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Learning about one's own type: a search model with two-sided uncertainty

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Abstract

This paper examines the movement of an individual's reservation level over time in a two-sided search model with two-sided imperfect self-knowledge, where agents are vertically heterogeneous and do not know their own types. Agents who do not know their own types update their beliefs about their own types through the offers or rejections they receive from others. The results in this paper show that an agent with imperfect self-knowledge revises his or her reservation level downward when the agent receives a rejection that has some information about his or her own type. In contrast, an agent with imperfect self-knowledge revises his or her reservation level upward when the agent receives an offer from an agent of the opposite sex who is of lower type than the reservation level. This upward revision of an agent's reservation level is due to the environment of two-sided imperfect self-knowledge.

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1 Introduction

The potential sources of declining reservation wages have received much attention in the search literature (see Burdett and Vishwanath (1988)). In particular, the sequence of reservation wage, which completely describes the behavior of agents when search is a sequential process, declines with the duration of search (see Gronau (1971), Salop (1973), Sant (1977), and Burdett and Vishwanath (1988)). However, in empirical studies, the effect of search duration on reservation wage is yet to be well-understood. It is generally ambiguous as to whether or not declining reservation wages are monotonic and it is difficult to measure the effect of search duration on reservation wage.[1] Several empirical studies show that this decline is monotonic only when certain conditions on the variables hold in the model (Kiefer and Neumann (1981), Lancaster (1985), Addison, Centeno, and Portugal (2004), and Brown and Taylor (2009)).

This paper examines the movement of an individual's reservation level over time in a two-sided search model with two-sided imperfect self-knowledge. We construct a model in which searchers of two types (high-type and low-type) do not know their own types, but know others' types.[2] They then update their beliefs about their own types when they receive offers or rejections from others. For example, workers in search of an employer are evaluated by employers on their abilities when they meet. When a worker is young in terms of experience, his or her self-assessment is based on limited experience. On the other hand, employers may have considerable experiences in evaluating workers. At this time, when a young worker observes an offer or a rejection from an employer, he or she learns something about his or her own type.[3] The key feature of this study is that others have better information about agents' types than the agents themselves. Similarly, in the search for a marriage partner, a single agent is evaluated with regard to his or her marital charms by a member of the opposite sex when they meet. When an agent is young, his or her self-assessment is based on limited experience, such as academic achievement and family background. However, because marital charm is composed of various elements, an individual of the opposite sex may have better assessments of agents' charm than the agents themselves.[4] Hence, when an agent observes an offer or a rejection from a member of the opposite sex, he or she infers something about his or her own type.

Our study is close in spirit to Burdett and Vishwanath (1988), who show that when workers learn the unknown wage distribution, the reservation wage of an unemployed worker declines with his or her unemployment spell. In their model, a high offer results in the worker

[1] This is because both variables are determined simultaneously if reservation wages are flexible.

[2] Though we construct the model with two types of agents to simplify the analysis, the qualitative features of our main results will not change under a model with n types of agents. If we consider a model in which there are n types of agents, the learning process becomes very complex. This issue is my next research work.

[3] Of course, when an experienced worker searches for a new job that is very similar to his or her previous job, he or she may have a more accurate self-view of his or her ability than employers. However, such situations are not considered in this paper.

[4] Marital charm is defined by various elements, including quality, attraction, intelligence, height, age, education, income, position at work, social status, and family background, in much of the literature regarding marriage.

getting employed, whereas an offer much lower than expected leads the worker to perceive the jobs available to him or her as jobs offering low wages and then the worker revises his or her reservation wage downward. Unlike their model, ours is a two-sided search model and agents know the type distribution but do not know their own types. Specifically, in two-sided search with imperfect self-knowledge, receiving an offer is likely to lead to an increase in the reservation level. In fact, this paper shows that an agent with imperfect self-knowledge revises his or her reservation level upward when he or she receives an offer lower than his or her reservation level.

We consider the basic framework of Burdett and Coles (1997), which is a two-sided search model with complete information. Though our model focuses on marriage, one could apply the ideas and techniques of the present paper to other two-sided search frameworks, such as the labor market, the housing market, and other markets in which heterogeneous buyers and sellers search for the right trading partner.[5] Using the marriage market interpretation, the model is described as follows. Single agents are vertically heterogeneous, i.e., there exists a ranking of marital charm (types). Single men or women enter the market in order to look for a marital partner. When a man and a woman meet, an opponent's type can be recognized. The agent's optimal search strategy has the reservation-level property, i.e., he or she continues searching until he or she meets a member of the opposite sex who is at least as good as the predetermined threshold, called the "reservation level," which depends on the agent's search cost and the type distribution of agents. If a man and a woman meet and both agents propose, they marry and leave the market. If at least one of the two decides not to propose, they separate and continue to search for another partner. Given these settings, the marriage pattern (i.e., who marries whom) in the market is determined. This marriage pattern becomes a kind of positive assortative matching.[6]

The results in this paper show that an agent with imperfect self-knowledge revises his or her reservation level downward when the agent receives a rejection that has some information about his or her own type. In contrast, an agent with imperfect self-knowledge revises his or her reservation level upward when the agent receives an offer from an agent of the opposite sex who is of lower type than the reservation level. These results imply that a series of rejections gradually reduces the reservation level of an agent through the duration of search.

Moreover, in a model with two-sided imperfect self-knowledge, the following two factors affect the reservation utility level of an agent. The first factor is the *assigning of the probability of an agent's own type*. Because an agent with imperfect self-knowledge assigns probabilities

[5]In this paper, we assume the *non-transferable utility:* there is no bargaining for the division of the total utility. In the labor market, utility is generally transferable. However, for example, when the worker is enthusiastic about a job because of its location, or the employer is attracted by the worker because of his or her personality, their utilities can be considered to be non-transferable. Furthermore if the worker offers to work for a reduced wage, this wage might be restricted to above some lower bound determined outside of the match, like a legislated minimum wage or an industry-wide union relationship (see Burdett and Wright (1998)). In this way, when wages and all other terms of the relationship are fixed in advance and there is nothing for the pair to negotiate after they meet, their utilities can be viewed as non-transferable utility.

[6]Positive assortative matching is said to hold if the characteristics (types and marital charm) of those who match are positively correlated. Becker (1973) found strong empirical evidence of a positive correlation between the characteristics of partners.

to his or her own possible type, the reservation level of a high-type agent is always lower than his or her reservation level when he or she has perfect knowledge about his or her own type.

The second factor is the *existence of others of the opposite sex with imperfect self-knowledge*. This factor has the following two effects. The first is the effect of the *chance of learning*. If the share of agents of the opposite sex with imperfect self-knowledge increases, an agent's chance of learning about his or her own type increases. The increase in the chance of learning raises the opportunity of receiving offers for a high-type agent with imperfect self-knowledge, which raises his or her reservation level. In contrast, the increase in the chance of learning raises the opportunity of receiving rejections for a low-type agent with imperfect self-knowledge. This reduces his or her reservation level. The second effect of the existing of others of the opposite sex with imperfect self-knowledge is the effect of a *few number of others of the opposite sex with perfect self-knowledge*. The share of agents of the opposite sex with imperfect self-knowledge increases and accordingly the share of agents of the opposite sex with *perfect* self-knowledge decreases. When low-type agents with imperfect self-knowledge reject low-type agents, the sparse existence of low-type agents of the opposite sex with perfect self-knowledge delays the marriages of low-type agents who learned their own types and then accept low-type agents. As a result, the reservation level of an agent with imperfect self-knowledge is raised to reject low-type agents because he or she prefers to learn about his or her own type rather than to accept a low-type agent before learning.

In particular, the upward revision of an agent's reservation level caused by a received offer occurs because of two-sided imperfect self-knowledge. Under one-sided imperfect self-knowledge, the upward revision of an agent's reservation level does not occur. Maruyama (2010) investigates the movement of an agent's reservation over time in a two-sided search model with one-sided imperfect self-knowledge. In Maruyama (2010), men know their own types but women do not. If a man proposes to a woman and then she revises her reservation level upward to reject his (actual) type by her learning, he cannot marry her. At this time, the man can predict that his offer leads him to be rejected by her because he knows his own type. Consequently, the man chooses his strategy so as to not change the reservation level of the woman. In contrast, under two-sided imperfect self-knowledge (i.e., both men and women do not know their own types), even if a man can predict that upon proposing to a woman, she may raise her reservation level to reject his (actual) type, he chooses to propose to her. This is because he cannot know whether he is accepted or rejected by her because of his imperfect self-knowledge. These results are also supported when sex is reversed.

Moreover, this paper shows that the opportunity of upward revision of an agent's reservation level caused by a received offer depends on the knowledge about the opponent's belief. When a man (woman) cannot recognize the opponent's belief about her (his) own type when they meet, the opportunity of revision of his (her) reservation level upward decreases relative to the case in which he (she) can recognize the opponent's belief. This is because the lack of knowledge about the opponent's belief reduces the agent's chance of learning. Therefore, the lack of knowledge about the opponent's belief accelerates the decline in reservation level through the duration of search under two-sided imperfect self-knowledge.

This paper also shows that in a steady state, the learning of low-type agents reduces the matching rate of high-type agents to high-type agents of the opposite sex, although the learning of high-type agents does not affect their own matching rate. The learning of low-type agents delays their own time until marriage relative to the case of perfect self-knowledge. Because they match relatively slowly, this steady state implies that the share of low-type agents in the market is greater than that in the case of perfect self-knowledge. In other words, the share of high-type agents is smaller than that in the case of perfect self-knowledge. This reduces the matching rate of high-type agents to high-type agents of the opposite sex.

Related literature

Our paper relates to a large number of works that study individual search behavior with incomplete information (e.g., Rothchild (1974), Morgan (1985), Burdett and Vishwanath (1988), Bikhechandani and Sharma (1996), Adam (2001), and Dubra (2004)). Most previous papers focus on the uncertainty about market condition in terms of the shape of the wage distribution.[7]

The idea of imperfect self-knowledge with learning is termed "looking-glass self" in sociology and social psychology.[8] Although there is much literature on the "looking-glass self" in the field of sociology and social psychology for the development of the self, the topic has received little attention in economics. However, recent works have introduced the idea of imperfect self-knowledge in the principal-agent model (e.g., Bénabou and Tirole (2003)).

Another strand of the literature, exemplified by the works of Zábojník (2003), Köszegi (2006), and Andolfatto, Morgan, and Myers (2009), models the biased self-esteem generated by agents' beliefs about their own abilities. Their models are mainly concerned with explaining how people may rationally become overconfident. However, agents in this paper have correct beliefs in the sense that they process information rationally, have prior beliefs that are consistent with the distribution in the market, and signals do not have noise.

In search literature of imperfect self-knowledge with learning, there are few studies that have given attention to the imperfect self-knowledge. In Gonzalez and Shi (2010), agents learn their own job-finding abilities by observing offers or rejections from firms. In the directed search model with two types of agents, they show that learning from search can induce the desired wages (the wage in the chosen submarket) and reservation wages to decline with unemployment duration. In particular, the value function of an unemployed worker strictly increases in the worker's belief in their model because a worker's (or a firm's) search decision is to choose the submarket to search. Hence, the reservation wage strictly decreases

[7] In search literature with Bayesian learning, agents initially do not know the types of opponents, but later, they learn them (e.g., Jovanovic (1979), MacDonald (1982), Chade (2006), and Anderson and Smith (2010)). There also exists search literature with Bayesian learning where agents are assumed to learn the unknown offer distribution (e.g., Rothchild (1974), McLennan (1984), Burdett and Vishwanath (1988), Adam (2001), and Dubra (2004)).

[8] The idea, attributed to Cooley (1902), is that people form their self-views by observing how others treat them. That is, others are significant as the "mirrors" that reflect images of the self.

over search spell as the worker's beliefs about his or her own ability become progressively worse. In contrast, our model is the random two-side search model with two-sided imperfect self-knowledge. An agent with imperfect self-knowledge decides the reservation utility by considering the composition of each (belief) type in the market and her future learning process fully. As a result, the value function is not monotonic with respect to agent's belief.

This paper is organized as follows. Section 2 is a description of the basic framework of our analysis. In Section 3, first, we consider *partial rational expectations equilibria with perfect self-knowledge* as a benchmark case. Section 4 examines the case of imperfect self-knowledge with knowledge about an opponent's belief. More concretely, we investigate the effect of learning on the reservation level and then the effect of duration of search on the reservation level. In Section 5, we consider steady state equilibria. In Section 6, we consider the case of lack of knowledge about the opponent's belief. Section 7 concludes.

2 Basic Framework

In this section, we present a basic framework of our analysis.

Let us assume that there are a large and equal number of men and women in a marriage market. Let N denote the participating men or women in this market. An agent in the market wishes to marry a member of the opposite sex.

Finding a marriage partner always involves a time cost. It is difficult for agents to meet someone of the opposite sex in the market. Let α denote the rate at which a single individual contacts a member of the opposite sex, where α is the parameter of the Poisson process. As α is assumed to not depend on N, we have what is termed constant returns in the matching function.

It is assumed that agents are ex-ante heterogeneous and all agents have the same ranking about a potential partner in the marriage market. Let x_k be a real number that denotes the type (charm) of a k-type single man or woman in the market.

When both sexes meet, each agent can instantly recognize the opponent's type and then decide whether or not to propose. We assume that both agents submit their offers or rejections simultaneously in order to simplify our analysis. If at least one of the two decides not to propose, they return to the marriage market and search for another partner. If both agents propose, they marry and leave the marriage market permanently. We assume that, if a couple marries, he or she obtains a utility flow equal to the spouse's type per unit of time and vice versa. That is, utilities are *non-transferable*: there is no bargaining for the division of the total marital utility. Let us assume that people live forever and that there is no divorce.

Let $F_i(x), i = m, w$ denote the stationary distribution of (actual) types among men (m) or women (w) in the market. That is, $F_i(x, t) = F_i(x)$ for all x and all t, where t denotes the period. Let us assume that x_k is drawn from $F_i(x)$. Let $\underline{x}_i$ and $\bar{x}_i$ indicate the infimum and supremum of F_i, respectively, where $\underline{x}_i > 0$. Here, we assume that $F_m(x)$ and $F_w(x)$ are symmetric among men and women to simplify the analysis. All agents know $F_m(x)$ and $F_w(x)$.

3 Stationary Market Environment: Perfect Self-knowledge

In this section, we assume that all agents believe that the market can be characterized by the stationary distribution of type among men or women. First, we investigate "who marries whom" under perfect self-knowledge as a benchmark in this section.[9] In the later section, we study cases with *imperfect self-knowledge* (i.e., agents do not know their own types perfectly) and compare these cases with the benchmark cases to show the influence of two-sided imperfect self-knowledge on a market.

Given (F_m, F_w) and the strategies of single agents, the set of single agents of the opposite sex who will agree to marry a k-type agent is well-defined. Let $H_m(.|x_k)$ denote the distribution of type among men who will agree to marry a k-type woman. Further, the arrival rate of such proposals faced by a k-type woman, $\alpha_w(x_k)$, is also well defined.

