Efficiency Predictions by Fuzzy Piecewise Auto-regression in Dynamic Network System

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Abstract: Since efficiency prediction can help managers to monitor future performance and detect potential failures, it is important for production and operation management. Data envelopment analysis is comprehensively applied to evaluate the relative performance in various areas. However, only few studies try to forecast the relative performance estimated by data envelopment analysis. We propose a performance forecasting model that integrates the multi-activity dynamic network data envelopment analysis and fuzzy piecewise auto-regression. The proposed approach constructs a dynamic performance measurement with the network structure to calculate the catching-up efficiency index. The catching-up efficiency index is further decomposed into the technical efficiency change and dynamic efficiency change to capture the effect of carry-over items. The fuzzy piecewise auto-regression is applied to regress the possibility and necessity estimation models by catching-up efficiency index for forecasting efficiency. In this paper, a data from banks in Taiwan from 2006 to 2012 are applied. The results indicate that the proposed approach has highly accuracy rate.

Keyword: Multi-activity dynamic network Data envelopment analysis, fuzzy piecewise auto-regression, catching-up efficiency index, banking performance

1. INTRODUCTION

To maintain or promote the competitive advantage, it is important for firms to utilize resources efficiently to generate outcomes. Performance analysis provides a method for managers to diagnose and analyze the level of resource utilization. By comparing the relative performance of each firm in adjacent periods, the managers can identify potential performance losses, and then identify the direction of resource adjustment. The performance prediction is also important for production and operation management, because it can monitor future performance and detect potential failures. It uses a forecasting model to anticipate the possible paths for a specific time horizon. The forecast information can help managers to avoid potential poor performance in the future.

Although various financial indicators are used to assess the performance (Caves, 1980; Megginson et al., 1994), they only consider single or parts of operational factors. Even if a performance is evaluated by aggregating various financial indicators, the appropriate weights are difficult to determine. However, data envelopment analysis (DEA) allow for the evaluation of multiple inputs and multiple outputs without predefining any weights (Charnes et al., 1978). Thus, DEA has been comprehensively applied to assess the relative performance in various areas, such as the banks (Chen and Yeh, 2000; Rezvanian and Mehdian, 2002), bus transit firms (Nolan et al., 2001; Odeck, 2006), government (Hsu and Hsueh, 2008), and schools (Tyagi et al., 2009). However, conventional DEA methods treat a decision making unit (DMU) as a "black box". They not only ignore the internal structure of the operational process, but also exclude the effect of carry-over items between two consecutive terms. Since the structure of a firm may include multiple activities and multiple processes and the operation of a firm is not independent among periods, the effects of inter-connected activities and processes as well as carry-over items should be considered when evaluating the performance. In response to these operational characteristics of firms, Yu et al. (2015) proposed a multi-activity dynamic network DEA (MDNDEA) model to incorporate multiple activities and multiple processes into a unified framework with considering the carry-over items. In order to obtain a more accurate performance measures, the MDNDEA model should be adopted.

In terms of performance prediction, some studies have tried to predict the efficiency by combining DEA with other predicting techniques. Sueyoshi (2000) proposed a stochastic DEA and used the stochastic and conventional efficiencies to decide the future efficiency. Kao and Liu (2004) introduced fuzzy concepts into DEA to forecast bank efficiency. Wu et al. (2006) integrated DEA and neural networks (NN) to forecast the efficiency of branch offices of Canadian banks. Hsiao et al. (2010) integrated DEA and fuzzy piecewise auto-regression analyses to forecast relative efficiency. Hsu (2014) forecasted the inter-fab performance by integrating DEA and back-propagation neural network (BPNN). The above methods have their specialty and uniqueness, in which the fuzzy regression technique can resolve the non-linear problems and is a good forecast method even if the available information is vague. However, the conventional fuzzy regression model is sensitive to outliers in possibility analysis (Redden and Woodall, 1994), and the necessity area cannot be obtained because of the large variation in data in necessity analysis (Tanaka et al., 1982; Tanaka and Ishibuchi, 1992; Yu et al., 1999, 2001). Yu et al. (1999, 2001) proposed the fuzzy piecewise regression models to avoid these two problems. Hence, it has the applicability to solve the forecasting problems in the real world.

In addition, the efficiency values evaluated by DEA models are censored at zero and one. The censored data will increase the complexity of the performance forecasting model. In order to avoid this problem, the catching-up index (CIE) can be applied. The CIE is the measure of efficiency change (EC) between any two adjacent periods. Hence, the values of CIE are not limited. However, the conventional CIE ignores the internal structure and carry-over items. Lei et al. (2013) built a dynamic Malmquist model with network structure to explore the black box performance. They decomposed the dynamic Malmquist productivity index into the EC and dynamic technical changes (DTC), in which OEC can be decomposed into technical efficiency change (TEC) and network efficiency change (NEC). Since this paper focuses on the effect of carry-over items, we will modify the decomposition process of EC to obtain TEC and dynamic efficiency change (DEC).

In order to account for the appropriate forecasting method, this paper proposes a performance forecasting model, which integrates MDNDEA and fuzzy piecewise auto-regression analyses. Our model includes three phases. First, the MDNDEA model proposed by Yu et al. (2015) is used to estimate the operational efficiency over various periods. Next, the CIE, that is the product of TEC and DEC, is applied to calculate the change of operational efficiency in adjacent periods. Finally, the fuzzy piecewise auto-regression is used to forecast the future performance.

The contributions of this paper are twofold. First, we propose a novel performance forecasting model, which integrates the MDNDEA model and the fuzzy piecewise auto-regression. Second, we decompose the CIE into TEC and DEC.

The rest of this paper is organized as follows. Section

2 presents the proposed performance forecasting model. Section 3 provides the application of 27 Taiwanese banks and describes the results. Finally, Section 4 presents the conclusions.

2. PERFORMANCE FORECASTING METHODOLOGY

This paper proposes a three-phase performance forecasting model to predict efficiency and to help in strategic decision-making. In the first phase, the MDNDEA model is used to evaluate the operational efficiency of each DMU in each period. In the second phase, the CIE of each DMU in two adjacent periods is calculated by dividing the efficiency of each DMU at the calculation period to its efficiency at the base period. In the third phase, the CIE values of each DMU in the training sample are applied to forecast its future efficiency by the fuzzy piecewise auto-regression. The notations used in the proposed model are shown in Table 1.

