Mobility of Bloch walls via the collective coordinate method

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We have studied the problem of the dissipative motion of Bloch walls considering a totally anisotropic one-dimensional spin chain in the presence of a magnetic field. Using the so-called ''collective coordinate method'' we construct an effective Hamiltonian for the Bloch wall coupled to the magnetic excitations of the system. It allows us to analyze the Brownian motion of the wall in terms of the reflection coefficient of the effective potential felt by the excitations due to the existence of the wall. We find that for finite values of the external field the wall mobility is also finite. The spectrum of the potential at large fields is investigated and the dependence of the damping constant on temperature is evaluated. As a result we find the temperature and magnetic-field dependence of the wall mobility.

I. INTRODUCTION

It is a well-known fact that ultimately due to magnetic dipole interaction, different domains are formed in magnetic systems.[1] In many situations, the physical region separating two different magnetic domains—the domain wall—must be treated as a physical entity because it has a characteristic behavior when acted by external agents. For instance, it is known that the response of a magnetic system to a frequency-dependent external magnetic field depends on whether domain walls are present.[2] Domain walls can also move dissipatively.[3]

A particularly interesting kind of domain wall is commonly found in low-dimensional ferromagnetic systems. These are the so-called Bloch walls.[4] It is known that these walls perform dissipative motion[3] due to the presence of the elementary excitations which can be scattered by the wall as it moves and the momentum transferred to them reduces the speed of the wall.

The primary aim of this work is to study the influence of finite temperatures in the mobility of these Bloch walls. For this purpose we start by considering a microscopic model for a one-dimensional ferromagnet containing hard and easy-axis anisotropies and subject to an external magnetic field. A semiclassical picture provides us with the localized solutions for the spin configurations which are the solitons corresponding to the walls.

Making use of a recently developed method for the analysis of the dissipative dynamics of solitons,[5,6] in which the ''collective coordinate method''[7] is used to transform the original Hamiltonian into one of a particle coupled to an infinite set of modes, we show that the Bloch wall behaves like a Brownian particle. The advantage of using this method is that we keep closer contact with the microscopic details of the system and the mobility is naturally calculated as a function of the temperature. The information from the microscopic scattering processes between the Bloch wall and the residual modes can be obtained from the knowledge of the phase shifts of the associated spectral problem. In the case of reflectionless potentials, as it happens for vanishing anisotropies or external field, the motion of the wall is undamped. If this is not the case, the reflection coefficient does not vanish and the mobility is finite.

The outline of this paper is as follows. In Sec. II we present the model. The dynamics of its static solution is investigated in Sec. III and there we also show how to obtain an effective Hamiltonian for the Bloch wall coupled to the residual magnetic excitations. In Sec. IV the mobility of the Bloch wall is studied in terms of the scattering phase shifts of the second variation problem. The case of large external fields is investigated in Sec.V where the phase shifts and the damping constant are explicitly evaluated. Finally, we present our conclusions in Sec. VI.

II. THE MODEL AND ITS STATIC SOLUTIONS

In this work we consider a one-dimensional magnetic system composed by an array of spins lying along the $\hat{z}$ direction. Furthermore, let us assume that there is an easy-plane anisotropy which tries to keep the spins on the x-y plane and, on top of this, an in-plane anisotropy tending to align them along the $\hat{x}$ direction. This is a totally anisotropic model which is described by a XYZ model of magnetic systems defined by the Hamiltonian

$$H=-\sum_{\langle ij\rangle}\left(J_x S_i^{(x)}S_j^{(x)}+J_y S_i^{(y)}S_j^{(y)}+J_z S_i^{(z)}S_j^{(z)}\right)-\frac{\mu}{\hbar}B\sum_i S_i^{(x)}, \tag{1}$$

where $J_x > J_y > J_z > 0$, $S_j^{(\alpha)}$ is the α component ($\alpha = x,y,z$) of the ith spin of the system, μ is the modulus of the magnetic moment on each site, and B is the external magnetic field. The ferromagnetic XYZ model is actually defined for $B=0$ and this is the starting point of our analysis. As we can see from Eq. (1), the ground state of this system is the configuration where all the spins are aligned in the $\hat{x}$ direction. However, there is another possible configuration which is a local minimum of the energy functional and cannot be obtained from the previous uniform configuration by any finite energy operation.

Let us imagine that we describe our spins classically by vectors

$$\mathbf{S}_i = S(\sin\theta_i \cos\varphi_i\,,\ \sin\theta_i \sin\varphi_i\,,\ \cos\theta_i), \tag{2}$$

where θ_i and φ_i are the polar angles of the ith spin. In this representation the above-mentioned configuration consists of all θ_i's equal to $\pi/2$ and φ_i's equal to zero or π. However, there are other configurations in which $\theta_i = \pi/2, \varphi_i = 0$ if $i \to -\infty$ and $\varphi_i = \pi$ if $i \to \infty$ which are approximately (only because θ_i's may slightly vary[8]) local minima of the energy functional of the system. So, $\mathbf{S}_i$ winds around the $\hat{z}$ direction starting at $(\theta,\varphi) = (\pi/2, 0)$ and ending at $(\theta,\varphi) = (\pi/2, \pi)$. The so-called π-Bloch wall[9] is one example of these configurations where φ_i varies from 0 to π without making a complete turn around the $\hat{z}$ axis. Later on we will see the specific form of this configuration when we consider the system in the continuum limit. It will then be shown that Bloch walls are related to solitonlike solutions of the nonlinear equations which control the spin dynamics in the semiclassical approximation.

If we now turn the external field B on it happens that the degeneracy between $\varphi = 0$ and $\varphi = \pi$ is broken. For $B>0$ it is clear from Eq. (1) that $\varphi = 0$ has lower energy than $\varphi = \pi$ which is now a metastable configuration of the system. In this circumstance the system still presents a local minimum of the energy functional. The only difference is that whereas θ_i is still approximately $\pi/2$, φ_i starts and ends at zero as $-\infty < i < \infty$. The 2π-Bloch wall is now the configuration where φ_i winds only once around $\hat{z}$.

