Model for phonetic changes driven by social interactions

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We propose a stochastic model to study phonetic changes as an evolutionary process driven by social interactions between two groups of individuals with different phonological systems. Particularly, we focus on the changes in the place of articulation, inspired by the drift $/\Phi/ \rightarrow /\text{h}/$ observed in some words of Latin root in the Castilian language. In the model, each agent is characterized by a variable of three states, representing the place of articulation used during speech production. In this frame, we propose stochastic rules of interactions among agents which lead to phonetic imitation and consequently to changes in the articulation place. Based on this, we mathematically formalize the model as a problem of population dynamics, derive the equations of evolution in the mean-field approximation, and study the emergence of three nontrivial global states, which can be linked to the pattern of phonetic changes observed in the language of Castile and in other Romance languages.

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I. INTRODUCTION

Oral communication, as the process of transmitting concepts and ideas from one individual to another by word of mouth, has been a main feature of human kind since the first primitive societies. Historically, research in this field has been faced by anthropologists and linguistics. However, in recent years the interest for the development of new technologies related to automatic speech recognition, especially in artificial intelligent systems [1], has made it an active multidisciplinary area of research [2–5].

In particular, in the field of the physics of complex systems, the topic has been faced from the point of view of competition and evolution. In Ref. [6], for instance, by performing agent-based model simulations, Castelló *et al.* analyzed the competition between two socially equivalent languages and studied the dynamics in structured populations in the frame of complex network theory. Similarly, in Ref. [7], Stauffer and Schulze focused on the concepts of evolution, analyzing the rise and fall of languages using both macroscopic differential equations and microscopic Monte Carlo simulations. In Ref. [8], likewise, Baronchelli *et al.* focused on the analysis of the emergence of grammatical constructions, reporting an order-disorder transition where the system goes through a sharp symmetry-breaking process to reach a shared set of conventions. Moreover, the coevolution of symbols and meanings has been studied through elementary language games by Puglisi *et al.* [9], showing the emergence of a hierarchical category structure.

In the frame of linguistic theories, on the other hand, speech production might be thought of as the combination of several cognitive processes: the selection of the proper words to express an idea, the suitable choice of a grammatical form, and the production of sounds via the motor system and the vocal apparatus [10]. In this work we focus on the last; therefore, in the following we describe the most relevant concepts regarding the production of sounds.

Formally, phonemes are the minimal units of either vocalic or consonant sounds needed to produce words. In this regard, the set of phonemes which encompasses all the sounds needed to produce every word in a given language defines a phonological system (PS). It is particularly important to emphasize that phonemes are not sounds but formal abstractions of speech sounds. Any phoneme in a PS might be a representation for a family of sounds, technically called phones, which are recognized by speakers and linked to a specific sound during oral communications [11]. Physiologically, the process by which the vocal apparatus produces sounds is called phonation [12]. Through this mechanism, humans are able to produce a wide range of sounds, usually divided into two groups: vowels and consonants [13]. Let us focus on consonant production. In this case, the phonetic apparatus uses a combination of tongue, lips, teeth, and the soft palate in order to shape the different air obstructions needed to produce the sounds. The point inside the vocal cavity where the obstruction occurs is called the articulation place (AP), and the manner in which the obstruction is shaped is called the articulation mode (AM). These two dimensions, AP and AM, are commonly used to categorize the main features of the consonants in a particular PS.

Since languages are in continuous evolution, it is well known that, under certain conditions, this process might lead to changes in the PS [14]. In particular, in this work we are interested in studying phonetic changes in consonants related to variations in the AP. In this regard, it has been observed that these changes are enhanced when two or more groups of people with different languages are forced to socially interact [15–19]. For instance, when a group invades another group's territory, or when two groups establish economic relations (trade, exchange of services, etc.). The linguistic, phonetic,

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TABLE I. Some examples of Latin root words which show the change $/\Phi/\to/h/$ in Castilian (columns 1 and). In column 3, we show the translation into Portuguese in order to show the change in this case did not happen. In column 4, we show the translation into English as a reference. In order to clarify, note that, in column 1, the phonetic transcription of the letter f is the bilabial phoneme $/\Phi/$, and in column 2, the phonetic transcription of the letter h is the glottal phoneme $/h/$.