Table 1: Description of notations

Variable /Notation	Definition/Item
J	Number of DMUs.
T	Number of periods.
P	Number of change points.
n_a	Number of common input variables.
m_c, m_e, m_g	Number of desirable intermediate output variables in the investment, loans and others activities, respectively.
m_f	Number of undesirable intermediate output variables in the loans activity.
s_q	Number of desirable output variables in the profitability process.
r_i	Number of undesirable carry-over items in the loans activity.
r_l	Number of discretionary carry-over items in the profitability process.
$x^t_{aj,s}$	The *a*th common input variable of DMU_j in *t*th period.
$y^t_{cj,OI}$, $y^t_{ej,OL}$, $y^t_{gj,OO}$	The *c*th desirable intermediate output variable of DMU_j in *t*th period in the investment, loans and others activities, respectively.
$b^t_{fj,OL}$	The *f*th undesirable intermediate output variable of DMU_j in *t*th period in the loans activity.
$y^t_{qj,P}$	The *q*th desirable output variable of DMU_j in *t*th period in the profitability process..
$u^{(t,t+1)}_{ij,OL}$	The *i*th undesirable carry-over item of DMU_j carries from *t*th period to *t+1*th period in the loans activity.
$d^{(t,t+1)}_{lj,P}$	The *l*th discretionary carry-over item of DMU_j carries from *t*th period to *t+1*th period in the profitability process.
j,k ($j,k=1,...,N$)	Indexes for DMUs.
t ($t=1,..,T$)	Indexes for periods.
a ($a=1,..,n_a$)	Indexes for common input variables.
c ($c=1,..,m_c$), e ($e=1,..,m_e$), g ($g=1,..,m_g$),	Indexes for desirable intermediate output variables in the investment, loans and others activities, respectively.
f ($f=1,..,m_f$)	Indexes for undesirable intermediate output variables in the loans activity.
q ($q=1,..,s_q$)	Indexes for desirable output variables in the profitability process.
i ($i=1,..,r_i$)	Indexes for undesirable carry-over items in the loans activity.
l ($l=1,..,r_l$)	Indexes for discretionary carry-over items in the profitability process.
p ($p=1,..,P$)	Indexes for change points.
$\beta^t_{k,OI}$, $\beta^t_{k,OL}$, $\beta^t_{k,OO}$, $\beta^t_{k,P}$	Inefficiency scores of the investment, loans and others activities and the profitability process of *k*th DMU in *t*th period, respectively.
$\lambda^t_{j,OI}$, $\lambda^t_{j,OL}$, $\lambda^t_{j,OO}$, $\lambda^t_{j,P}$	Intensity variables of the investment, loans and others activities and the profitability process for projecting DMU_j in *t*th period, respectively.
$\delta^{t,t+1}_k$	Indexes for technical efficiency change and dynamic efficiency change of *k*th DMU in *t*th and *t+1*th period, respectively.
$\eta^{t,t+1}_{k,TEC}$, $\eta^{t,t+1}_{k,DEC}$	Catching-up index of *k*th DMU in *t*th and *t+1*th period.
$\gamma^L_{k,TEC}$, $\gamma^L_{k,DEC}$, $\gamma^L_{k,CIE}$,	The lower bounds of possibility regression predicting TEC, DEC and CIE values of *k*th DMU, respectively.
$\gamma^U_{k,TEC}$, $\gamma^U_{k,DEC}$, $\gamma^U_{k,CIE}$,	The upper bounds of possibility regression predicting TEC, DEC and CIE values of *k*th DMU, respectively.
$\pi^L_{k,TEC}$, $\pi^L_{k,DEC}$, $\pi^L_{k,CIE}$	The lower bounds of necessity regression predicting TEC, DEC and CIE values of *k*th DMU, respectively.
$\pi^U_{k,TEC}$, $\pi^U_{k,DEC}$, $\pi^U_{k,CIE}$,	The upper bounds of necessity regression predicting TEC, DEC and CIE values of *k*th DMU, respectively.
$\upsilon^L_{k,t}$	The lower bound of possibility regression predicting *t*th period's efficiency values of *k*th DMU.
$\upsilon^U_{k,t}$	The upper bound of possibility regression predicting *t*th period's efficiency values of *k*th DMU.
$\varpi^L_{k,t}$	The lower bound of necessity regression predicting *t*th period's efficiency values of *k*th DMU.
$\varpi^U_{k,t}$	The upper bound of necessity regression predicting *t*th period's efficiency values of *k*th DMU.

2.1 Phase I: Efficiency Evaluations

In this phase, the operational efficiency of each DMU in different periods is generated by using the MDNDEA model proposed by Yu et al. (2015). However, Yu et al.'s (2015) model is constructed based on the operational characteristics of bus transit firms. Since the operational characteristics among different industries are different, the MDNDEA model should be modified to suit the bank industry that is applied to illustrate the issue of performance prediction in this paper. Before modifying the MDNDEA model, the operational framework of a bank should be described. Following Chao et al. (2015), the operation of a bank mainly includes two processes: operating process and profitability process. The operating process can be further divided into three activities: investment activity, loans activity and others activity. The original common inputs are shared among activities in the operating process. The intermediate outputs produced by individual activities flow into the profitability process, including the undesirable outputs produced in the loans activity. In the profitability process, the final outputs are generated. In addition, the carry-over items exist in the loans activity and profitability process. However, in the loans activity, the carry-over items are undesirable. The operational framework is shown in Figure 1.

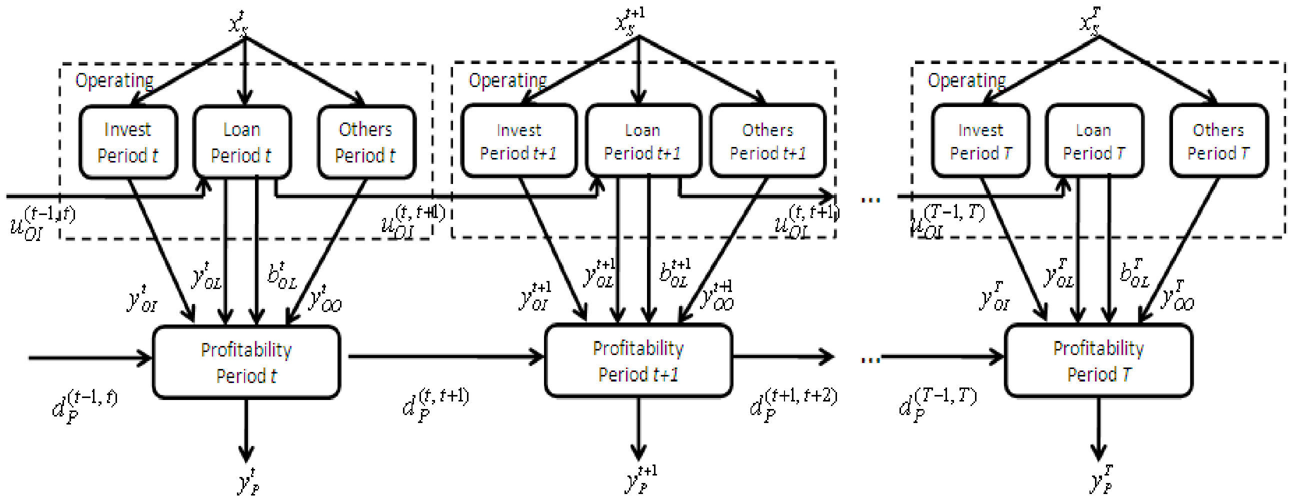

Figure 1: The operational framework of bank

Based on the notations in Section 2.1, the operational inefficiency for DMU k can be estimated by solving the following MDNDEA model based on the directional distance function:

$$\max \ \beta_k = \sum_{t=1}^{T} W^t \begin{bmatrix} w^O (w^{OI} \cdot \beta^t_{k,OI} + w^{OL} \cdot \beta^t_{k,OL} + w^{OO} \beta^t_{k,OO}) \\ + w^P \cdot \beta^t_{k,P} \end{bmatrix} \tag{1}$$

