DC four point resistance of a double barrier quantum pump

Federico Foieri[1], Liliana Arrachea[1,2] and María José Sánchez[3].
[1]*Departamento de Física "J. J. Giambiagi" FCEyN, Universidad de Buenos Aires, Ciudad Universitaria Pab.I, (1428) Buenos Aires, Argentina,*
[2] *BIFI, Universidad de Zaragoza, Pedro Cerbuna 12, 50009, Zaragoza, Spain,*
[3]*Centro Atómico Bariloche and Instituto Balseiro, Bustillo 9500 (8400), Bariloche, Argentina.*
(Dated: April 29, 2009)

We investigate the behavior of the dc voltage drop in a periodically driven double barrier structure (DBS) sensed by voltages probes that are weakly coupled to the system. We find that the four terminal resistance R_{4t} measured with the probes located outside the DBS results identical to the resistance measured in the same structure under a stationary bias voltage difference between left and right reservoirs. This result, valid beyond the adiabatic pumping regime, can be taken as an indication of the universal character of R_{4t} as a measure of the resistive properties of a sample, irrespectively of the mechanism used to induce the transport.

PACS numbers: 72.10.-Bg,73.23.-b,73.63.Nm

Quantum transport induced by time dependent fields attracts presently an impressive amount of research. A phase coherent conductor subjected to two periodically varying voltages becomes a paradigmatic example of a quantum pump, in which a dc current can be generated in the absence of a net external bias[1,2,3,4].

After the works of Landauer and Buttiker,[5,6] the four point resistance R_{4t} is considered as the proper measure of the genuine resistive behavior of a mesoscopic sample, free from the effects of the contact resistance. Several theoretical works have been devoted to study the details of the voltage drop between the contacts in systems where the transport is induced by means of a stationary dc voltage bias[7]. Furthermore, the striking feature that R_{4t} can be negative in a coherent conductor has been experimentally observed in semiconductors[8] and in carbon nanotubes[9]. However, the behaviour of R_{4t} in the case of a quantum pump has not been so far analyzed.

The aim of the present work is to investigate to what an extent the concept of R_{4t} could depend on the underlying driving mechanism. To this end, we consider as a model of the quantum pump, a quantum wire coupled to left and right reservoirs at a fixed chemical potential and with two narrow gates to which oscillating voltages are applied with a phase-lag. This set up (see Fig.1 upper plot) mimics the actual double barrier structure (DBS) used in Ref.1 where two of such ac potentials were applied at the walls confining a quantum dot. Experimentally, the dc response is actually inferred from the measurement of the voltage drop between two extra probes, one located at the left and the other at the right of the DBS. Accordingly, we consider non-invasive voltage probes weakly coupled to the wire. The chemical potential μ_i $(i = P, P')$ of each probe is adjusted to maintain zero net current through the respective contact. Our goal is to investigate the dc four terminal resistance for the quantum pump defined as the dc voltage drop $\Delta\mu_{PP'}$ sensed by the two probes P, P' connected at two arbitrary points along the sample, divided by the dc component of the current J^{dc} flowing through the device :

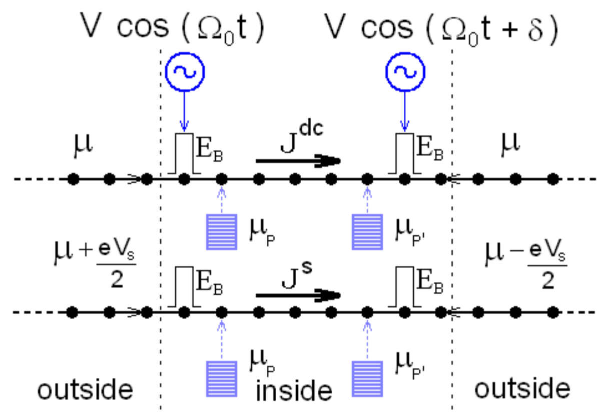

FIG. 1: (Color on-line) Scheme of the set up. The central device is a wire with two barriers of height E_B connected to L and R reservoirs. Two voltage probes P and P' sense the voltage drop. Upper plot: Pumping set up in which a dc current J^{dc} is induced by two local ac voltages. The L and R reservoirs are at the same chemical potential μ. Lower plot: Stationary set up, in which the current J^s is induced by a dc voltage difference V_s between L and R reservoirs. See text for more details.

$$R_{4t} = \frac{\Delta\mu_{PP'}}{J^{dc}}. \tag{1}$$

We will also compare this quantity with the four terminal resistance obtained when the current through the DBS is induced by a slight stationary voltage difference V_s between the left (L) and right (R) reservoirs, as indicated in the lower panel of Fig.1.

