Redundancy of classical and quantum correlations during decoherence

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(Received 31 March 2009; published 22 October 2009)

We analyze the time dependence of entanglement and total correlations between a system and fractions of its environment in the course of decoherence. For the quantum Brownian motion model, we show that the entanglement and total correlations have rather different dependence on the size of the environmental fraction. Redundancy manifests differently in both types of correlations and can be related with induced classicality. To study this, we present a measure of redundancy and compare it to the existing one.

DOI: 10.1103/PhysRevA.80.042111 PACS number(s): 03.65.Yz, 03.67.Mn, 03.65.Ta

I. INTRODUCTION

Decoherence is a physical process taking place when a system interacts with its environment. This process is crucial to understand the origin of the classical domain from a fundamentally quantum substrate [1]. Absence of macroscopic superpositions is one of the essential features of the classical realm which is explained by decoherence as a consequence of an effective superselection rule that prevents the stable existence of the vast majority of states in the Hilbert space. In recent years, important developments enabled us to understand the origin of another defining property of classical systems: the fact that they exist in an objective state, a state of their own. Zurek and co-workers made this idea precise by noticing that the emergence of classicality is connected with the existence of redundant records of the state of the system imprinted in the environment. Redundancy turns out to be the key notion to define objectivity and classicality. A system behaves classically when different fractions of any environment correlate with it in the same way (i.e., when correlations become redundant). Such consensus about the state of the system characterizes the classical realm and enables a definition of objectivity. In their pioneering work, Zurek and co-workers [2–6] analyzed the emergence of redundancy in the total correlations between the system and the environment which can be measured by using information theoretic tools such as mutual information. Here, we will examine how does redundancy manifest in purely quantum correlations (entanglement) established between the system and fractions of the environment in the course of decoherence. We will present a definition of redundancy based on the entanglement and show that the way in which redundancy manifests in total and quantum correlations is rather different.

We will focus on the paradigmatic quantum Brownian motion (QBM) model [7,8] where a particle $\mathcal{S}$ interacts with an environment $\mathcal{E}$ that can be split into a fraction $\mathcal{E}_f$ and its complement $\mathcal{E}_{1-f}$ (f parameterizes the size of the fraction $\mathcal{E}_f$ with respect to $\mathcal{E}$ ranging between 0 and 1). Total correlations between $\mathcal{S}$ and $\mathcal{E}_f$ can be measured with the mutual information $\mathcal{I}(\mathcal{S},\mathcal{E}_f)$ (computed from the reduced state obtained after tracing out the complement $\mathcal{E}_{1-f}$). On the other hand, quantum correlations can be quantified with an entanglement measure between $\mathcal{S}$ and $\mathcal{E}_f$, denoted as $E^{(f)}$. Here we will study both $\mathcal{I}(\mathcal{S},\mathcal{E}_f)$ and $E^{(f)}$ as a function of time and f for random choices of the components of $\mathcal{E}_f$ out of the total $\mathcal{E}$.

The paper is organized as follows. In Sec. II, we present the model and describe the temporal evolution of the correlations between the system and the different portions of the environment. We consider two types of splittings of the environment in different fractions. First we analyze a splitting into bands of oscillators (characterized by their frequencies) and then we analyze grouping of such bands into random fractions (characterized by their size). In Sec. III, we derive analytical results for the mutual information and entanglement in the nondissipative regime. In Sec. IV, we study the evolution of redundancy of total correlations and quantum correlations. In Sec. V, we summarize our results.