Latin	Castilian	Portuguese	English translation
facere	hacer	fazer	to do, to make
femina	hembra	fêmea	female
ferru	hierro	ferro	iron
filiu	hijo	filho	son
folia	hoja	folha	leaf
fumu	humo	fumaça	smoke

and grammatical mutual influences produced by the interaction among the groups define a linguistic stratum (LS) [20,21], where the persistent social interaction over time, in a process of oral communication, guides the evolution to a common PS and to a common new language.

A notable example of phonetic change in the AP due to a LS, and the main inspiration of this work, is the case of the glottalization of the bilabial fricative phoneme $/\Phi/$ towards the glottal fricative $/h/$ (hereafter referred to as the change $/\Phi/\to/h/$), and its subsequent disonorization in some Latin root words of Castilian language (see Table I). It is thought that the social process which led to the LS in this particular case was related to the social interactions among the prehispanic tribes (Iberics, Asturians, and mainly Vascons) and the Romans, which were forced to socially interact in the Iberian peninsula—during the period of Rome's domain—from the second to the ninth century, A.D. [22–24]. In this particular LS there were groups of people with a PS (based on Celtic language) socially interacting with another group of people with a total different PS (based on Latin language).

In this context, it is thought that the change $/\Phi/\to/h/$ is related to prehispanic tribe speakers performing changes in the AP during fricative production, trying to improve their communication skills with Latin speakers [25]. Notably, these changes are not observed in other Romance languages on the Iberian peninsula, as in the case of Portuguese or Catalan, which emerged from a similar LS as Castilian. This fact led researchers to theorize about the properties of this particular LS, and additionally to propose alternative theories [26,27]. Until now, it seems there is not a total consensus regarding the causes which led to these different evolutions of the Romance languages; therefore, the case is currently considered by linguistics as an open problem.

Motivated by the historical observations regarding the changes $/\Phi/\to/h/$, the aim of this work is to propose a model of language competition [28–32] which captures the internal dynamics of a LS. The model considers two social groups having different PSs, where the changes in the articulation places are guided by social interactions based on rules of phonetic imitation.

We face the problem in the frame of population dynamics, where we study the evolution of the changes in both groups, and the emergence of general states of pronunciation in the LS.

This paper is divided into three main sections: in Sec. II we mathematically formalize the model, define the main variables, propose the rules of the interactions, and describe the dynamics; in Sec. III we derive from first principles the equations of evolution; and, last, in Sec. IV we analyze the emergence of global states by performing an analysis of both the evolution equations and agent-based model simulations. We found that by tuning the parameters related to the social interactions in the LS, our model shows the emergence of three general global states which capture qualitatively the observations of the emergent Romance languages of the Iberian peninsula, reinforcing from our mathematical approach the stratum-based theories present in the literature.

II. THE MODEL

We aim to model a process of phonetic imitation which leads to changes in the AP during consonant production. Accordingly, we have made the following simplifications: (i) we limit our analysis to the changes in the AP, neglecting any change in the AM; (ii) we propose there are only three possible APs in the vocal cavity, a *front* place (bilabial, labiodental), a *middle* place (dental, alveolar, and postalveolar), and a *back* place (palatal, velar, uvular, and glottal); (iii) we suppose there are two PSs in the LS, one which favors front and middle production, and another which favors middle and back; and (iv) we study the evolution of the changes in the pronunciation of a single word.

In this frame, we define the main elements of the model as follows:

(1) A and B are two groups of agents in a stratum LS.

(2) N_A and N_B are the number of agents in A and B and $N = N_A + N_B$ the total number of agents in the LS.

(3) S is the state of an agent in the LS at time t, where $S \in \{1, 2, 3\}$ represents the AP of agent i, such that $1 =$ front, $2 =$ middle, and $3 =$ back.

