GRIPS Discussion Paper 11-15

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November 2011

GRIPS
NATIONAL GRADUATE INSTITUTE FOR POLICY STUDIES

National Graduate Institute for Policy Studies
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Abstract
In the conventional social productive efficiency measurement, a DEA-based non-parametric method is typically employed to identify the piece-wise-linear production possibility frontier. Applying the directional distance-function approach a-la Luenberger (1992) to the production possibility frontier obtained in this fashion can, however, lead to an underestimation of inefficiency for a DMU with relatively large undesirable outputs. This underestimation becomes more acute if the sample size is small or data are clustered. This paper reveals the mechanism behind this underestimation bias, and then quantifies the degree of underestimation using nine-year panel data of rail and aviation sectors in Japan. Through a comparative analysis between parametric and non-parametric methods, we find, among others, that the underestimation of the aviation sector's productive inefficiency is as large as 80%, which the non-parametric method failed to detect.

Acknowledgement
We thank Kenneth Button and David Gillen for constructive comments and suggestions on an earlier version of the paper. We also thank Hiroyoshi Fujimoto and Kaoru Koide for data collection. This work is partly supported by JR Central and the Ministry of Education of Japan (Grant-in-Aid for Scientific Research #21730224).

1. Introduction

Social efficiency benchmarking, unlike the ordinary benchmarking, deals with the situation where production of goods and services involves the production of undesirable outputs. These undesirable outputs include emission pollution, congestion delays, noise, risk of accidents, and so forth. When measuring the production efficiency by taking into account these undesirable outputs, one cannot apply the ordinary distance function approach where all outputs are treated symmetrically. Since the amount of goods and bads can be increased together given the same level of inputs, output-oriented efficiency is not well-defined in the ordinary distance function approach. Moreover, in the ordinary distance function approach, the input-oriented efficiency cannot capture the undesirableness of bad outputs as it merely shrinks the input vector, holding the amount of both good and bad outputs. That is, a decision-making unit (DMU) with large amount of undesirable output relative to its desirable output amount can mark a very high efficiency score if its use of inputs is relatively less than other DMUs.

In the conventional literature of social productive efficiency measurement such as Pathomsiri et al. (2008) and Ha, Yoshida, and Zhang (2011), it is typical to employ a DEA-based non-parametric method in identifying the piece-wise-linear production possibility frontier (PPF), and then to apply the directional distance-function approach a-la Luenberger. However, there can potentially be a severe underestimation bias of inefficiency measured in this manner for a DMU with relatively large undesirable outputs, especially when the sample size is small or data are not well-scattered. When the sample size is small or data are not well-scattered, the estimated piece-wise-linear frontier truncates a substantial part of the true production possibility set. This results in the underestimation of inefficiency for those DMUs with relatively large undesirable outputs, as the directional distance-function approach measures the distance to the frontier in the direction in which desirable outputs are increasing but the undesirable outputs are decreasing.

To quantitatively measure the significance of this underestimation bias due to the non-parametric approach in a more practical case, we conduct a social efficiency measurement of air and rail sectors in Japan through parametric estimation of production technology. Ha, Yoshida, and Zhang (2011; HYZ hereafter) measured social efficiency of these sectors using the same data via the directional distance function approach with non-parametric PPF identification, and concluded that aviation sector's inefficiency is almost negligible. However, taking into account that the aviation sector has relatively large undesirable output while its desirable output level is just comparable to that of the major railway companies, this favorable result of aviation sector in Japan would most likely suffer the above-mentioned underestimation bias. We thus compare our parametric results to the non-parametric results obtained by HYZ and identify the magnitude of underestimation of social productive inefficiency in Japan's intercity transport sectors. Our empirical results actually show that the aviation sector's inefficiency is as large as around 80%, which was not detected in the non-parametric approach.

We begin, in Section 2, by illustrating the mechanism behind this underestimation bias in the directional distance function approach coupled with the non-parametric PPF identification. Section 3 then conducts a social efficiency measurement via a parametric estimation of production transformation function using nine-year panel data of rail and aviation sectors in Japan. Finally, Section 4 concludes.