II. DYNAMICS OF CORRELATIONS IN QUANTUM BROWNIAN MOTION

In the QBM model, a central particle $\mathcal{S}$ is coupled to an environment $\mathcal{E}$ formed by harmonic oscillators q_n ($n=1,\ldots,N$). The total Hamiltonian is $H=H_{\mathcal{S}}+H_{\mathcal{S}\mathcal{E}}+H_{\mathcal{E}}$, where $H_{\mathcal{S}}=p^2/2m+m\omega_{\mathcal{S}}^2x^2/2$, $H_{\mathcal{E}}=\Sigma_n(\frac{\pi_n^2}{2m_n}+\frac{m_n}{2}w_n^2q_n^2)$, and $H_{\mathcal{S}\mathcal{E}}=x\Sigma_nc_nq_n$ (we use $\hbar=1$ throughout). The spectral density $J(\omega)=\Sigma_nc_n^2\delta(\omega-\omega_n)/2m_n\omega_n$ determines the effect of $\mathcal{E}$ on $\mathcal{S}$. We consider a realistic family of environments where $J(\omega)=2m\gamma_0\omega(\omega/\Lambda)^{n-1}\theta(\Lambda-\omega)/\pi$ that includes Ohmic ($n=1$), super-Ohmic, ($n>1$), and sub-Ohmic ($n<1$) members (Λ is a high-frequency cutoff, γ_0 a coupling strength, and $\theta(x)$ the Heaviside step function). We assume that the initial state of $\mathcal{E}$ is the ground state of $H_{\mathcal{E}}$ and that system-environment correlations vanish initially; when the interaction is switched on correlations are created between system and environment [9]. Quantum mutual information (MI) $\mathcal{I}(\mathcal{S},\mathcal{E}_f)$ [10,11] is defined from the von Neumann entropies of the corresponding states $\mathcal{I}(\mathcal{S},\mathcal{E}_f)=\mathcal{H}(\mathcal{S})+\mathcal{H}(\mathcal{E}_f)-\mathcal{H}(\mathcal{S},\mathcal{E}_f)$ (here, $\mathcal{H}(\mathcal{S})=-\mathrm{Tr}[\rho_{\mathcal{S}}\ln\rho_{\mathcal{S}}]$, etc.). The reduced state $\rho_{\{\mathcal{S},\mathcal{E}_f\}}$ is obtained after tracing out the complementary environment $\mathcal{E}_{1-f}$. We restrict to initial Gaussian states which, due to linearity, remain Gaussian for all times and can be described efficiently by a covariance matrix. In such case, analytic expressions for $\mathcal{I}(\mathcal{S},\mathcal{E}_f)$ can be obtained. In order to quantify the quantum correlations, we will use the multipartite logarithmic negativity [12] as a measure of entanglement between $\mathcal{S}$ and $\mathcal{E}_f$. This is defined as the maximum between zero and $-\Sigma_{i:\nu_i<1/2}\log(2\tilde{\nu}_i)$, where $\tilde{\nu}_i$ are the symplectic eigenvalues of