(4) PS^A and PS^B are the phonological systems of A and B, indicating that A has $S = 1, 2$ as preferential states (front, middle) and, conversely, B has $S = 2, 3$ (middle, back).

In the evolutionary process, at time t, we randomly take from the set $A \cup B$ an active agent and a reference agent. The state of the former will change according to the state of the latter, guided by the following imitation rules which we summarize by using the usual chemical reactions notation,

$$A_2 + B_3 \xrightarrow{q} A_3 + B_3, \tag{1a}$$

$$B_2 + A_1 \xrightarrow{q} B_1 + A_1, \tag{1b}$$

$$A_3 + A_2 \xrightarrow{p} 2A_2, \tag{1c}$$

$$B_1 + B_2 \xrightarrow{p} 2B_2, \tag{1d}$$

$$A_1 \overset{r}{\rightleftharpoons} A_2, \tag{1e}$$

$$B_2 \overset{r}{\rightleftharpoons} B_3, \tag{1f}$$

where A_i and B_i are agents of group A and B, respectively, in state $S = i$. For example, Eq. (1c) states that a member of the population A_2 can interact with a member of A_3 and that the result of the interaction are two members of the population A_2.

The rules (1a)–(1d) are introduced in order to emulate a process of imitation, where interactions between agents of different groups lead to nonpreferential states of pronunciation, and interactions between agents of the same group, conversely, reinforce the preferential states of the group. In this respect, probabilities q, p define the interaction strength between agents of different groups, and the interaction strength between agents of the same group, respectively. Moreover, the noisy component expressed by rules (1e) and (1f) captures the variations caused by both random phonetic changes and the production of possible allophones in both PS^A and PS^B, respectively. On the other hand, note that in the frame of the proposed imitation rules, the changes in the states occur only when the AP distance, between the referent and the active agents, is equal to one (i.e., $|\Delta S_{ij}| = 1$, with i, j reference and active agents). The idea here is to model the changes in the context of close or similar sounds [33–35] between the different PSs, neglecting the contribution to the changes of any other possible interactions.

In this theoretical frame, our proposal has been inspired by the models of opinion formation dynamics [36,37], where social interactions, in the context of a social debate, drive the population to emergent states of consensus or polarization [38–40].

Macroscopically, in the frame of evolutionary dynamics, the global state of the system can be analyzed by counting the number of agents in A, B, in the states $S = 1, 2, 3$. In the following section, based on this idea we introduce the master equation of the process and derive the evolution equations for the first moments, or mean-field approximation.

III. THE EQUATIONS OF EVOLUTION

Let N_i^A and N_i^B be the numbers of agents in A and B in state $S = i$, with $i = 1, 2, 3$ (hereafter referred to as the occupation numbers). The master equation of the system is given by

$$\frac{\partial P(\vec{x};t)}{\partial t} = \sum_{\vec{y}\neq\vec{x}} T(\vec{x}|\vec{y})P(\vec{y};t) - \sum_{\vec{x}\neq\vec{y}} T(\vec{y}|\vec{x})P(\vec{x};t), \tag{2}$$

where $\vec{x} = (N_1^A, N_2^A, N_3^A; N_1^B, N_2^B, N_3^B)$ is the so-called occupation vector; $P(\vec{x};t)$ is the probability to find the system with an occupation vector $\vec{x}$ at time t, and $T(\vec{x}|\vec{y})$ is a transition probability from a global state given by an occupation vector $\vec{y}$ to another given by $\vec{x}$. In this approach, we consider a fixed population in both groups; therefore, for all t we have

$$\begin{aligned} N_A &= N_1^A + N_2^A + N_3^A, \\ N_B &= N_1^B + N_2^B + N_3^B, \\ N &= N_A + N_B. \end{aligned} \tag{3}$$