Subject to

(Investment activity)

$$\sum_{j=1}^{J} \mu^t_{a,OI} \lambda^t_{j,OI} x^t_{aj,s} \le (1-\beta^t_{k,OI}) \mu^t_{a,OI} x^t_{ak,s}, \quad a=1,\ldots,n_a,\ t=1,\ldots,T \tag{1.1}$$

$$L^t_{a,OI} < \mu^t_{a,OI} < U^t_{a,OI}, \quad a=1,\ldots,n_a, t=1,\ldots,T \tag{1.2}$$

$$\sum_{j=1}^{J} \lambda^t_{j,OI} y^t_{cj,OI} = \sum_{j=1}^{J} \lambda^t_{j,P} y^t_{cj,OI}, \quad c=1,\ldots,m_c,\ t=1,\ldots,T \tag{1.3}$$

$$\sum_{j=1}^{J} \lambda^t_{j,OI} y^t_{cj,OI} = y^t_{ck,OI} - S^{t,\,free}_{ck,\,OI}, \quad c=1,\ldots,m_c, t=1,\ldots,T \tag{1.4}$$

(Loans activity)

$$\sum_{j=1}^{J} \mu^t_{a,OL} \lambda^t_{j,OL} x^t_{aj,s} \le (1-\beta^t_{k,OL}) \mu^t_{a,OL} x^t_{ak,s}, \quad a=1,\ldots,n_a,\ t=1,\ldots,T \tag{1.5}$$

$$L^t_{a,OL} < \mu^t_{a,OL} < U^t_{a,OL}, \quad a=1,\ldots,n_a, t=1,\ldots,T \tag{1.6}$$

$$\sum_{j=1}^{J} \lambda^t_{j,OL} y^t_{ej,OL} = \sum_{j=1}^{J} \lambda^t_{j,P} y^t_{ej,OL}, \quad e=1,\ldots,m_e,\ t=1,\ldots,T \tag{1.7}$$

$$\sum_{j=1}^{J} \lambda_{j,OL}^{t} y_{ej,OL}^{t} = y_{ek,OL}^{t} - S_{ek,\,OL}^{t,\,free}, \quad e = 1,\ldots,m_e, t = 1,\ldots,T \tag{1.8}$$

$$\sum_{j=1}^{J} \lambda_{j,OL}^{t} b_{fj,OL}^{t} = \sum_{j=1}^{J} \lambda_{j,P}^{t} b_{fj,OL}^{t}, \quad f = 1,\ldots,m_f, t = 1,\ldots,T \tag{1.9}$$

$$\sum_{j=1}^{J} \lambda_{j,OL}^{t} b_{fj,OL}^{t} \le b_{fk,OL}^{t}, \quad f = 1,\ldots,m_f, t = 1,\ldots,T \tag{1.10}$$

$$\sum_{j=1}^{J} \lambda_{j,OL}^{t} u_{ij,OL}^{(t,t+1)} \le u_{ik,OL}^{(t,t+1)}, \quad i = 1,\ldots,r_i, t = 1,\ldots,T-1 \tag{1.11}$$

$$\sum_{j=1}^{J} \lambda_{j,OL}^{t} u_{ij,OL}^{(t,t+1)} = \sum_{j=1}^{J} \lambda_{j,OL}^{t+1} u_{ij,OL}^{(t,t+1)}, \quad i = 1,\ldots,r_i, t = 1,\ldots,T-1 \tag{1.12}$$

(Others activity)

$$\sum_{j=1}^{J} (1-\mu_{a,OI}^{t} - \mu_{a,OL}^{t}) \lambda_{j,OO}^{t} x_{aj,s}^{t} \le (1-\beta_{k,OO}^{t})(1-\mu_{a,OI}^{t} - \mu_{a,OL}^{t}) x_{ak,s}^{t}, \quad a = 1,\ldots,n_a, t = 1,\ldots,T \tag{1.13}$$

$$\sum_{j=1}^{J} \lambda_{j,OO}^{t} y_{gj,OO}^{t} = \sum_{j=1}^{J} \lambda_{j,P}^{t} y_{gj,OO}^{t}, \quad g = 1,\ldots,m_g, = 1,\ldots,T \tag{1.14}$$

$$\sum_{j=1}^{J} \lambda_{j,OO}^{t} y_{gj,OO}^{t} = y_{gk,OO}^{t} - S_{gk,\,OO}^{t,\,free}, \quad g = 1,\ldots,m_g, t = 1,\ldots,T \tag{1.15}$$

(Profitability production process)

$$\sum_{j=1}^{J} \lambda_{j,P}^{t} y_{qj,P}^{t} \ge (1+\beta_{k,P}^{t}) y_{qk,P}^{t}, \quad q = 1,\ldots,s_q, t = 1,\ldots,T \tag{1.16}$$

$$\sum_{j=1}^{J} \lambda_{j,P}^{t} d_{lj,P}^{(t,t+1)} = d_{lk,P}^{(t,t+1)} - S_{lk,P}^{(t,t+1),\,free}, \quad l = 1,\ldots,r_l, t = 1,\ldots,T-1 \tag{1.17}$$

$$\sum_{j=1}^{J} \lambda_{j,P}^{t} d_{lj,P}^{(t,t+1)} = \sum_{j=1}^{J} \lambda_{j,P}^{t+1} d_{lj,P}^{(t,t+1)}, \quad l = 1,\ldots,r_l, t = 1,\ldots,T-1 \tag{1.18}$$

(Initial conditions)

$$\sum_{j=1}^{J} \lambda_{j,OL}^{1} u_{ij,OL}^{(1,1)} = u_{ik,OL}^{(0,1)}, \quad i = 1,\ldots,r_i \tag{1.19}$$

$$\sum_{j=1}^{J} \lambda_{j,P}^{1} d_{lj,P}^{(0,1)} = d_{lk,P}^{(0,1)}, \quad l = 1,\ldots,r_l \tag{1.20}$$

$$\sum_{t=1}^{T} W^{t} = 1 \tag{1.21}$$

$$w^{O} + w^{P} = 1 \tag{1.22}$$

$$w^{OI} + w^{OL} + w^{OO} = 1 \tag{1.23}$$

$$\lambda_{j,OI}, \lambda_{j,OL}, \lambda_{j,OO}, \lambda_{j,P} \ge 0, \quad j = 1,\ldots,J \tag{1.24}$$

$$W^{t}, w^{O}, w^{P}, w^{OI}, w^{OL}, w^{OO} \ge 0, \quad t = 1,\ldots,T \tag{1.25}$$

$$S_{ck,OI}^{t,\,free}, S_{ek,\,OL}^{t,\,free}, S_{gk,\,OO}^{t,\,free} : free, \quad c = 1,\ldots,m_c, \; e = 1,\ldots,m_e, \; g = 1,\ldots,m_g, t = 1,\ldots,T \tag{1.26}$$