Let $V_w(x_k)$ denote a k-type woman's expected discounted lifetime utility when single. Standard dynamic programming arguments imply that

$$V_w(x_k) = \tfrac{1}{1+rdt}\left[\alpha_w(x_k)\,dtE\left[\max\left\{\tfrac{\tilde{x}_k}{r}, V_w(x_k)\right\}|x_k\right] + (1-\alpha_w(x_k)\,dt)\,V_w(x_k)\right]$$

where given that an offer is made, $\tilde{x}_k$ has distribution $H_m(\tilde{x}_k|x_k)$. Manipulating and then letting $dt \to 0$ yields

$$rV_w(x_k) = \alpha_w(x_k)\left[E\left[\max\left\{\tfrac{\tilde{x}_k}{r}, V_w(x_k)\right\}|x_k\right] - V_w(x_k)\right].$$

The strategy that maximizes a single agent's expected discounted lifetime utility takes the form of a reservation match strategy—a k-type woman will marry a man on contact if and only if his type is at least as great as $R_w(x_k) \equiv rV_w(x_k)$.

As the situation is the same for men, the expected discounted lifetime utility of a single k-type man, $V_m(x_k)$, satisfies

$$rV_m(x_k) = \alpha_m(x_k)\left[E\left[\max\left\{\tfrac{\tilde{x}_k}{r}, V_m(x_k)\right\}|x_k\right] - V_m(x_k)\right]. \tag{1}$$

where $\tilde{x}_k$ has distribution $H_w(\tilde{x}_k|x_k)$. From this equation, we can obtain the reservation match strategy of a k-type man: $R_m(x_k) \equiv rV_m(x_k)$.

The equilibrium concept for this subsection is as follows.

Definition 1 *When all agents know their own types, an equilibrium is a* partial rational expectations equilibrium with perfect self-knowledge (PEP)*:*

(PEP-i) all agents maximize their expected discounted utilities given that they have correct expectations about the strategies of all other agents in the market.

Equilibrium strategies in this section are derived from condition (PEP-i). This "partial rational expectations equilibrium" is named by Burdett and Coles (1997), and here, "partial"

[9] We consider the basic framework of Burdett and Coles (1997).

means that it may not be necessarily that the inflow of agents in the market equals the outflow of agents.

In this paper, let us assume that there are two types of men or women according to their charm: high (H) and low (L).[10] A participant in the marriage market belongs to one of these types. Let x_k/r denote the (discounted) utility of marrying a k-type agent ($k = H, L$), where $r > 0$ is the discount rate. We assume that $x_H > x_L > 0$. That is, in any equilibrium, all agents would like to marry an H-type agent of the opposite sex. Both sexes are assumed to obtain zero utility flow while they are single.

Let λ denote the share of H-type men or women in the marriage market. Hence, $(1 - \lambda)$ denotes the share of L-type men or women in the marriage market.

In this paper, we restrict our attention to the two equilibria (PEPs) mentioned below in order to show the influence of two-sided imperfect self-knowledge on a marriage market.

Definition 2 *In* the elitist equilibrium*, high-type agents marry within their group, as do low-type agents.*

Definition 3 *In* the mixing equilibrium*, all types accept each other.*

In the elitist equilibrium, men and women of the same type marry. Therefore, we can consider that in this equilibrium, H-type agents who marry within their group form the first cluster of marriages, and L-type agents who marry within their group form the second cluster of marriages. In contrast, in the mixing equilibrium, all agents marry the first person of the opposite sex whom they meet. Hence, we can consider that agents who marry form one cluster of marriages in the mixing equilibrium.

We now define the following situations as benchmark cases: if all agents know their own types, a PEP is elitist (E-PEP) or mixing (M-PEP). The following proposition shows the sufficient conditions for an E-PEP and an M-PEP.

Proposition 1 *Let us assume that all agents recognize their own types. The economy is at an E-PEP if*

$$x_L < R^*(x_H) \equiv \frac{\alpha\lambda x_H}{\alpha\lambda + r}. \tag{2}$$

In contrast, if

$$x_L \geq R^*(x_H) \equiv \frac{\alpha\lambda x_H}{\alpha\lambda + r}. \tag{3}$$

the economy is at an M-PEP.

Proof. See, Appendix A. ■

Proposition 1 means that, given constant α, if the share of H-type agents of the opposite sex is large enough or if the difference between x_H and x_L is large enough ($\alpha\lambda > \frac{rx_L}{(x_H - x_L)}$), an H-type agent turns down an L-type opposite sex agent in the market. At this time, an

[10] Though we construct a model with two types of agents to simplify the analysis, the qualitative features of our results will not change under a model with n types of agents. In order to illustrate the main findings of this paper, one does not need n types of agents, as this complicates matters without adding intuition.

L-type agent always accepts an L-type opposite sex agent. (Otherwise, he or she cannot marry.) As a result, the economy is at an E-PEP.

Conversely, if there are sufficiently few H-type opposite sex agents or if $(x_H - x_L)$ is small enough ($\alpha\lambda \leq \frac{rx_L}{(x_H - x_L)}$), an H-type agent accepts an L-type opposite sex agent. Therefore, the economy is at an M-PEP. At this time, all agents obtain the same expected discounted lifetime utility: $V(x_L) = V(x_H) < \frac{x_L}{r}$.

If $r = 0$, then $x_L < R^*(x_H)$ holds. Therefore, the equilibrium is an E-PEP when $r = 0$.

In the next section, we introduce imperfect knowledge about agents' own types into the benchmark cases. To investigate the influence of two-sided imperfect self-knowledge on a market, in the following sections, we consider the case in which (2) or (3) holds when all agents know their own types.

4 Two-sided Imperfect Self-knowledge

In this subsection, we introduce imperfect self-knowledge into the benchmark cases. Let us assume that all agents understand the (actual) type distributions $F_m(x)$ and $F_w(x)$. However, no agent initially knows his or her own type upon entering the marriage market. An agent with imperfect self-knowledge may learn something about his or her actual type by observing the offers or rejections by agents of the opposite sex. Thus, an agent's belief about his or her own type depends on the agents of the opposite sex whom he or she met in the past.[11] Moreover, we assume that an agent can recognize an opponent's actual type and an opponent's belief when they meet in this section.[12] The case of lack of knowledge about the opponent's belief is investigated in Section 6.

For the explanation, let us suppose that a k-type woman ($k = H, L$) is at the start of period $t = \{0, 1, ...\}$ of single. Let Ξ denote a set of actual types of women and $b_k \in \Delta(\Xi)$ denote a belief of a k-type woman, where $\Delta(\Xi)$ is the set of probability distributions over Ξ. Let $b_k^t\left(b_k^0, h_k^t\right)$ denote a k-type woman's belief at the start of period t about her own type after history h_k^t given the prior belief $b_k^0 \in \Delta(\Xi_0)$, where $b_k^0\left(b_k^0, h_k^0\right) = b_k^0$ and Ξ_0 is a set of all actual types of women. In this paper, we assume that b_k^0 is the distribution of belief among those who have not updated their beliefs yet in the market.[13] Here, $b_0 \equiv b_k^0$ because we assume in the following that the distribution of belief $G_i(x,t)$, $i = m, w$ is common knowledge. Moreover, $h_k^t = \left(\left(\tilde{x}_k, a_m(x_k), \tilde{b}_k\right)^0, ..., \left(\tilde{x}_k, a_m(x_k), \tilde{b}_k\right)^{t-1}\right)$ is the k-type woman's history

[11] As a result, there are different kinds of men (women) with different beliefs even if they belong to the same actual type.

[12] If a male (female) agent can observe the belief of his female (male) opponent when they meet, he (she) can know her (his) action before he (she) observes her (his) action. This is because agents decide their strategies on the basis of their beliefs. Therefore, though we assume that agents submit their offers or rejections simultaneously in this study, the results obtained in this section are the same as those in the case of a sequential move in which a woman proposes to a man in the first move and he proposes or rejects her in the next move.

[13] Gonzalez and Shi (2009) assume that the initial prior expectation of ability for a new worker is calculated from the distribution of new workers over the levels of ability. However, because we assume exogenous inflow in the later section, the distribution of new single agents over the levels of charm may not equal the distribution of belief among those who have not updated their own types yet in the market.

up to, but not including period t. Here, $a_m(x_k) \in A = \{a, a^-\}$ is the action of a k-type man $(\tilde{x}_k^t)$ observed by the k-type woman (x_k) as the result of a search outcome in period $t(= 0, \ldots, t-1)$, where a indicates that the man proposed to the k-type woman and a^- indicates that he rejected her. Furthermore, $\tilde{b}_k$ is the belief of that k-type man. If a k-type woman observes $\left(\tilde{x}_k, a, \tilde{b}_k\right)^t$ (or $\left(\tilde{x}_k, a^-, \tilde{b}_k\right)^t$) at period t, she knows that a k-type man with the belief $\tilde{b}_k$ accepted (rejected) her and then she updates her belief about her own type. In Section 6, where we investigate the case of lack of knowledge about the opponent's belief, a k-type woman observes $(\tilde{x}_k, a)^t$ (or $(\tilde{x}_k, a^-)^t$) at period t and then she updates her belief about her own type.

In this paper, we use the term "action" to distinguish it from the reservation "strategy." Specifically, in our model with discrete types, even if an agent lowers his (her) reservation utility strategy, this does not guarantee that he (she) accepts a woman (man) whom he (she) has rejected previously. Therefore, in the following analysis, the statement that an agent changes his (her) *action* means that he (she) changes the type of women (men) whom he (she) is willing to accept.

Let $b_k^t\left(b_0, h_k^t\right)(x_k)$ denote a k-type woman's probability assigned to the particular type $x_k \in \Xi$. This probability is determined by Bayes' rule. The k-type woman's posterior belief $b_k^{t+1}\left(b_0, h_k^t | \left(\tilde{x}_k, a_m(x_k), \tilde{b}_k\right)^t\right)(x_k)$ after observing $\left(\tilde{x}_k, a_m(x_k), \tilde{b}_k\right)^t$ at period t given her current belief $b_k^t\left(b_0, h_k^t\right)$ is given by

$$b_k^{t+1}\left(b_0, h_k^t | \left(\tilde{x}_k, a_m(x_k), \tilde{b}_k\right)^t\right)(x_k) = \frac{b_k^t\left(b_0, h_k^t\right)(x_k) \Pr\left(\left(\tilde{x}_k, a_m(x_k), \tilde{b}_k\right)^t | x_k\right)}{\sum_{x_k \in \Xi} b_k^t\left(b_0, h_k^t\right)(x_k) \times \Pr\left(\left(\tilde{x}_k, a_m(x_k), \tilde{b}_k\right)^t | x_k\right)},$$

where $b_k^{t+1}\left(b_0, h_k^{t+1}\right)(x_k) = b_k^{t+1}\left(b_0, h_k^t | \left(\tilde{x}_k, a_m(x_k), \tilde{b}_k\right)^t\right)(x_k)$. In what follows, because b_0 is fixed in this paper, we omit explicitly writing b_0.

Because we consider only pure strategies when self-knowledge is perfect in the model presented here, $\Pr\left(\left(\tilde{x}_k, a_m(x_k), \tilde{b}_k\right)^t | x_k\right) = 0$ or 1 when a k-type woman observes $\left(\tilde{x}_k, a_m(x_k), \tilde{b}_k\right)^t$ given the strategies of men. Because $\Pr\left(\left(\tilde{x}_k, a_m(x_k), \tilde{b}_k\right)^t | x_k\right) = 0$ or 1 and an agent's optimal strategy has the reservation-level property, a k-type woman knows that her actual type does not belong to a type set. Let $\Phi_k^t\left(\left(\tilde{x}_k, a_m(x_k), \tilde{b}_k\right)^t, h_k^t\right)$ denote the impossible type set of a k-type woman, which she recognizes by observing $\left(\tilde{x}_k, a_m(x_k), \tilde{b}_k\right)^t$ given h_k^t. Then, we can define the set of the k-type woman's remaining possible types at period $t+1$ recursively. Let $\Xi^{t+1} = \Xi^t \backslash \Phi^t\left(\left(\tilde{x}_k, a_m(x_k), \tilde{b}_k\right)^t, h_k^t\right)$ denote the set of the k-type woman's remaining possible types at the start of period $t+1$, where $\Xi^0 = \Xi_0$. It is noteworthy that set Ξ^t can be interpreted as an information set in a sequential-move game. Therefore, from now on, we write $b_k^t\left(h_k^t\right)$ as $b_k\left(\Xi^t\right)$ for the sake of simplicity because the changes in belief over time can be represented by the elements of Ξ^t.

Let $V_w\left(b_k\left(\Xi^t\right)\right)$ denote the lifetime expected discounted utility of a k-type woman at the start of period t conditional on her belief $b_k\left(\Xi^t\right)$. Thus,

$$V_w\left(b_k\left(\Xi^t\right)\right)=\sum_{x_k\in\Xi^t}\left[b_k\left(\Xi^t\right)(x_k)\right]V_w\left(b_k\left(\Xi^t\right)\right)$$

$$=\frac{1}{1+rdt}\sum_{x_k\in\Xi^t}\left[b_k\left(\Xi^t\right)(x_k)\right]\left[\begin{array}{l}\left(1-\alpha_w\left(b_k\left(\Xi^t\right)\right)dt\right)V_w\left(b\left(\Xi^t\right)\right)\\+\alpha_w\left(b_k\left(\Xi^t\right)\right)dtE\left(\max\left\{\frac{\tilde{x}_k}{r},V_w\left(b_k\left(\Xi^t|\left(\tilde{x}_k,a_m\left(x_k\right),\tilde{b}_k\right)^t\right)\right)\right\}|x_k\right)\end{array}\right].$$

where $\tilde{x}_k$ has distribution $H_m\left(\tilde{x}_k|x_k\right)$. Manipulating and then letting $dt\to 0$ yields

$$rV_w\left(b_k\left(\Xi^t\right)\right)=\sum_{x_k\in\Xi_k}\left[b_k\left(\Xi^t\right)(x_k)\right]\alpha_w\left(b_k\left(\Xi^t\right)\right)\left[E\left(\max\left\{\frac{\tilde{x}_k}{r},V_w\left(b_k\left(\Xi^{t+1}\right)\right)\right\}|x_k\right)-V_w\left(b_k\left(\Xi^t\right)\right)\right].\tag{4}$$

where $b_k\left(\Xi^{t+1}\right)\equiv b_k\left(\Xi^t|\left(\tilde{x}_k,a_m\left(x_k\right),\tilde{b}_k\right)^t\right)$. The expectation operator E is taken with respect to the belief distribution of men $G_m\left(x,t\right)$.[14] Because $V_w\left(b_k\left(\Xi^t\right)\right)$ depends only on $b_k\left(\Xi^t\right)$, women in the same information set Ξ at period t face the same decision problem regardless of their actual types.

As the situation is the same for men, the lifetime expected discounted utility of a k-type man, $V_m\left(b_k\left(\Xi^t\right)\right)$, satisfies

$$rV_m\left(b_k\left(\Xi^t\right)\right)=\sum_{x_k\in\Xi_k}\left[b_k\left(\Xi^t\right)(x_k)\right]\alpha_m\left(b_k\left(\Xi^t\right)\right)\left[E\left(\max\left\{\frac{\tilde{x}_k}{r},V_m\left(b_k\left(\Xi^{t+1}\right)\right)\right\}|x_k\right)-V_m\left(b_k\left(\Xi^t\right)\right)\right],$$

where $\tilde{x}_k$ has distribution $H_w\left(\tilde{x}_k|x_k\right)$ and $b_k\left(\Xi^{t+1}\right)\equiv b_k\left(\Xi^t|\left(\tilde{x}_k,a_w\left(x_k\right),\tilde{b}_k\right)^t\right)$. The expectation operator E is taken with respect to the belief distribution of women $G_w\left(x,t\right)$.