For sake of clarity we start considering just one voltage probe, which is modeled as a third reservoir coupled to the central system at the position P (the extension to more probes is trivial). The corresponding Hamiltonian for the full system reads:

$$H = H_{leads} + H_P + H_C(t) - w_L(a_L^\dagger c_1 + H.c.) \\ -w_R(a_R^\dagger c_N + H.c.) - w_P(a_P^\dagger c_p + H.c.) \qquad (2)$$

with $H_C(t)$ denoting the Hamiltonian for the central piece that we model as a 1d tight-binding chain of length N with two barriers located at sites A and B:

$$H_C(t) = V\cos(\Omega_0 t + \delta) c_A^\dagger c_A + \sum_{l=1}^{N} \varepsilon_l c_l^\dagger c_l \\ -w_h \sum_{l=1}^{N} (c_l^\dagger c_{l+1} + H.c) + V\cos(\Omega_0 t) c_B^\dagger c_B, \qquad (3)$$

with w_h the hopping parameter and the profile $\varepsilon_A = \varepsilon_B = E_b$, $\varepsilon_l = 0, l = 1, \ldots, N \neq A, B$, defining a double barrier structure. The time dependent ac potentials act locally at the position of the barriers and have amplitude V, frequency Ω_0 and oscillate with a phase difference δ. We denote with H_{leads} the Hamiltonians of two semi-infinite tight-binding chains with hopping w_l at the same chemical potential μ, which play the role of the L and R reservoirs. These two leads are connected to the central device at sites $1, N$ respectively. Similarly, H_P is the hamiltonian of the voltage probe P that we also model as a particle reservoir with a chemical potential μ_P that is fixed to satisfy the condition of net zero dc current through its contact to the central system[6]. The contacts between the central system and the L and R leads and the probe P are described by the last three terms of Eq.(2), where the fermionic operators a_α ($\alpha = L, R, P$) denote degrees of freedom for the L, R and P reservoirs respectively.

We employ the formalism of Keldysh non equilibrium Green's functions, which is a convenient tool in transport theory on multiterminal structures driven by time-periodic fields. Following Ref.4 we use the Floquet representation $G^R_{l,l'}(t,\omega) = \sum_{k=-\infty}^{\infty} e^{-ik\Omega_0 t} \mathcal{G}_{l,l'}(k,\omega)$, where $G^R_{l,l'}(t,\omega)$ is the Fourier transform with respect to $t - t'$ of the retarded Green's function. The dc component of the charge current flowing through the contact between the central system and the probe P, can be written (in units of e/h) as[4]:

$$J_P^{dc} = \sum_{\alpha=L,P,R} \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \{\Gamma_\alpha(\omega)\Gamma_P(\omega + k\Omega_0) \\ |\mathcal{G}_{l_P,l_\alpha}(k,\omega)|^2 \left[f_\alpha(\omega) - f_P(\omega + k\Omega_0)\right]\}, \qquad (4)$$

where l_α are the sites of the central system at which the reservoirs $\alpha = L, R, P$ are attached, while $\Gamma_\alpha(\omega) = |w_\alpha|^2 \rho_\alpha(\omega)$ is the spectral function associated to the self-energies due to the coupling to these reservoirs, $\rho_\alpha(\omega)$ is the corresponding density of states and $f_\alpha(\omega) = 1/(e^{\beta_\alpha(\omega-\mu_\alpha)} + 1)$ is the Fermi function of the reservoir α, which we assume to be at the temperature $1/\beta_\alpha = 0$.