2. Underestimation Mechanism of Social Productive Efficiency via Directional Distance Function Approach with Non-parametric PPF Identification

In the social efficiency benchmarking, the following axioms for the production technology are assumed (see for example, Kuosmanen and Podinovski, 2009):

(i) inputs and good outputs are freely disposable;

(ii) desirable and undesirable outputs are weakly disposable, i.e., they can be decreased proportionally; and

(iii) production possibility set is a convex set.

After specifying the production technology as above, directional distance function approach is typically used to measure the social efficiency. There the distance from the combination of actual outputs and inputs to the production frontier is measured in a specific direction, which in turn gives the inefficiency score of each DMU as the ratio of this distance and the length of the input-output vector. There are mainly three different ways of defining this direction of measuring the distance to the production frontier. First is the direction such that the vector of undesirable outputs is shrunk while the amount of desirable outputs and inputs are held constant; the second is such that the undesirable outputs are decreased and at the same time desirable outputs are increased while inputs are held constant; and finally the third is the direction in that undesirable outputs and inputs are decreased while desirable output is increased.

In the following, we take the second approach (as in HYZ) to show that using the nonparametric method in empirical identification of the production possibility set can lead to a significant underestimation of social inefficiency measurement for the DMUs with high level of undesirable output when the sample size is small or data are not well-scattered.

[INSERT FIGURE 1 HERE.]

Figure 1 sketches the mechanism behind this underestimation bias intuitively for the case where there are three DMUs producing one desirable output and one undesirable output, using the same level of inputs. As one can see, DMU B and DMU C are very inefficient as they generate much more undesirable output than the desirable output. The production frontier is formed as a piece-wise linear lines connecting the origin and these three DMUs, A, B, and C in this order, when nonparametric method is used. Then the directional distance function measures the distance from each DMU to this estimated piece-wise-linear frontier in the direction that desirable output is increasing and undesirable output is decreasing. Then only C becomes inefficient, and both A and B is measured as completely efficient DMUs. However, this is clearly not true, as the true production frontier is way outside of the estimated frontier. DMU B is clearly inefficient, and the inefficiency of DMU C is underestimated as well. This bias will be exaggerated for the DMUs producing more undesirable outputs (i.e., those on the right side in the figure) especially when the data set is small or not scattered well so that the large part (of the upper side in the figure) of the production set is truncated by applying the non-parametric method.

3. Empirical Measurement of the Underestimation Bias

To quantify the significance of above-mentioned underestimation bias in a practical case, we measure the social efficiency of rail and air industries in Japan by using parametric identification of the production technology. Using the same data set as ours, HYZ followed the methodology of Pathomsiri et al. (2008), i.e., a DEA-based non-parametric method of production frontier identification coupled with the directional distance function approach to compute the social inefficiency. After parametrically estimating the production transformation function, we apply the directional distance function approach in the same way as HYZ. Following sections provide description of our data set, and then the comparison of the results obtained through parametric estimation of social productive efficiency with that of non-parametric method by HYZ.

3.1. Data Description

Our data set includes three railway companies namely JR East, JR Central, and JR West, as well as one aggregated DMU for the aviation sector as a whole, for the years from 1999 to 2007; thus 36 observations for four DMUs. Included variables are labor, capital, other variable costs, and aggregated users' time costs as inputs; total passenger-and-cargo kilometers as a desirable output; and the life-cycle CO_2 emission (per each year) as an undesirable output. As for rail, labor and capital are shared between high-speed rail and commuting rail services. Due to this inability of decomposing inputs into these two different uses, data for JR companies include both inter- and intra-city transport services. The aviation industry in our analysis refers to the domestic air-transport services provided by three airlines, Japan Air Lines (JAL), All Nipon Airways (ANA), and Japan Air System (which was merged with JAL, and became part of JAL, in 2002) including their subsidiary airlines. The remaining airlines are excluded from the analysis as these airlines are rather small, covering less than 10 percent of the domestic air transport market. Airline companies do not assume ownership of airport facilities. Airports are developed and operated by the central or regional governments under an airport improvement special account of the Ministry of Land, Infrastructure, Transport and Tourism (MLIT), and they are included as inputs for the aviation sector in our analysis. Also, international and domestic air transport services provided are separated according to the passenger volume.