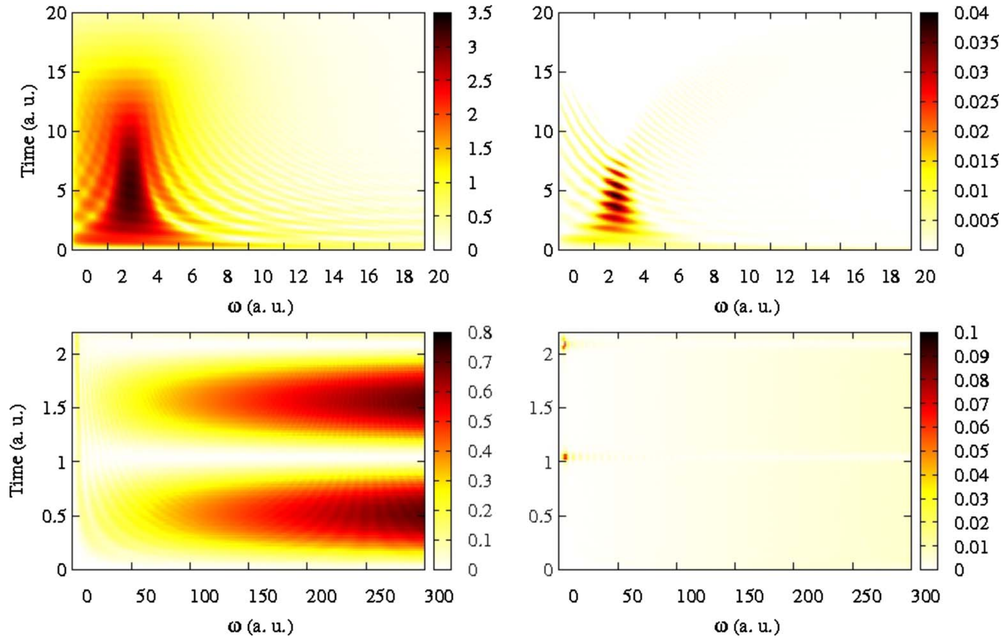

FIG. 1. (Color online) Mutual information (left) and logarithmic negativity (right) between a Brownian particle and bands of oscillators of the environment of frequency ω. The initial state of the system is squeezed in position ($r=-5$). Plots at the top correspond to a sub-Ohmic environment while the ones at the bottom correspond to the super-Ohmic case. Ohmic environment behaves like sub-Ohmic one.

the partially transposed covariance matrix corresponding to the Gaussian state $\rho_{\{\mathcal{S},\mathcal{E}_f\}}$.

We present first numerical results and later we show analytic expressions that reproduce them with high accuracy. To do numerics, we use a discrete version of the model [5,13] with 600 modes (we checked that our results are stable as this number is increased) and coupling strengths c_n^2 proportional to the spectral density. We set all masses to unity and take $\gamma_0=0.1$, the renormalized frequency $\Omega_{\mathcal{S}}=3$, the cutoff frequency $\Lambda=20$ for dissipative environments ($n=1,1/2$), and we use a high cutoff, $\Lambda=300$, for the nondissipative one ($n=3$). The initial state of the system is a pure Gaussian squeezed state with a squeezing parameter $r=\ln(m\Omega_{\mathcal{S}}\Delta x/\Delta p)=-5$ (unless it is specified) and $\Delta x\Delta p=1/2$.

A. Correlations with bands of oscillators

Mutual information and entanglement between the system and environmental bands of frequency ω are shown in Fig. 1. MI develops fast and for short times concentrates at high frequencies. For dissipative environments (sub-Ohmic case, $n=1/2$) at later times, the resonant band is dominant. After a few relaxation times, the resonant peak is washed out and all oscillators in the environment tend to play a similar role. The entanglement has a similar behavior except for the super-Ohmic case. The resonant band of the environment is dominant for entanglement and is also washed out by dissipation (contrary to the case of mutual information, entanglement with nonresonant oscillators seems to be always negligible). One of our findings is that the super-Ohmic environment displays recoherence effects which manifest as oscillatory behavior both for the mutual information and entanglement. In this case, entanglement only develops with resonant oscillators. Thus, the super-Ohmic environment does not induce truly dissipative effects but is responsible for a renormalization (dressing) of the system as well as for coherent oscillations which will also be visible in our forthcoming studies. It is worth pointing out that the existence of a perfect reversion for the super-Ohmic environment only takes place for some initial states of the system. If instead of the above state we choose one with a large momentum squeezing, complete recoherence is lost (see below). This was already noticed in [14] and can be attributed to a process taking place at ultrashort time scales, as will be evident below.

B. Correlations with fractions of the environment

We use "partial information plots" (PI plots) and "partial entanglement plots" (PE plots) to study the evolution of correlations between $\mathcal{S}$ and fractions $\mathcal{E}_f$ of variable size f. They are plots of the average mutual information (average of MI over random choices of fractions $\mathcal{E}_f$ of the same size) and average entanglement between the $\mathcal{S}$ and $\mathcal{E}_f$, respectively. The evolution of the PI plots for different spectral densities are shown in Fig. 2 (we verified that for the Ohmic and sub-Ohmic cases, the results are similar), where for simplicity we subtract the system's entropy and plot $\mathcal{I}(\mathcal{S},\mathcal{E}_f)-\mathcal{H}(\mathcal{S})$.