In the case of the π-Bloch wall mentioned above, there is no way we could spend a finite energy to transform the wall into an uniform configuration. We would need to turn an infinite number of spins over an anisotropy energy barrier. However the 2π-Bloch case is different, it is known that above a certain critical field the spin configuration becomes unstable favoring the totally polarized state.[10,11]

Another important point is that Eq. (1) is translation invariant and this is reflected by the translation invariance of the Bloch wall. This means that the region about which the spins wind up can be centered anywhere on the $\hat{z}$ axis. In reality they can even move with constant speed along that direction.

These structures can be obtained by mapping the original Hamiltonian (1) into a $(1+1)$-field-theoretical model such as the φ^4, sine-Gordon or any other appropriate model. This can be done starting from the equation of motion for the spin components

$$\frac{dS^{(\alpha)}}{dt} = \frac{1}{i\hbar}[S^{(\alpha)}, H]. \tag{3}$$

Substituting Eq. (1) in Eq. (3) we get

$$\dot{S}^x(z,t) = a^2(J_z S^{z''} S^y - J_y S^{y''} S^z) + 2(J_z S^z S^y - J_y S^y S^z), \tag{4}$$

$$\dot{S}^y(z,t) = a^2(J_x S^{x''} S^z - J_z S^{z''} S^x) + 2(J_x S^x S^z - J_z S^z S^x) + \frac{\mu B}{\hbar} S^z, \tag{5}$$

$$\dot{S}^z(z,t) = a^2(J_y S^{y''} S^x - J_x S^{x''} S^y) + 2(J_y S^y S^x - J_x S^x S^y) - \frac{\mu B}{\hbar} S^y, \tag{6}$$

which are a generalization of the Landau-Lifshitz equation for the totally anisotropic case. Now, proceeding a bit further with the semiclassical description for the spin, we write the equations of motion (4)–(6) in terms of $\theta_i(t)$ and $\varphi_i(t)$ introduced in Eq. (2).

After having done that we take the continuum limit $\theta_i \to \theta(z,t)$, $\varphi_i(t) \to \varphi(z,t)$ and write

$$\theta(z,t) \approx \frac{\pi}{2} + \alpha(z,t), \tag{7}$$

where $\alpha(z,t) \ll 1$. Assuming that the variations of φ and α from site to site of the spin chain are small and linearizing the equations of motion with respect to α one obtains

$$\dot{\varphi} = \alpha 2S(J_x \cos^2\varphi + J_y \sin^2\varphi - J_z), \tag{8}$$

$$\dot{\alpha} = a^2 S(J_x \sin^2\varphi + J_y \cos^2\varphi)\frac{\partial^2\varphi}{\partial z^2} - \sin\varphi\left[\frac{\mu B}{\hbar} + 2S(J_x - J_y)\cos\varphi\right], \tag{9}$$

where a is the lattice spacing. Then, eliminating α from these equations, we get an effective equation of motion for $\varphi(z,t)$ of the form

$$\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} - \frac{\partial^2\varphi}{\partial z^2} = -A_1 \sin\varphi - A_2 \sin 2\varphi, \tag{10}$$

where

$$c^2 \cong 2a^2 S^2 J_x J_y \left(1 - \frac{J_z}{J_y}\right), \tag{11}$$

$$A_1 = \frac{\mu B}{a^2 S J_x \hbar}, \tag{12}$$

and

$$A_2 = \frac{1}{a^2}\left(1 - \frac{J_y}{J_x}\right). \tag{13}$$

Actually Eq. (10) is not the complete story. In order to be more rigorous we would need to keep terms of fourth order in the position derivatives of φ if we want to recover the correct dispersion relation for ferromagnetic magnons when $k\rightarrow 0$; namely,

$$\omega^2=(K_1+K_2k^2)^2. \tag{14}$$

In its present form, Eq. (10) will provide us with a dispersion relation of the form

$$\omega^2=A+Bk^2, \tag{15}$$

which clearly reminds us of that of an antiferromagnetic spin wave. Therefore, if we (only due to mathematical convenience) keep on employing Eq. (10) as the correct dynamical equation for φ, one should bear in mind that our future expression for $\omega(k)$ will only be valid up to terms of the order of k^2.

Notice that if $J_x=J_y$ one has $A_2=0$ whereas if $B=0$ it turns out that $A_1=0$. So, as we can derive the right-hand side (rhs) of Eq. (10) from a potential energy density $U(\varphi)$ given by

$$U(\varphi)=A_1(1-\cos\varphi)+\frac{A_2}{2}(1-\cos 2\varphi), \tag{16}$$

we see that A_1 controls the potential energy barrier due to the presence of $B\neq 0$ and A_2 controls the anisotropy energy barrier.

The static solutions $(\partial\varphi/\partial t=0)$ of Eq. (10) are obtained using that[7]

$$z-z_0=\int_{\varphi(z_0)}^{\varphi(z)}\frac{d\varphi'}{\sqrt{2U(\varphi')}} \tag{17}$$

are the solitons of the system. In particular, the examples of Bloch walls we gave above are the solitons

$$\varphi(z)=2\tan^{-1}[\exp\sqrt{2A_2}(z-z_0)]\quad \text{if}\ \ A_1=0, \tag{18}$$

$$\varphi(z)=4\tan^{-1}[\exp\sqrt{A_1}(z-z_0)]\quad \text{if}\ \ A_2=0. \tag{19}$$

Notice that the solitonic solution (18), which corresponds to the case of zero magnetic field, reproduces the π-Bloch walls properties: $\varphi(-\infty)=0$ and $\varphi(+\infty)=\pi$, while the second solution (19) corresponding to the zero anisotropy case, describes the 2π-Bloch walls $[\varphi(-\infty)=0,\varphi(+\infty)=2\pi]$ in agreement with the symmetry breaking due to the magnetic field.