Desirable output in our analysis is the transport volume and distance, which we measure as work-load-unit kilometers (WLU-km), where one WLU is one passenger or 100 kg of cargo. The data for passenger-km and cargo ton-km are available from annual financial reports. As for the undesirable output, CO2 emissions from operations, infrastructure construction, construction of rolling stocks and aircraft are computed and summed. Unit values are available from *Suji de Miru Koku* (A Quantitative Look at Railways) and the input–output table published by the Center for Global Environmental Research of Japan, and they are:

(i) 113 gram/passenger-km for air transport and 18 gram/passenger-km for railway from their operations;

(ii) 3.612 ton/million Japanese yen (JPY) for the construction of buildings and other infrastructure except railway trackage which is 4.791 ton/million JPY excluding land acquisition costs; and

(iii) 1.549 ton/million JPY for aircraft construction and 4.746 ton/million JPY for construction of train cars.

There are four inputs in our data set. As for the labor input, we use the number of employees found in the transportation business section of annual financial reports. Since this number includes both domestic and international operations for the aviation sector, it is separated according to the share of WLU in domestic services. Capital input is the sum of depreciation found in the balance sheet and the opportunity cost of capital assuming rental rate of 4%, deflated by the domestic corporate good price index. Other variable costs capture the rest of all operational costs, excluding labor costs, depreciation, and lease fees; and its major component is fuel and electricity costs. Other variable costs are calculated based on financial reports, deflated by the domestic corporate goods price index as well. We also include the aggregate users' time cost of travel as an input. For air transport it is the sum of travel time multiplied by the number of passengers on all routes, found in the *Koku Yuso Tokei Chosa* (Annual Report of Air Transport Statistics) by MLIT. For rail, we use the average speed of 200 km/hour for high-speed rail and 60 km/hour for local trains to compute the counterpart from the transport volume data. Table 1 summarizes the data set.

[INSERT TABLE 1 HERE.]

3.2. Estimation and Results

We now turn to the parametric estimation of the production frontier and apply the directional distance function approach to measure the inefficiency scores. For the production transformation function, we assume the following specification that satisfies the axioms presented above:

$$\begin{aligned} & f\left(y_{jt}, b_{jt}, x_{jt}\right) \\ & = \alpha\left(\frac{\tilde{y}_{1jt}}{\tilde{b}_{1jt}}\right)^{\rho} + \beta \tilde{b}_{1jt}{}^{\rho} - X_{jt} \\ & = 0 \end{aligned} \tag{1}$$

for parameters $\alpha, \beta > 0$ and $\rho > 1$, where y_{jt}, b_{jt}, and x_{jt} are, respectively, desirable output, undesirable output, and input vectors for DMU j in time t; X_{jt} is the input index consisted of four inputs in our data; and tilde (~) indicates normalization of the raw data. In our analysis, desirable and undesirable output vectors y_{jt} and b_{jt} are consisted of one variable each, namely y_{1jt} and b_{1jt}, and input vector x_{jt} include four variables x_{ijt} for i=1,…,4. Due to the limited number of data observations and hence to minimize the number of parameters, after normalization we compute the geometric average of four inputs to consist the input index X_{jt}. The normalization is done as follows:

$$\tilde{x}_{ijt} = \exp\left[\frac{\ln x_{ijt} - \overline{\ln x_i}}{s_{\ln x_i}}\right]$$

where upper bar indicates the mean and s indicates the standard deviation for all t and j. The same applies to outputs y_{1jt} and b_{1jt}.

In actual estimation we replace the right-hand side of (1) above with an error term ε_{it} i.e.,

$$\alpha\left(\frac{\tilde{y}_{1jt}}{\tilde{b}_{1jt}}\right)^{\rho} + \beta \tilde{b}_{1jt}{}^{\rho} - X_{jt} - \varepsilon_{it} = 0.$$

Table 2 summarizes the parameter estimates, which satisfy the sign requirements by either 5% or 10% level of statistical significance.

[INSERT TABLE 2 HERE.]