In this paper, because we consider the two-types case, an agent learns about his or her own type at most one time and therefore there are three kinds of information sets: $\Xi_0\equiv\{x_H,x_L\}$, $\Xi_H\equiv\{x_H\}$ and $\Xi_L\equiv\{x_L\}$. In the following analysis, let us call a "k_l-type agent" and a "k-type agent" as an agent whose actual type is $k\in\{H,L\}$ with belief $b_k\left(\Xi_l\right),l\in\{0,H,L\}$, and an agent whose actual type is $k\in\{H,L\}$ with any belief, respectively. Moreover, we write b_l and $b_{l|\left(\tilde{x}_k,a_m(x_k),\tilde{b}_k\right)^t}$ instead of $b_k\left(\Xi_l\right)$ and $b_k\left(\Xi_l|\left(\tilde{x}_k,a_m\left(x_k\right),\tilde{b}_k\right)^t\right)$, respectively.[15]

Let $G_m\left(x\right)$ and $G_w\left(x\right)$ denote the stationary distribution of men's belief and that of women's belief, respectively. That is, $G_i\left(x,t\right)=G_i\left(x\right)$ for all x and all t. Let us assume that $G_m\left(x\right)$ and $G_w\left(x\right)$ are symmetric and that all agents know $G_m\left(x\right)$ and $G_w\left(x\right)$ (we later show that $G_m\left(x\right)$ and $G_w\left(x\right)$ depend on α and $F_i\left(x\right)$, which are common knowledge among all agents) and believe the market to be characterized by $\left(G_m,G_w\right)$. However, the particular assignment of beliefs to agents need not be known. If all agents know their own types, $F_i\left(x\right)=G_i\left(x\right),\ i=m,w$.

In this section, we introduce the next equilibrium concept for our model with imperfect

[14] If all agents of the opposite sex know their own types, the expectation operator E is taken with respect to the actual type distribution of agents of the opposite sex $F_i\left(x,t\right)$.

[15] When $t=0$, $b^0=b_0$.

self-knowledge. Though each agent's belief (state) changes over time, we first focus on the market in a stationary environment.

Definition 4 *In a* partial rational expectations equilibrium with imperfect self-knowledge (PEI)*:*

(PEI-i) agents' strategies satisfy sequential rationality; and

(PEI-ii) agents' beliefs on the sets of remaining possible types (information sets) along the equilibrium path are consistent with Bayesian updating given the equilibrium strategies.

In a PEI, it is not necessary that the outflow of the market equals the inflow.[16] In the following subsection, we consider the following two PEIs: one is a mixing PEI (M-PEI) and the other is an elitist PEI (E-PEI). An M-PEI and an E-PEI satisfy (PEI-i)–(PEI-ii). By characterizing an M-PEI and an E-PEI for any (G_m, G_w), Section 5 will identify those (G_m, G_w) which imply that the outflow distribution equals the inflow distribution, thereby identifying two possible steady state equilibria.

4.1 Matching strategies and PEIs

Agents with imperfect self-knowledge decide their optimal strategies given (G_m, G_w) and other agents' actions (who accepts (rejects) whom) in the market. The strategy (and then action) of each type is common knowledge among all agents, because all agents know $G_i(x)$, $i = m, w$.

4.1.1 An M-PEI

First, we investigate the optimal strategies of an agent with imperfect self-knowledge in an M-PEI.

In an M-PEI, any agent accepts an L-type agent of the opposite sex. Therefore, there is no learning in an M-PEI because an offer from an agent of the opposite sex does not have information about the agent who receives that offer. Therefore, there are H_0-type agents and L_0-type agents in an M-PEI. The stationary distribution (G_m, G_w) is given from the share of H_0-type agents and of L_0-type agents.

Agents decide their optimal strategies given (G_m, G_w) and other agents' actions. The following lemma applies to a k_0-type agent.

Lemma 1 *Let us assume that any agent accepts an L-type agent of the opposite sex. If*

$$x_L < (\geq) \; R^{MI}(b_0) = \tfrac{\alpha\lambda x_H}{(r+\alpha\lambda)} = R^*(x_H),$$

a k_0-type agent rejects (accepts) an L-type agent of the opposite sex.

[16] When the outflow of the market equals the inflow, a PEI becomes a steady state equilibrium, which we consider in Section 5.

Proof. See, Appendix A. ■

This lemma implies that when any agent accepts an L-type agent of the opposite sex, the decision of a k_0-type agent is the same as that of an H-type agent in the case of perfect self-knowledge. Hence, if $R^*(x_H) > (\leq) x_L$, k_0-type agents reject (accept) L-type agents of the opposite sex in the case of imperfect self-knowledge. Therefore, there is no M-PEI when $R^*(x_H) > x_L$. Hence, the existence of an M-PEI requires that $R^*(x_H) \leq x_L$ holds. We then immediately obtain a sufficient condition for an M-PEI.

Proposition 2 *Suppose that $x_L \geq R^*(x_H)$. At this time, the economy is at an M-PEI, in which all agents form a cluster of marriages.*

Proof. See, Appendix A. ■

The implications of Proposition 2 are as follows: Because there are few enough H-type men or H-type women ($R^{MI}(b_0) \leq x_L$), any agent accepts an L-type agent of the opposite sex.

4.1.2 An E-PEI

Next, let us consider an E-PEI, in which k_0-type agents reject L-type agents of the opposite sex, H_H-type agents reject L-type agents of the opposite sex, and L_L-type agents accept L-type agents of the opposite sex.[17] At this time, the learning processes of men (women) are as in Figure 1. The outline box for each type in Figure 1 represents the share of each type of men (women): λ or $(1-\lambda)$. For example, if an H_0-type woman meets an H-type man, then she learns that her actual type is H-type, leaving the market with him. After another H_0-type woman meets an L-type man, she becomes an H_H-type woman. As a result, there are two kinds of H-type women according to different beliefs. Here, let $\phi_i \in (0,1)$, $i = m, w$ denote the share of H_0-type agents ($i = m$, men; $i = w$ women) in all agents i whose actual types are H. Likewise, if an L_0-type man (woman) meets an H-type woman (man), then he (she) learns that he (she) is an L_L-type. Then, $\eta_i \in (0,1)$, $i = m, w$ denote the share of L_0-type agents ($i = m$, men; $i = w$ women) in all agents i whose actual types are L. Because $G_m(x)$ and $G_w(x)$ are symmetric, $\phi_m = \phi_w = \phi$ and $\eta_m = \eta_w = \eta$. From λ and these shares (ϕ, η), the stationary distribution (G_m, G_w) is obtained.

The optimal strategies of agents are obtained in the next lemma.

Lemma 2 *Let us assume that k_0-type agents reject L-type agents of the opposite sex, H_H-type agents reject L-type agents of the opposite sex, and L_L-type agents accept L-type agents of the opposite sex. If*

$$x_L < (\geq)\ R^{EI}(b_0) \equiv \frac{b_0(x_H)\alpha\lambda x_H(r+\alpha(1-\lambda)(1-\eta))(r+\alpha\lambda+\alpha\eta(1-\lambda))}{(r+\alpha\lambda)(b_0(x_H)(r+\alpha(1-\lambda)(1-\eta))(r+\alpha\lambda+\alpha\eta(1-\lambda))+rb_0(x_L)(1-\eta)(r+\alpha))} < R^*(x_H),$$

[17] If k_0-type agents accept L-type agents of the opposite sex, an E-PEI does not occur because men and women of different types marry.

where $b_0(x_H) = \frac{\phi\lambda}{\phi\lambda+\eta(1-\lambda)}$ *and* $b_0(x_L) = \frac{\eta(1-\lambda)}{\phi\lambda+\eta(1-\lambda)}$*, then a* k_0*-type agent rejects (accepts) an* L*-type agent of the opposite sex. If*

$$x_L < (\geq)\, R^{EI}(b_L) = \frac{\alpha(1-\eta)(1-\lambda)x_L}{r+\alpha(1-\eta)(1-\lambda)} < R^*(x_L),$$

an L_L*-type agent rejects (accepts) an* L*-type agent of the opposite sex. Moreover,*

$$R^{EI}(b_H) = \frac{\alpha\lambda x_H}{r+\alpha\lambda} = R^*(x_H).$$

Proof. See, Appendix A. ■

Lemma 2 means that a k_0-type agent rejects (accepts) an L-type agent of the opposite sex if there are enough (few enough) H-type agents of the opposite sex or if $b_0(x_H)$ is sufficiently large (sufficiently small).

From Lemma 2, an agent with imperfect self-knowledge revises his or her reservation level downward when the agent receives a rejection that has some information about his or her own type. In contrast, an agent with imperfect self-knowledge revises his or her reservation level upward when the agent receives an offer from an agent of the opposite sex who is lower than his or her reservation level. These results imply that a series of rejections gradually reduces the reservation level of an agent through the duration of search.

From Lemma 2, the following two factors affect the reservation utility level of an agent.

The first factor is the *assigning of the probability of an agent's own type.* Because an H_0-type agent assigns probabilities to his or her own types, his or her reservation level is always lower than the reservation level when he or she has perfect knowledge about his or her own type.[18] In contrast, the reservation level of an L_0-type agent is lower or higher than his or her reservation level with perfect self-knowledge, depending on the parameter values.

The second factor is the *existence of others of the opposite sex with imperfect self-knowledge.* The existence of others of the opposite sex with imperfect self-knowledge has the following two effects.

The first is the effect of the *chance of learning.* The effect of the chance of learning on the reservation level of an agent with imperfect self knowledge is generally ambiguous. When the share of L_0-type agents of the opposite sex increases, $\alpha\eta(1-\lambda)$ in $R^{EI}(b_0)$ increases. This increases the chance of learning of an H_0-type agent, who learns that he or she is of type H_H by an offer from an L_0-type agent of the opposite sex. As a result, the value of a single k_0-type agent who rejects an L-type agent $(V^r(b_0))$ increases and therefore the reservation level of a k_0-type agent increases. The increase in the share of L_0-type agents of the opposite sex also increases the chance of learning of an L_0-type agent, who learns that he or she is of type L_L by a rejection from an L_0-type agent of the opposite sex. This increase in the chance of learning of an L_0-type agent decreases the reservation level of a k_0-type agent. In our model with two types of agents, the increase in $R^{EI}(b_0)$ because of an H_0-type agent's

[18] The share of H_0-type agents ϕ_i affects the reservation utility level R_i, whereas the share of H_0-type agents of the opposite sex ϕ_j does not affect R_i $(i, j = m, w,\ i \neq j)$. This is because H-type agents of the opposite sex reject L-type agents regardless of their beliefs at an E-PEI.

learning is greater than the decrease in $R^{EI}(b_0)$ because of an L_0-type agent's learning in absolute value.

The second effect of the existing of others of the opposite sex with imperfect self-knowledge is the effect of a *few number of others of the opposite sex with perfect self-knowledge*. This effect raises the reservation level of an agent with imperfect self-knowledge. If the share of L_0-type agents of the opposite sex increases, the share of L_L-type agents of the opposite sex $(1-\eta)$ decreases. This delays the marriages of L_L-type agents, when a k_0-type agent rejects L-type agents. Hence, the value of a match to an L_L-type agent of the opposite sex after learning $(V(b_L))$ decreases. Furthermore, when a k_0-type agent accepts L-type agents, the value of a match to an L_L-type agent of the opposite sex before learning $(V^a(b_0))$ also decreases because there are few L_L-type agents of the opposite sex in the market. As a result, the reservation utility level $R^{EI}(b_0)$ increases. In other words, a k_0-type agent prefers to learn about his or her own type rather than to accept an L-type agent before learning when there are few L_L-type agents of the opposite sex. It is noteworthy that the share of H_0-type agents of the opposite sex ϕ_j does not affect the reservation level of a k_0-type agent i. This is because H-type agents of the opposite sex reject L-type agents regardless of their beliefs at an E-PEI.

From these two effects, the existence of others of the opposite sex with imperfect self-knowledge raises the reservation level of a k_0-type agent in our model with two types of agents.

The upward revision of an agent's reservation level caused by a received offer occurs because of two-sided imperfect self-knowledge. Under one-sided imperfect self-knowledge, the upward revision of an agent's reservation level does not occur. In Maruyama (2010), men know their own types but women do not (one-sided imperfect self-knowledge). In Maruyama (2010), if a man proposes to a woman and then she revises her reservation level upward to reject his (actual) type by her learning, he cannot marry her. At this time, the man can predict that his offer leads him to be rejected by her because he knows his own type. Consequently, the man chooses his strategy so as not to change the reservation level of the woman. In contrast, under two-sided imperfect self-knowledge, even if a man can predict that upon proposing to a woman, she may raise her reservation level and may reject his (actual) type, he chooses to propose to her. This is because he cannot know whether he is accepted or rejected by her because of his imperfect self-knowledge. Therefore, the upward revision of an agent's reservation level caused by a received offer occurs. (These results are also supported when sex is reversed.)

There is no E-PEI when $R^*(x_H) \leq x_L$ from $R^*(x_H) > R^{EI}(b_0)$ in Lemma 2. Hence, the existence of an E-PEI requires that $R^*(x_H) > x_L$ holds. We then immediately obtain sufficient conditions for an E-PEI.

Proposition 3 *Suppose that $x_L < R^*(x_H)$. If $x_L < R^{EI}(b_0)$, the economy is at an E-PEI, in which men and women of the same type marry.*

Proof. See, Appendix A. ■

The implications of Proposition 3 are as follows. If there are enough H-type agents or if there are enough L_0-type agents of the opposite sex ($x_L < R^{EI}(b_0)$), a k_0-type ($k = H, L$) or an H_H-type agent rejects an L-type agent of the opposite sex. Because an L_0-type agent rejects an L-type agent of the opposite sex, he or she becomes an L_L-type agent sooner or later because of rejection from an H-type agent of the opposite sex. Then, an L_L-type agent accepts an L-type agent of the opposite sex. As a result, an E-PEI occurs.

The first cluster of marriages is not influenced by agents who are unaware of their own types: the learning of L_0-type agents in the E-PEI delays their own time until marriage relative to that in the E-PEP, whereas the learning of H_0-type agents in the E-PEI does not affect their own time until marriage. This is because when an H-type agent meets an H-type agent of the opposite sex, they always marry regardless of their beliefs in the E-PEI.

5 Steady State Equilibria

In a PEP and a PEI, it is not necessary that the outflow of the market equals the inflow. When the outflow of the market equals the inflow, a PEP and a PEI become a steady state equilibrium.

A PEP or a PEI implies that given any steady state values $((G_m, G_w), N)$, we can know who will marry whom and can compute the exit rates of H-type and L-type agents. We now turn to the problem of steady state equilibria to determine G_w, G_m, and N where the corresponding PEP or PEI matching strategies imply a steady sate.

We assume that there are exogenous inflows—the inflow of single men (and women) is $g\Delta$ per interval Δ, where π is the share of single agents of H type.[19]

5.1 Case of perfect self-knowledge

In the case of perfect self-knowledge, $G_i = F_i\,(i = m, w)$. Therefore, the results in this subsection are the same as in the two-types case of Burdett and Coles (1997). Then, we investigate the values of λ and N where the corresponding M-PEP or E-PEP matching strategies imply a steady sate. The equilibrium concept for this subsection is as follows.

Definition 5 *When all agents know their own types, an equilibrium is a* steady state equilibrium with perfect self-knowledge (SEP)*:*

(PEP-i) all agents maximize their expected discounted utility given that they have correct expectations about the strategies of all other agents in the market; and

(SEP-i) the inflow and outflow of each type are balanced.

[19] Burdett and Coles (1999) give four typical assumptions of "inflow" in search literature. To investigate the influence of imperfect self-knowledge compared to that of the case of perfect self-knowledge, we assume *exogenous inflows*, as in Burdett and Coles (1997, 1999). This assumption is more reasonable than the *cloning assumption*, in which if a pair marries and leaves the market, two identical agents enter the market at once (see Burdett and Coles (1999)).