In the general case of finite anisotropy and magnetic field, the localized solution is the 2π-Bloch wall, as was mentioned earlier, and has the form

$$\varphi(z)=2\tan^{-1}\left[\frac{\cosh\rho}{\sinh(z/\lambda)}\right], \tag{20}$$

where we defined

$$\lambda=1/\sqrt{A_1+2A_2}, \tag{21}$$

$$\cosh\rho=\sqrt{1+\frac{2A_2}{A_1}}. \tag{22}$$

The soliton (20) can be expressed as a superposition of two twisted π-Bloch walls[11] with arguments $\lambda^{-1}(z-z_0)\pm\rho$. We mention here there is another static solution of Eq. (10), the so-called *nucleus*[12] which corresponds to a superposition of two untwisted π-Bloch walls.[11] This solution is topologically distinct from the previous one.

So, until now, the continuous model introduced for the one-dimensional totally anisotropic ferromagnet allowed us to describe, from the topological point of view, the presence of the π- and 2π-Bloch walls depending on the presence of the magnetic field.

III. DYNAMICS OF BLOCH WALLS

The quantum dynamics of our spin system can be analyzed by studying the quantum mechanics of the field theory described by the action

$$S[\varphi]=JS^2\int_{-\infty}^{+\infty}\int_0^t dz\,dt\left\{\frac{1}{2c^2}\left(\frac{\partial\varphi}{\partial t}\right)^2-\frac{1}{2}\left(\frac{\partial\varphi}{\partial z}\right)^2-U(\varphi)\right\}. \tag{23}$$

The next step is to quantize the system described by Eq. (23). The standard way to carry this program forward is to evaluate[7]

$$G(t)=\mathrm{tr}\int\mathcal{D}\varphi\exp\frac{i}{\hbar}S[\varphi], \tag{24}$$

where the functional integral has the same initial and final configurations and tr means to evaluate it over all such configurations.

As the functional integral in Eq. (24) is impossible to be evaluated for a potential energy density as in Eq. (16) we must choose an approximation to do it. Since we are already considering large spins $(S\gg\hbar/2)$, and consequently in the semiclassical limit, let us take this approximation as the appropriate one for our case.

The semiclassical limit $(\hbar\rightarrow 0)$ turns out to be very easily tractable within the functional integral formulation of quantum mechanics.[7] It is simply the stationary phase method applied to Eq. (24). Moreover, since we are only interested in static solutions, the functional derivative of S happens to be the equation of motion (10) when $\partial\varphi/\partial t=0$. Its solutions can be either constant (uniform magnetization) or the solitons (Bloch walls) we mentioned in Eqs. (18)–(20). Since we are interested in studying the magnetic system in the presence of walls it is obvious that we must pick up one of those localized solutions as the stationary ''point'' in the configuration space and the second functional derivative of Eq. (23) should be evaluated at this configuration.

When this is done we are left with an eigenvalue problem that reads

$$\left\{-\frac{d^2}{dz^2}+U''(\varphi_s)\right\}\psi_n(z-z_0)=k_n^2\psi_n(z-z_0), \tag{25}$$

where φ_s is denoting the solitonlike solution about which we are expanding $\varphi(z,t)$.

Now one can easily show that $d\varphi_s/dz$ is a solution of Eq. (25) with $k_n=0$. The existence of this mode is related to the translation invariance of the Lagrangian in Eq. (23) and this makes the functional integral in Eq. (24) blow up in the semiclassical limit (Gaussian approximation).

The way out of this problem is the so-called collective coordinate method which was developed by field theorists in the 1970s (see Ref. 7 and references therein). It consists of keeping the expansion of the field configurations about $\varphi_s(z)$ as

$$\varphi(z,t)=\varphi_s[z-z_0(t)]+\sum_{n=1}^{\infty} c_n\psi_n[z-z_0(t)] \tag{26}$$

but regarding the c number z_0 as a position operator. Equation (11) is then substituted in the Hamiltonian

$$H=JS^2a\int dx\left\{\frac{c^2\Pi^2}{2}+\frac{1}{2}\left(\frac{d\varphi}{dz}\right)^2+U(\varphi)\right\}, \tag{27}$$

where $\Pi=(1/c)(\partial\varphi/\partial t)$, which can be transformed into[13]

$$H=\frac{1}{2M_s}\left(P-\sum_{mn}\hbar g_{mn}b_n^+b_m\right)^2+\sum\hbar\Omega_n b_n^+b_n, \tag{28}$$

where $\Omega_n\equiv ck_n$.

In the Hamiltonian (28), P stands for the momentum canonically conjugated to z_0,

$$M_s=\frac{2JS^2a}{c^2}\int_{-\infty}^{+\infty}dz\,U(\varphi_s(z)) \tag{29}$$

is the soliton mass[7] and the coupling constants g_{mn} are given by

$$g_{mn}=\frac{1}{2i}\left[\left(\frac{\Omega_m}{\Omega_n}\right)^{1/2}+\left(\frac{\Omega_n}{\Omega_m}\right)^{1/2}\right]\int dz\,\psi_m(z)\frac{d\psi_n(z)}{dz}. \tag{30}$$

The operators b^+ and b are, respectively, the creation and annihilation operators for the excitations of the magnetic system (magnons) in the presence of the wall. In fact, the term

$$\sum_{mn}\hbar g_{mn}b_n^+b_m, \tag{31}$$

can be interpreted as the total linear momentum of the magnons of the system and therefore we are left with a problem in which the momentum associated to the Bloch wall is now coupled to the magnons' momenta. This effective model suggests that, as the population of magnons is a temperature-dependent quantity, the mobility of the Bloch wall will be strongly related to the temperature of the system and its dynamics [determined by Eq. (28)] will be nontrivial.