We then find the maximum value of the errors $\varepsilon_{jt} \; \forall j,t$, denoted by, say, $\hat{\varepsilon}^{max}$, and subtract it from *f* to form the production frontier a-la corrected OLS (see for example, Coelli and Perelman (2000)). That is, production possibility set $P = \{(x, y, b) : x \text{ can produce } y \text{ and } b\}$ is such that

$$P = \left\{ (y,b,x) : \alpha \left(\frac{\tilde{y}_1}{\tilde{b}_1} \right)^{\rho} + \beta \tilde{b}_1^{\rho} - X - \hat{\varepsilon}^{max} \leq 0 \right\}$$

with some subscripts abbreviated for notational simplicity. Finally, we adopt the directional distance function approach, as in HYZ, to measure the inefficiency score β^* as follows:

$$\beta^* = \left\{ \max \beta : \left(x, (1+\beta) y, (1-\beta) b \right) \in P \right\}.$$

Pathomsiri et al. as well as HYZ carried out the above analysis in one step by using linear programming. Table 3 presents our inefficiency scores along with the scores obtained via non-parametric method by HYZ.

[INSERT TABLE 3 AND FIGURE 2 HERE.]

Figure 2 compares the inefficiency scores via these two methods diagrammatically. These results conclude that the inefficiency of the aviation sector is heavily underestimated in the non-parametric method. Aviation sector, estimated as extremely efficient in the non-parametric method, is actually as much as 80% inefficient with the use of parametric method. Our data set shows that aviation sector has its CO2 emission several times greater than rail companies, while its output level is comparable to JR East. This indicates that the aviation sector corresponds to the DMUs B or C in Figure 2, and thus that its inefficiency is heavily underestimated due to the above-mentioned limitation of non-parametric method.

Time trends of the inefficiency scores via parametric method seem to be highly correlated with those via non-parametric method. However, for JR Central parametric inefficiency is less volatile than non-parametric inefficiency. This is partly due to increased (econometric) efficiency of production frontier estimation via parametric method, while in the non-parametric method JR Central is essentially benchmarking its inefficiency to itself in year 2007 and not referencing to other DMUs.

It is worth noting that it is not always the case that non-parametric method underestimates inefficiency. JR West, measured inefficient by almost 50% via non-parametric method turned out to be rather efficient via parametric approach. This is resulted from the fact that the estimated production transformation function exhibits, though rather weak, increasing returns to scale, which by construction cannot be captured in the non-parametric approach.

4. Conclusions

We have shown that applying the directional distance-function approach to the non-parametrically identified production possibility frontier causes underestimation of social inefficiency for DMUs with relatively large undesirable outputs, especially when the sample size is small or data are not well-scattered. This is due to the truncation of the true production possibility set with an assumption of piece-wise linear PPF. We proceeded to an empirical analysis to quantify the magnitude of this underestimation in the actual social efficiency benchmarking analysis by parametrically identifying the production transformation function. We used the same data for Japanese domestic inter-city transport services as in Ha, Yoshida, and Zhang (2011), where they utilized a DEA-based non-parametric method of PPF identification instead. We then applied the directional distance function approach just as in HYZ and compared the results. Our parametric approach showed the result, inter alia, that the social productive inefficiency of the aviation sector is as large as 80%, which was not detected by the non-parametric method.

References:

Coelli, T. J. and S. Perelman, 2000, "Technical efficiency of European railways: A distance function approach," *Applied Economics*, Vol. 32, pp. 1967-1976.

Ha, H. K., Y. Yoshida and A. Zhang, 2011, “Social efficiency benchmarking of Japanese domestic transport services: A comparison of rail and air," *Transportation Research Part D*, Vol. 16, pp. 554-561

Kuosmanen, T. and V. Podinovski, 2009, "Weak disposability in nonparametric production analysis: Reply to Färe and Grosskopf," *American Journal of Agricultural Economics*, Vol. 91(2), pp. 539-545.

Luenberger, D. G., 1992, "Benefit functions and duality," *Journal of Mathematical Economics*, Vol. 21, Issue 5, pp. 461-81.

Pathomsiri, S., A. Haghani, M. Dresner and R. J. Windle, 2008, "Impact of undesirable outputs on the productivity of US airports," *Transportation Research Part E*, Vol. 44, pp. 235–259.