$$S_{lk,\,P}^{(t,t+1),\,free} : free, \quad l = 1,\ldots,r_l, t = 1,\ldots,T-1 \tag{1.27}$$

where $\mu_{a,OI}^{t}$ and $\mu_{a,OL}^{t}$ are the proportions of common input *a* shared to the investment and loans activities in period *t*, respectively. $W^{t}, w^{OI}, w^{OL}, w^{OO}, w^{O}$ and w^{P} are the weights on period *t*, the investment activity, loans activity, others activity, operating process and profitability process respectively, and indicate the relative importance of these periods, activities and processes. $S_{ck,OI}^{t,\,free}, S_{ek,\,OL}^{t,\,free}, S_{gk,\,OO}^{t,\,free}$ and $S_{lk,\,P}^{(t,t+1),\,free}$ are slack variables. L and U are the lower bound and upper bound on the shared proportion of the various common inputs. Constraints (1.3), (1.4), (1.7), (1.8), (1.14) and (1.15) indicate free links between the operating process and the profitability process. Constraints (1.9) and (1.10) show bad links between the loans activity and the profitability process. Constraints (1.11) and (1.12) represent undesirable links between period *t* and *t+1* in the loan activity. Constraints (1.17) and (1.18) indicate free links between period *t* and *t+1* in the profitability process. Constraints (1.12) and (1.18) impose the continuity condition between two consecutive periods. Constraints (1.19) and (1.20) account the initial conditions which are given and fixed. Based on the above the measures of various inefficiencies, the operational efficiency score in period *t* can be shown as follows:

$$\theta_{k}^{t} = 1 - [w^{O}(w^{OI}\cdot\beta_{k,OI}^{t} + w^{OL}\cdot\beta_{k,OL}^{t} + w^{OO}\cdot\beta_{k,OO}^{t}) + w^{P}\cdot\beta_{k,P}^{t}], \quad k = 1,\ldots,J \tag{2}$$

If DMU *k* is operationally efficient in the *t*th period, θ_{k}^{t} is equal to one.

2.2 Phase II: CIE

By Model (1), the operational efficiency scores can be calculated from period 1 to period *T*. Hence, we can further compute the CIE between any two adjacent periods. Following Lei et al. (2013), the CIE can be formed as

$$\delta_k^{t,t+1} = \frac{D^{t+1}(x_{ak,s}^{t+1}, y_{ck,OI}^{t+1}, y_{ek,OL}^{t+1}, b_{fk,OL}^{t+1}, y_{gk,OO}^{t+1}, u_{ik,OL}^{(t,t+1)}, d_{lk,P}^{(t,t+1)}, y_{qk,P}^{t+1}, u_{ik,OL}^{(t+1,t+2)}, d_{lk,P}^{(t+1,t+2)})}{D^{t}(x_{ak,s}^{t}, y_{ck,OI}^{t}, y_{ek,OL}^{t}, b_{fk,OL}^{t}, y_{gk,OO}^{t}, u_{ik,OL}^{(t-1,t)}, d_{lk,P}^{(t-1,t)}, y_{qk,P}^{t}, u_{ik,OL}^{(t,t+1)}, d_{lk,P}^{(t,t+1)})} = \frac{\theta_k^{t+1}}{\theta_k^t} \quad (3)$$

CIE is greater than one as efficiency increases; otherwise, it represents efficiency decreases. CIE can be further decomposed as

$$\begin{aligned}\delta_k^{t,t+1} &= \frac{D^{t+1}(x_{ak,s}^{t+1}, y_{ck,OI}^{t+1}, y_{ek,OL}^{t+1}, b_{fk,OL}^{t+1}, y_{gk,OO}^{t+1}, y_{qk,P}^{t+1})}{D^{t}(x_{ak,s}^{t}, y_{ck,OI}^{t}, y_{ek,OL}^{t}, b_{fk,OL}^{t}, y_{gk,OO}^{t}, y_{qk,P}^{t})} \\ &\times \frac{D^{t+1}(x_{ak,s}^{t+1}, y_{ck,OI}^{t+1}, y_{ek,OL}^{t+1}, b_{fk,OL}^{t+1}, y_{gk,OO}^{t+1}, u_{ik,OL}^{(t,t+1)}, d_{lk,P}^{(t,t+1)}, y_{qk,P}^{t+1}, u_{ik,OL}^{(t+1,t+2)}, d_{lk,P}^{(t+1,t+2)}) \big/ D^{t+1}(x_{ak,s}^{t+1}, y_{ck,OI}^{t+1}, y_{ek,OL}^{t+1}, b_{fk,OL}^{t+1}, y_{gk,OO}^{t+1}, y_{qk,P}^{t+1})}{D^{t}(x_{ak,s}^{t}, y_{ck,OI}^{t}, y_{ek,OL}^{t}, b_{fk,OL}^{t}, y_{gk,OO}^{t}, u_{ik,OL}^{(t-1,t)}, d_{lk,P}^{(t-1,t)}, y_{qk,P}^{t}, u_{ik,OL}^{(t,t+1)}, d_{lk,P}^{(t,t+1)}) \big/ D^{t}(x_{ak,s}^{t}, y_{ck,OI}^{t}, y_{ek,OL}^{t}, b_{fk,OL}^{t}, y_{gk,OO}^{t}, y_{qk,P}^{t})} \\ &= \frac{\rho_k^{t+1}}{\rho_k^t} \frac{\theta_k^{t+1} \big/ \rho_k^{t+1}}{\theta_k^t \big/ \rho_k^t} \\ &= \eta_{k,TEC}^{t,t+1} \cdot \eta_{k,DEC}^{t,t+1}\end{aligned} \quad (4)$$

where ρ_k^t and ρ_k^{t+1} are the measures of operational efficiencies for DMU k in period t and period t+1 respectively, without considering the effects of carry-over items. This two operational efficiencies can be obtained by applying the objective function: $\max \rho_k^t = w^O(w^{OI} \cdot \beta_{k,OI}^t + w^{OL} \cdot \beta_{k,OL}^t + w^{OO}\beta_{k,OO}^t) + w^P \cdot \beta_{k,P}^t$, and the constraints identified in Equations (1.1)-(1-10), (1.13)-(1.16) and (1.22)-(1.27). $\rho_k^{t+1} \big/ \rho_k^t$ is used to measure the TEC with network structure; $(\theta_k^{t+1} \big/ \rho_k^{t+1}) \big/ (\theta_k^t \big/ \rho_k^t)$ is used to measure the DEC in order to capture the effects of carry-over items.

2.3 Phase III: Fuzzy Piecewise Auto-Regression

After Phase II, number of $T-1$ TEC and DEC data can be obtained respectively. The TEC and DEC of each DMU will be forecasted by applying fuzzy piecewise auto-regression, in which number of $T-2$ TEC and DEC data are treat as independent variables of fuzzy piecewise auto-regression respectively, and the Tth data are dependent variables respectively. Fuzzy piecewise auto-regression will find two ranges. The first range is estimated by the possibility estimation model, indicating that the predicted values should be included in the regression range. The second range is calculated by the necessity estimation model, indicating that the predicted values should be excluded in the regression range. Hence, we can respectively obtain four TEC and DEC coefficients for each DMU. By mixing these TEC and DEC coefficients, we can further calculate the CIE coefficients for each DMU, and then forecast the future operational efficiency for each DMU.