Condition (SEP-i) requires finding a steady state number and distribution of types in the market so that the corresponding equilibrium strategies defined in condition (PEP-i) generate an exit flow for each type equal to the inflow of that type.

In a mixing SEP (M-SEP),

$$\begin{aligned} \pi g &= \alpha \lambda N, \\ (1-\pi) g &= \alpha (1-\lambda) N, \end{aligned}$$

hold. This implies that $N = g/\alpha$ and

$$\lambda = \pi. \tag{5}$$

This is consistent with an M-PEP if and only if $\frac{\pi\alpha}{(r+\pi\alpha)} x_H \leq x_L$.

In contrast, in an elitist SEP (E-SEP),

$$\begin{aligned} \pi g &= \alpha \lambda^2 N, \\ (1-\pi) g &= \alpha (1-\lambda)^2 N. \end{aligned}$$

This implies that the steady state share of H-type, which is denoted by $\tilde{\lambda}(\pi)$, is

$$\frac{\pi}{(1-\pi)} = \frac{\tilde{\lambda}(\pi)^2}{\left(1-\tilde{\lambda}(\pi)\right)^2},$$

where $\tilde{\lambda}(0) = 0$. Moreover, we obtain $\tilde{N} = \frac{\pi g}{\alpha \tilde{\lambda}^2} = \frac{\pi g}{\alpha \left(\frac{1}{2\pi-1}\left(\pi - \sqrt{-\pi(\pi-1)}\right)\right)^2}$. The steady state share of H-type is strictly increasing in π where there exists a threshold value $\tilde{\pi} = 0.5$ such that if $\pi < (>) \tilde{\pi}$, $\tilde{\lambda}(\pi) > (<) \pi$. The intuition is that if π is small and the equilibrium is elitist, then the exit rate of H-type agents is less than the that of L-type agents. This implies that the number of H-type agents builds up relative to the number of L-type agents, and in a steady state, $\tilde{\lambda}(\pi) > \pi$.[20] This is also consistent with an E-SEP if and only if $\frac{\tilde{\lambda}(\pi)\alpha}{\left(r+\tilde{\lambda}(\pi)\alpha\right)} x_H > x_L$.

Moreover, Burdett and Coles (1997) show that there exist parameter values where both an E-SEP and an M-SEP exist.

Proposition 4 *(Burdett and Coles (1997)) Both an E-SEP and an M-SEP exist if and only if* $0 < \pi < \tilde{\pi} = 0.5$ *and* $\frac{\pi\alpha}{(r+\pi\alpha)} x_H \leq x_L < \frac{\tilde{\lambda}(\pi)\alpha}{\left(r+\tilde{\lambda}(\pi)\alpha\right)} x_H$*, where* $\tilde{\lambda}(\pi) > \pi$.

Proof. Omitted. ■

The intuition of multiple equilibria is as follows. In an M-SEP, the share of H-type agents is π. However, in an E-SEP, the share of H-type agents is greater than π (but less than $\tilde{\pi} = 0.5$). At the relevant parameters this higher share of H-type agents justifies them being more selective. In contrast, at an E-SEP when $\pi > \tilde{\pi} = 0.5$, the share of H-type agents is lower than π. Therefore, the share of H-type agents cannot support an M-SEP at the same time.

[20] However, if $\pi < (\geq) \tilde{\pi}$, $\tilde{\lambda}(\pi) < (\geq) \left(1 - \tilde{\lambda}(\pi)\right)$.

The welfare implications of the E-SEP and the M-SEP are as follows. Let us consider the case in which $\pi < \tilde{\pi} = 0.5$ and $\frac{\pi\alpha}{(r+\pi\alpha)} x_H \leq x_L < \frac{\tilde{\lambda}(\pi)\alpha}{\left(r+\tilde{\lambda}(\pi)\alpha\right)} x_H$. When an H-type agent accepts an L-type agent, he or she matches quickly relative to the E-SEP. At this time, a steady state implies that H-type agents are relatively few in number. Therefore, H-type agents prefer to match with L-type agents rather than to continue searching for H-type agents. Namely, an H-type agent makes other H-type agents worse off by accepting an L-type agent. Conversely, if an H-type agent rejects an L-type agent, this behavior makes other H-type agents better off. As a result, the E-SEP and the M-SEP are not Pareto rankable: H-type agents prefer the E-SEP and L-type agents prefer the M-SEP.

5.2 Case of imperfect self-knowledge

In this subsection, we investigate the values of λ, ϕ, η, and N where the corresponding matching strategies in an M-PEI or an E-PEI imply a steady sate. The equilibrium concept for this subsection is as follows.

Definition 6 *In a* steady state equilibrium with imperfect self-knowledge (SEI)*:*

(PEI-i); agents' strategies satisfy sequential rationality;

(PEI-ii); agents' beliefs on the sets of remaining possible types (information sets) along the equilibrium path are consistent with Bayesian updating given the equilibrium strategies; and,

(SEI-i) for each state $k_l (k = H, L,\ l{=}0, H, L)$, the inflow and outflow of agents are balanced.

(SEI-i) means that the exit flow for each state k_l equals the inflow of that state. As a result of (SEI-i), for each actual type k, the inflow and outflow of agents are balanced.

In a mixing SEI (M-SEI), the results are the same as in an M-SEP because there is no learning.

In contrast, in an elitist SEI (E-SEI), the steady state requires that

$$\pi g = \phi\lambda\left(\alpha\lambda + \alpha\eta\left(1-\lambda\right)\right)N = \left(\phi\lambda + \left(1-\phi\right)\lambda\right)\alpha\lambda N$$
$$\left(1-\pi\right)g = \eta\left(1-\lambda\right)\left(\alpha\lambda + \alpha\eta\left(1-\lambda\right)\right)N = \alpha\left(1-\eta\right)^2\left(1-\lambda\right)^2 N$$

hold in Figure 1. From these equations, the steady state share of H-type agents, which is denoted as $\hat{\lambda} \equiv \hat{\lambda}\left(\pi\right)$, is defined as

$$\frac{\pi}{(1-\pi)} = \frac{\hat{\lambda}^2}{\left(\hat{\lambda}-1\right)^2 (\hat{\eta}-1)^2} = \frac{\hat{\lambda}^2 \left(\hat{\lambda}-2\right)^2}{\left(\hat{\lambda}-1\right)^2}. \tag{6}$$

Moreover, we obtain,

$$\begin{aligned}\hat{N} &= \frac{\pi g}{\alpha \hat{\lambda}^2} = \frac{\pi g}{\alpha\left(1-\frac{1}{2}\sqrt{2}\sqrt{\frac{1}{\pi-1}\left(\pi+\sqrt{-\pi(3\pi-4)}-2\right)}\right)^2}, \\ \hat{\eta} &\equiv \hat{\eta}(\pi) = \frac{1-\hat{\lambda}}{2-\hat{\lambda}}, \qquad (7) \\ \hat{\phi} &\equiv \hat{\phi}(\pi) = \hat{\lambda}\left(2-\hat{\lambda}\right). \qquad (8)\end{aligned}$$

It is noteworthy that $\hat{\lambda}(0) = 0$, $\hat{\phi}(0) = 0$, and $\hat{\eta}(1) = 0$ and that $\hat{\lambda}$ and $\hat{\phi}$ are strictly increasing in π, and $\hat{\eta}$ is strictly decreasing in π. From (6), there exists a threshold value $\hat{\pi} = 0.245$ such that if $\pi < (>)\, \hat{\pi}$, $\hat{\lambda} > (<)\, \pi$. If π is small and the equilibrium is elitist, then the exit rate of H-type agents is less than that of L-type agents. This implies that the number of H-type agents builds up relative to the number of L-type agents, and in a steady state, $\hat{\lambda}(\pi) > \pi$. This is also consistent with an E-SEI if and only if $R^{ES}(b_0) \equiv \frac{\alpha\hat{\lambda}^3 x_H(\hat{\lambda}-2)\left((1-\hat{\lambda})\alpha^2+r(\hat{\lambda}-2)^2\alpha+(r\hat{\lambda}-2r)^2\right)}{\left(-\hat{\lambda}^2(\hat{\lambda}-1)(\hat{\lambda}-2)\alpha^2+r\left(2\hat{\lambda}-9\hat{\lambda}^2+12\hat{\lambda}^3-6\hat{\lambda}^4+\hat{\lambda}^5-1\right)(r+\alpha)\right)(r+\alpha\hat{\lambda})} > x_L$, where $2\hat{\lambda} - 9\hat{\lambda}^2 + 12\hat{\lambda}^3 - 6\hat{\lambda}^4 + \hat{\lambda}^5 - 1 < 0$ because of $\hat{\lambda} \in (0, 1)$.[21]

The learning of L_0-type agents in the E-SEI delays their own time until marriage relative to that in the E-SEP. Because L-type agents match relatively slowly, the steady state implies that the number of agents N and the share of L-type agents $(1-\hat{\lambda})$ in the E-SEI are greater than in the E-SEP. As a result, the share of H-type agents in the E-SEI $\hat{\lambda}$ is smaller than that in the E-SEP $\tilde{\lambda}(\pi)$, even if the number of agents in the E-SEI is larger than that in the E-SEP. In other words, the learning of L_0-type agents reduces the matching rate of H-type agents, although the learning of H_0-type agents does not affect their own matching rate. As a result, the threshold $\hat{\pi}$ is smaller than $\tilde{\pi}$ because $\hat{\lambda}(\pi) < \tilde{\lambda}(\pi)$.

An E-SEI and an M-SEI do not exist at the same time, unlike in the case of perfect self-knowledge. Finally, we show that there do not exist parameter values where both an E-SEI and an M-SEI exist.

Proposition 5 *If $x_L < R^{ES}(b_0)$ under any π, there exists an E-SEI. If $x_L \geq \frac{\pi\alpha}{(r+\pi\alpha)} x_H$ under any π, there exists an M-SEI. There do not exist parameter values where both an E-SEI and an M-SEI exist.*

Proof. See, Appendix A. ■

The intuition is as follows. In an M-SEI, the steady state share of H-type agents is π.

At an E-SEI when $\pi > \hat{\pi} = 0.245$, $\hat{\lambda} < \pi$. At an E-SEI, the probability that an agent's own type is H increases the reservation level of an agent because the share of H_0-type agents is greater than that of H_H-type agents ($\hat{\phi} > 1 - \hat{\phi}$). However, the share of H-type agents in an E-SEI is smaller than that in an M-SEI ($\hat{\lambda} < \pi$). This lower share of H-type agents ($\hat{\lambda} < \pi$) justifies them being less selective. As a result, the reservation level of a k_0-type agent is lowered enough to $R^{ES}(b_0) < \frac{\pi\alpha}{(r+\pi\alpha)} x_H$.

[21] Because $\frac{d}{d\lambda} R^{ES}(b_0) > 0$, $R^{ES}(b_0)$ is strictly increasing in λ.

At an E-SEI when $\pi < \hat{\pi} = 0.245$, the share of H-type agents is greater than that in the M-SEI π (but less than $\hat{\pi} = 0.245$). This higher share of H-type agents justifies them being more selective. However, at an E-SEI when $\pi < \hat{\pi} = 0.245$, there are fewer H-type agents than L-type agents ($\hat{\lambda} < 1 - \hat{\lambda}$) and the share of H_0-type agents is smaller than that of H_H-type agents in the market ($\hat{\phi} < 1 - \hat{\phi}$). Consequently, the reservation level of a k_0-type agent is lowered enough to $R^{ES}(b_0) < \frac{\pi\alpha}{(r+\pi\alpha)} x_H$.[22]

From these results, $R^{ES}(b_0) < \frac{\alpha\pi x_H}{\alpha\pi + r}$ under any π, and then, there do not exist parameter values where both an E-SEI and an M-SEI exist. If all agents know their own types and if $0 < \pi < \tilde{\pi} = 0.5$, multiple equilibria arise. In contrast, multiple equilibria cannot arise in that range when agents do not know their own types. The reservation level of an agent with imperfect self-knowledge is affected by the steady state distribution and the assigning of the probability of an agent's own type.[23] Consequently, when $0 < \pi < \hat{\pi} = 0.245$, the lowered reservation level does not generate multiple equilibria.[24]

The welfare implications of steady state equilibria are as follows. The welfare of each actual type in the M-SEI is the same as that in the M-SEP because the number of marriages of each actual type is the same. Similarly, the welfare of each actual type in the E-SEI is also the same as that in the E-SEP.[25]

6 Lack of Knowledge about the Opponent's belief

Though we assume that an agent can know the opponent's belief when they meet in the above sections, we relax this assumption in this section. That is, we assume that an agent can know the opponent's actual type but not the opponent's belief.

6.1 Matching strategies and PEIs

First, let us investigate the optimal strategies of agents and PEIs.

In a mixing PEI (M-PEI2), the results are the same as in an M-PEP because there is no learning.

In contrast, in an elitist PEI (E-PEI2), a k_0-type agent ($k = H, L$) rejects an L-type member of the opposite sex. At this time, the learning processes of men (women) are as in Figure 2. Specifically, even if an H_0-type woman (man) meets an L-type man (woman), she (he) remains an H_0-type woman (man) after the meeting. This is because from the offer

[22] At this time, $\hat{\phi}$ is low enough to satisfy (20) and (21) in a steady sate.

[23] When the share of L_0-type agents η increases, the existence of others with imperfect self-knowledge raises the reservation level of an agent with imperfect self-knowledge, from Lemma 2. However, because $\hat{\eta} < (1 - \hat{\eta})$ under any π since $\hat{\eta} \in (0, 0.5)$ from (7), the effect of the existence of L_0-type agents is small relative to the case in which $\hat{\eta} > (1 - \hat{\eta})$.

[24] Even if we assume the cloning assumption, there do not exist parameter values where both an E-SEI and an M-SEI exist. This is because the share of H-type agents in the E-SEI is the same as that in the M-SEI under the cloning assumption. Therefore, from Lemma 2, $R^{EI}(b_0)$ is always lower than the reservation level of a k_0-type agent in an M-SEI, which equals the reservation level of an H-type agent in an M-SEP $R^*(x_H)$.

[25] The rejections because of learning simply reduce the total discounted flow of utility.

which she (he) received, she (he) cannot know whether the L-type man (woman) whom she (he) met is of type L_0 or type L_L.

When an L_0-type agent meets an H_0-type agent of the opposite sex, she (he) becomes an L_L-type agent. Hence, there are two kinds of L-type women (men) according to different beliefs. Let $\eta_i \in (0,1)$ denote the share of L_0-type agents $i\,(=m,w)$ in all agents i whose actual types are L. From λ and the share $\eta = \eta_i = \eta_j$ $(i,j=m,w,\ i\neq j)$, the stationary distribution (G_m, G_w) is obtained.

In an E-PEI2, a k_0-type agent rejects an L-type agent of the opposite sex and L_L-type agents accept L-type agents of the opposite sex. Therefore, the optimal strategies of agents are as in the next lemma.