From master equation (2), we can derive the evolutionary equations for the first moment of the occupation numbers (mean-field approximation). For instance, for N_2^A we have

$$\frac{d}{dt}\langle N_2^A\rangle = \langle T(N_2^A + 1|N_2^A)\rangle - \langle T(N_2^A - 1|N_2^A)\rangle, \tag{4}$$

and the transitions T are defined by the rules proposed in the last section and depend on probabilities p, q, and r. For the case by which the system increases one agent in N_2^A, T can be written as

$$T(N_2^A + 1|N_2^A) = \frac{N_1^A}{N}r + \frac{N_3^A}{N}\frac{N_2^A}{N-1}p, \tag{5}$$

where the first term is the probability of finding an agent of group A in state 1, times the probability it randomly goes to state 2; the second term is the probability of one interaction between an active agent of group A in state 3 and a reference agent of the same group in state 2, leading the former to imitate the latter with probability p.

For the case in which the occupation number N_2^A decreases by one agent, the transition is given by

$$T(N_2^A - 1|N_2^A) = \frac{N_2^A}{N}\frac{N_3^B}{N-1}q + \frac{N_2^A}{N}r, \tag{6}$$

where the first term shows the loss of an active agent of group A in state 2 due to the interaction with a reference agent of group B in state 3, and the second term stands for the random loss of an agent in 2 who moves to 1.

Replacing Eqs. (5) and (6) in Eq. (4), we obtain

$$\begin{aligned} \frac{d}{dt}\langle N_2^A\rangle &= \left\langle \frac{N_1^A}{N}r + \frac{N_3^A}{N}\frac{N_2^A}{N-1}p\right\rangle - \left\langle \frac{N_2^A}{N}\frac{N_3^B}{N-1}q + \frac{N_2^A}{N}r\right\rangle, \\ &= \frac{r}{N}\langle N_1^A\rangle + \frac{p}{N(N-1)}\langle N_3^A N_2^A\rangle \\ &\quad - \frac{q}{N(N-1)}\langle N_2^A N_3^B\rangle - \frac{r}{N}\langle N_2^A\rangle. \end{aligned} \tag{7}$$

From now on, we consider that the evolution of the numbers are uncorrelated; then $\langle N_2^A N_3^B\rangle = \langle N_2^A\rangle\langle N_3^B\rangle$ and $\langle N_3^A N_2^A\rangle = \langle N_3^A\rangle\langle N_2^A\rangle$.

Moreover, we rescale the time as $t \to \frac{t}{N}$, use the approximation for large population $(N-1) \approx N$, and define the occupation number as fractions of the total population: $a_1 = \langle\frac{N_1^A}{N}\rangle$, $a_2 = \langle\frac{N_2^A}{N}\rangle$, $a_3 = \langle\frac{N_3^A}{N}\rangle$, and $b_3 = \langle\frac{N_3^B}{N}\rangle$. Using the notation and approximations introduced above, we can write Eq. (7) as follows:

$$\dot{a_2} = ra_1 + pa_2a_3 - qa_2b_3 - ra_2. \tag{8}$$

Similarly, it is possible to obtain the equations for the evolution of the other occupation numbers, which define the following system of coupled differential equations:

$$\begin{aligned} \dot{a_1} &= r(a_2 - a_1), \\ \dot{a_2} &= -r(a_2 - a_1) + pa_2a_3 - qa_2b_3, \\ \dot{a_3} &= -pa_2a_3 + qa_2b_3, \\ \dot{b_1} &= -pb_1b_2 + qb_2a_1, \\ \dot{b_2} &= pb_1b_2 - qb_2a_1 - r(b_2 - b_3), \\ \dot{b_3} &= r(b_2 - b_3). \end{aligned} \tag{9}$$

It is important to highlight that, since we are now working with the fractions of the occupation numbers, the constraints related to the fixed population become $n_A = a_1 + a_2 + a_3$, $n_B = b_1 + b_2 + b_3$, and $n_A + n_B = 1$, where $n_A = \frac{N_A}{N}$ and $n_B = \frac{N_B}{N}$. The constraints also show that it is possible to reduce the rank of the system to four; however, for the sake of clarity, we decided to keep the six equations for a more detailed analysis.