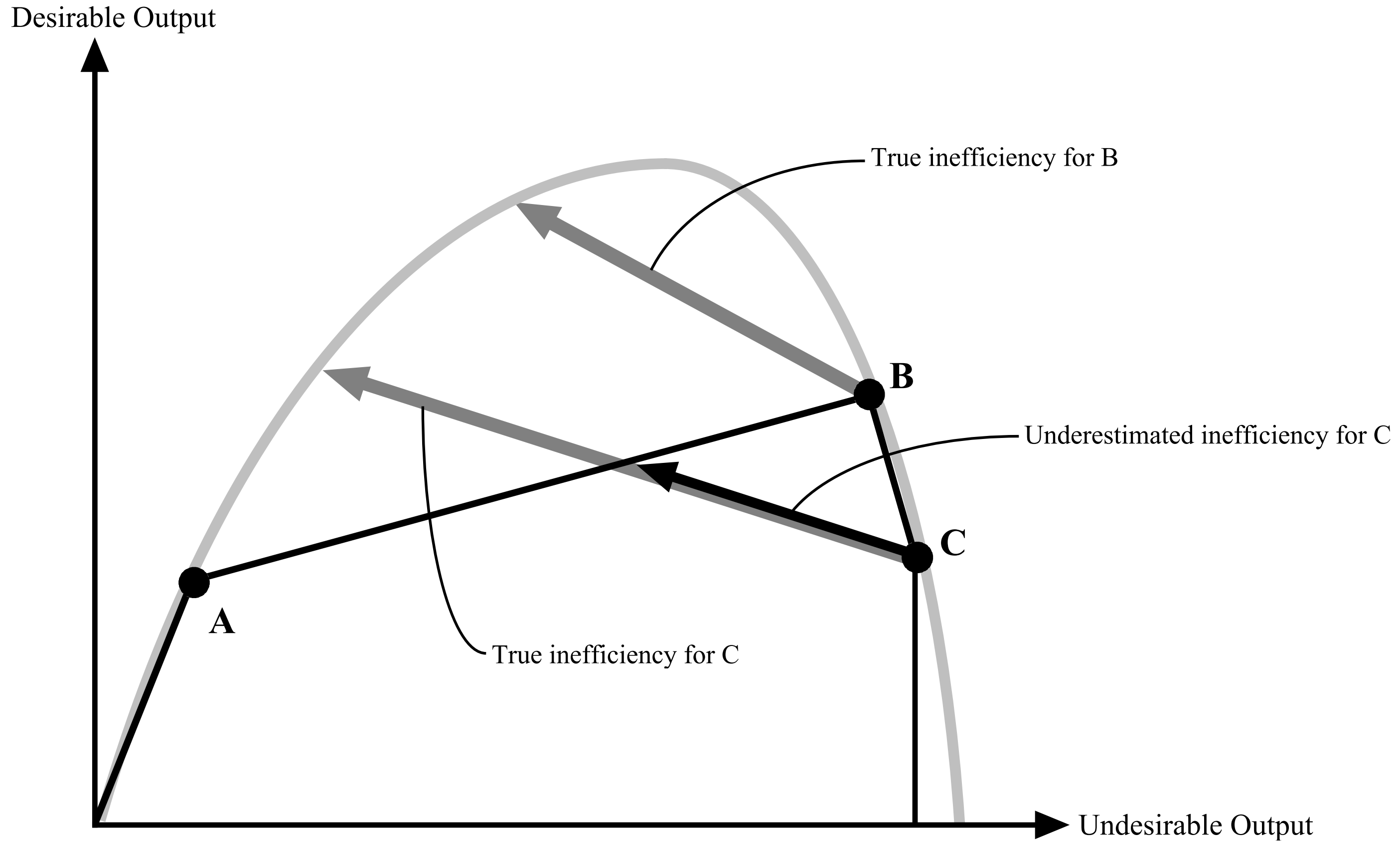

Figure 1. Smooth curve in thick grey color is the true production frontier for a given input level and mix. In the figure there are three DMUs, A, B, and C. Piecewise-linear line represents the production frontier obtained via non-parametric method. Arrows are the measures of inefficiencies.