The voltage profile sensed by the probe can be exactly evaluated under general conditions from the solution μ_P that satisfies $J_P^{dc} = 0$ in the above expression. It is, however, instructive to analyze first the case of low driving frequency Ω_0 and small pumping amplitude V, which corresponds to the so called adiabatic pumping regime[2]. As we already mentioned, we are considering "non-invasive probes". This corresponds to probes weakly coupled to the central system, in such a way that they do not introduce neither inelastic nor elastic scattering processes for the electronic propagation between L and R reservoirs. Below we derive an analytical expression, valid under these conditions, for the voltage profile μ_P.

For low V, the perturbative solution of the Dyson's equation up to the second order in this parameter contains only the following Floquet components that contribute to the dc current[4]:

$$\mathcal{G}_{l,l'}(\pm 1, \omega) \sim \frac{V}{2}[G^0_{l,A}(\omega \mp \Omega_0) G^0_{A,l'}(\omega) \\ + e^{\pm i\delta} G^0_{l,B}(\omega \mp \Omega_0) G^0_{B,l'}(\omega)]. \qquad (5)$$

For weak coupling to the probes, Eq.(4) is evaluated with Green's functions up to the 1st order in w_P. This corresponds to consider the functions $G^0_{l,l'}(\omega)$ in (5), as the equilibrium retarded Green's functions of the central system attached only to the L and R reservoirs. For perfect matching to the reservoirs ($w_L = w_R = w_l = w_h$) and for barriers with low amplitude $E_B \leq w_h$, these functions can be written in the following simple form: $G^0_{l,l'}(\omega) = g_{l,l'}(\theta) + E_B \sum_{j=A,B} g_{l,j}(\theta) g_{j,l'}(\theta)$, with $g_{l,l'}(\theta) = i e^{-i|l-l'|\theta}/(2w_h \sin\theta)$, being $\omega = 2w_h\cos\theta$[10]. Using them to evaluate (5), substituting the result in (4) and considering the adiabatic ($\propto \Omega_0$) contribution in the resulting J_P^{dc} we get two different results depending on the place at which the probe is connected:

$$\begin{aligned} \mu_P &= \mu \pm \Omega_0 V^2 \sin\delta[\alpha^o(k_F) \\ &\quad + E_B \beta^o(k_F)], \quad x_P > x_B, x_P < x_A, \\ \mu_P &= \mu + \Omega_0 V^2 \sin\delta[\alpha^i(k_F, x_P) \\ &\quad + E_B \beta^i(k_F, x_P)], \quad x_A < x_P < x_B, \end{aligned} \qquad (6)$$

with $x_j (j = A, B, P)$ denoting the position of the barriers and probe in units of the lattice parameter of the tight-binding model. The upper and lower sign of the first identity corresponds, respectively, to the voltage probe located at the left ($x_P < x_A$) and right ($x_P > x_B$) side of the DBS, while the second identity corresponds to the voltage probe located between the two barriers. We have defined the Fermi vector (in units of the lattice parameter) as $k_F \equiv \theta(\mu)$ as well as the following functions:

$$\begin{aligned} \alpha^o(k_F) &= \frac{\sin[2k_F(x_A - x_B)]}{2(w_h \sin k_F)^2}, \\ \beta^o(k_F) &= \frac{\sin^2[k_F(x_A - x_B)]}{4(w_h \sin k_F)^3}, \\ \alpha^i(k_F, x_P) &= \sin[k_F(2x_P - x_A - x_B)]\alpha^o(k_F), \\ \beta^i(k_F, x_P) &= \sin[k_F(2x_P - x_A - x_B)]\beta^o(k_F), \end{aligned} \qquad (7)$$

where the superscripts $o, (i)$ stress that the probe senses points outside (inside) the region where all the scattering