It should also be stressed that Eq. (28) is not an exact result. It is only valid in the limit $\hbar\to 0$ or, to be more precise, when $g^2\hbar\to 0$ where $g^2\equiv 1/JS^2a$ is the coupling constant that originally appears in $U(g,\varphi)$. It must also be emphasized that we have neglected inelastic terms such as b^+b^+ or bb because these are only important if the wall moves at high speed $(v>c)$ originating Cherenkov-like radiation of the elementary excitations of the medium. This approximation also means that the number of excitations in the medium is conserved.

IV. MOBILITY OF THE BLOCH WALL

At this point we are ready to start to study properties such as the mobility of the wall because we have been able to map that problem into the Hamiltonian (28) which on its turn has been recently used to study the mobility of polarons, heavy particles, and solitons in general. We shall not discuss this specific problem in this paper and urge those interested in the details of this calculation to follow them in Refs. 5,13–16.

The result that can be obtained reads[15]

$$\gamma(t)=\frac{\hbar}{2M}\int_0^\infty\int_0^\infty d\omega d\omega' S(\omega,\omega')(\omega-\omega')\times[n(\omega)-n(\omega')]\cos(\omega-\omega')t, \tag{32}$$

where $\gamma(t)$ is the damping function (the inverse of the mobility),

$$n(\omega)=\frac{1}{e^{\beta\hbar\omega}-1} \tag{33}$$

is the Bose function and

$$S(\omega,\omega')=\sum_{mn}|g_{mn}|^2\delta(\omega-\Omega_n)\delta(\omega'-\Omega_m) \tag{34}$$

is the so-called scattering function.

In the long-time limit $\gamma(t)$ can, to a good approximation, be written as

$$\gamma(t)\cong\overline{\gamma}(T)\delta(t) \tag{35}$$

and $\overline{\gamma}(T)$ is given by[15]

$$\overline{\gamma}(T)=\frac{1}{2\pi M_s}\int_0^\infty dE\,\mathcal{R}(E)\frac{\beta E e^{\beta E}}{(e^{\beta E}-1)^2}, \tag{36}$$

where $\mathcal{R}(E)$ is the reflection coefficient of the ''potential'' $U''(\varphi_s)$ in the Schrödinger-like equation (25). Notice that Eq. (36) is only valid if the states involved in Eq. (34) are scattering states (see Sec. V A below for details). One important point that should be emphasized here is that there are parameters of the nonlinear field equations for which the localized solutions render $U''(\varphi_s)$ a reflectionless potential. These are genuine solitons and for these the mobility is infinite. One may realize this is what happens for the Bloch walls Eqs. (18) and (19). In these cases, the ''potential'' appearing in Eq. (25) can be written as

$$U''(z)=\eta^2(1-2\,\mathrm{sech}^2\,\eta z), \tag{37}$$

where $\eta^2=A_1$ for vanishing anisotropy and $\eta^2=2A_2$ for vanishing external field. The spectrum of Eq. (37) contains a bound state with zero energy

$$\psi_0=\sqrt{\frac{\eta}{2}}\,\mathrm{sech}(\eta z),\quad k_0^2=0, \tag{38}$$

which constitutes the translation mode of the domain wall (Goldstone mode), and a continuum of quasiparticles modes (magnons) given[17] by

$$\psi_n(x)=\frac{1}{\sqrt{L}}\left[\frac{k_n+i\eta\tanh(\eta z)}{k_n+i\eta}\right]e^{ik_nz}\quad k\geqslant\eta, \tag{39}$$

where

$$k_n=\frac{2n\pi}{L}-\frac{\delta(k_n)}{L},\quad \delta(k)=\arctan\left[\frac{2\eta k}{k^2-\eta^2}\right]. \tag{40}$$

It is known that the reflection coefficient $\mathcal{R}$ for a general symmetric potential can be expressed in terms of the corresponding phase shifts as[15]

$$\mathcal{R}(k)=\sin^2(\delta^e(k)-\delta^o(k)), \tag{41}$$

where δ^e and δ^o are the even and odd scattering phase shifts, respectively. Then, reexpressing Eq. (39) in terms of even and odd defined parity states, it is easy to prove that this potential belongs to the class of reflectionless because its phase shifts are given by

$$\delta^{e,o}(k)=\arctan(\eta/k), \tag{42}$$

that do not distinguish between odd and even parities.

Nevertheless, when both the anisotropy and external field are finite, the reflection coefficient is nonvanishing and consequently the 2π-Bloch wall (20) has a finite mobility. In this case, the spectral problem (25) can be rewritten as

$$\left\{-\frac{d^2}{dz^2}+V(z)\right\}\psi_n(z)=k_n^2\psi_n(z), \tag{43}$$

where the potential $V(z)$ is expressed as (a similar expression was first reported in Ref. 11)

$$V(z)=\frac{1}{\lambda^2}\left[1-2\,\mathrm{sech}^2\left(\frac{z}{\lambda}+\rho\right)-2\,\mathrm{sech}^2\left(\frac{z}{\lambda}-\rho\right)+2\,\mathrm{sech}\left(\frac{z}{\lambda}+\rho\right)\mathrm{sech}\left(\frac{z}{\lambda}-\rho\right)\right]. \tag{44}$$

The second and the third terms on the rhs of Eq. (44) are the potentials (37) of the noninteracting π-Bloch walls located at $z/\lambda=\pm\rho$ whereas, the last term describes the interaction of the two π-Bloch walls at $z/\lambda=\pm\rho$, respectively.

Now, for all finite values of λ and ρ, the translational invariance of the system persists and as a consequence the potential (44) has a zero-energy state that is given by

$$\psi_0\propto\mathrm{sech}\left(\frac{z}{\lambda}+\rho\right)+\mathrm{sech}\left(\frac{z}{\lambda}-\rho\right), \tag{45}$$

which is nothing but the Goldstone mode of the Bloch wall for finite anisotropy and external field.