As the total state is pure, mutual information is $\mathcal{I}(\mathcal{S},\mathcal{E}_f)=\mathcal{H}(\mathcal{S})+\mathcal{H}(\mathcal{S},\mathcal{E}_{1-f})-\mathcal{H}(\mathcal{S},\mathcal{E}_f)$, i.e., it is symmetric around $f=1/2$ [4]. This means that by adding the mutual information between the system and two complementary fractions of the environment, one always obtains the maximal available information $\mathcal{I}(\mathcal{S},\mathcal{E})=2\mathcal{H}(\mathcal{S})$ [i.e., $\mathcal{I}(\mathcal{S},\mathcal{E}_f)+\mathcal{I}(\mathcal{S},\mathcal{E}_{1-f})=2\mathcal{H}(\mathcal{S})$]. Redundancy clearly manifests in the PI plots. There is a sharp growth of the mutual information for small f followed by a plateau that indicates that mutual informa-

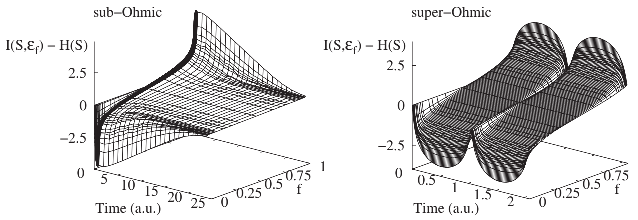

FIG. 2. PI plots for a dissipative (sub-Ohmic) environment (left) and nondissipative (super-Ohmic) one (right). Initial state is squeezed in momentum with $r=-5$. The super-Ohmic case displays clear recoherence events for which the system and the environment loose their correlations.

tion becomes almost independent of the fraction f.

As PI plots are not completely flat around $f=1/2$, there are also some nonredundant information $\mathcal{I}_{NR}$ [4] that can be defined as the slope of $\mathcal{I}(f)$ at $f=1/2$ (see below). Our results show that for dissipative environments (both Ohmic and sub-Ohmic), $\mathcal{I}_{NR}$ is dramatically reduced, much faster than the total available information. The behavior of the super-Ohmic environment is rather different: redundancy develops but this effect is reversible. Decoherence is followed by recoherence and decorrelation. Thus, the super-Ohmic environment does not induce truly dissipative effects but is responsible for a renormalization (dressing) of the system as well as for coherent oscillations which will also be visible in our forthcoming studies. Perfect reversion for the super-Ohmic environment only takes place for some initial states of the system. If instead of the above state we chose one with a large momentum squeezing, complete recoherence is lost. This was already noticed in [14] and is due to transitory effects due to the instantaneous switching of the interaction.

PE plots in Fig. 3 show that for small (and moderate) values of f, entanglement scales almost linearly with f and stays very small. For larger fractions, the rapid growth of entanglement indicates that it is established between the system and the environment "as a whole." The effect of dissipation is displayed in the PE plots: dissipation reduces the maximum attainable entanglement and flattens the initial slope of the PE plot, suppressing even more the quantum correlations with small fractions of the environment. Super-Ohmic PE plot is dramatically different from the one corresponding to the dissipative case. In this nondissipative environment, revivals are evident in the PE plot. Redundancy of quantum correlations is related with the flatness of the PE plot as it indicates that many small environmental pieces become entangled as a whole with the system $\mathcal{S}$. A way to quantify this notion of redundancy will be presented below.

III. ANALYTICAL RESULTS

A simple model enables us to reproduce and understand the numerical results. We assume that $\mathcal{S}$ is massive and underdamped, justifying a Born-Oppenheimer approximation [5]. Under this approximation, the evolution of the joint state of $\mathcal{S}-\mathcal{E}$ has a branch structure that can be written as

$$|\Psi\rangle_{\mathcal{S}\mathcal{E}} = \int \psi(x,t)|x\rangle_{\mathcal{S}} \otimes |\psi_1(x,t)\rangle_{\mathcal{E}_1} \cdots \otimes |\psi_N(x,t)\rangle_{\mathcal{E}_N} dx, \tag{1}$$

