpha_1, \alpha_2, \alpha_3, \delta_1, \delta_2,$ and $\beta_i$ are parameters to be estimated. The variables MAN and SER are the percentage contributions of the manufacturing industry and the service industry relative to GDP. We exclude the share of agriculture to avoid collinearity between the three shares. The MAN and SER variables are those used to identify the relationship between industry structure and efficiency. The indicator variables $dummy_i \quad (i = 2007, 2008)$ are time dummies with $dummy_{2009}$ deleted. While Otsuka et al. (2010) used the distance estimates[3] $d_{jk}$ from the Central Research Institute of Electric Power Industry, we use estimates of $d_{jk}$ derived from the Ministry of Land, Infrastructure, Transport and Tourism.

In (14), the estimates of $PPE$ derived from the first stage estimation of the model represented by equations (1) to (11) are correlated with the discretionary explanatory variables used in the second stage regression. Simar and Wilson (2007) proposed bootstrap truncated regression analysis to overcome this problem.

Therefore, we employ Simar and Wilson's (2007) approach to generate a set of bias-corrected PPE estimates $\hat{PPE}$ and confidence intervals for the regression coefficients. Once bias-corrected productive efficiency scores are obtained from the bootstrap algorithm, they are then regressed on the set of environmental factors using the regression (14).

Table 3 gives the bootstrap regression results based on two models. Except for the time indicator variables all the coefficients for the independent variables are positive and statistically significant at the 1% level. Density (DEN) has a positive and significant correlation with prefectural efficiency, a finding that has a significant policy implication. Japan's population is aging rapidly and since 2004 more than 20% of Japanese are sixty-five years or older. In addition, fertility rates are below replacement which will lead to a rapidly shrinking population. The situation is particularly severe in nonurban areas. In 2015 the Japanese government embarked on a five-year comprehensive strategy to combat the population shrinkage. However, any such attempts to reduce the decline in population will be slow. Therefore, local governments working with the central government must combat declining population by increasing market access, through greater fiscal transfers, or by encouraging greater rural to urban migration to increase urban population density.

While Otsuka et al. (2010) reported FT has a negative effect on productive efficiency, our estimates show a positive significant effect. This indicates that the funds received from the central government are a significant driver for improving the productive efficiency of the local economies through for example local construction businesses which provided job opportunities to the region. This result is in sharp contrast to that

[3] We used a linear interpolation technique to interpolate the values between 2007 and 2009 because the ministry only report the travelling time estimates every five years.

of Otsuka et al. (2010). In addition, prefectures with greater density and market access have higher productive efficiency consistent with the existence of agglomeration economies.

Our finding shows that a 1% increase in the relative share of manufacturing increases prefectural efficiency by about 0.5% and a 1% increase in the relative share of services increases prefectural efficiency by approximately 0.4.

**Table 3: Bootstrap Regression Results**

| | Coefficient | | Standard Error | z | Prob. z>Z* | 95% Confidence Interval | |
|---|---|---|---|---|---|---|---|
| $\alpha$ | -0.2215 | * | 0.1241 | -1.79 | 0.0742 | -0.4648 | 0.0217 |
| $DEN$ | 0.0045 | *** | 0.0010 | 4.41 | 0 | 0.0025 | 0.0065 |
| $MA$ | 0.0019 | ** | 0.0008 | 2.34 | 0.0195 | 0.0003 | 0.0035 |
| $FT$ | 0.0348 | *** | 0.0118 | 2.95 | 0.0032 | 0.0117 | 0.0580 |
| $MAN$ | 0.4908 | *** | 0.1215 | 4.04 | 0.0001 | 0.2528 | 0.7289 |
| $SER$ | 0.4047 | *** | 0.1243 | 3.26 | 0.0011 | 0.1614 | 0.6479 |
| $dummy_{2007}$ | -0.0052 | *** | 0.0018 | -2.86 | 0.0042 | -0.0088 | -0.0016 |
| $dummy_{2008}$ | -0.0014 | | 0.0022 | -0.69 | 0.4909 | -0.0054 | 0.0026 |

Legend: ***, **, * indicates significance at 1%, 5%, and 10% level.

## 5. Conclusions

In this chapter we developed a dynamic network DEA model to estimate the productive efficiency of Japanese prefectures. The model assumes that two separate sectors–a human capital sector and a physical capital sector–jointly produce a prefecture's contribution to real GDP. In addition to its contribution to real GDP the physical capital sector links production between various periods in that previous capital formation is also used to produce capital as an intermediate output which is used in a future period. Average prefectural productive efficiency is 93%. In addition, we find not only that market access and population density are positively related to productive efficiency, but also that the roles played by manufacturing and service industry are of great importance to improve productive efficiency. We confirm that policy makers in charge of regional development at the prefecture level should consider industry structure and agglomeration economies.

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