In order to obtain an expression for the damping constant (36) we need an expression for the odd and even phase shifts of Eq. (44). Unfortunately, their analytical evaluation is very complicated for all finite values of λ and ρ, and we study the situation of large fields in what follows.

V. 2π-BLOCH WALLS FOR LARGE FIELDS

In this section we evaluate the scattering phase shifts in the situation of large external fields and provide an explicit expression for the damping constant. As it was mentioned before there is a critical magnetic field above which the 2π-Bloch wall becomes unstable. The instabilities of this kind of configuration were investigated in details by Magyari *et al.*[10] and independently by Braun.[11]

Following the same approach presented in Ref. 10 it is possible to estimate the critical value of the external magnetic field, as a function of the coupling constants J_x, J_y, and J_z, which render the 2π-Bloch wall unstable. The relation can be explicitly deduced from the Hamiltonian (see Ref. 10 for details)

$$H=\sum_i\left(-\frac{1}{2}J\mathbf{S}_i\cdot\mathbf{S}_{i+1}+D(S_i^z)^2-A(S_i^x)^2-g\mu_BBS_i^x\right) \tag{46}$$

as

$$b_c=\frac{1}{2}\left(1+\frac{a}{5}\right)\quad\text{for}\quad a\ll 1, \tag{47}$$

$$b_c=\frac{1}{2}\left(1-\frac{1}{2a}\right)\quad\text{for}\quad a\gg 1, \tag{48}$$

where

$$a=\frac{A}{D},\quad b=\frac{g\mu_BB}{2DS}. \tag{49}$$

Mapping our Hamiltonian (1) into the model (46), these constants read

$$b=\frac{\mu B}{2\hbar(J_y-J_z)S},\quad a=\frac{J_x-J_y}{J_y-J_z}. \tag{50}$$

As we are interested in the stable 2π-Bloch wall for $\rho\ll 1$ ($B\gg J_x-J_y$), the magnetic field cannot exceed the critical value given by the expression (47). Explicitly,

$$B_c=\frac{\hbar S}{5\mu}(J_x+4J_y-5J_z), \tag{51}$$

where we have used the definitions (50).

So, keeping in mind the limit (51) for the magnetic field, we can carry on the study of the 2π-Bloch wall for the case $\rho\ll 1$.

A. Scattering phase shifts

In the case of large fields ($\rho\ll 1$) the Schrödinger-like equation (43) can be written as

$$\left\{-\frac{d^2}{dz^2}+V(z)\right\}\psi_n(z)=\kappa_n^2\psi_n(z),\quad \kappa_n^2=k_n^2-\frac{1}{\lambda^2}-\frac{\rho^2}{\lambda^2}, \tag{52}$$

where the potential (44) is now reduced to the sum of the reflectionless contribution and a perturbation coming from the presence of the large field. Explicitly,

$$V(z)=V_0(z)+\left(\frac{\rho}{\lambda}\right)^2V_1(z), \tag{53}$$

where

$$V_0(z)=-2\,\mathrm{sech}^2\left(\frac{z}{\lambda}\right) \tag{54}$$

and

$$V_1(z)=-8\,\tanh^2\left(\frac{z}{\lambda}\right)\mathrm{sech}^2\left(\frac{z}{\lambda}\right). \tag{55}$$

In order to obtain the even and odd scattering phase shifts $\Delta^{e,o}$ corresponding to a particle in a one-dimensional (1D) symmetric potential like Eq. (53), we will use of a 1D version of the Fredholm theory,[18] which states that

$$\pi A^{e,o}(E)\cot(\Delta^{e,o})=1+\mathcal{P}\int_0^\infty dE\,\frac{A^{e,o}(E')}{E-E'}, \tag{56}$$

where $\Delta^{e,o}$ are the phase shifts originated by both contributions, the first coming from the reflectionless potential V_0, and the other associated to the high-field perturbation V_1. On the other hand, the even and odd spectral functions, $A^{e,o}(E)$, can be calculated from the series expansion (see Ref. 18 for details)

$$A(E)=-\langle E|V(z)|E\rangle +\mathcal{P}\int_0^\infty \frac{dE_1}{E-E_1}\begin{vmatrix}\langle E|V(z)|E\rangle & \langle E|V(z)|E_1\rangle\\ \langle E_1|V(z)|E\rangle & \langle E_1|V(z)|E_1\rangle\end{vmatrix}+\cdots, \tag{57}$$

where $\mathcal{P}$ stands for the Cauchy principal value. Clearly, expression (57) cannot be analytically evaluated to all orders. On the other hand, making use of Eq. (42) and considering that ρ is small enough, the expression for the the phase shifts (56) can be written up to first order in ρ^2 as

$$\tan\Delta^{e,o}=\frac{1}{\lambda k}+\left(\frac{\rho}{\lambda}\right)^2\frac{\pi A_1^{e,o}}{1+2B_0^{e,o}}, \tag{58}$$

where

$$A_1=-\langle E|V_1(z)|E\rangle, \tag{59}$$

and

$$B_0^{e,o}=\mathcal{P}\int_{-\infty}^{\infty}\frac{A_0^{e,o}(k')k'dk'}{k^2-k'^2},\quad A_0=-\langle E|V_0(z)|E\rangle. \tag{60}$$

Using a convenient basis set, the three expressions given by Eqs. (59) and (60) can be analytically evaluated (see the Appendix) yielding

$$A_0^{e,o}(k)=\frac{2M}{\hbar^2}\left[\frac{1}{\pi k\lambda}\pm\frac{1}{\sinh(\pi k\lambda)}\right], \tag{61}$$

$$A_1^{e,o}(k)=\frac{8\rho^2M}{3\hbar^2}\left[\frac{1}{\pi k\lambda}\mp(2k^2-\lambda^{-2})\frac{\lambda^2}{\sinh(\pi k\lambda)}\right], \tag{62}$$

$$B^{e,o}=\pm\frac{4M}{\hbar^2}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{[(k\lambda)^2+n^2]}. \tag{63}$$