where for each environmental oscillator, the state $|\psi_x(t)\rangle_{\mathcal{E}_n}$ depends parametrically on the system's position x. Its time evolution can be found by noticing that each oscillator feels an external force $F_n(t)=c_n x(t)$. Here, $x(t)$ parametrically depends on x and has a rather simple form for large squeezings. For positive (negative) r, a trajectory starts from $x(0)=x[x(0)=0]$ with velocity $\dot{x}(0)=0[\dot{x}(0)=\Omega_{\mathcal{S}}x]$. Thus, in general, $x(t)=\theta(r)x\cos(\Omega_{\mathcal{S}}t)+\theta(-r)x\sin(\Omega_{\mathcal{S}}t)$. Using this, the fate of initial squeezed Gaussian states can be found by computing all elements of the covariance matrix of the full state. The entanglement $E^{(f)}$ can be analytically computed and turns out to depend on the function $d^{(\mathcal{E}_f)}(t)=\Sigma_{n\in\mathcal{E}_f}d_n=\Sigma_{n\in\mathcal{E}_f}c_n^2[w_n^2a_n^2(t)+\dot{a}_n^2(t)]/4m_nw_n$, where

$$a_n(t) = \frac{\theta(r)}{(w_n^2-\Omega_{\mathcal{S}}^2)}\left[\frac{\Omega_{\mathcal{S}}}{w_n}\sin(w_nt)-\sin(\Omega_{\mathcal{S}}t)\right] + \frac{\theta(-r)}{(w_n^2-\Omega_{\mathcal{S}}^2)}[\cos(\Omega_{\mathcal{S}}t)-\cos(w_nt)].$$

On average, this satisfies $d^{(\mathcal{E}_f)}(t)=fd(t)$ and

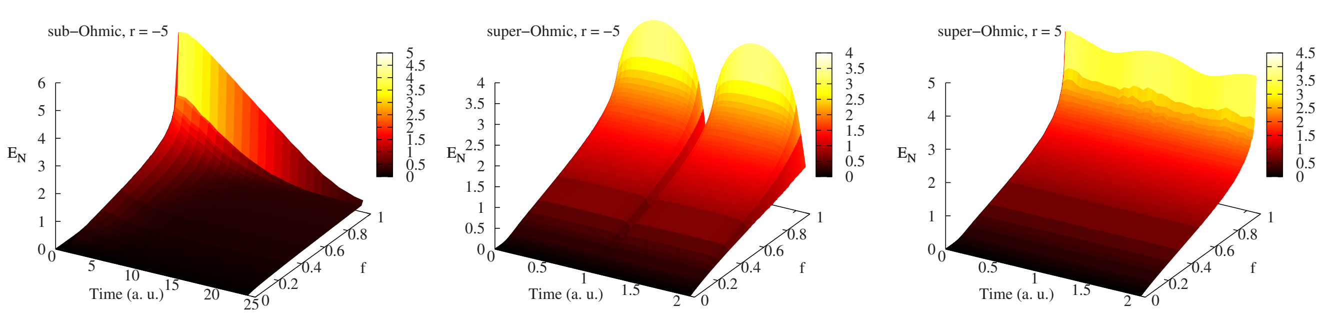

FIG. 3. (Color online) PE plot for sub-Ohmic (left) and super-Ohmic environments (center and right). The initial state of the system is squeezed in position with $r=-5$ except for the right plot where the squeezing is in momentum $r=5$. In that case, recoherence is suppressed. Entanglement grows very slowly as a function of the fraction size f, which is evidence of the branch structure of the total state.