Fuzzy regression analysis can be interpreted as an interval estimation of dependent variables (Yu et al., 1999; Tanaka and Ishibuchi, 1992; Tanaka and Lee, 1998). First, an interval, that covers all training data, is computed. Then, and a membership function is constructed based on this interval. We adopt the quadratic form in Phase III for illustrating the forecasting process. Based on the observed period T, there are one dependent variable, $\eta_{k,h}^{t-1,t}$, and $t-2$ independent variables, $h = TEC, DEC$. The linear interval regression model for DMU k with independent variables using interval parameters A_i ($i = 0, \ldots, t-2$) is shown as follows

$$\eta_{k,h}^{t-1,t} = A_0 + A_1\eta_{k,h}^{t-2,t-1} + \ldots + A_{t-2}\eta_{k,h}^{1,2} \quad (5)$$

where $\eta_{k,h}^{t-1,t}$ is the predicted interval for DMU k corresponding to the input vector ($\eta_{k,h}^{t-2,t-1}, \eta_{k,h}^{t-3,t-2}, \ldots, \eta_{k,h}^{1,2}$), which is a one-dimensional input vector for DMU k, and t is the index for time ($t = 1, \ldots, T$). An interval defined by the ordered pair in brackets is written as follows:

$$A = [a_L, a_R] = [a : a_L < a < a_R] \quad (6)$$

where a_L and a_R denote the left and right limits of A, respectively. Interval A can be also denoted by its center and radius as

$$A = (a_c, a_w) = \{a : a_c - a_w \le a \le a_c + a_w\} \quad (7)$$

where a_c and a_w denote the center and the radius, respectively. Hence, Equation (5) can be represented in detail as follows:

$$\begin{aligned} \eta_{k,h}^{t-1,t} &= A_0 + A_1\eta_{k,h}^{t-2,t-1} + \ldots + A_{t-2}\eta_{k,h}^{1,2} \\ &= (a_{0c,k,h}, a_{0w,k,h}) + (a_{1c,k,h}, a_{1w,k,h})\eta_{k,h}^{t-2,t-1} \\ &+ \ldots + (a_{t-2c,k,h}, a_{t-2w,k,h})\eta_{k,h}^{1,2} \\ &= (Y_{ck,h}, Y_{wk,h}) \end{aligned} \quad (8)$$

where

$$Y_{ck,h} = a_{0c,k,h} + a_{1c,k,h}\eta_{k,h}^{t-2,t-1} + \ldots + a_{t-2c,k,h}\eta_{k,h}^{1,2} \quad (9)$$

$$Y_{wk,h} = a_{0w,k,h} + a_{1w,k,h}\left|\eta_{k,h}^{t-2,t-1}\right| + \ldots + a_{t-2w,k,h}\left|\eta_{k,h}^{1,2}\right| \quad (10)$$

where $Y_{ck,h}$ and $Y_{wk,h}$ represent the center and the radius of predicted interval $\eta_{k,h}^{t-1,t}$ of DMU *k*. Then, the possibility and necessity estimation models are explored. First, the possibility estimation model can be expressed as follows:

$$\begin{aligned} (\eta_{k,h}^{t-1,t})^* &= A_0^* + A_1^*(\eta_{k,h}^{t-2,t-1})^* + \ldots + A_{t-2}^*(\eta_{k,h}^{1,2})^* \\ &= (a_{0c,k,h}^*, a_{0w,k,h}^*) + (a_{1c,k,h}^*, a_{1w,k,h}^*)\eta_{k,h}^{t-2,t-1} \\ &+ \ldots + (a_{t-2c,k,h}^*, a_{t-2w,k,h}^*)\eta_{k,h}^{1,2} \\ &= (Y_{ck,h}^*, Y_{wk,h}^*) \end{aligned} \quad (11)$$

which satisfies the following conditions:

$$\eta_{k,h}^{t-1,t} \subseteq (\eta_{k,h}^{t-1,t})^* \quad , \quad t = 1,..,T \quad (12)$$

In the possibility analysis, the width of the predicted interval $(\eta_{k,h}^{t-1,t})^*$ is minimized and includes all observed data. Second, the necessary estimate model can be written as follows:

$$\begin{aligned} (\eta_{k,h}^{t-1,t})_* &= A_{0*} + A_{1*}\eta_{k,h}^{t-2,t-1} + \ldots + A_{t-2*}\eta_{k,h}^{1,2} \\ &= (a_{0c*,k,h}, a_{0w*,k,h}) + (a_{1c*,k,h}, a_{1w*,k,h})\eta_{k,h}^{t-2,t-1} \\ &+ \ldots + (a_{t-2c*,k,h}, a_{t-2w*,k,h})\eta_{k,h}^{1,2} \\ &= (Y_{ck,h*}, Y_{wk,h*}) \end{aligned} \quad (13)$$

which satisfies the following conditions:

$$(\eta_{k,h}^{t-1,t})_* \subseteq \eta_{k,h}^{t-1,t} \quad , \quad t = 1,..,T \quad (14)$$

In the necessity analysis, the width of the predicted interval $(\eta_{k,h}^{t-1,t})_*$ is maximized and is included by all observed data. The relations of possibility model and necessity model can be expressed as follows:

$$(\eta_{k,h}^{t-1,t})_* \subseteq \eta_{k,h}^{t-1,t} \subseteq (\eta_{k,h}^{t-1,t})^* \quad (15)$$

Furthermore, the fuzzy regression can be extended to the fuzzy piecewise regression. We use the quadratic programming formulation to determine the necessity area by the piecewise linear interval regression model as shown in Equation (16).

$$(\eta_{k,h}^{t-2,t-1})_* = h(\eta_{k,h}^{t-2,t-1}) + \sum_{p=1}^{P-1}\left\{\frac{B_{p,h*}}{2}\left(\left|\eta_{k,h}^{t-2-p,t-1-p} - P_{p,h}\right| + \eta_{k,h}^{t-2-p,t-1-p} - P_{p,h}\right)\right\} \quad (16)$$

where $h(\eta_{k,h}^{t-2,t-1}) = a_{0*} + a_{1*}\eta_{k,h}^{t-2,t-1}$. $B_{p,h*}$ is the interval of the necessity estimation model of B_p. $B_{p,h} = (B_{pc,h}, B_{pw,h})$ represents the center and radius of $B_{p,h}$. Similarly, the possibility area can be obtained as Equation (16) by substituting $B_{p,h}^*$ to $B_{p,h*}$, where $B_{p,h}^*$ is the interval of the possibility estimation model of B_p.