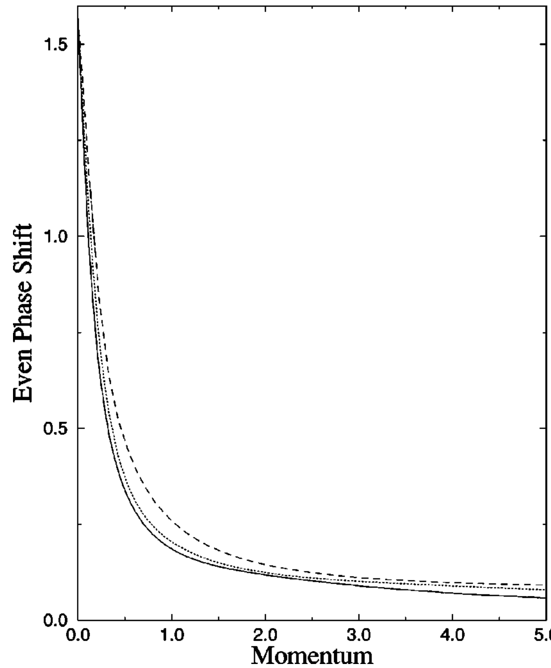

FIG. 1. The even phase shift as a function of the momentum for three different situations. The continuous line corresponds to $\rho=0.14$, the dotted line to $\rho=0.31$, and the dashed line to $\rho=0.50$.

Now we can finally write down an expression for the phase shifts $\Delta^{e,o}$ by substituting Eqs. (62) and (63) in Eq. (58). In so doing one gets

$$\tan\Delta^{e,o}(k)=\frac{1}{k\lambda}+\frac{8\pi M\hbar^{-2}\rho^2}{1\pm 8M\hbar^{-2}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{(k\lambda)^2+n^2}}\times\left[\frac{1}{3k\pi\lambda}\mp\frac{2k^2\lambda^2-1}{3\sinh(k\pi\lambda)}\right]. \tag{64}$$

Looking at Eq. (64) we realize that, whereas $\Delta^e(k)$ remains almost unchanged as a function of k for $\rho\neq 0$ (see Fig. 1), $\Delta^o(k)$ presents a completely different structure. In fact, the behavior of $\Delta^o(k)$, within this approximation, does not reproduce the correct values of the phase shifts for low energies. As was demonstrated by Kivshar *et al.*[20] in the study of the small-amplitude modes or fluctuations around the localized solution of the double sine-Gordon equation, there is always an odd bound state for this kind of systems. Therefore, using the 1D version of the Levinson's theorem[19] for one-dimensional symmetric potentials, which establishes that

$$\Delta^e(k=0)=\pi\left(n^e-\frac{1}{2}\right),$$

$$\Delta^o(k=0)=\pi n^o, \tag{65}$$

where n^e and n^o are the number of even and odd parity bound states, we expect that $\Delta^e(k=0)=\pi/2$, as it was shown above for any finite value of ρ, and $\Delta^o(k=0)=\pi$.

The existence of the even parity bound state is in complete agreement with the translational invariance of the system and corresponds to a Goldstone mode. However, the odd

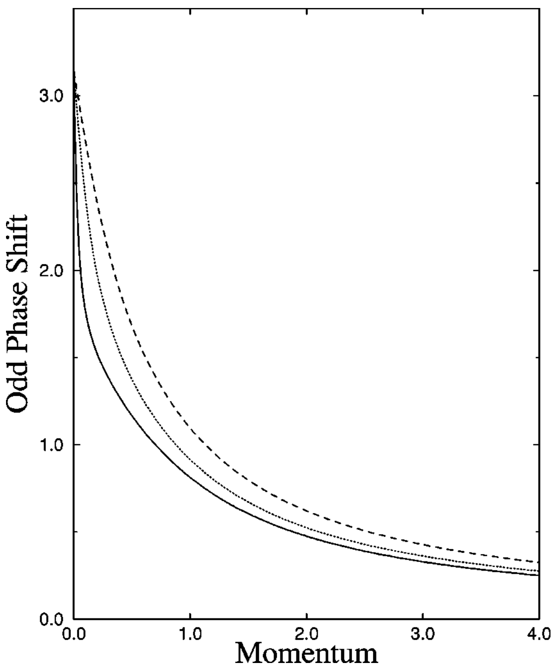

FIG. 2. The odd phase shift as a function of the momentum. The continuous line corresponds to $\rho=0.14$, the dotted line to $\rho=0.31$, and the dashed line to $\rho=0.50$.

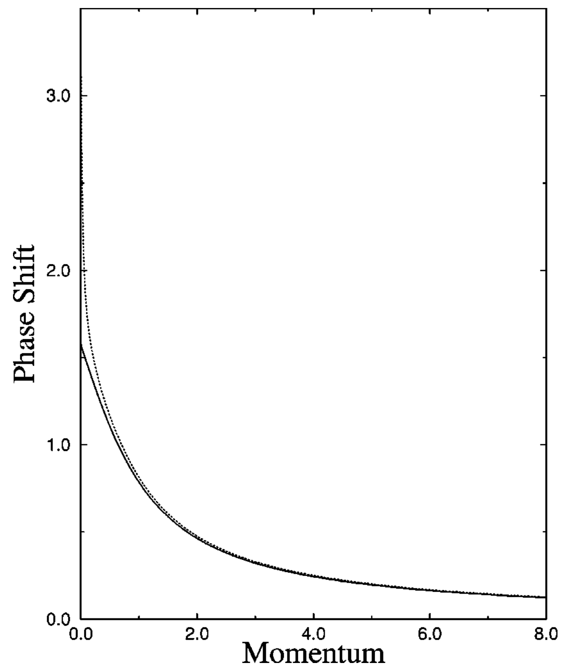

FIG. 3. The even and odd phase shifts when $\rho=0.14$. As can be seen they approach each other as the ratio A_1/A_2 increases. This means that, in the limit $\rho\rightarrow 0$, the only difference between them comes from the singular point ($k=0$) in the odd phase shift contribution.