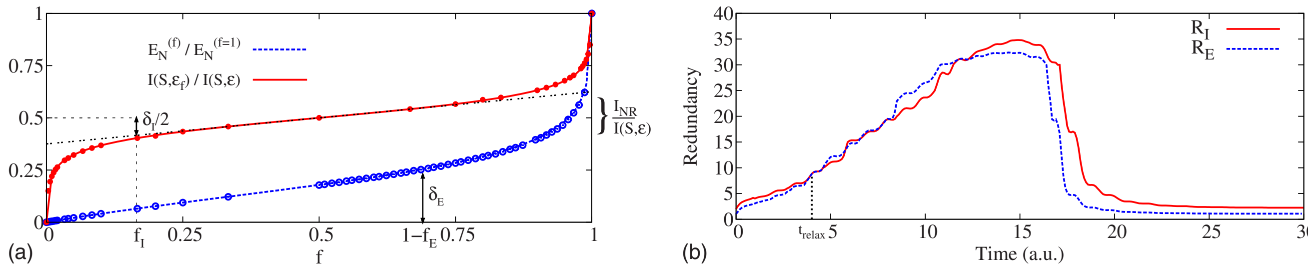

FIG. 4. (Color online) (Top) Normalized PE plot and PI plot for a fixed time. Analytical estimates given by expressions in Eqs. (2) and (3) (dashed and solid lines, respectively) show a remarkable agreement with numerical data (dots). The relevant fractions f_E and f_I to compute information and entanglement redundancies R_E and R_I are shown in the figure. (Bottom) R_E and R_I as a function of time are shown for a dissipative environment ($n=1/2$), where the relaxation-time scale is $t_{relax}\approx 3$ (with a deficit $\delta_E=2\delta_I=0.2$).

$$E^{(f)}(t) = -\frac{1}{2}\ln\left\{1 + 4d(t)\,\delta x^2(1+3f) - 4\sqrt{\left(\frac{f}{1+3f} + d(t)\,\delta x^2(1+3f)\right)^2 - \left(\frac{f}{1+3f}\right)^2}\right\}, \tag{2}$$

where δx is defined in terms of the absolute value of the squeezing parameter $|r|=\ln(\delta x/\delta x_0)$ being δx_0 the ground-state variance $\delta x_0=(1/2m\Omega_S)^{1/2}$. This result reproduces with high accuracy the numerical data before dissipation sets in (see Fig. 4) and replacing $f\rightarrow d_n(t)/d(t)$, we obtain the entanglement with the band w_n. Before analyzing this, we present a similar result for the mutual information [5] that turns out to be

$$\mathcal{I}(\mathcal{S},\mathcal{E}_f) = h[\chi(1)] + h[\chi(f)] - h[\chi(1-f)]. \tag{3}$$

Here, $h(\chi)$ is the entropy function $h(\chi)=(\chi+1/2)\ln(\chi+1/2)-(\chi-1/2)\ln(\chi-1/2)$ and its argument $\chi(f)$ satisfies

$$\chi(f) = \sqrt{\frac{1}{4} + 2fd(t)\,\delta x^2}. \tag{4}$$

The three terms in the Eq. (3) are, respectively, the entropies for the system, the environment, and the joint $\mathcal{S}-\mathcal{E}_f$ ensemble [notice that $\chi(f=1)$ is the symplectic area of $\mathcal{S}$]. It is worth noticing that this simple model enables us to explain the peculiar super-Ohmic case (with high cutoff) for which $d(t)$ is

$$d(t) \approx \frac{m\gamma_0}{2\pi}(\theta(-r)\sin^2\Omega t + \theta(r)(1+\cos^2\Omega t)). \tag{5}$$

We can use this in the above formulas for $E^{(f)}$ and see that a dramatically different behavior for positive and negative squeezings arises, fully reproducing the numerical data. For initial states delocalized in position, the sudden switch of the interaction produces a kick for each environmental oscillator whose effect cannot be undone later (the environmental oscillators remain out of phase and no recoherence is possible). On the other hand, for initial states delocalized in momentum, the initial kick is negligible and the transient effects are minimized. An equivalent behavior is observed if the interaction is adiabatically switched on. The existence of recoherence is not a novel result [14], but is remarkable that it shows up in such a clear way for the entanglement and the mutual information.