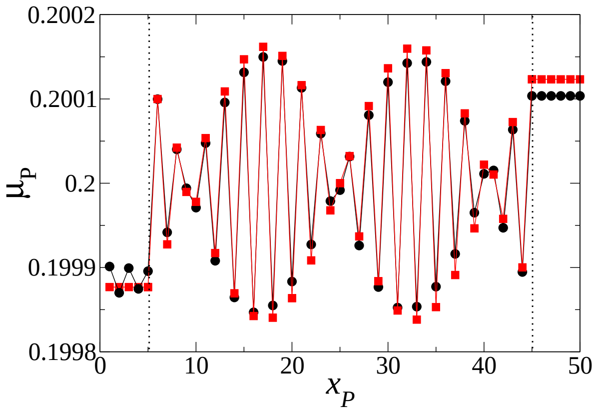

FIG. 2: (Color on-line) Local voltage μ_P sensed by the voltage probe P as a function of the probe position x_P along a DBS composed by $N = 50$ sites with two barriers of height $E_B = 0.2$ located at $x_A = 5$ and $x_B = 45$ as indicated by the vertical dashed lines. The pumping parameters are $V = 0.01$, $\Omega_0 = 0.01$ and $\delta = \pi/2$. Red squares correspond to Eq.(6), the analytical solution for the adiabatic pumping regime and a weakly connected probe. Black circles correspond to the exact numerical solution obtained equating Eq. (4) to zero with $w_P = 0.01$. The chemical potential is $\mu = 0.2$, which corresponds to $k_F = 1.47$.

processes (dynamical as well as stationary) take place. In the simple model we are considering, with a perfect matching between the central system and the reservoirs, this coincides with the DBS, as indicated in Fig. 1. Equations (6) and (7) tell us that the local voltage sensed by a probe is constant outside the region defined by the DBS, while it presents the characteristic pattern of Friedel oscillations[7] with a period $2k_F$ at positions lying between the two oscillating barriers. Fig. 2 shows the benchmark of the analytical result, Eq.(6), against the exact voltage profile obtained numerically from Eq. (10) in the regime of weak V, Ω_0 and w_P, and a moderate E_B. A good agreement of the qualitative behavior is observed. In particular, the exact profile μ_P exhibits Friedel oscillations with period $2k_F$ as a function of the probe position x_P as predicted by Eq.(6) and only a slight disagreement is found in the amplitude of the envelope function.

To the lowest order of perturbation in the coupling constant w_P, the effect of an additional second voltage probe P' is completely uncorrelated from the first one, since the associated interference effects involve second order processes in w_P. At this level of approximation let us call $\mu_{P'}$ the local voltage sensed by the additional probe at P' and $\Delta\mu_{PP'} \equiv \mu_{P'} - \mu_P$ the corresponding voltage drop. In a set up with the probe P (P') located at the left (right) side of the DBS, the voltage drop between both probes is from Eq.(6):

$$\Delta^o\mu_{PP'} = 2\Omega_0 V^2 \sin\delta[\alpha^o(k_F) + E_B\beta^o(k_F)]. \quad (8)$$

Another possible measurement corresponds to locate the voltage probes P and P' inside the DBS. In this case, the voltage drop between the two probes explicitly depends on the probe positions x_P and $x_{P'}$ as follows:

$$\Delta^i\mu_{PP'} = 2\Omega_0 V^2 \sin\delta \left[\frac{\alpha^o(k_F) + E_B\beta^o(k_F)}{\sin(k_F(x_A - x_B))}\right] \times \quad (9)$$
$$\cos\{k_F[(x_P - x_A) + (x_{P'} - x_B)]\} \sin[k_F(x_{P'} - x_P)] \, ,$$

where, as before, we have employed the superscripts o, (i) to distinguish configurations with the probes outside (inside) the DBS.

Under the conditions assumed in the derivation of Eq.(6), i.e. low V, Ω_0, w_P and E_B, the dc current flowing through the DBS reads:

$$J^{dc} \cong 2\Gamma_L^0\Gamma_R^0\Omega_0 V^2 \sin\delta \left[\frac{\alpha^o(k_F) + E_B\beta^o(k_F)}{(2w_h \sin k_F)^2}\right], \quad (10)$$

with $\Gamma_\alpha^0 \equiv \Gamma_\alpha^0(\mu), (\alpha = L, R)$. We can now compute the dc four terminal resistance R_{4t} in an adiabatic weakly driven pumping process. For a set up in which the probes are located outside the DBS ($x_P < x_A, x_B < x_{P'}$) it reads:

$$R_{4t}^o = \frac{\Delta^o\mu_{PP'}}{J^{dc}} = \frac{(2w_h \sin k_F)^2}{\Gamma_L^0\Gamma_R^0} \, , \quad (11)$$

while it is:

$$R_{4t}^i = \frac{\Delta^i\mu_{PP'}}{J^{dc}}$$
$$= E_B R_{4t}^o \frac{\sin[k_F(x_{P'} - x_P)]}{\sin[k_F(x_A - x_B)]}$$
$$\times \cos[k_F(x_P - x_A + x_{P'} - x_B)] \, , \quad (12)$$

for a set up in which the probes are located inside the DBS ($x_A < x_P < x_{P'} < x_B$).