Let $P_{p,h}$ be a change-point. Then, the operation of piecewise term can be written as follows:

$$\frac{\left(\left|\eta_{k,h}^{t-2-p,t-1-p} - P_{p,h}\right| + \eta_{k,h}^{t-2-p,t-1-p} - P_{p,h}\right)}{2} = \begin{cases} \eta_{k,h}^{t-2-p,t-1-p} - P_{p,h}, & \text{if } \eta_{k,h}^{t-2-p,t-1-p} \ge P_{p,h} \\ 0 & , \text{if } \eta_{k,h}^{t-2-p,t-1-p} < P_{p,h} \end{cases} \quad (17)$$

where $P_{p,h} = \{P_{1,h}, .., P_{P,h}\}$ are the values of variables $\eta_{k,h}^{t-2-p,t-1-p}$ and are subject to an ordering constraint $P_1 < P_2 < \ldots < P_P$, $P \le N-1$.

Hence, the fuzzy piecewise auto-regression quadratic programming formulation is shown as follows:-

$$\min \sum_{k=1}^{N}\left\{a_{0w,h*} + a_{1w,h*}\eta_{k,h}^{t-2,t-1} + \sum_{p=1}^{P-1}\left\{B_{pw,h*}\frac{(|\eta_{k,h}^{t-2-p,t-1-p} - P_{p,h}| + \eta_{k,h}^{t-2-p,t-1-p} - P_{p,h})}{2}\right\}\right\}^2 \quad (18)$$

Subject to

(Possibility constraints)

$$a^*_{0c,h} + a^*_{1c,h}\eta^{t-2,t-1}_{k,h} + \sum_{p=1}^{P-1}\left\{\frac{B^*_{pc,h}}{2}(\left|\eta^{t-2-p,t-1-p}_{k,h} - P_{p,h}\right| + \eta^{t-2-p,t-1-p}_{k,h} - P_{p,h})\right\}$$
$$-\left\{a^*_{0w,h} + a^*_{1w,h}\eta^{t-2,t-1}_{k,h} + \sum_{p=1}^{P-1}\left\{\frac{B^*_{pw,h}}{2}(\left|\eta^{t-2-p,t-1-p}_{k,h} - P_{p,h}\right| + \eta^{t-2-p,t-1-p}_{k,h} - P_{p,h})\right\}\right\} \le \eta^{t-1,t-2}_{k,h} + \varepsilon, \quad (18.1)$$

$$a^*_{0c,h} + a^*_{1c,h}\delta^{t-2,t-1}_{k,h} + \sum_{p=1}^{P-1}\left\{\frac{B^*_{pc,h}}{2}(\left|\eta^{t-2-p,t-1-p}_{k,h} - P_{p,h}\right| + \eta^{t-2-p,t-1-p}_{k,h} - P_{p,h})\right\}$$
$$+\left\{a^*_{0w,h} + a^*_{1w,h}\delta^{t-2,t-1}_{k,h} + \sum_{p=1}^{P-1}\left\{\frac{B^*_{pw,h}}{2}(\left|\eta^{t-2-p,t-1-p}_{k,h} - P_{p,h}\right| + \eta^{t-2-p,t-1-p}_{k,h} - P_{p,h})\right\}\right\} \ge \eta^{t-1,t-2}_{k,h} - \varepsilon, \quad (18.2)$$

$P \le T-2, \forall\ \mathrm{t} = 1,\ldots,T-2$

(Necessity constraints)

$$a_{0c,h*} + a_{1c,h*}\eta^{t-2,t-1}_{k,h} + \sum_{p=1}^{P-1}\left\{\frac{B_{pc,h*}}{2}(\left|\eta^{t-2-p,t-1-p}_{k,h} - P_{p,h}\right| + \eta^{t-2-p,t-1-p}_{k,h} - P_{p,h})\right\}$$
$$-\left\{a_{0w,h*} + a_{1w,h*}\eta^{t-2,t-1}_{k,h} + \sum_{p=1}^{P-1}\left\{\frac{B_{pw,h*}}{2}(\left|\eta^{t-2-p,t-1-p}_{k,h} - P_{p,h}\right| + \eta^{t-2-p,t-1-p}_{k,h} - P_{p,h})\right\}\right\} \ge \eta^{t-1,t}_{k,h} + \varepsilon, \quad (18.3)$$

$$a_{0c,h*} + a_{1c,h*}\eta^{t-2,t-1}_{k,h} + \sum_{p=1}^{P-1}\left\{\frac{B_{pc,h*}}{2}(\left|\eta^{t-2-p,t-1-p}_{k,h} - P_{p,h}\right| + \eta^{t-2-p,t-1-p}_{k,h} - P_{p,h})\right\}$$
$$+\left\{a_{0w,h*} + a_{1w,h*}\eta^{t-2,t-1}_{k,h} + \sum_{p=1}^{P-1}\left\{\frac{B_{pw,h*}}{2}(\left|\eta^{t-2-p,t-1-p}_{k,h} - P_{p,h}\right| + \eta^{t-2-p,t-1-p}_{k,h} - P_{p,h})\right\}\right\} \le \eta^{t-1,t}_{k,h} - \varepsilon, \quad (18.4)$$

$P \le N, \forall\ \mathrm{t} = 1,\ldots,T-2$

where ε is defined as a very small number.

By calculating $a^*_{0c,h}$, $a^*_{1c,h}$, $B^*_{pc,h}$, and $B^*_{pw,h}$, the lower bound, $\gamma^L_{k,h}$, and the upper bound, $\gamma^U_{k,h}$, of $\eta^{t-1,t}_{k,h}$ for DMU *k* can be determined by the following equations.

$$\gamma^L_{k,h} = (a^*_{0c,h} + a^*_{1c,h}\eta^{t-2,t-1}_{k,h} + \sum_{p=1}^{P-1} B^*_{pc,h}\eta^{t-2-p,t-1-p}_{k,h}) - \left\{a^*_{0w,h} + a^*_{1w,h}\eta^{t-2,t-1}_{k,h} + \sum_{p=1}^{P-1} B^*_{pw,h}\eta^{t-2-p,t-1-p}_{k,h}\right\} \quad (19)$$

$$\gamma^U_{k,h} = (a^*_{0c,h} + a^*_{1c,h}\eta^{t-2,t-1}_{k,h} + \sum_{p=1}^{P-1} B^*_{pc,h}\eta^{t-2-p,t-1-p}_{k,h}) + \left\{a^*_{0w,h} + a^*_{1w,h}\eta^{t-2,t-1}_{k,h} + \sum_{p=1}^{P-1} B^*_{pw,h}\eta^{t-2-p,t-1-p}_{k,h}\right\} \quad (20)$$

Any $\eta^{t-1,t}_{k,h}$ will lie on $[\gamma^L_{k,h}, \gamma^U_{k,h}]$.