$E^{(f)}$ and $\mathcal{I}(\mathcal{S},\mathcal{E}_f)$ have similarities but also remarkable differences. $E^{(f)}$ grows slowly for small f and fast for $f\approx 1$. For $d(t)\,\delta x^2 \gg 1$, we have

$$E^{(f)} \approx \frac{1}{2}\ln\left(\frac{(1+3f)^3}{(1-f)(1+3f)^2 + 2f/d(t)\,\delta x^2}\right). \tag{6}$$

Numerical results are compared to the analytic estimate of $E^{(f)}$ in Fig. 4. Notably, in the limit of large squeezing and $f<1$, the entanglement $E^{(f)}$ becomes independent of the environmental spectral density that enters the above expressions through the function $d(t)$. In fact, in such case, we have

$$E^{(f<1)} \approx \frac{1}{2}\ln\left(\frac{1+3f}{1-f}\right). \tag{7}$$

A similar behavior was observed for Gaussian Greenberger-Horne-Zeilinger (GHZ)-type states [15]. In the presence of dissipation, $E^{(f)}$ is reduced so that the above universal expression is really an upper bound for $E^{(f)}$. A remarkably similar result arises for the mutual information. The analysis in this case is slightly different since MI depends on the squeezing through the combinations $fd(t)\,\delta x^2$ and $(1-f)d(t)\,\delta x^2$. Thus, both for small and large fractions, the first derivative of $\mathcal{I}(\mathcal{S},\mathcal{E}_f)$ is singular [due to the singularity in $h(\chi)$]. In the limit $d(t)\,\delta x^2 \gg 1$, $\mathcal{H}(\mathcal{S})\approx\ln(2e^2 d(t)\,\delta x^2)/2$ and mutual information behaves as

$$\mathcal{I}(\mathcal{S},\mathcal{E}_{0<f<1}) \approx \mathcal{H}(\mathcal{S}) + \frac{1}{2}\ln\left(\frac{f}{1-f}\right), \tag{8}$$

where $\mathcal{I}(\mathcal{S},\mathcal{E}_f)$ grows fast for small f and approaches a plateau with a slope related to the so-called nonredundant information $\mathcal{I}_{NR}$. This also turns out to be universal since $\mathcal{I}_{NR} \equiv \partial\mathcal{I}(\mathcal{S},\mathcal{E}_f)/\partial f|_{f=1/2}\approx 2$ (see Fig. 4).

IV. REDUNDANCY OF CORRELATIONS

Our results for $E^{(f)}$ enable us to define a measure quantifying the redundancy of quantum correlations. We define the *entanglement redundancy*, $R_E=1/f_E$, as the number of environmental fractions of size f_E such that by ignoring one of them, we induce a decay of the average entanglement to a fraction δ_E of the maximal one. Roughly speaking, a large entanglement redundancy is obtained when by ignoring a small fraction of the environment, one looses a large fraction of the total entanglement. This is the case if f_E turns out to be small when δ_E is small. GHZ states have large entanglement redundancy since by tracing over one of its constituents, the entanglement is lost. Indeed, large entanglement redundancy characterizes the fragility of the multipartite entanglement. Computing R_E from the PE plots is simple: as shown in Fig. 4, we should find the smallest fraction f_E such that $E^{(1-f_E)} = \delta_E E^{(f=1)}$. Since R_E is the number of such fractions in the environment $\mathcal{E}$, we have $R_E = 1/f_E$. As $E^{(f)}$ is a monotonous function of f, a large redundancy is expected for branch states Eq. (1). In that case $E^{(f)}$ would grow very slowly with f for small and moderate values of f. Remarkably, our analytic results before dissipation enable us to obtain a good estimate for R_E,

$$R_E = \frac{1}{f_E} \approx \exp(4\delta_E E^{(f=1)}) \approx A(t)^{2\delta_E}, \tag{9}$$

where $A(t) = d(t)\delta x^2/\delta_0 x^2$ is the symplectic area of the system's state (in the limit of large squeezings) and $E^{(f=1)} \approx \ln[32 d(t)\delta x^2]/2$ is the maximal entanglement (which is proportional to the entropy of the system).