We now turn to compare the value of R_{4t} obtained for the quantum pump with the resistance of the same DBS when the transport is induced through a stationary bias voltage V_s established by a difference in the electrochemical potentials of L and R reservoirs $\mu_L = \mu + eV_s/2$ and $\mu_R = \mu - eV_s/2$, as is depicted in the lower panel of Fig.1. Under the conditions of non-invasive probes and for linear response in V_s, it is possible to implement the same kind of perturbative procedure as before, but for the stationary Green's functions. Assuming again perfect matching between the DBS and the L and R reservoirs and $E_B < w_h$, we derive the following simple expression for the current flowing through the device,

$$J^s = \frac{\Gamma_L^0\Gamma_R^0 V_s}{(2w_h \sin k_F)^2} \, , \quad (13)$$

which is the stationary counterpart of Eq.(10). ¿From the above expression we compute V_s/J^s and arrive immediately to the important identity:

$$R_{4t}^o \equiv \frac{V_s}{J^s} \, , \quad (14)$$

which tells that the dc four point resistance measured in the quantum pump when the probes are connected outside the DBS R_{4t}^o (Eq.(11)) exactly coincides with the *total resistance* of the structure measured under stationary bias, provided that the driving condition corresponds to linear response in the stationary setup and the adiabatic regime in the pumping setup. At this point, it is important to recall that in the outside configuration the two probes enclose the whole region where all the scattering processes and, therefore, the full voltage drop V_s applied in the stationary setup takes place.

On the other hand, the counterpart of Eq. (12) for the stationary configuration reads:

$$R_{4t}^{s,i} \equiv R_{4t}^{i} \frac{\sin[k_F(x_A - x_B)]}{\sin k_F}, \qquad (15)$$

which means that inside the DBS the dc resistance for the pumping setup differs from that under stationary driving just in a geometrical factor.

Besides the relevance of the analytical results it is valuable to analyze the response of the system beyond the adiabatic pumping condition. For that purpose we have performed extensive numerical calculations of the dc current flowing through the system J^{dc} and the potential drop sensed by the voltage probes along the DBS, as a function of the pumping frequency Ω_0. In the left panel of Fig.(3) we plot the chemical potential μ_P as a function of Ω_0 for different locations of the probe x_P. In addition, in the right panel we have plotted the voltage drop $\Delta\mu = \mu_{P'} - \mu_P$ between two probes located outside the DBS (circles) and inside the DBS (squares).

When the frequency Ω_0 is close to the energy difference between two neighbouring levels of the DBS, the latter become mixed by the pumping potential which causes an inversion in the sign of the dc current. A rough estimate for the frequency at which such a resonant condition is achieved in the example of Fig. 3 casts $\Omega_0 \sim 0.22$. In good agreement, we find an inversion in the sign of $\Delta^o\mu_{PP'}$ for probes connected outside the DBS at $\Omega_0 \sim 0.25$ (plots in circles of Fig. 3). Moreover, the voltage drop between two points outside the DBS in this case results identical to the product of the dc current J^{dc} times the value of the four point resistance R_{4t}^o obtained in the adiabatic pumping regime in Eq.(11). In other words, our results indicate that even for pumping frequencies Ω_0 beyond the adiabatic regime, it is still possible to unambiguously define R_{4t}^o as the value obtained under the adiabatic approximation (Eq.(11)). On the other hand, the voltage drop measured inside the DBS coincides with the product of R_{4t}^i times J^{dc} only within the adiabatic pumping regime, *i.e* when $\Delta^i\mu_{PP'}$ depends linearly on Ω_0.

In conclusion we have shown that the four terminal resistance R_{4t}^o measured in a pumping set up *for probes located outside the DBS* coincides with the total resistance of the structure measured under stationary bias. This result can be taken as an indication of the universal character of R_{4t}^o as a concept to characterize the resistive properties of a system. Our calculation could be extended to the case where the probes themselves are subjected to ac voltages.