phase shift calculated up to second order in ρ Eq. (64) goes to $3\pi/2$ for $k\rightarrow 0$ in contradiction with Levinson's statement. Clearly this wrong result is due to the impossibility of going further in analytically computing $\Delta^o(k)$. To get the correct behavior of the odd phase shift, numerical calculations were performed in which we solved explicitly the Schrödinger like equation (52). As it can be seen in Fig. 2, the results are now in agreement with the presence of an odd parity bound state as predicted in Ref. 20. On the other hand, for large fields (small values of ρ) the odd parity phase shift approaches the even parity values, as can be seen in Fig. 3, in agreement with the reflectionless behavior of the nonperturbed potential.

Therefore, the spectrum of Eq. (53) is composed by (i) the ψ_0 solution (45) corresponding to the translation mode of the wall (Goldstone mode), (ii) an internal mode which appears when the system is perturbed, and (iii) the ψ_k solutions which constitute the continuum modes and correspond to magnons.

B. The damping coefficient

In order to find the damping coefficient we must compute $\mathcal{R}(k)$. This can be done by inserting the numerical results of the even and odd phase shifts into the general expression

$$\mathcal{R}(k)=\sin^2[\Delta^e(k)-\Delta^o(k)]. \tag{66}$$

In Fig. 4, we have plotted $\mathcal{R}(k)$ for different values of the perturbation parameter ρ defined by Eq. (22) for the whole range of k. As it can be seen the major contribution for the reflection coefficient comes from the low-energy states, in agreement with the well behaved potentials (54) and (55). [Notice that we have not considered the second bound state of the potential (53) in computing the damping coefficient because in evaluating the scattering matrix (34), only elastic terms are taken into account (Refs. 5,6,14 and 15).]

Having done that, one can immediately integrate this function in expression (36) which finally allows us to describe the damping as a function of the temperature as shown in Fig. 5. As it can be seen, the damping constant is linear for high temperatures. This result can be obtained directly from Eq. (36). In fact, for T high enough the damping constant can be approximated by

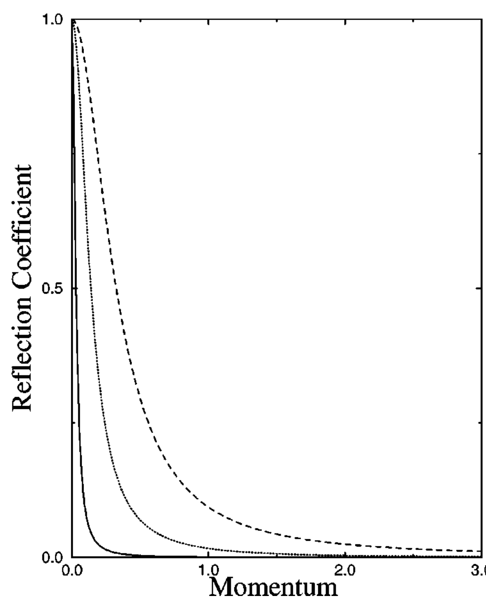

FIG. 4. The reflection coefficient as a function of the momentum. The continuous line for $\rho=0.14$, the dotted line for $\rho=0.31$, and the dashed line for $\rho=0.50$.

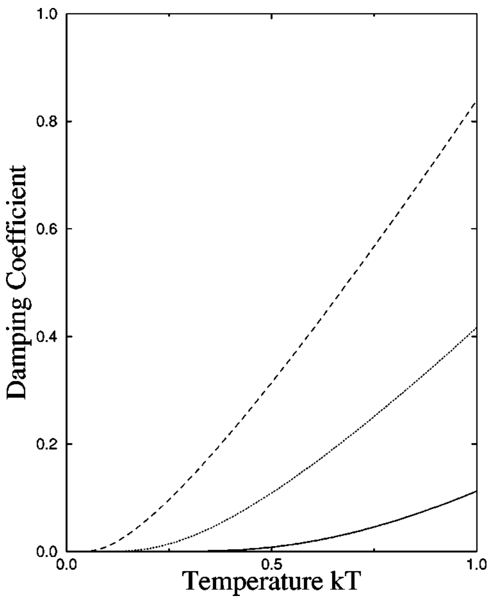

FIG. 5. The damping coefficient as a function of the temperature for different values of ρ. The continuous line for $\rho=0.14$, the dashed line for $\rho=0.31$, and the dotted line for $\rho=0.50$.

$$\overline{\gamma}(T)\simeq\frac{1}{2\pi M_s\beta}\int_0^{\infty}dE\frac{\mathcal{R}(E)}{E}\propto T, \tag{67}$$

which is linear on T, independently of the explicit form of $\mathcal{R}(E)$. In the low-temperature regime we can write

$$\overline{\gamma}(T)\simeq\frac{1}{2\pi M_s}\int_0^{\infty}dE\mathcal{R}(E)\beta E e^{-\beta E}, \tag{68}$$

where E always presents a gap determined by the presence of the magnetic field and/or the anisotropy. Here we shall not attempt to write an approximate expression for Eq. (68) because the correct behavior of the reflection coefficient was only numerically determined. As it is shown in Fig. 5, for low enough temperatures, the damping coefficient drops exponentially to zero due to the existence of the gap. As the temperature increases the damping coefficient rises following a power-law behavior until it becomes linear for high enough temperatures. On the other hand, when the ratio anisotropy-magnetic field (ρ) goes to zero, we recover the case of the reflectionless potential $(A_2=0)$ in which the mobility of the Bloch wall goes to infinity.