It is interesting to notice that entanglement redundancy R_E can be related with *information redundancy* R_I, defined in [5] as the number of environmental pieces, $R_I = 1/f_I$, that carry a fraction $(1-\delta_I)$ of the available classical information $\mathcal{H}(\mathcal{S})$ (see Fig. 4). In Fig. 4, we can see that both redundancy measures, R_E and R_I, behave in a similar way. Notably, both R_E and R_I grow for times much longer than the relaxation-time scale. This is a consequence of the fact that when dissipation becomes effective, the quantum correlations with fractions of the environment are almost erased. Consequently, PE plots and PI plots flatten well before equilibration. Finally, in the asymptotic limit, redundancy almost disappears as do all types of correlations when the system approaches equilibrium. Our results also show that the amount of nonredundant information is intimately related with the entanglement created between $\mathcal{S}$ and fractions of the environment. Besides, we can show that both redundancies numerically coincide when the deficits δ_E and δ_I are such that $\delta_E \approx \delta_I \mathcal{H}(\mathcal{S})/E^{(f=1)} + E^{(f=1/2)}/E^{(f=1)}$. Thus, in the limit of large squeezing, both deficits become identical (this is the case because $E^{(f=1/2)}$ is bounded from above as $E^{(f=1/2)} \leq \ln\sqrt{5}$).

V. CONCLUSIONS

In this paper, we studied the evolution of quantum correlations (characterized by the entanglement) and total correlations (measured by the mutual information) between a system and its environment. First, we analyzed the evolution of such correlations as a function of the frequency of the environmental oscillators. In this case, we showed that for dissipative environments, the resonant bands are always the ones that correlate the most with the system. On the other hand, for the nondissipative environments, the high-frequency bands dominate the evolution inducing short-time effects that can leave a long-time footprint for some initial states. Other initial states lead to revivals of the correlations. Our analytic and numerical studies of the evolution of correlations for QBM show that both mutual information and entanglement do become redundant at a time scale which is connected with the one of decoherence. Indeed, the function $d(t)$ determining the behavior of entanglement and mutual information is the same one that characterizes the suppression of the quantum interference effects. This relation can be seen both in the PI plots and PE plots. Our results also show the special nature of the state that is dynamically produced for the QBM model. Such state has a branch structure compatible with large entanglement redundancy and truly multipartite entanglement between $\mathcal{S}$ and $\mathcal{E}$. As we said, our analytical results are accurate for times shorter than the relaxation-time scale. In such regime, the environment effectively measures the system, the total state has a branch structure, and the entanglement is established only between the system and the environmental oscillators. The situation becomes more complicated when dissipation sets in. In that regime, the system evolves toward equilibrium and it mediates an effective interaction between the environmental oscillators. As a consequence of this interaction, the environmental oscillators also become entangled between each other. Clearly, a different analytical model should be used to describe this regime. However, by means of our numerical results, we were able to analyze some aspects of this regime. Indeed, we have shown that dissipation rapidly erases quantum correlations with fractions of the environment and flattens both PI plots and PE plots. Thus, redundancy is dramatically increased by the action of dissipation. Furthermore, we have shown that on average, the fractions of the environment that carry almost all the available classical information, $\mathcal{H}(\mathcal{S})$, have low entanglement with the system. Our work shows that the existence of redundant records of the state of the system imprinted in the environment can be connected with the classicality of the system. Similar ideas were analyzed before in the context of the consistent histories approach to quantum mechanics [16]. In fact, in such approach, the goal is to define a good criterion for classicality and the consistency between histories was shown to be insufficient for such purpose [17]. Thus, the existence of some sort of redundancy in the records of the state of the system imprinted in the environment was discussed as a possible additional criterion for classicality [16]. Our approach analyzes this issue from a different, but related, perspective. By inspecting the evolutions of quantum and total correlations, we obtain an idea of the capability of an environmental fraction to gather information about the state of the system at a given time. In this sense, quantum mutual information between the system and a fraction of the environment quantifies the entropy that can be reduced by doing the optimal measurement over such a fraction. In this case, a redundant record of the system is dynamically produced by decoherence. Meanwhile, information about the initial state of the system is lost nonlocally in the environment.

ACKNOWLEDGMENTS

The authors acknowledge support of CONICET, UBACYT, and ANPCyT (Argentina).

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