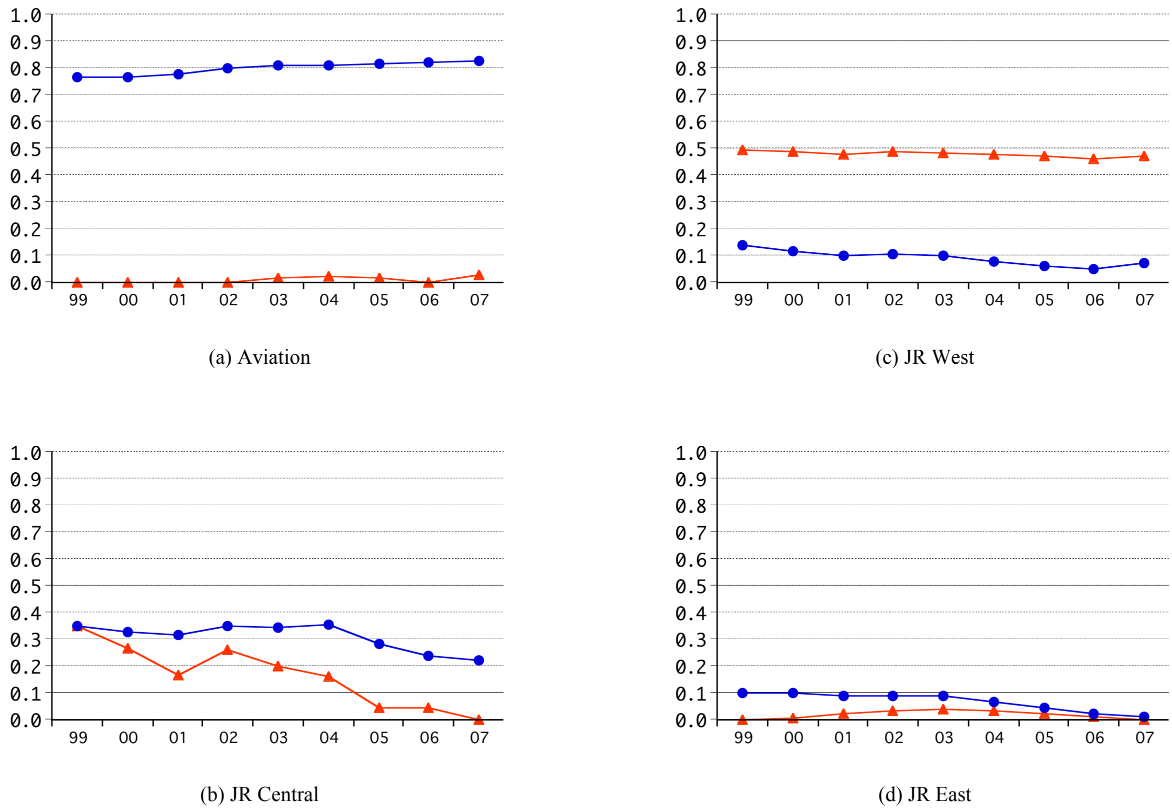

Figure 2. Lines with circles are inefficiency scores obtained via parametric method while those with triangles are for non-parametric method.

Table 1. Data.