Lemma 3 *Let us assume that k_0-type agents reject L-type agents of the opposite sex and L_L-type agents accept L-type agents of the opposite sex. If*

$$x_L < (\geq)\ R^{EI2}(b_0) \equiv \frac{b_0(x_H)\alpha\lambda x_H(\alpha(1-\eta)(1-\lambda)+r)}{(\alpha(1-\eta)(1-\lambda)+r)(r+b_0(x_H)\alpha\lambda)+(\alpha\lambda+\alpha\eta(1-\lambda))b_0(x_L)r} < R^*(x_H),$$

where $b_0(x_H) = \frac{\lambda}{\lambda+\eta(1-\lambda)}$ and $b_0(x_L) = \frac{\eta(1-\lambda)}{\lambda+\eta(1-\lambda)}$, then k_0-type agents reject (accept) L-type agents of the opposite sex. If

$$x_L < (\geq) R^{EI2}(b_L) = \frac{\alpha(1-\eta)(1-\lambda)x_L}{r+\alpha(1-\eta)(1-\lambda)} < R^*(x_L),$$

L_L-type agents reject (accept) L-type agents of the opposite sex.

Proof. See, Appendix A. ■

Lemma 3 implies that a k_0-type agent rejects (accepts) an L-type agent of the opposite sex if there are enough (few enough) H-type agents of the opposite sex or if $b_0(x_H)$ is sufficiently large (sufficiently small).

From Lemma 3, the following two factors affect the reservation utility level of an agent.

The first factor is the *assigning of the probability of an agent's own type.* As in an E-PEI, because an H_0-type agent assigns probabilities to his or her own type, his or her reservation level is always lowered. In contrast, the reservation level of an L_0-type agent is lowered or raised depending on the parameter values.

The second factor is the *existence of others of the opposite sex with imperfect self-knowledge.* This factor has the following two effects.

The first effect is the effect of the *chance of learning.* When the share of L_0-type agents of the opposite sex increases, $\alpha\eta(1-\lambda)$ in $R^{EI2}(b_0)$ increases. That is, the chance of learning of k_0-type agents increases. However, this effect decreases the reservation level of a k_0-type agent unlike in the case in which agents can recognize opponents' beliefs. This is because lack of knowledge about the opponent's belief eliminates the chance of upward revision of the reservation level of a k_0-type agent.

The second effect is the effect of a *few number of others of the opposite sex with perfect self-knowledge.* If the share of L_0-type agents of the opposite sex increases, $(1-\eta)$ in $R^{EI2}(b_0)$

decreases. At this time, the value of match to an L_L-type agent of the opposite sex after learning $(V(b_L))$ and the value of a match to an L_L-type agent before learning $(V^a(b_0))$ decrease because there are few L_L-type agents of the opposite sex. As a result, the the reservation level of a k_0-type agent increases.

From these two effects, we get that an increase in the existence of L_0-type agents of the opposite sex decreases the reservation level of a k_0-type agent in our model with two types of agents.

Compared to the case in which an agent can recognize his or her opponent's belief, we can say that the lack of knowledge about the opponent's belief accelerates the decline in reservation level through the duration of search. When a man (woman) cannot recognize the opponent's belief about her (his) own type when they meet, the opportunity of revising his (her) reservation level upward decreases relative to the case in which he can recognize the opponent's belief. This is because the lack of knowledge about the opponent's belief reduces the agent's chance of learning.

From $R^*(x_H) > R^{EI2}(b_0)$ in Lemma 3, there is no E-PEI2 when $R^*(x_H) \leq x_L$. Hence, the existence of an E-PEI2 requires that $R^*(x_H) > x_L$ holds. We then immediately obtain sufficient conditions for an E-PEI2.

Proposition 6 *Suppose that $x_L < R^*(x_H)$. If $x_L < R^{EI2}(b_0)$, the economy is at an E-PEI2 in which men and women of the same type marry.*

Proof. See, Appendix A. ■

The implications of Proposition 6 are as follows: if there are enough H-type agents or there are enough L_0-type agents of the opposite sex $(x_L < R^{EI2}(b_0) < R^*(x_H))$, a k_0-type agent $(k = H, L)$ or an H_H-type agent reject an L-type agent of the opposite sex. Because an L_0-type agent rejects an L-type agent of the opposite sex, he or she becomes an L_L-type agent sooner or later because of rejection from an H-type agent of the opposite sex. Then, an L_L-type agent accepts an L-type agent of the opposite sex. As a result, an E-PEI2 occurs.

6.2 Steady state equilibria

In this subsection, we investigate the values of λ, η, and N, where the corresponding M-PEI2 or E-PEI2 matching strategies imply a steady sate.

In a mixing SEI (M-SEI2), the results are the same as in an M-SEP because there is no learning.

In contrast, in an elitist SEI (E-SEI2), the steady state requires that

$$\begin{aligned}
&\pi g = N\alpha\lambda^2 \\
&(1-\pi)\,g = \eta(1-\lambda)(\alpha\lambda + \alpha\eta(1-\lambda))\,N = \alpha(1-\eta)^2(1-\lambda)^2 N
\end{aligned}$$

hold in Figure 2. From these equations, the steady state share of H-type agents, which is denoted by $\bar{\lambda} \equiv \bar{\lambda}(\pi)$, is given as

$$\frac{\pi}{(1-\pi)} = \frac{\bar{\lambda}^2}{\left(\bar{\lambda}-1\right)^2(\bar{\eta}-1)^2} = \frac{\bar{\lambda}^2\left(\bar{\lambda}-2\right)^2}{\left(\bar{\lambda}-1\right)^2}. \tag{9}$$

It is noteworthy that $\bar{\lambda} = \hat{\lambda}$. Moreover, we obtain

$$\begin{aligned} \bar{N} &= \frac{\pi g}{\alpha\bar{\lambda}^2} = \frac{\pi g}{\alpha\left(\frac{1}{2(\pi-1)}\left(2\pi+\sqrt{2}\sqrt{\pi^{\frac{3}{2}}\sqrt{4-3\pi}-\sqrt{\pi}\sqrt{4-3\pi}-3\pi+\pi^2+2}-2\right)\right)^2}, \\ \bar{\eta} &\equiv \bar{\eta}(\pi) = \frac{1-\bar{\lambda}}{2-\bar{\lambda}}. \end{aligned} \tag{10}$$

It is noteworthy that $\bar{\lambda}(0) = 0$ and $\hat{\eta}(1) = 0$ and that $\bar{\lambda}$ is strictly increasing in π and $\bar{\eta}$ is strictly decreasing in π. Here, there exists a threshold value $\bar{\pi} = 0.245$ such that if $\pi < (>)\, \bar{\pi}$, $\bar{\lambda}(\pi) > (<)\, \pi$. This threshold value is the same as in the case in which an agent knows the opponent's belief. If π is small and the equilibrium is elitist, then the exit rate of H-type agents is less than the that of L-type agents. This implies that the number of H-type agents builds up relative to the number of L-type agents and in a steady state, $\bar{\lambda}(\pi) > \pi$. This is also consistent with an E-SEI2 if and only if $R^{ES2}(b_0) \equiv \frac{\left(\bar{\lambda}\left(2-\bar{\lambda}\right)\right)\left(\left(2-\bar{\lambda}\right)r+\alpha\left(1-\bar{\lambda}\right)\right)\alpha\bar{\lambda}x_H}{\left(\left(2-\bar{\lambda}\right)r+\alpha\left(1-\bar{\lambda}\right)\right)\left(\alpha\bar{\lambda}\left(\bar{\lambda}\left(2-\bar{\lambda}\right)\right)+r\right)+r\left(\bar{\lambda}-1\right)^2\alpha}$ $> x_L$.[26]

Similarly to the case with knowledge of opponents' beliefs, the learning of L_0-type agents reduces the matching rate of H-type agents, although the learning of H_0-type agents does not affect their own matching rate. The learning of L_0-type agents in the E-SEI2 delays their own time until marriage relative to that in the E-SEP. Because L-type agents match relative slowly, steady state implies that the number of agents N and the share of L-type agents $(1-\bar{\lambda})$ in the E-SEI2 are greater than in the E-SEP. As a result, the share of H-type agents in the E-SEI2 $\bar{\lambda}$ is smaller than in the E-SEP $\tilde{\lambda}(\pi)$.[27] Therefore, the threshold $\bar{\pi}$ is smaller than $\tilde{\pi}$.

An E-SEI2 and an M-SEI2 do not exist at the same time, unlike in the case of perfect self-knowledge.

Proposition 7 *Under any π, there do not exist parameter values where both an E-SEI2 and an M-SEI2 exist. If $x_L < R^{ES2}(b_0)$ under any π, there exists an E-SEI2. If $x_L \geq \frac{\pi\alpha}{(r+\pi\alpha)}x_H$ under any π, there exists an M-SEI2.*

Proof. See, Appendix A. ■

The intuition is as follows. In an M-SEI2, the share of H-type agents is π.

At an E-SEI2 when $\pi > \bar{\pi} = 0.245$, $\bar{\lambda}$ is lower than π. At this time, the probability that a k_0-type agent's actual type is H_0 can be larger than the probability that the agent's actual type is L_0 if $\pi > 0.333$.[28] Moreover, the share of H-type agents can also be larger

[26] The reservation level $R^{ES2}(b_0)$ is strictly increasing in λ, because $\frac{d}{d\lambda}R^{ES2}(b_0) = r\alpha\lambda\frac{x_H\left(2(\lambda-1)^2\alpha^2+2r(\lambda-1)(\lambda-3)\alpha-r^2(3\lambda-4)\right)}{\left(r^2-r\alpha\lambda^3+2r\alpha\lambda^2-r\alpha\lambda+r\alpha-\alpha^2\lambda^3+\alpha^2\lambda^2\right)^2} > 0$.

[27] It is noteworthy that the number of agents in the E-SEI2 is larger than in the E-SEP.

[28] More concretely, if $\pi < (\geq)\, 0.3333$, $\frac{\bar{\eta}\left(1-\bar{\lambda}\right)}{\bar{\lambda}+\bar{\eta}\left(1-\bar{\lambda}\right)} > (\leq)\, \frac{\bar{\lambda}}{\bar{\lambda}+\bar{\eta}\left(1-\bar{\lambda}\right)}$.

than that of L-type agents ($\lambda > 1-\lambda$) when $\pi > 0.692$.[29] Hence, the reservation level of a k_0-type agent is raised when $\pi > 0.692$. However, even under such a circumstance, the smaller share of H-type agents as compared to in an M-SEI2 ($\tilde{\lambda}(\pi) < \pi$) justifies agents being less selective. As a result, the reservation level of a k_0-type agent is lowered enough to $R^{ES2}(b_0) < \frac{\pi\alpha}{(r+\pi\alpha)} x_H$.

At an E-SEI2 when $\pi < \bar{\pi} = 0.245$, the share of H-type agents $\bar{\lambda}$ is greater than in an M-SEI2 π (but less than $\bar{\pi} = 0.245$). However, at an E-SEI2 when $\pi < \bar{\pi} = 0.245$, the probability that a k_0-type agent's actual type is L_0 is larger than the probability that the agent's actual type is H_0 ($\frac{\bar{\eta}(1-\bar{\lambda})}{\bar{\lambda}+\bar{\eta}(1-\bar{\lambda})} > \frac{\bar{\lambda}}{\bar{\lambda}+\bar{\eta}(1-\bar{\lambda})}$). Moreover, the share of H-type agents is smaller than that of L-type agents ($\lambda < 1-\lambda$). Consequently, although the higher share of H-type agents ($\tilde{\lambda}(\pi) > \pi$) justifies them being more selective, the reservation level of a k_0-type agent is lowered enough to $R^{ES2}(b_0) < \frac{\pi\alpha}{(r+\pi\alpha)} x_H$.

From the above results, $R^{ES2}(b_0) < \frac{\alpha\pi x_H}{\alpha\pi+r}$ under any π; then, there do not exist parameter values where both an E-SEI2 and an M-SEI2 exist unlike in the case of perfect self-knowledge.[30]

The welfare implications of steady state equilibria are as follows. Similar to the case in which agents can recognize their opponents' beliefs, the welfare of each actual type in the M-SEI2 is the same as that in the M-SEP because the number of marriages of each actual type is the same. Similarly, the welfare of each actual type in the E-SEI2 is also the same as that in the E-SEP.

7 Concluding Remarks

We analyze a two-sided search model in which we presume that agents initially do not know their own types and learn about their own types from the offers or rejections by agents of the opposite sex. With this learning process, the two-sided aspect of a search problem generates a significant interest. We show that an agent with imperfect self-knowledge revises his or her reservation level downward when the agent receives a rejection that has some information about his or her own type. In contrast, an agent with imperfect self-knowledge revises his or her reservation level upward when the agent receives an offer from a lower type agent of the opposite sex. These results imply that a series of rejections gradually reduces the reservation level of an agent through the duration of search. Specifically, this upward revision of an agent's reservation level is generated by the environment of two-sided imperfect self-knowledge. Moreover, the upward revision of an agent's reservation level is affected by the knowledge about his or her opponent's belief. When a male (female) agent cannot recognize his female (her male) opponent's belief about her (his) own type when they meet, the opportunity of revising his (her) reservation level upward decreases relative

[29] If $\pi < (\geq)$ 0.69231, $\lambda < (\geq)(1-\lambda)$.

[30] Even if we assume the cloning assumption, there do not exist parameter values where both an E-SEI2 and an M-SEI2 exist. This is because the share of H-type agents in the E-SEI2 is the same as that in the M-SEI2 under the cloning assumption. Therefore, from Lemma 3, $R^{ES2}(b_0)$ is always lower than the reservation level of a k_0-type agent in the M-SEI2, which equals the reservation level of an H-type agent in the M-SEP.

to the case in which he (she) can recognize the opponent's belief. This is because the lack of knowledge about the opponent's belief reduces the agent's chance of learning. Therefore, the lack of knowledge about the opponent's belief accelerates the decline in reservation level through the duration of search under two-sided uncertainty.

This paper also shows that when all agents know their own types under the assumption of exogenous inflow, multiple equilibria arise in some parameter ranges (see Burdett and Coles (1997)). However, the results in this study with imperfect self-knowledge show that multiple equilibria cannot arise in the ranges where multiple equilibria arise in the case of perfect self-knowledge. This is mainly due to the assigning of the probability of an agent's actual type.

We conclude with a discussion of some possible further extensions of this model. First, we assume two types of agents. If we consider a model in which there are n types of agents and many clusters of marriages, the learning process about one's own type will be very complex. This issue is my next research work. However, if there are n types of agents and two clusters of marriages are generated by a large enough α, our results still hold.

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Appendix A

Proof of Proposition 1: First, we consider the decision of an H-type agent. He (she) decides whether to accept or not a woman (a man) of the L-type. From (1), the expected discounted lifetime utility of a single H-type agent $V(x_H)$, becomes

$$rV(x_H) = \alpha\lambda_H\left(\tfrac{x_H}{r} - V(x_H)\right) + \alpha\lambda_L\left[\max\left(V(x_H), \tfrac{x_L}{r}\right) - V(x_H)\right]. \tag{11}$$

An H-type agent meets an H-type agent of the opposite sex with probability $\alpha\lambda_H$, and they marry. However, if an H-type agent meets an L-type agent of the opposite sex with probability $\alpha\lambda_L$, he or she compares x_L/r and $V(x_H)$ and then decides whether or not to propose.