Finally, note that, if there are equilibrium points in system (9), in the frame of our model, these may be related to the population reaching consensus about a common general way of pronunciation in both groups. In the following section we probe the existence of four equilibrium points, and show that the stability of these emergent states strongly depends on the intra- and intergroup interaction rates.

IV. THE EMERGENT STATES

At time $t \to \infty$, if an equilibrium exists, it must satisfy $\dot{a}_i = 0$ and $\dot{b}_i = 0$. In these conditions, from Eqs. (9) and using the population constraints, it is possible to probe the existence of four nontrivial equilibrium points:

(I) $\vec{n}_{\rm eq} = (0, 0, n_A; n_B, 0, 0)$,

(II) $\vec{n}_{\rm eq} = (0, 0, n_A; 0, \frac{n_B}{2}, \frac{n_B}{2})$,

(III) $\vec{n}_{\rm eq} = (\frac{n_A}{2}, \frac{n_A}{2}, 0; n_B, 0, 0)$,

(IV) $\vec{n}_{\rm eq} = (\alpha, \alpha, Q\beta; Q\alpha, \beta, \beta)$, where $\alpha = \frac{2n_A - Qn_B}{4-Q^2}$, $\beta = \frac{2n_B - Qn_A}{4-Q^2}$, and $Q = \frac{q}{p}$.

The first equilibrium can only be reached if the initial conditions are set at this point; later we show this equilibrium is unstable. Equilibria II and III are the cases where the system loses the back and the frontal AP, respectively. Equilibrium IV, on the other hand, shows a mixed final state, where the system reaches a balance among the three states of pronunciation.

The stability analysis around the equilibrium can be performed by analyzing the eigenvalues of system (9) (see the Appendix).

We are particularly interested in studying the evolution of the system as a function of Q since this parameter controls the interaction between A and B, and also rules the population state of the equilibrium point IV. To this purpose, we set a constant equal population $n_A = n_B$, $r = 0.5$, and $q = 0.5$ and vary the parameter p, in order to focus only on the study of the effect of $Q = q/p$, on the equilibrium conditions of the system. In this frame, we have calculated the eigenvalues as a function of Q and studied the sign of the real part of the eigenvalues to evaluate the stability conditions.

The plots in Fig. 1 show the curves for the real part of the largest nontrivial eigenvalue, $\mathrm{Re}(\lambda_1)$ vs Q for the four equilibrium points. Figure 1(a) shows the calculation for state I. We can see that for all Q there is an eigenvalue with positive real part, which means this equilibrium state is always unstable. In Figs. 1(b) and 1(c) we can observe that states II and III behave similarly to each other. This is expected because these two states are symmetric. States II and III are unstable for $Q < 2$ since they have at least one eigenvalue with positive real part. Finally, Fig. 1(d) shows the calculation for equilibrium state IV; in this case, complementary to states II and III, the equilibrium is unstable for $Q > 2$.

Clearly, coefficient Q, which measure the relative intra- and intergroup imitation rates, determines the stability of the different equilibrium states of the system. We can understand this phenomenon by reasoning as follows: At $Q = 2$, the intergroup interactions equal the sum of the interaction rates within each group ($Q = 2 \to q = p + p$), and equilibrium