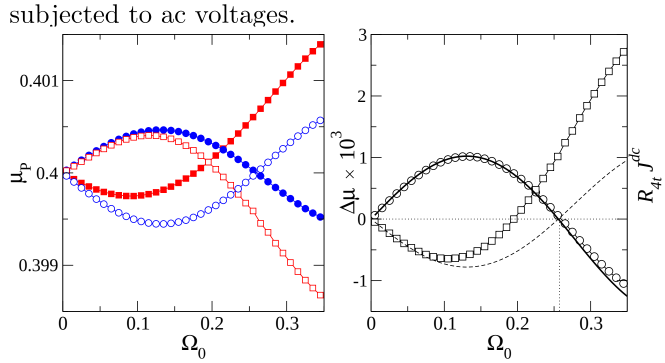

FIG. 3: (Color on-line) Left panel: Exact local voltage μ_P sensed at $x_P = 2, 14$ (empty symbols) and $x_{P'} = 26, 38$ (solid symbols), as a function of the pumping frequency Ω_0 obtained numerically equating Eq. (4) to zero. Circles (squares) correspond to points outside (inside) the DBS. Parameters are: $\mu = 0.4$, $N = 40$, $E_B = 0.2$, $x_A = 10$ and $x_B = 30$. Right panel: The corresponding potential drops $\Delta^{o,i}\mu_{PP'}$. The products $R_{4t}^{o,i} J^{dc}$ (solid and dashed lines, respectively) between the exact current J^{dc} and the resistances evaluated from Eqs. (11) and (12), respectively. The vertical dotted line indicates the frequency of the pumping at which current inversion occurs.

We acknowledge support from CONICET, PICT 0313829 (MJS), UBACYT (FF and LA), Argentina.

[1] M. Switkes, *et al* Science **283**, 1905 (1999).

[2] L. DiCarlo, *et al*, Phys. Rev. Lett. **91**, 246804 (2003); M. D. Blumenthal, *et al* , Nature Physics **3**, 343 (2007); P. W. Brouwer, Phys. Rev. B **58**, R10135 (1998); I.L. Aleiner and A.V. Andreev, Phys. Rev. Lett. **81**, 1286 (1998); F. Zhou, *et al* , Phys. Rev. Lett. **82**, 608 (1999); J.E. Avron, *et al* , Phys. Rev. Lett. **87**, 236601 (2001); V. Kashcheyevs *et al*, Phys. Rev. B **69**, 195301 (2004); J. Splettstoesser, *et al*, Phys. Rev. Lett. **95**, 246803 (2005); S. Kim *et al.*, Phys. Rev. B **73**, 075308 (2006); E. Faizabadi, Phys. Rev. B **76**, 075307 (2007); V. Moldoveanu *et al* , Phys. Rev. B **76**, 165308 (2007); Amit Agarwal *et al.*, Phys. Rev. B **76**, 035308 (2007).

[3] M. Moskalets and M. Büttiker, Phys. Rev. B **66**, 205320 (2002); Phys. Rev. B **69**, 205316 (2004); Phys. Rev. B **78**, 035301 (2008).

[4] L. Arrachea, Phys. Rev. B **72**, 125349 (2005); L. Arrachea and M. Moskalets, Phys. Rev. B **74**, 245322 (2006).

[5] R. Landauer, Philos. Mag. **21**, 863 (1970).

[6] M. Büttiker, Phys. Lett. **57**, 1761 (1986). M. Büttiker, IBM J. Res. Dev, **32**, 317 (1988).

[7] J. L. D'Amato and H. M. Pastawski, Phys. Rev. B **41**, 7411 (1990); V. A. Gopar *et al., ibid* **50**, 2502 (1994); T. Gramespacher and M. Büttiker, Phys. Rev. B **56**, 13026 (1997); L. Arrachea *et al*, Phys. Rev. B **77**, 233105 (2008).

[8] R. de Picciotto, et al , Nature **411**, 51 (2001).

[9] B. Gao, et al , Phys. Rev. Lett. **95** 196802 (2005).
[10] F. Sols at al., J. App. Phys. **66**, 3892 (1989).