Similarly, by computing $a_{0c,h*}$, $a_{1c,h*}$, $B_{pc,h*}$, and $B_{pw,h*}$, the lower bound, $\pi^L_{k,h}$, and the upper bound, $\pi^U_{k,h}$, of $\eta^{t-1,t}_{k,h}$ for DMU *k* can be determined by the following equations.

$$\pi^L_{k,h} = a_{0c,h*} + a_{1c,h*}\eta^{t-2,t-1}_{k,h} + \sum_{p=1}^{P-1} B_{pc,h*}\eta^{t-2-p,t-1-p}_{k,h} + \left\{a_{0w,h*} + a_{1w,h*}\eta^{t-2,t-1}_{k,h} + \sum_{p=1}^{P-1} B_{pw,h*}\eta^{t-2-p,t-1-p}_{k,h}\right\} \quad (21)$$

$$\pi^U_{k,h} = a_{0c,h*} + a_{1c,h*}\eta^{t-2,t-1}_{k,h} + \sum_{p=1}^{P-1} B_{pc,h*}\eta^{t-2-p,t-1-p}_{k,h} + \left\{a_{0w,h*} + a_{1w,h*}\eta^{t-2,t-1}_{k,h} + \sum_{p=1}^{P-1} B_{pw,h*}\eta^{t-2-p,t-1-p}_{k,h}\right\} \quad (22)$$

All $\eta^{t-1,t}_{k,h}$ could not lie on $[\pi^L_{k,h}, \pi^U_{k,h}]$.

For any DMU *k*, we check whether these values satisfy the conditions that

$\gamma^U_{k,TEC} \ge \pi^U_{k,TEC} \ge \pi^L_{k,TEC} \ge \gamma^L_{k,TEC}$ and

$\gamma^U_{k,DEC} \ge \pi^U_{k,DEC} \ge \pi^L_{k,DEC} \ge \gamma^L_{k,DEC}$.

If these two conditions are satisfied simultaneously, we can further calculate the possibility and necessity areas of CIE as

$$\gamma^L_{k,CIE} = \gamma^L_{k,TEC} \cdot \gamma^L_{k,DEC} \quad (23)$$

$$\gamma^U_{k,CIE} = \gamma^U_{k,TEC} \cdot \gamma^U_{k,DEC} \quad (24)$$

$$\pi^L_{k,CIE} = \pi^L_{k,TEC} \cdot \pi^L_{k,DEC} \quad (25)$$

$$\pi^U_{k,CIE} = \pi^U_{k,TEC} \cdot \pi^U_{k,DEC} \quad (26)$$

where $\gamma^L_{k,CIE}$ and $\gamma^U_{k,CIE}$ are the lower and upper bounds of possibility areas of $\delta^{t-1,t}_k$ for DMU k, while $\pi^L_{k,CIE}$ and $\pi^U_{k,CIE}$ are the lower and upper bounds of necessity areas of $\delta^{t-1,t}_k$ for DMU k. Then, the four operational efficiency values, $\upsilon^L_{k,t}$, $\varpi^L_{k,t}$, $\varpi^U_{k,t}$, and $\upsilon^U_{k,t}$, in the t period can be obtained by multiplying the operational efficiency value, θ^{t-1}_k, in the t-1 period to $\gamma^L_{k,CIE}$, $\pi^L_{k,CIE}$, $\pi^U_{k,CIE}$, and $\gamma^U_{k,CIE}$, respectively. Moreover, we check $\theta^t_k \in [\upsilon^L_{k,t}, \varpi^L_{k,t}]$ or $\theta^t_k \in [\varpi^U_{k,t}, \upsilon^U_{k,t}]$. After validation, the time horizon is shifted from t to $t+1$ period to forecast the operational efficiency of each DMU.

3. FORECAST RESULTS

3.1 The Data

To demonstrate the validity of the proposed approach, we conduct an empirical study to analyze the data of 27 banks from 2006 to 2012 in Taiwan. The panel data set collects from the Taiwan Economic Journal Data Bank. Following Chao et al. (2015), we select operating costs (OC) and capital utilization expense (CUE) as common inputs, which are shared among the investment activity, loans activity, others activity and profitability process. Investments (I), performing loans (PL) and business volume (BV) are treated as the desirable intermediate outputs flowing from the investment, loans and others activities to the profitability process, respectively. Write-offs (WO) is the undesirable intermediate output flowing from the loans activity to the profitability process. Interest income (IN), non-interest income (NIN) and earnings per share (EPS) are selected as the final outputs for the profitability process. In addition, non-performing loans (NPL) is the undesirable carry-over item in the loans activity. Net worth (NW) is the discretionary carry-over item in the profitability process. Table 2 presents the descriptive statistics of all variables used in this paper.

Table 2: Summary statistics of inputs and outputs

Unit: million NTD

Variable	Mean	Std. Dev.	Max	Min
OC	22,262	29,876	362,799	2,106
CUE	5,008	390	18,372	248
I	284,115	514,066	3,841,931	2,232
PL	629,715	560,472	2,183,508	69,284
WO	7,371	5,746	26,542	574
NPL	6,472	6,596	37,452	118
BV	3,568,241	9,734,540	78,394,200	56
IN	22,045	18,380	86,859	2,218
NIN	11,054	7,494	37,915	49
EPS(NTD)	4.90	2.68	12.89	0.02
NW	69,030	55,307	263,734	15,522

3.2 Efficiency Prediction

First, the operational efficiency scores from 2006 to 2012 are evaluated by Model (1). All banks don't have full efficiency during the sample period. Then, the CIE, TEC and DEC can be calculated by Equations (3) and (4). [1] Finally, the fuzzy piecewise auto-regression is used to forecast the efficiency of 27 banks in 2012 in Taiwan.

Since the CIE ranges are obtained from the TEC and DEC ranges, the possibility and necessity areas of TEC and DEC should be calculated. The possibility and necessity estimation models of TEC obtained from Model (18) is written as follows:

$$\begin{aligned}\eta^{2010,2011}_{k,TEC} &= [0.8870, 0, 0.1739] + [0, 0, 0.0057]\eta^{2006,2007}_{k,TEC} \\ &+ [0, 0, 0.0161]\eta^{2008,2009}_{k,TEC} \\ &+ [0, 0, 0.0058]\eta^{2009,2010}_{k,TEC} + ESP_1\end{aligned} \quad (27)$$

where $ESP_1 = \sum_{p=1}^{P-1} \left\{ \begin{array}{l} \frac{B_{pc,TEC}}{2}(\left|\eta^{t-2-p,t-1-p}_{k,TEC} - P_{p,TEC}\right| \\ +\eta^{t-2-p,t-1-p}_{k,TEC} - P_{p,TEC}) \end{array} \right\}$ [2].

The first value in the square bracket represents the center, the second value represents the necessity radius, and the final value is the possibility radius.

Similarly, the possibility and necessity estimation models of DEC obtained from Model (18) is show as

[1] Due to limited space, the values of operational efficiency, CIE, TEC and DEC are not presented. They are available from the author upon request.

[2] The values of ESP_1 among 27 banks are different. Due to limited space, the values of ESP_1 are not presented. They are available from the author upon request.

follows:

$$\eta_{k,DEC}^{2010,2011} = \left[0.3351, 0, 0.1359\right] + \left[0.1711, 0, 0.0022\right]\eta_{k,TEC}^{2006,2007} + \left[0.2768, 0, 0.0281\right]\eta_{k,TEC}^{2007,2008} + \left[0.1129, 0, 0.0169\right]\eta_{k,TEC}^{2008,2009} + \left[0, 0, 0.0168\right]\eta_{k,TEC}^{2009,2010} + ESP_2 \quad (28)$$

where $ESP_2 = \sum_{p=1}^{P-1} \left\{ \frac{B_{pc,DEC}}{2} \left(\left| \eta_{k,DEC}^{t-2-p,t-1-p} - P_{p,DEC} \right| + \eta_{k,DEC}^{t-2-p,t-1-p} - P_{p,DEC} \right) \right\}$ [3].