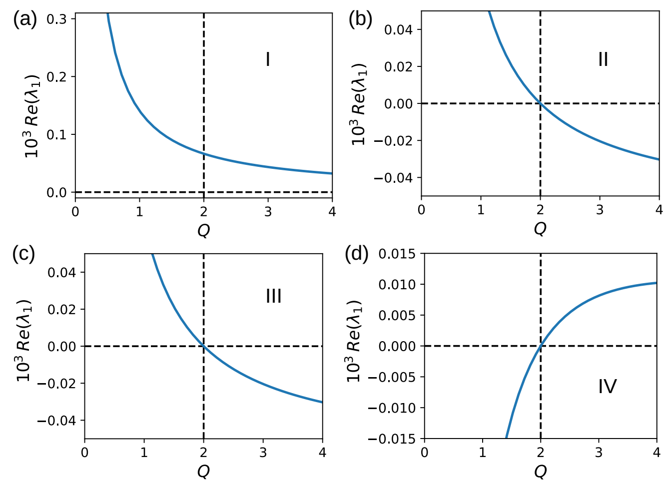

FIG. 1. Stability analysis. Real part of the largest nontrivial eigenvalue of the Jacobian matrix (λ_1), as a function of the parameter Q. (a)–(d) Numerical calculation for the equilibria I, II, III, and IV, respectively. Note that when $\mathrm{Re}(\lambda_1) > 0$ the system is unstable; therefore, equilibrium I will be unstable for all values of Q; equilibria II and III will be unstable for $Q < 2$, and equilibrium IV for $Q > 2$.

IV becomes $\vec{n}_{\rm eq} = (\alpha, \alpha, Q\beta; Q\alpha, \beta, \beta) = (\frac{1}{8}, \frac{1}{8}, \frac{1}{4}; \frac{1}{4}, \frac{1}{8}, \frac{1}{8})$; i.e., there is a balance between the number of agents in preferential and nonpreferential states of pronunciation in both groups ($n_1^A + n_2^A = n_3^A$ and $n_1^B = n_2^B + n_3^B$). For $Q > 2$, the system loses the balance: the number of agents in nonpreferential states becomes larger than the number of agents in preferential states, and hence equilibrium IV becomes unstable and, depending on the initial conditions, the system evolves to equilibrium II or III. Furthermore, in the stochastic version of the model and balanced initial conditions fluctuations can determine the final equilibrium state, which occurs with equal probability for states II and III.

Finally, a more general stability analysis of the mean-field model, which includes unbalanced conditions $n_A \neq n_B$, is conducted in the Appendix. The main results of this analysis are summarized in the phase diagram shown in Fig. 2. There

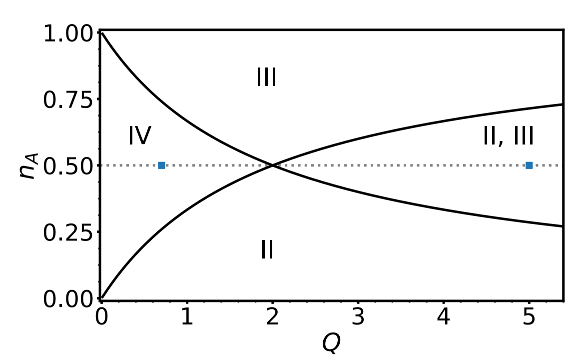

FIG. 2. Projection of the phase diagram in the plane (n_A, Q). The curves $n_A = 2/(2+Q)$ and $n_A = Q/(2+Q)$ divide the plane into four regions with different stability. The labels for each region correspond to the equilibria that are stable inside the region. The two blue markers represent the parameters that were used in the main text.