If an H-type agent turns down an L-type agent of the opposite sex, $V(x_H) > \frac{x_L}{r}$. From (11), this H-type agent's discounted lifetime utility when he or she is single becomes

$$rV^r(x_H) = \alpha\lambda_H\left(\tfrac{x_H}{r} - V^r(x_H)\right).$$

On the other hand, when he or she accepts an L-type agent, i.e., $\frac{x_L}{r} \geq V(x_H)$, his or her value function is[31]

$$rV^a(x_H) = \alpha\lambda_H\left(\tfrac{x_H}{r} - V^a(x_H)\right) + \alpha\lambda_L\left(\tfrac{x_L}{r} - V^a(x_H)\right).$$

If $V^r(x_H) > V^a(x_H)$ is satisfied, an H-type agent refuses an L-type opposite sex agent. This inequality $V^r(x_H) > V^a(x_H)$ means that

$$x_L < R_i^*(x_H) \equiv \tfrac{\alpha\lambda_H x_H}{\alpha\lambda_H + r}.$$

If $x_L \geq R^*(x_H)$, an H-type agent proposes to an L-type agent.□

Proof of Lemma 1: When k_0-type women accept low-type men, from (4), the expected discounted lifetime utility of a single k_0-type man becomes

$$rV(b_0) = \alpha\lambda_H\left(\tfrac{x_H}{r} - V(b_0)\right) + \alpha\lambda_L\left(\max\{\tfrac{x_L}{r}, V(b_0)\} - V(b_0)\right).$$

Therefore, we obtain the reservation level of a k_0-type man:

$$R^{MI}(b_0) = \tfrac{\alpha\lambda x_H}{(r+\alpha\lambda)} = R^*(x_H).$$

Even if sex is reversed, these results hold.□

Proof of Proposition 2: From Lemma 1, $R^{MI}(b_0) = R^*(x_H)$. Therefore, $R^{MI}(b_0) \leq x_L$, when $x_L \geq R^*(x_H)$. Hence, all types accept each other.□

[31] If an H-type agent proposes to an L-type agent ($V_H \leq x_L/r$), the H- and L-type agents receive at least the same number of offers. Hence, $V_H \geq V_L$, and then we have $V_L \leq x_L/r$. Namely, an L-type agent wishes to marry another L-type agent.

Proof of Lemma 2: If a k_0-type man (woman) rejects an L-type woman (man), the expected discounted lifetime utility of a single k_0-type man (woman) becomes

$$\begin{aligned}
rV^r(b_0) &= b_0(x_H)\left[\alpha\lambda\left(\tfrac{x_H}{r}-V^r(b_0)\right)+\alpha\eta(1-\lambda)(V(b_H)-V^r(b_0))\right] \\
&\quad +b_0(x_L)\left[(\alpha\lambda+\alpha\eta(1-\lambda))(V(b_L)-V^r(b_0))\right]. \\
rV(b_H) &= \alpha\lambda\left(\tfrac{x_H}{r}-V(b_H)\right) \qquad (12)\\
rV(b_L) &= \alpha(1-\eta)(1-\lambda)\left(\tfrac{x_L}{r}-V(b_L)\right) \qquad (13)\\
b_0(x_H) &= \tfrac{\phi\lambda}{\phi\lambda+\eta(1-\lambda)},\ b_0(x_L)=1-\tfrac{\phi\lambda}{\phi\lambda+\eta(1-\lambda)} \qquad (14)
\end{aligned}$$

In contrast, if he (she) accepts an L-type woman (man), his (her) expected discounted lifetime utility becomes

$$\begin{aligned}
rV^a(b_0) &= b_0(x_H)\left[\alpha\lambda\left(\tfrac{x_H}{r}-V^a(b_0)\right)+\alpha(1-\lambda)\left(\tfrac{x_L}{r}-V^a(b_0)\right)\right] \\
&\quad +b_0(x_L)\left[(\alpha\lambda+\alpha\eta(1-\lambda))(V(b_L)-V^a(b_0))+\alpha(1-\eta)(1-\lambda)\left(\tfrac{x_L}{r}-V^a(b_0)\right)\right].
\end{aligned}$$

Therefore, we obtain the reservation level of a k_0-type man (woman):

$$R^{EI}(b_0)\equiv\tfrac{b_0(x_H)\alpha\lambda x_H(r+\alpha(1-\lambda)(1-\eta))(r+\alpha\lambda+\alpha\eta(1-\lambda))}{(r+\alpha\lambda)(b_0(x_H)(r+\alpha(1-\lambda)(1-\eta))(r+\alpha\lambda+\alpha\eta(1-\lambda))+rb_0(x_L)(1-\eta)(r+\alpha))}. \qquad (15)$$

From (12) and (13),

$$\begin{aligned}
R^{EI}(b_H) &= \tfrac{\alpha\lambda x_H}{r+\alpha\lambda}=R^*(x_H), \\
R^{EI}(b_L) &= \tfrac{\alpha(1-\eta)(1-\lambda)x_L}{r+\alpha(1-\eta)(1-\lambda)}\ (<x_L). \qquad (16)
\end{aligned}$$

Then, we obtain

$$R^{EI}(b_L)-R^*(x_L)=\tfrac{(\lambda-1)r\alpha\eta x_L}{(\alpha(1-\lambda)+r)(r+\alpha(1-\eta)(1-\lambda))}<0.$$

Therefore,

$$\begin{aligned}
&R^{EI}(b_0)-R^*(x_H) \\
&= -\tfrac{b_0(x_L)r\alpha\lambda x_H(1-\eta)(r+\alpha)}{(r+\alpha\lambda)(b_0(x_H)(r+\alpha(1-\lambda)(1-\eta))(r+\alpha\lambda+\alpha\eta(1-\lambda))+rb_0(x_L)(1-\eta)(r+\alpha))}<0, \qquad (17)
\end{aligned}$$

and

$$\begin{aligned}
&R^{EI}(b_0)-R^{EI}(b_L) \\
&= -\alpha\tfrac{(\eta-1)(\lambda-1)(r+\alpha\lambda)\Psi x_L-b_0(x_H)\lambda x_H(r+\alpha\lambda+\alpha\eta(1-\lambda))(r+\alpha(1-\eta)(1-\lambda))^2}{(r+\alpha\lambda)(r+\alpha(1-\eta)(1-\lambda))(b_0(x_H)(r+\alpha(1-\lambda)(1-\eta))(r+\alpha\lambda+\alpha\eta(1-\lambda))+rb_0(x_L)(1-\eta)(r+\alpha))},
\end{aligned}$$

where $\Psi\equiv[b_0(x_H)(r+\alpha(1-\lambda)(1-\eta))(r+\alpha\lambda+\alpha\eta(1-\lambda))+rb_0(x_L)(1-\eta)(r+\alpha)]$. Here, iff $x_L<(\geq)\frac{b_0(x_H)\lambda x_H(r+\alpha\lambda+\alpha\eta(1-\lambda))(r+\alpha(1-\eta)(1-\lambda))^2}{(\eta-1)(\lambda-1)(r+\alpha\lambda)(b_0(x_H)(r+\alpha(1-\lambda)(1-\eta))(r+\alpha\lambda+\alpha\eta(1-\lambda))+rb_0(x_L)(1-\eta)(r+\alpha))}=\frac{r+\alpha(\eta-1)(\lambda-1)}{\alpha(\eta-1)(\lambda-1)}R^{EI}(b_0)$, $R^{EI}(b_0)>(\leq)R^{EI}(b_L)$. Therefore, if $R^{EI}(b_0)>x_L$, $R^{EI}(b_0)>R^{EI}(b_L)$ holds.[32]

[32] Likewise, if $x_L<(\geq)\frac{r+\alpha(\lambda-1)}{\alpha(\lambda-1)}R^{EI}(b_0)$, $R^{EI}(b_0)>(\leq)R^*(x_L)$. Hence, if $R^{EI}(b_0)>x_L$, $R^{EI}(b_0)$

It is noteworthy that from (17), $R^{EI}(b_0) \to R^*(x_H)$, when $\phi = \phi_i = \phi_j \to 0$ and $\eta = \eta_i = \eta_j \to 0$ $(i, j = m, w,\ i \neq j)$. Here, in order to investigate the effect of assigning the probability of an agent's own type on the reservation utility level, let us now suppose that $\phi_i > 0$ and $\eta_i > 0$ under $\phi_j \to 0$ and $\eta_j \to 0$. At this time, from (15) and (16), we obtain[33]

$$\begin{aligned} R_i^{EI}(b_0)|_{\phi_j,\eta_j\to 0} &\equiv \tfrac{b_0(x_H)\alpha\lambda x_H(r+\alpha(1-\lambda))(r+\alpha\lambda)}{(r+\alpha\lambda)(b_0(x_H)(r+\alpha(1-\lambda))(r+\alpha\lambda)+rb_0(x_L)(r+\alpha))}, \\ R_i^{EI}(b_L)|_{\phi_j,\eta_j\to 0} &= \tfrac{\alpha x_L(1-\lambda)}{r+\alpha(1-\lambda)} = R^*(x_L). \end{aligned}$$

Therefore, we obtain

$$R_i^{EI}(b_0)|_{\phi_j,\eta_j\to 0} - R^*(x_H) = \tfrac{-b_0(x_L)r\alpha\lambda x_H(r+\alpha)}{(r+\alpha\lambda)(b_0(x_H)(r+\alpha(1-\lambda))(r+\alpha\lambda)+rb_0(x_L)(r+\alpha))} < 0.$$

This difference between $R_i^{EI}(b_0)|_{\phi_j,\eta_j\to 0}$ and $R^*(x_k)$, $k = H, L$ represents the effect of assigning the probability of an agent's own type. Moreover,

$$R_i^{EI}(b_0)|_{\phi_j,\eta_j\to 0} - R^*(x_L) = \tfrac{b_0(x_H)\lambda x_H(r+\alpha-\alpha\lambda)^2-(1-\lambda)(r(r+\alpha)b_0(x_L)+b_0(x_H)(r+\alpha-\alpha\lambda)(r+\alpha\lambda))x_L}{(r+\alpha(1-\lambda))(b_0(x_H)(r+\alpha-\alpha\lambda)(r+\alpha\lambda)+b_0(x_L)r(r+\alpha))}.$$

Here, iff $\frac{p\lambda x_H(r+\alpha-\alpha\lambda)^2}{(1-\lambda)(r(r+\alpha)q+p(r+\alpha-\alpha\lambda)(r+\alpha\lambda))} = \frac{r+\alpha-\alpha\lambda}{\alpha(1-\lambda)} R_i^{EI}(b_0)|_{\phi_j,\eta_j\to 0} > (\leq) x_L$, $R_i^{EI}(b_0)|_{\phi_j,\eta_j\to 0} > (\leq) R^*(x_L)$. Therefore, if $R_i^{EI}(b_0)|_{\phi_j,\eta_j\to 0} > (\leq) x_L$, $R_i^{EI}(b_0)|_{\phi_j,\eta_j\to 0} > (\leq) R^*(x_L)$ holds.

Next, let us investigate the effect of the existence of others of the opposite sex with imperfect self-knowledge on the reservation utility level. This effect is represented by the difference between $R_i^{EI}(b_0)|_{\phi_j,\eta_j\to 0}$ and $R^{EI}(b_0)$ under $\phi_i > 0$ and $\eta_i > 0$ (then, $b_0(x_H) > 0$ and $b_0(x_L) > 0$). Therefore, we obtain

$$\begin{aligned} & R^{EI}(b_0) - R_i^{EI}(b_0)|_{\phi_j,\eta_j\to 0} \\ = & \tfrac{r\alpha\lambda\eta b_0(x_H)b_0(x_L)x_H(r+\alpha)\left(r(r+\alpha)+\alpha^2(\lambda-1)^2(1-\eta)\right)}{(r+\alpha\lambda)((b_0(x_H)(r+\alpha(1-\lambda)(1-\eta))(r+\alpha\lambda+\alpha\eta(1-\lambda))+b_0(x_L)r(1-\eta)(r+\alpha))(b_0(x_H)(r+\alpha(1-\lambda))(r+\alpha\lambda)+rb_0(x_L)(r+\alpha)))} > 0. \end{aligned}$$

From the above results, we obtain

$$R_i^{EI}(b_0)|_{\phi_j,\eta_j\to 0} < R^{EI}(b_0) < R^*(x_H).$$

The difference between $R_i^{EI}(b_L)|_{\phi_j,\eta_j\to 0}$ and $R^{EI}(b_L)$ represents the effect of delay in marriage because of refusals by L_0-type agents. Therefore,

$$R^{EI}(b_L) - R_i^{EI}(b_L)|_{\phi_j,\eta_j\to 0} = \tfrac{r\alpha\eta x_L(\lambda-1)}{(r+\alpha(1-\lambda))(r+\alpha(1-\eta)(1-\lambda))} < 0.$$

It is noteworthy that when agents are patient ($r = 0$), $R^{EI}(b_0) = R_i^{EI}(b_0)|_{\phi_j,\eta_j\to 0} = R^*(x_H) = x_H$ and $R^{EI}(b_L) = R_i^{EI}(b_L)|_{\phi_j,\eta_j\to 0} = R^*(x_L) = x_L$ hold.

$> R^*(x_L)$ holds.

[33] At this time, it is noteworthy that $b_0(x_H) > 0$ and $b_0(x_L) > 0$ because $\phi_w > 0$ and $\eta_w > 0$ in (14).

Note that

$$\frac{\partial R^{EI}(b_0)}{\partial \eta} = \frac{b_0(x_H)b_0(x_L)r\alpha\lambda x_H(r+\alpha)\left(r(r+\alpha)+\alpha^2(\eta-1)^2(\lambda-1)^2\right)}{(r+\alpha\lambda)(b_0(x_H)(r+\alpha(1-\lambda)(1-\eta))(r+\alpha\lambda+\alpha\eta(1-\lambda))+rb_0(x_L)(1-\eta)(r+\alpha))^2} > 0.$$

Moreover, substituting (14) into (15), we obtain

$$\begin{aligned}
\frac{\partial R^{EI}(b_0)}{\partial \phi} &= \frac{(r+\alpha-\alpha\lambda-\alpha\eta+\alpha\lambda\eta)r\alpha\lambda^2\eta x_H(\lambda-1)(\eta-1)(r+\alpha)(r+\alpha\lambda+\alpha\eta-\alpha\lambda\eta)}{(r+\alpha\lambda)(\phi\lambda(r+\alpha(1-\lambda)(1-\eta))(r+\alpha\lambda+\alpha\eta(1-\lambda))+r\eta(1-\lambda)(1-\eta)(r+\alpha))^2} > 0. \\
\frac{\partial R^{EI}(b_0)}{\partial \eta} &= \frac{r\alpha\lambda^2\phi x_H(\lambda-1)(r+\alpha)\left[\lambda(\eta-1)^2(1-\lambda)\alpha^2+r(1-2\eta)(r+\alpha)\right]}{(r+\alpha\lambda)(\phi\lambda(r+\alpha(1-\lambda)(1-\eta))(r+\alpha\lambda+\alpha\eta(1-\lambda))+r\eta(1-\lambda)(1-\eta)(r+\alpha))^2} < 0.
\end{aligned}$$

Therefore, $R^{EI}(b_0)$ is strictly increasing in ϕ whereas $R^{EI}(b_0)$ is strictly decreasing in η if $\eta < \frac{1}{2}$. $\square$

Proof of Proposition 3: From Lemma 2, if $x_L < R^{EI}(b_0)\,(< R_m^*(x_M))$, an k_0-type agent rejects an L-type agent of the opposite sex. Moreover, an H_H-type agent rejects an L-type agent of the opposite sex ($R^{EI}(b_H) = R^*(x_H) > x_L$). Therefore, L_L-type agents always accept L-type agents ($0 < R^{EI}(b_L) < x_L$) (otherwise, they cannot marry). As a result, there exists an E-PEI, where men and women of the same type marry.$\square$

Proof of Proposition 5: From (5) and (6), we obtain

$$\frac{\alpha\pi x_H}{\alpha\pi+r} = \frac{\hat{\lambda}^2\alpha}{\left(\hat{\lambda}^2+\left(\hat{\lambda}-1\right)^2(\hat{\eta}-1)^2\right)r+\hat{\lambda}^2\alpha}x_H.$$