VI. CONCLUSIONS

In the foregoing sections we have shown that the continuum approximation to treat the totally anisotropic one-dimensional ferromagnet allows us to describe the Bloch wall as the localized solution of an effective field theory. The advantage of this procedure is the fact that employing the collective coordinates method to quantize the system we reformulated the problem in such a way that the new Hamiltonian takes into account the Bloch wall-magnons collisions. However, one should keep in mind that for realistic systems our limit of large external field should never reach values that would render the localized solutions unstable.

Finally, the formulation presented in this paper provides a systematic way to calculate the mobility, basically the inverse of the damping parameter, of the Bloch wall as a function of temperature and magnetic field. Although we have considered the limit of high magnetic fields, there is no reason why one should not apply the same methods to the low-field case. The only difference is that the scattering problem with which one has to deal is more straightforward in the high-field case.

It would be desirable to compare our results to experimentally measured values of the mobility of Bloch walls in anisotropic chainlike ferromagnets in order to shed light on the discussion of the relevance of the spin-wave scattering to the motion of these objects.

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APPENDIX

In this appendix we show how to obtain the expressions of the even and odd spectral functions (60) and (59). Suppose we have a particle in a one-dimensional symmetricpotential of the form $V=V_0+gV_1$ confined to a region $(-L,+L)$ with L much larger than the range of the potential V. The asymptotic form of the wave, then functions for $V\neq 0$ are given by

$$|z\rangle^e=\sqrt{\frac{1}{L}}\cos[k|z|+\Delta^e(k)],$$

$$|z\rangle^o=\sqrt{\frac{1}{L}}\mathrm{sgn}(z)\sin[k|z|+\Delta^o(k)], \tag{A1}$$

for $|z|\to\infty$. If $V=0$ the wave functions have the same structure as in Eq. (A1) with $\Delta^{e,o}=0$. Because the wave functions must vanish at $z=\pm L$ one realizes that

$$\frac{\delta E_n}{\Delta E_n}=-\frac{1}{\pi}\Delta^{e,o},$$

where $\Delta E_n=E^0_{n+1}-E^0_n$ and $\delta E_n=E_n-E^0_n$. Following closely the prescriptions given in[18] for the 1D case, the spectral functions (60) and (59) are given by

$$A_i^{e,o}(E)=-\int_{-\infty}^{+\infty}dzV_i(z)\langle z|E\rangle_{e,o}\langle E|z\rangle_{e,o}, \quad i=0,1, \tag{A2}$$

where the states $\langle E|z\rangle_{e,o}$ can be obtained by taking

$$\langle z|E\rangle_{e,o}=\lim_{L\to\infty}\frac{|z\rangle}{\sqrt{\Delta E_n}}. \tag{A3}$$

Explicitly,

$$\langle z|E\rangle=\frac{1}{\sqrt{2\pi k}}\begin{cases}\cos(kx), & \text{for even parity}\\ \sin(kx), & \text{for odd parity.}\end{cases} \tag{A4}$$

Inserting Eqs. (A4) and (55) in Eq. (A2) we have

$$A_1^e+A_1^o=\frac{8M\rho^2}{\pi\hbar^2 k}\int_{-\infty}^{+\infty}\text{sech}^2\left(\frac{z}{\lambda}\right)\tanh^2\left(\frac{z}{\lambda}\right)dz, \tag{A5}$$

which can be easily evaluated with the substitution $y=\tanh z/\lambda$, yielding

$$A_1^e+A_1^o=\frac{16\rho^2 M}{\pi\hbar^2\lambda k}. \tag{A6}$$

On the other hand, we have

$$A_1^e-A_1^o=\frac{8M\rho^2}{\pi\hbar^2 k}\int_{-\infty}^{+\infty}\text{sech}^2\left(\frac{z}{\lambda}\right)\tanh^2\left(\frac{z}{\lambda}\right)\cos(2k\lambda)dz, \tag{A7}$$

that can be written as

$$A_1^e-A_1^o=\frac{8M\rho^2}{\pi\hbar^2 k}\int_{-\infty}^{+\infty}\left[\text{sech}^2\left(\frac{z}{\lambda}\right)-\text{sech}^4\left(\frac{z}{\lambda}\right)\right]\cos(2k\lambda)dz, \tag{A8}$$

which can be analytically evaluated[21] yielding

$$A_1^e-A_1^o=\frac{16\rho^2 M}{\hbar^2\sinh(\pi k\lambda)}\left[\frac{1}{3}-\frac{2k^2\lambda^2}{3}\right]. \tag{A9}$$

Therefore, combining Eqs. (A6) and (A9) we have Eq. (62). In the same fashion it can be shown that

$$A_0^e+A_0^o=\frac{4M}{\pi\hbar^2 k\lambda}\quad\text{and}\quad A_0^e-A_0^o=\frac{4M}{\hbar^2\sinh(\pi k\lambda)}, \tag{A10}$$

which immediately gives Eq. (61). Now we can evaluate the Cauchy principal value in Eq. (60) which reads

$$B_0^{e,o}=\frac{2M}{\hbar^2}\mathcal{P}\int_{-\infty}^{+\infty}\left[\frac{1}{\pi k'\lambda}\pm\frac{1}{\sinh(\pi k'\lambda)}\right]\frac{k'dk'}{k^2-k'^2}. \tag{A11}$$

The first term on the right-hand side of Eq. (A11) is clearly zero. Therefore, using the product expansion of the $\sinh(\pi z)$ function[21] its second term becomes

$$B_0^{e,o}=\pm\frac{2M}{\pi\hbar^2}\mathcal{P}\int_{-\infty}^{+\infty}\frac{dq}{k^2-q^2}\prod_{n=1}^{\infty}\frac{n^2}{n^2+q^2}, \tag{A12}$$

where $q=k/\lambda$. Going to the complex plane, the previous expression can be analytically evaluated as

$$B_0^{e,o}=\pm\frac{4M}{\hbar^2}\sum_{n=1}^{\infty}\frac{(-1)^n n}{(k\lambda)^2+n^2}. \tag{A13}$$

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