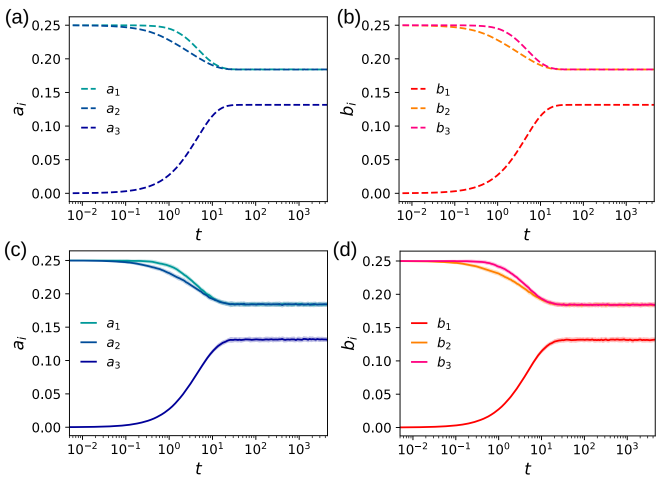

FIG. 3. The case when equilibrium IV is stable. For $Q = 0.7$, first moment evolution of the numbers related to (a) group A (a_1, a_2, a_3) and (b) group B (b_1, b_2, b_3), respectively. The curves are the outcome of the evolution of Eqs. (9), which were solved by performing a Runge Kutta eighth-order method. (c), (d) Average curves over 100 realizations, of the numbers in the ABM. The shading indicates the standard deviation around the mean value. In these conditions, the population evolves to equilibrium IV, a mixed state of pronunciation where the system reaches $\vec{n}_{\rm eq} = (\frac{5}{27}, \frac{5}{27}, \frac{7}{54}; \frac{7}{54}, \frac{5}{27}, \frac{5}{27}) \approx (0.18, 0.18, 0.13; 0.13, 0.18, 0.18)$.

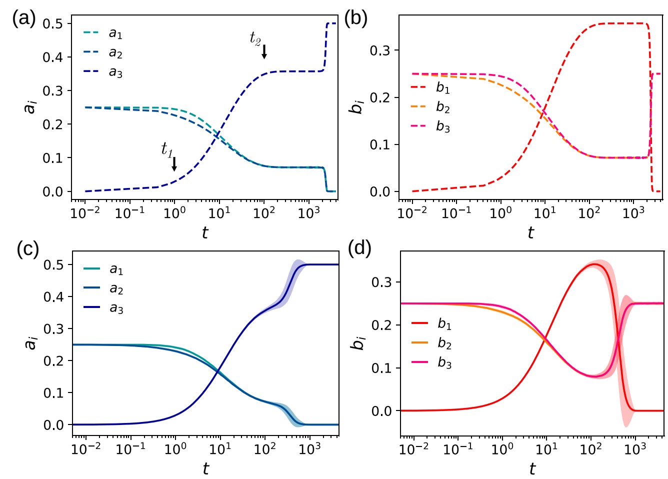

FIG. 4. The case where equilibrium II is stable. For $Q = 5$, the first moment evolution of the numbers related to (a) group A and (b) group B, respectively. (c), (d) The average curves over 100 realizations, of the numbers in the ABM. The shading in the plots indicates the standard deviation, which we see increasing when the system reaches the unstable equilibrium IV. Note how at the beginning the system seems to evolve toward IV, but it escapes toward equilibrium II, which is stable in these conditions [$\vec{n}_{\rm eq} = (0, 0, \frac{1}{2}; 0, \frac{1}{4}, \frac{1}{4})$]. The arrows in (a) indicate the time scales obtained from the eigenvalues analysis: $t_1 \approx 1$ and $t_2 \approx 10^2$.

are four stability regions in the n_A-Q plane: one for point IV at low values of Q, another two for points II and III, respectively, and one region where points II and III coexist. Our analysis is restricted to two representative values of Q in this phase diagram.

In order to visualize the time evolution of the occupation number in the system, and to study the convergence to the equilibrium points, we solved numerically the set of coupled differential equations given in Eqs. (9), and also performed numerical simulations of the stochastic agent-based model (ABM)—using the rules proposed in Sec. II—in order to test the effect of fluctuations in the dynamics of the system. In a first approach we explore two limit cases for the singular point $Q = 2$: the case of $Q = 0.7$ where the intergroup interaction domain governs the dynamics, and the case of $Q = 5$ where, conversely, the dynamic is governed by the intragroup interactions. For this purpose, we set (i) the initial conditions on $\vec{n}(t = 0) = (\frac{n_A}{2}, \frac{n_A}{2}, 0; 0, \frac{n_B}{2}, \frac{n_B}{2})$ (preferential states for both groups), (ii) the fraction of agents such that $n_A = n_B$ (equal population in both groups), (iii) the parameters $r = 0.5$ and $q = 0.5$, and (iv) for the ABM simulations, a population size of $N = 10^4$. The results are summarized in the plots of Figs. 3 and 4, which we describe in the following. Figure 3 shows the evolution of the occupation numbers for $Q = 0.7$ where state IV is stable. Figures 3(a) and 3(b) show the evolution in the mean-field approximation [Eqs. (9)], whereas Figs. 3(c) and 3(d) show the outcome of the ABM; here we have averaged the results over 100 performed simulations. We can see that, as expected, the system evolves toward equilibrium IV and the mean reaches the value $\vec{n}_{\rm eq} = (\alpha, \alpha, Q\beta; Q\alpha, \beta, \beta) = (\frac{5}{27}, \frac{5}{27}, \frac{7}{54}; \frac{7}{54}, \frac{5}{27}, \frac{5}{27})$.