Therefore,

$$\begin{aligned}
&R^{ES}(b_0) - \frac{\alpha\pi x_H}{\alpha\pi+r} \\
=& \frac{\frac{\hat{\phi}\hat{\lambda}}{\hat{\phi}\hat{\lambda}+\hat{\eta}\left(1-\hat{\lambda}\right)}\alpha\hat{\lambda}x_H\left(r+\alpha\left(1-\hat{\lambda}\right)(1-\hat{\eta})\right)\left(r+\alpha\hat{\lambda}+\alpha\hat{\eta}\left(1-\hat{\lambda}\right)\right)}{\left(r+\alpha\hat{\lambda}\right)\left[\frac{\hat{\phi}\hat{\lambda}}{\hat{\phi}\hat{\lambda}+\hat{\eta}\left(1-\hat{\lambda}\right)}\left(r+\alpha\left(1-\hat{\lambda}\right)(1-\hat{\eta})\right)\left(r+\alpha\hat{\lambda}+\alpha\hat{\eta}\left(1-\hat{\lambda}\right)\right)+r\frac{\hat{\eta}\left(1-\hat{\lambda}\right)}{\hat{\phi}\hat{\lambda}+\hat{\eta}\left(1-\hat{\lambda}\right)}(1-\hat{\eta})(r+\alpha)\right]} - \frac{\hat{\lambda}^2\alpha}{\left(\hat{\lambda}^2+\left(\hat{\lambda}-1\right)^2(\hat{\eta}-1)^2\right)r+\hat{\lambda}^2\alpha}x_H \\
=& \frac{r\alpha\hat{\lambda}^2x_H\left(\hat{\lambda}-1\right)\left[\left(\hat{\eta}(\hat{\eta}-2)\left(\hat{\lambda}-1\right)+2\hat{\lambda}-1\right)\left(r+\alpha(\hat{\eta}-1)\left(\hat{\lambda}-1\right)\right)\left(r+\alpha\left(\hat{\lambda}+\hat{\eta}\left(1-\hat{\lambda}\right)\right)\right)\hat{\phi}+\hat{\eta}(1-\hat{\eta})(r+\alpha)\left(r+\alpha\hat{\lambda}\right)\right]}{\left(r+\alpha\hat{\lambda}\right)\left[\left(\hat{\eta}\left(\hat{\lambda}-1\right)^2(\hat{\eta}-2)+\left(2\hat{\lambda}^2-2\hat{\lambda}+1\right)\right)r+\alpha\hat{\lambda}^2\right]\left[\hat{\lambda}\left(r+\alpha(\hat{\eta}-1)\left(\hat{\lambda}-1\right)\right)\left(r+\alpha\left(\hat{\lambda}+\hat{\eta}\left(1-\hat{\lambda}\right)\right)\right)\hat{\phi}+r\hat{\eta}(\hat{\eta}-1)\left(\hat{\lambda}-1\right)(r+\alpha)\right]}. \quad (18)
\end{aligned}$$

From (7), $\left(\hat{\eta}(\hat{\eta}-2)\left(\hat{\lambda}-1\right)+2\hat{\lambda}-1\right) = \frac{5\hat{\lambda}-4\hat{\lambda}^2+\hat{\lambda}^3-1}{\left(\hat{\lambda}-2\right)^2}$. Moreover, $\left(5\hat{\lambda}-4\hat{\lambda}^2+\hat{\lambda}^3-1\right)$ $= -\frac{1}{4}\frac{-2\pi+2\sqrt{4\pi-3\pi^2}-\sqrt{2}\left(\frac{1}{\pi-1}\left(\pi+\sqrt{4\pi-3\pi^2}-2\right)\right)^{\frac{3}{2}}+\sqrt{2}\pi\left(\frac{1}{\pi-1}\left(\pi+\sqrt{4\pi-3\pi^2}-2\right)\right)^{\frac{3}{2}}}{\pi-1}$ because $\hat{\lambda} = 1-\frac{1}{2}\sqrt{2}\sqrt{\frac{1}{\pi-1}\left(\pi+\sqrt{-\pi(3\pi-4)}-2\right)}$ from (6). Then, if $\pi > 0.245 = \hat{\pi}$, $\left(5\hat{\lambda}-4\hat{\lambda}^2+\hat{\lambda}^3-1\right)$ > 0. Therefore, if $\pi > \hat{\pi}$, $R^{ES}(b_0) < \frac{\alpha\pi x_H}{\alpha\pi+r}$.

In contrast, if $\pi < \hat{\pi}$, $\left(5\hat{\lambda}-4\hat{\lambda}^2+\hat{\lambda}^3-1\right) < 0$. At this time, if

$$\hat{\phi} - \frac{-\hat{\eta}(1-\hat{\eta})(r+\alpha)\left(r+\alpha\hat{\lambda}\right)}{\left(\hat{\eta}(\hat{\eta}-2)\left(\hat{\lambda}-1\right)+2\hat{\lambda}-1\right)\left(r+\alpha(\hat{\eta}-1)\left(\hat{\lambda}-1\right)\right)\left(r+\alpha\left(\hat{\lambda}+\hat{\eta}\left(1-\hat{\lambda}\right)\right)\right)} > 0, \quad (19)$$

in (18), $R^{ES}(b_0) > \frac{\alpha\pi x_H}{\alpha\pi + r}$. However, substituting (7) and (8) into (19),we obtain

$$
\begin{aligned}
& \hat{\phi} - \frac{-\hat{\eta}(1-\hat{\eta})(r+\alpha)\left(r+\alpha\hat{\lambda}\right)}{\left(\hat{\eta}(\hat{\eta}-2)\left(\hat{\lambda}-1\right)+2\hat{\lambda}-1\right)\left(r+\alpha(\hat{\eta}-1)\left(\hat{\lambda}-1\right)\right)\left(r+\alpha\left(\hat{\lambda}+\hat{\eta}\left(1-\hat{\lambda}\right)\right)\right)} \\
= & \left(\hat{\lambda}-2\right) \frac{\left(\hat{\lambda}-2\right)\left(3\hat{\lambda}-11\hat{\lambda}^2+13\hat{\lambda}^3-6\hat{\lambda}^4+\hat{\lambda}^5-1\right)r^2+\alpha\left(\hat{\lambda}-2\right)\left(2\hat{\lambda}-10\hat{\lambda}^2+13\hat{\lambda}^3-6\hat{\lambda}^4+\hat{\lambda}^5-1\right)r+\alpha^2\hat{\lambda}\left(1-\hat{\lambda}\right)\left(4\hat{\lambda}-4\hat{\lambda}^2+\hat{\lambda}^3+1\right)}{\left(5\hat{\lambda}-4\hat{\lambda}^2+\hat{\lambda}^3-1\right)\left(2r+\alpha-r\hat{\lambda}-\alpha\hat{\lambda}\right)\left(r\left(\hat{\lambda}-2\right)-\alpha\right)} \\
< & \; 0, \qquad (20)
\end{aligned}
$$

where under $\hat{\lambda} \in (0,1)$, $\left(3\hat{\lambda} - 11\hat{\lambda}^2 + 13\hat{\lambda}^3 - 6\hat{\lambda}^4 + \hat{\lambda}^5 - 1\right) < 0$, $\left(2\hat{\lambda} - 10\hat{\lambda}^2 + 13\hat{\lambda}^3 - 6\hat{\lambda}^4 + \hat{\lambda}^5 - 1\right) < 0$, and $\left(4\hat{\lambda} - 4\hat{\lambda}^2 + \hat{\lambda}^3 + 1\right) > 0$. Hence, $R^{ES}(b_0) < \frac{\alpha\pi x_H}{\alpha\pi + r}$, when $\pi < \hat{\pi}$.

Therefore, an E-SEI cannot support an M-SEI at the same time because $R^{ES}(b_0) < \frac{\alpha\pi x_H}{\alpha\pi + r}$ under any π.

Furthermore, $\hat{\phi} - \hat{\eta} = \frac{5\hat{\lambda} - 4\hat{\lambda}^2 + \hat{\lambda}^3 - 1}{2 - \hat{\lambda}}$. Hence, if $\pi > (<)\,\hat{\pi}$,

$$
\hat{\phi} > (<)\,\hat{\eta}. \qquad (21)
$$

□

Proof of Lemma 3: If a k_0-type man (woman) rejects an L-type woman (man), the expected discounted lifetime utility of a single k_0-type man (woman) becomes

$$
\begin{aligned}
rV^r(b_0) &= b_0(x_H)\left[\alpha\lambda\left(\tfrac{x_H}{r} - V^r(b_0)\right)\right] \\
&\quad + b_0(x_L)\left[(\alpha\lambda + \alpha\eta(1-\lambda))\left(V(b_L) - V^r(b_0)\right)\right]. \\
rV(b_L) &= \alpha(1-\eta)(1-\lambda)\left(\tfrac{x_L}{r} - V(b_L)\right) \qquad (22)
\end{aligned}
$$

In contrast, if he (she) accepts an L-type woman (man), his (her) expected discounted lifetime utility becomes

$$
\begin{aligned}
rV^a(b_0) &= b_0(x_H)\left[\alpha\lambda\left(\tfrac{x_H}{r} - V^a(b_0)\right) + \alpha(1-\lambda)\left(\tfrac{x_L}{r} - V^a(b_0)\right)\right] \\
&\quad + b_0(x_L)\left[(\alpha\lambda + \alpha\eta(1-\lambda))\left(V(b_L) - V^a(b_0)\right) + \alpha(1-\eta)(1-\lambda)\left(\tfrac{x_L}{r} - V^a(b_0)\right)\right].
\end{aligned}
$$

Therefore, we obtain the reservation level of a k_0-type man (woman):

$$
R^{EI2}(b_0) \equiv \frac{b_0(x_H)\alpha\lambda x_H(\alpha(1-\eta)(1-\lambda)+r)}{(\alpha(1-\eta)(1-\lambda)+r)(r+b_0(x_H)\alpha\lambda)+(\alpha\lambda+\alpha\eta(1-\lambda))b_0(x_L)r}. \qquad (23)
$$

From (22),

$$
R^{EI2}(b_L) = \frac{\alpha(1-\eta)(1-\lambda)x_L}{r+\alpha(1-\eta)(1-\lambda)}\,(< x_L).
$$

Therefore,

$$
R^{EI2}(b_L) - R^*(x_L) = \frac{(\lambda-1)r\alpha\eta x_L}{(\alpha(1-\lambda)+r)(r+\alpha(1-\eta)(1-\lambda))} < 0.
$$

Moreover,

$$\begin{aligned} & R^{EI2}(b_0) - R^*(x_H) \\ = \;& r\alpha\lambda x_H \tfrac{-(1-b_0(x_H))(r+\alpha(1-\eta)(1-\lambda))-(\alpha\lambda+\alpha\eta(1-\lambda))b_0(x_L)}{(r+\alpha\lambda)((\alpha(1-\eta)(1-\lambda)+r)(r+b_0(x_H)\alpha\lambda)+(\alpha\lambda+\alpha\eta(1-\lambda))b_0(x_L)r)} < 0, \end{aligned} \tag{24}$$

and

$$\begin{aligned} & R^{EI2}(b_0) - R^{EI2}(b_L) \\ = \;& \alpha \tfrac{\left(b_0(x_H)\lambda(r+\alpha(\eta-1)(\lambda-1))^2\right)x_H - x_L(\eta-1)(\lambda-1)[(\alpha(1-\eta)(1-\lambda)+r)(r+b_0(x_H)\alpha\lambda)+(\alpha\lambda+\alpha\eta(1-\lambda))b_0(x_L)r]}{(r+\alpha(\eta-1)(\lambda-1))[(\alpha(1-\eta)(1-\lambda)+r)(r+b_0(x_H)\alpha\lambda)+(\alpha\lambda+\alpha\eta(1-\lambda))b_0(x_L)r]}. \end{aligned}$$

Here, iff $x_L < (\geq) \frac{b_0(x_H)\lambda(r+\alpha(\eta-1)(\lambda-1))^2}{(\eta-1)(\lambda-1)((\alpha(1-\eta)(1-\lambda)+r)(r+b_0(x_H)\alpha\lambda)+(\alpha\lambda+\alpha\eta(1-\lambda))b_0(x_L)r)} = \frac{r+\alpha(\lambda-1)(\eta-1)}{\alpha(\lambda-1)(\eta-1)} R^{EI2}(b_0)$, $R^{EI2}(b_0) > (\leq) R^{EI2}(b_L)$. Therefore, if $R^{EI2}(b_0) \geq x_L$, $R^{EI2}(b_0) > R^{EI2}(b_L)$ holds.[34]

From (24), $R^{EI2}(b_0) \to R^*(x_H)$ when $\phi = \phi_i = \phi_j \to 0$ and $\eta = \eta_i = \eta_j \to 0$ ($i, j = m, w,\ i \neq j$). Here, in order to separate the effects of two-sided imperfect self-knowledge, first, let us consider the case in which there is no agent j with imperfect self-knowledge (i.e., $\eta_j \to 0$ holds in (23) under $\eta_i > 0$). In this case, we obtain the reservation level of a k_0-type agent i:

$$\begin{aligned} R_i^{EI2}(b_0)\,|_{\eta_j\to 0} &\equiv \tfrac{b_0(x_H)\alpha\lambda x_H(r+\alpha(1-\lambda))}{r^2+\alpha r+b_0(x_H)\alpha^2\lambda(1-\lambda)} = \tfrac{b_0(x_H)\alpha\lambda x_H(r+\alpha(1-\lambda))}{(r+\alpha(1-\lambda))(r+b_0(x_H)\alpha\lambda)+b_0(x_L)r\alpha\lambda}, \\ R_i^{EI2}(b_L)\,|_{\eta_j\to 0} &\equiv \tfrac{\alpha(1-\lambda)x_L}{r+\alpha(1-\lambda)} = R^*(x_L). \end{aligned}$$

The difference between $R_i^{EI2}(b_0)\,|_{\eta_j\to 0}$ and $R^*(x_k)$, $k = H, L$, represents the effect of assigning the probability of an agent's own type. Therefore,

$$\begin{aligned} & R_i^{EI2}(b_0)\,|_{\eta_j\to 0} - R^*(x_H) \\ = \;& r\alpha\lambda x_H \tfrac{(b_0(x_H)-1)(r+\alpha-\alpha\lambda)-b_0(x_L)\alpha\lambda}{(r+\alpha\lambda)((r+\alpha(1-\lambda))(r+b_0(x_H)\alpha\lambda)+b_0(x_L)r\alpha\lambda)} < 0, \end{aligned}$$

and

$$\begin{aligned} & R_i^{EI2}(b_0)\,|_{\eta_j\to 0} - R_i^{EI2}(b_L)\,|_{\eta_j\to 0} \\ = \;& -\alpha \tfrac{(1-\lambda)\left(r(r+\alpha)+b_0(x_H)\alpha^2\lambda(1-\lambda)\right)x_L - b_0(x_H)\lambda x_H(r+\alpha(1-\lambda))^2}{(r+\alpha(1-\lambda))(r(r+\alpha)+b_0(x_H)\alpha^2\lambda(1-\lambda))}. \end{aligned}$$

Here, iff $x_L > (\leq) \frac{b_0(x_H)\lambda x_H(r+\alpha(1-\lambda))^2}{(1-\lambda)(r(r+\alpha)+b_0(x_H)\alpha^2\lambda(1-\lambda))} = \frac{r+\alpha(1-\lambda)}{\alpha(1-\lambda)} R_i^{EI2}(b_0)\,|_{\eta_j\to 0}$, $R_i^{EI2}(b_0)\,|_{\eta_j\to 0} < (\geq) R_i^{EI2}(b_L)\,|_{\eta_j\to 0}$. Therefore, if $x_L \leq R_i^{EI2}(b_0)\,|_{\eta_j\to 0}$, $R_i^{EI2}(b_0)\,|_{\eta_j\to 0} > R_i^{EI2}(b_L)\,|_{\eta_j\to 0}$ holds.