Furthermore, using Equations (23)-(28), the lower and upper bounds of possibility and necessity areas ($\gamma_{k,CIE}^{L}$, $\gamma_{k,CIE}^{U}$, $\pi_{k,CIE}^{L}$ and $\pi_{k,CIE}^{U}$) of $\delta_k^{2010,2011}$ for DMU k can be obtained. The validating ranges of $\delta_k^{2010,2011}$ are represented in Table 3. The final row in Table 3 reports the actual data. The results identify the validation of the proposed forecasting approach.

Table 3: Validating CIE index

Bank	γ^L	π^L	π^U	γ^U	$\delta^{2010,2011}$
1	0.5924	0.9425	0.9425	1.3727	0.8659
2	0.5687	0.9140	0.9140	1.3407	0.9877
3	0.7203	1.1055	1.1055	1.5723	1.0010
4	0.7221	1.1034	1.1034	1.5652	1.2200
5	0.7172	1.0982	1.0982	1.5595	1.0401
6	0.7513	1.1363	1.1364	1.6007	1.1431
7	0.5961	0.9503	0.9503	1.3865	1.0491
8	0.5309	0.8624	0.8652	1.2772	0.9107
9	0.7074	1.0838	1.0838	1.5402	1.3020
10	0.6855	1.0593	1.0593	1.5142	1.0557
11	0.6780	1.0558	1.0558	1.5148	1.0158
12	0.7253	1.1093	1.1093	1.5745	1.0643
13	0.6814	1.0540	1.0540	1.5075	0.9572
14	0.6429	1.0064	1.0064	1.4505	0.9190
15	0.7228	1.1066	1.1066	1.5716	1.0584
16	0.8081	1.2063	1.2100	1.6884	0.9999
17	0.7378	1.1226	1.1226	1.5879	1.0186
18	0.6557	1.0238	1.0238	1.4729	1.0017
19	0.6964	1.0770	1.0770	1.5392	1.0274
20	0.6431	1.0101	1.0101	1.4595	1.0019
21	0.7440	1.1301	1.1301	1.5964	1.1209
22	0.7529	1.1455	1.1455	1.6185	1.1580
23	0.8141	1.2163	1.2163	1.6990	1.0318
24	0.6657	1.0415	1.0415	1.5006	0.8846
25	0.7115	1.0879	1.0905	1.5469	1.1382
26	0.6846	1.0595	1.0595	1.5149	0.9071
27	0.5094	0.8377	0.8377	1.2470	0.9008

After validation, the 2012 operational efficiency scores can be forecasted. $\upsilon_{k,2012}^{L}$, $\varpi_{k,2012}^{L}$, $\varpi_{k,2012}^{U}$, and $\upsilon_{k,2012}^{U}$ can be obtained by multiplying the operational efficiency value, θ_k^{2011} , to $\gamma_{k,CIE}^{L}$, $\pi_{k,CIE}^{L}$, $\pi_{k,CIE}^{U}$, and $\gamma_{k,CIE}^{U}$, respectively. The results of efficiency prediction are shown in Table 4. The final row reports the actual values of operational efficiency in 2012. The results show the accuracy rate is 100%. In addition, our approach can predict the trend of efficiency change. If the actual value of operational efficiency lies on $[\varpi_{2012}^{U}, \upsilon_{2012}^{U}]$, the trend is up; whereas if the actual value of operational efficiency lies on $[\upsilon_{2012}^{L}, \varpi_{2012}^{L}]$, the trend is down. The results indicate that 11 banks (1, 2, 6, 7, 8, 13, 14, 20, 24, 26 and 27) have the upward trends, but 16 banks (3, 4, 5, 9, 10, 11, 12, 15, 16, 17, 18, 19, 21, 22, 23 and 25) have the downward trends.

Table 4: The comparison of observed efficiency and predicted efficiency

Bank	υ_{2012}^{L}	ϖ_{2012}^{L}	ϖ_{2012}^{U}	υ_{2012}^{U}	θ^{2012}
1	0.4916	0.8092	0.8092	1.0000	0.9999
2	0.4902	0.8081	0.8081	1.0000	0.8758
3	0.6021	0.9464	0.9464	1.0000	0.8612
4	0.6300	0.9912	0.9912	1.0000	0.7958
5	0.6191	0.9808	0.9808	1.0000	0.8972
6	0.5648	0.8912	0.8912	1.0000	0.9173
7	0.4508	0.7370	0.7370	1.0000	0.8977
8	0.3877	0.6529	0.6529	0.9863	0.8458
9	0.6056	0.9572	0.9572	1.0000	0.8541
10	0.5901	0.9380	0.9380	1.0000	0.8895
11	0.5887	0.9343	0.9343	1.0000	0.8501
12	0.5975	0.9407	0.9407	1.0000	0.8563
13	0.5586	0.8870	0.8870	1.0000	0.8946
14	0.4805	0.7753	0.7753	1.0000	0.7917
15	0.5990	0.9410	0.9410	1.0000	0.8285
16	0.5679	0.8795	0.8795	1.0000	0.8066
17	0.6189	0.9680	0.9680	1.0000	0.8710
18	0.5682	0.9108	0.9108	1.0000	0.8475
19	0.5927	0.9446	0.9446	1.0000	0.8641
20	0.5133	0.8313	0.8313	1.0000	0.8604
21	0.5601	0.8825	0.8825	1.0000	0.7934
22	0.6383	1.0000	1.0000	1.0000	0.8558
23	0.6793	1.0000	1.0000	1.0000	0.8492
24	0.4930	0.7961	0.7961	1.0000	0.8749
25	0.5948	0.9482	0.9482	1.0000	0.8788
26	0.5037	0.8072	0.8072	1.0000	0.8471
27	0.3886	0.6557	0.6557	0.9917	0.9054

[3] Similarly, the values of ESP_2 are available from the author upon request.

4. CONCLUSIONS

This paper develops a performance forecasting model by integrating MDNDEA and fuzzy piecewise auto-regression. The advantages of the proposed approach are: First, the operational efficiency is evaluated under the consideration with the internal structure of operational process. Second CIE is calculated by the relative operational efficiencies in the two adjacent periods to avoid the limitation of efficiency data. Third, CIE is decomposed into TEC and DEC to excavate the effects of carry-over items. Finally, the interval estimation can be used to forecast efficiency and explore the trend of efficiency change. We conducted an empirical study using real data from 27 banks in Taiwan from 2006 to 2012 to demonstrate the validation of the proposed approach. The results indicate that the proposed approach for performance prediction has high accuracy.

However, there are some limitations in this paper. First, the effect of network structure is ignore, when CIE is decomposed in this paper. Second, the technical change could affect the performance movement. However, our performance prediction does not consider the effect of technical change. Future research can further investigate these issues.

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