	Labor persons	Variable mil. JPY	Capital input mil. JPY	Users' time costs 1,000 hours	WLU-km million	CO2 ton
Aviation 1999	32,782	1,749,463	563,434	127,230	83,139	9,299,170
Aviation 2000	31,313	1,821,488	580,376	126,880	83,108	9,303,329
Aviation 2001	31,616	1,839,533	632,479	129,289	83,878	9,513,629
Aviation 2002	37,192	1,881,454	659,446	127,259	84,367	9,416,551
Aviation 2003	38,682	1,848,044	713,025	124,324	82,835	9,291,921
Aviation 2004	37,011	1,880,306	699,403	121,480	81,568	9,119,615
Aviation 2005	36,854	2,022,876	714,409	124,014	82,561	9,252,501
Aviation 2006	39,067	2,099,588	734,824	124,675	84,018	9,431,629
Aviation 2007	38,998	1,990,249	783,546	121,131	81,861	9,281,719
JR Central 1999	17,244	305,048	412,997	338,815	47,892	1,901,390
JR Central 2000	16,852	319,675	403,871	342,490	48,674	1,896,969
JR Central 2001	16,228	340,944	408,492	346,137	49,533	1,911,445
JR Central 2002	16,696	370,566	412,811	339,997	48,467	1,883,661
JR Central 2003	16,450	391,305	415,351	344,557	49,273	1,911,276
JR Central 2004	16,385	382,737	429,553	350,272	50,478	2,048,525
JR Central 2005	16,428	388,743	400,646	364,061	52,880	2,012,918
JR Central 2006	16,791	411,707	369,357	366,555	53,533	1,929,866
JR Central 2007	17,174	406,325	373,799	380,280	55,812	2,029,520
JR West 1999	34,532	290,949	165,462	707,236	52,588	1,534,027
JR West 2000	33,060	297,552	159,351	704,360	52,551	1,515,529
JR West 2001	30,803	310,573	158,426	703,507	52,647	1,497,586
JR West 2002	29,528	324,156	157,451	691,398	51,674	1,472,760
JR West 2003	28,359	329,171	160,201	696,225	52,142	1,486,123
JR West 2004	27,371	328,101	156,283	697,942	52,544	1,490,077
JR West 2005	26,708	342,029	151,491	696,011	52,828	1,487,823
JR West 2006	26,390	351,914	149,763	706,272	53,679	1,507,854
JR West 2007	26,408	346,334	162,166	711,837	54,585	1,587,140
JR East 1999	63,597	569,649	447,129	1,894,465	125,998	3,928,715
JR East 2000	62,606	571,350	440,790	1,881,854	125,344	3,907,647
JR East 2001	60,087	593,453	441,365	1,873,999	124,916	3,866,778
JR East 2002	59,510	615,283	442,767	1,872,053	125,176	3,845,295
JR East 2003	58,900	630,085	440,435	1,876,137	125,752	3,869,641
JR East 2004	57,236	622,557	425,894	1,870,632	125,172	3,832,916
JR East 2005	55,616	638,442	412,439	1,881,151	126,142	3,812,684
JR East 2006	54,326	644,658	403,107	1,900,463	127,653	3,829,502
JR East 2007	53,511	658,036	409,133	1,942,432	130,558	3,940,599
Average	**35,064**	**803,176**	**416,430**	**766,873**	**78,217**	**4,162,454**

Table 2. Estimation results of the production transformation function.

Variable	Coefficient	(Std. Err.)	[95% Conf.	Interval]
α	0.4647**	(0.0214)	0.4212	0.5082
β	0.2709**	(0.0192)	0.2319	0.3099
ρ	1.0922**	(0.0488)	0.9930	1.1914
N	36			
Ajusted R^2	0.9995			

Table 3. Inefficiency scores obtained by parametric and non-parametric methods.

	Parametric	Non-Parametric
Aviation 1999	0.7667	0.0000
Aviation 2000	0.7676	0.0000
Aviation 2001	0.7802	0.0000
Aviation 2002	0.7977	0.0000
Aviation 2003	0.8129	0.0160
Aviation 2004	0.8106	0.0244
Aviation 2005	0.8146	0.0179
Aviation 2006	0.8204	0.0000
Aviation 2007	0.8297	0.0298
JR Central 1999	0.3525	0.3507
JR Central 2000	0.3294	0.2686
JR Central 2001	0.3151	0.1692
JR Central 2002	0.3496	0.2621
JR Central 2003	0.3436	0.2021
JR Central 2004	0.3575	0.1597
JR Central 2005	0.2814	0.0470
JR Central 2006	0.2374	0.0451
JR Central 2007	0.2205	0.0000
JR West 1999	0.1379	0.4939
JR West 2000	0.1178	0.4881
JR West 2001	0.0974	0.4799
JR West 2002	0.1065	0.4898
JR West 2003	0.0974	0.4856
JR West 2004	0.0766	0.4792
JR West 2005	0.0628	0.4729
JR West 2006	0.0499	0.4637
JR West 2007	0.0729	0.4744
JR East 1999	0.1025	0.0000
JR East 2000	0.0983	0.0072
JR East 2001	0.0916	0.0229
JR East 2002	0.0896	0.0319
JR East 2003	0.0875	0.0368
JR East 2004	0.0691	0.0355
JR East 2005	0.0439	0.0224
JR East 2006	0.0200	0.0097
JR East 2007	0.0112	0.0000