Next, we compare $R_i^{EI2}(b_0)\,|_{\eta_j\to 0}$ with $R^{EI2}(b_0)$. The difference between $R_i^{EI2}(b_0)\,|_{\eta_j\to 0}$ and $R^{EI2}(b_0)$ represents the effect of the existence of others of the opposite sex with imperfect

[34] Likewise, if $x_L < (\geq) \frac{r+\alpha(\lambda-1)}{\alpha(\lambda-1)} R^{EI2}(b_0)$, $R^{EI2}(b_0) > (\leq) R^*(x_L)$. Hence, if $R^{EI2}(b_0) > x_L$, $R^{EI2}(b_0) > R^*(x_L)$ holds.

self-knowledge.

$$
\begin{aligned}
& R^{EI2}(b_0) - R_i^{EI2}(b_0)\,|_{\eta_j\to 0} \\
= \; & \frac{(r+\alpha)b_0(x_H)b_0(x_L)r\alpha^2\lambda\eta x_H(\lambda-1)}{((r+\alpha(1-\lambda))(r+b_0(x_H)\alpha\lambda)+b_0(x_L)r\alpha\lambda)((\alpha(1-\eta)(1-\lambda)+r)(r+b_0(x_H)\alpha\lambda)+(\alpha\lambda+\alpha\eta(1-\lambda))b_0(x_L)r)} < 0.
\end{aligned}
$$

That is, the existence of others with imperfect self-knowledge decreases the reservation level of an agent.

From the above results, we obtain

$$R^*(x_H) > R_i^{EI2}(b_0)\,|_{\eta_j\to 0} > R^{EI2}(b_0).$$

The difference between $R_i^{EI2}(b_L)\,|_{\eta_j\to 0}$ and $R^{EI2}(b_L)$ represents the effect of delay in marriage because of refusals by L_0-type agents. Then,

$$R^{EI2}(b_L) - R_i^{EI2}(b_L)\,|_{\eta_j\to 0} = -\frac{r\alpha\eta x_L(1-\lambda)}{(r+\alpha(1-\lambda))(r+\alpha(1-\eta)(1-\lambda))} < 0,$$

It is noteworthy that when agents are patient ($r=0$), $R^{EI2}(b_0) = R_i^{EI2}(b_0)\,|_{\eta_j\to 0} = R^*(x_H)$ $= x_H$ and $R^{EI2}(b_L) = R_i^{EI2}(b_L)\,|_{\eta_j\to 0} = R^*(x_L) = x_L$ hold.□

Proof of Proposition 6: From Lemma 3, $0 < R^{EI2}(b_0)\,(< R^*(x_H))$. Therefore, if $x_L < R^{EI2}(b_0)$, k_0-type agents ($k = H, L$) reject L-type agents of the opposite sex and H_H-type agents reject L-type agents of the opposite sex. Because L_L-type agents always accept L-type agents of the opposite sex ($0 < R^{EI2}(b_L) < x_L$) (otherwise, they cannot marry), H-type agents marry within their group, as do L-type agents.□

Proof of Proposition 7: From (5) and (9), we obtain

$$\frac{\alpha\pi x_H}{\alpha\pi+r} = \frac{\bar{\lambda}^2\alpha}{\left(\bar{\lambda}^2+\left(\bar{\lambda}-1\right)^2(\bar{\eta}-1)^2\right)r+\bar{\lambda}^2\alpha}x_H.$$

Therefore,

$$
\begin{aligned}
& R^{ES2}(b_0) - \frac{\alpha\pi x_H}{\alpha\pi+r} \\
= \; & \frac{\frac{\bar{\lambda}}{\bar{\lambda}+\bar{\eta}\left(1-\bar{\lambda}\right)}\alpha\bar{\lambda}x_H\left(\alpha(1-\bar{\eta})\left(1-\bar{\lambda}\right)+r\right)}{\left(\alpha(1-\bar{\eta})\left(1-\bar{\lambda}\right)+r\right)\left(r+\frac{\bar{\lambda}}{\bar{\lambda}+\bar{\eta}\left(1-\bar{\lambda}\right)}\alpha\bar{\lambda}\right)+\left(\alpha\bar{\lambda}+\alpha\bar{\eta}\left(1-\bar{\lambda}\right)\right)\frac{\bar{\eta}\left(1-\bar{\lambda}\right)}{\bar{\lambda}+\bar{\eta}\left(1-\bar{\lambda}\right)}r} - \frac{\bar{\lambda}^2\alpha}{\left(\bar{\lambda}^2+\left(\bar{\lambda}-1\right)^2(\bar{\eta}-1)^2\right)r+\bar{\lambda}^2\alpha}x_H, \\
= \; & \frac{\alpha\bar{\lambda}^2\left\{\left(\alpha(1-\bar{\eta})\left(1-\bar{\lambda}\right)+r\right)r\left[\left(\bar{\lambda}^2+\left(\bar{\lambda}-1\right)^2(\bar{\eta}-1)^2\right)-\left(\bar{\lambda}+\bar{\eta}\left(1-\bar{\lambda}\right)\right)\right]-\left(\alpha\bar{\lambda}+\alpha\bar{\eta}\left(1-\bar{\lambda}\right)\right)\bar{\eta}\left(1-\bar{\lambda}\right)r\right\}}{\left[\left(\alpha(1-\bar{\eta})\left(1-\bar{\lambda}\right)+r\right)\left(\left(\bar{\lambda}+\bar{\eta}\left(1-\bar{\lambda}\right)\right)r+\bar{\lambda}\alpha\bar{\lambda}\right)+\left(\alpha\bar{\lambda}+\alpha\bar{\eta}\left(1-\bar{\lambda}\right)\right)\bar{\eta}\left(1-\bar{\lambda}\right)r\right]\left[\left(\bar{\lambda}^2+\left(\bar{\lambda}-1\right)^2(\bar{\eta}-1)^2\right)r+\bar{\lambda}^2\alpha\right]}x_H.
\end{aligned}
$$

Here, $\left[\left(\bar{\lambda}^2+\left(\bar{\lambda}-1\right)^2(\bar{\eta}-1)^2\right)-\left(\bar{\lambda}+\bar{\eta}\left(1-\bar{\lambda}\right)\right)\right] = \left(\bar{\lambda}-1\right)\frac{2\bar{\lambda}-3\bar{\lambda}^2+\bar{\lambda}^3+1}{\left(\bar{\lambda}-2\right)^2}$ from (10). Because $2\bar{\lambda}-3\bar{\lambda}^2+\bar{\lambda}^3+1 > 0$ under $\bar{\lambda}\in(0,1)$, we obtain $\left(\bar{\lambda}-1\right)\frac{2\bar{\lambda}-3\bar{\lambda}^2+\bar{\lambda}^3+1}{\left(\bar{\lambda}-2\right)^2} < 0$. Hence, $R^{ES2}(b_0) < \frac{\alpha\pi x_H}{\alpha\pi+r}$ holds under any π. Therefore, an E-SEI and an M-SEI cannot occur at the same time.

Moreover, $\bar{\eta}\left(1-\bar{\lambda}\right)-\bar{\lambda}=\frac{4\bar{\lambda}-2\bar{\lambda}^{2}-1}{\bar{\lambda}-2}$. Then, if $0.29289 \ >(<)\,\bar{\lambda}$, $\bar{\eta}\left(1-\bar{\lambda}\right)>(<)\,\bar{\lambda}$. Hence, if $0.333\,33>(<)\,\pi$, $\bar{\eta}\left(1-\bar{\lambda}\right)>(<)\,\bar{\lambda}$, from (9).□

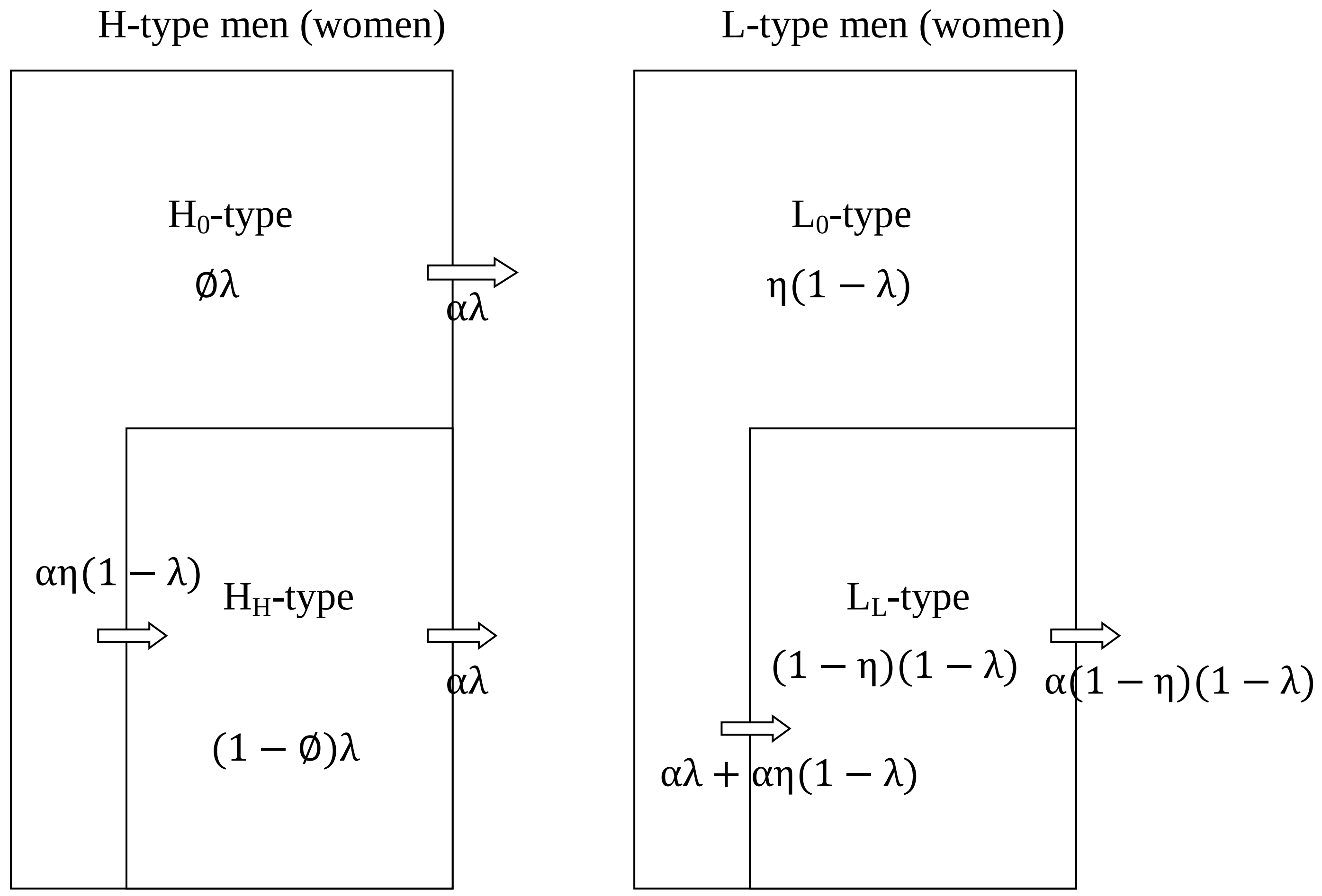

Figure 1: An E-PEI

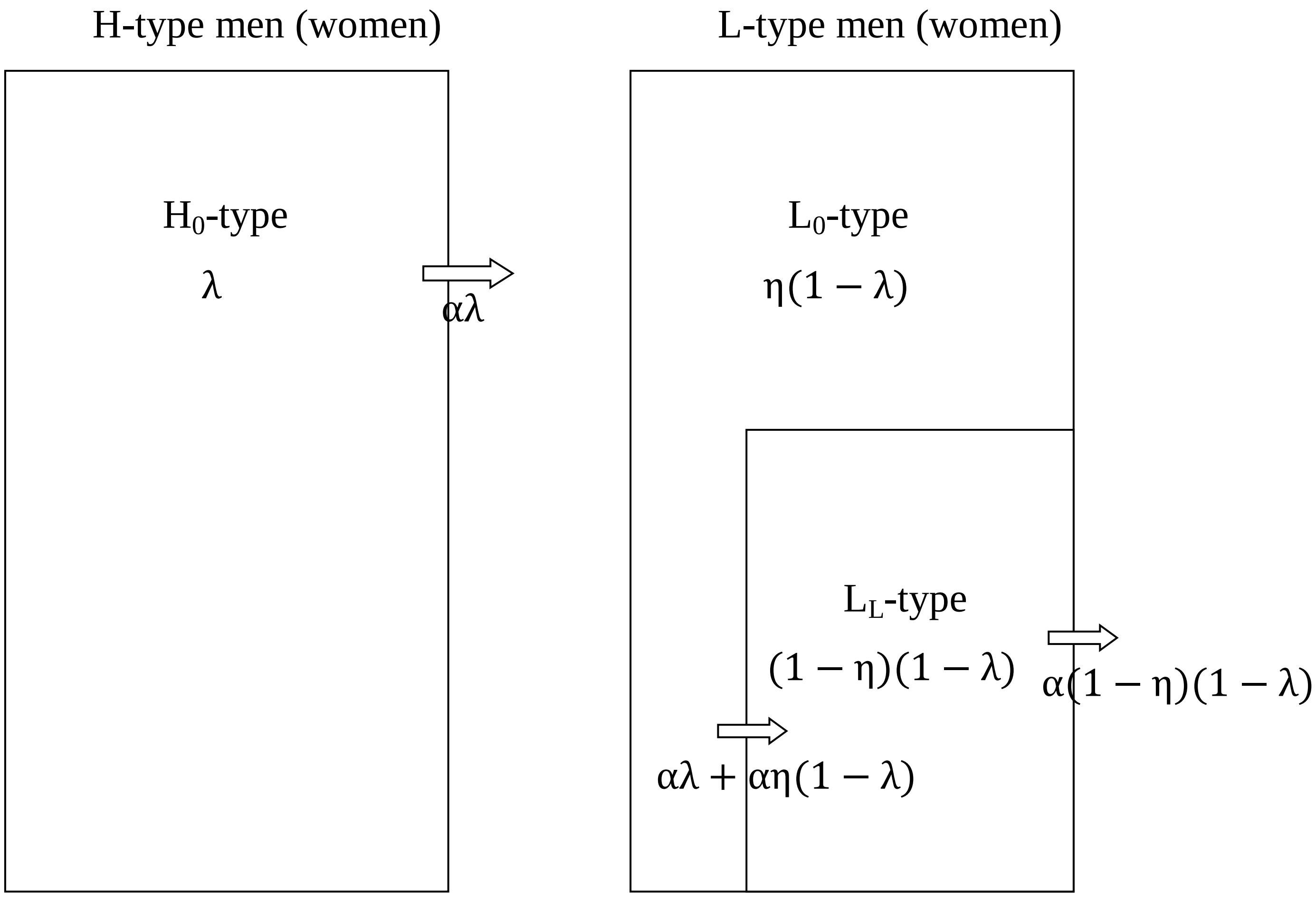

Figure2： An E-PEI2