Figure 4, by contrast, shows the evolution for $Q = 5$ where equilibria II and III are stable; in this case, the realization shown in the plots went to equilibrium II. Notably, at the beginning of the evolution, the system seems to stabilize at equilibrium IV, but at larger times moves toward equilibrium II, as expected. The arrows in Fig. 4(a) indicate the relaxation times operating at each regime, given by the inverse of the eigenvalues ($\propto 1/\lambda_i$).

In a second approach, we explore finite size effects in the system by performing an ABM simulation for different sizes (N) of the total population and comparing with the mean-field approximation. For the case $Q = 5$, we can see from Figs. 5(a) and 5(b) that the finite size effects become notable once the system reaches the unstable equilibrium IV. The larger the population the lower the fluctuations; therefore, the system will spend more time at the unstable equilibrium IV before fluctuations lead it to the stable state II.

In connection with that expressed above, we measured the distribution of times (T) needed for the system to reach the final equilibrium at $Q = 5$ by performing 2000 realizations of the stochastic model. Figure 5(c) shows the distribution obtained. We can see a nonsymmetric distribution with a peak around $T \approx 600$, and a tail at the right of the distribution. As we said before, in every realization the system stays in the unstable equilibrium IV before it reaches the final stable state; therefore, the total time T depends on the magnitude of the fluctuations which drive the system from the unstable equilibrium to the stable one. The latter explains the tail at the right of the distribution and also the differences in the final times observed among the stochastic simulations and the mean-field approach.

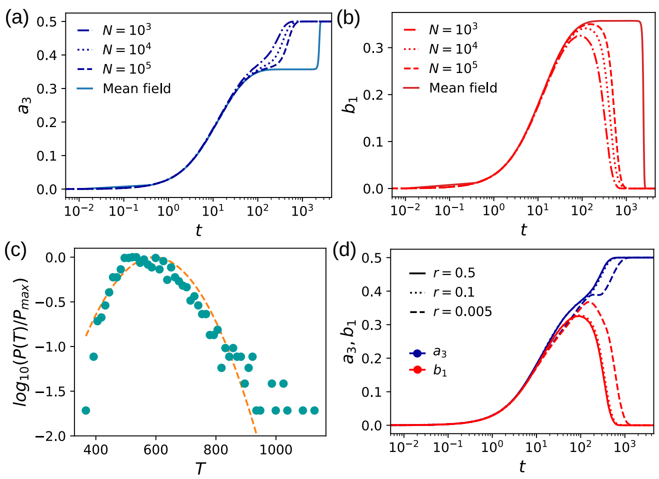

FIG. 5. Average curves for the evolution of the numbers (a) a_3 and (b) b_1, for different values of the population size N, compared with the mean-field approximation in order to see the finite size effect. (c) Distribution of final times T needed for the system to reach equilibrium II at $Q = 5$. The dashed line in the background indicates a Gaussian distribution with the same mean and standard deviation, which helps to visualize the presence of a large tail in distribution $P(T)$. (d) Average curves for the evolution of the numbers a_3 and b_1 for different values of the parameter r.

On the other hand, as we show above, the equilibrium points are not dependent on parameter r; however, since r is a stochastic parameter of the model, the dynamics towards the equilibrium should depend on it. In Fig. 5(d) we show the evolution of the numbers a_3 and b_1 for $r = 0.5$, 0.1, and 0.005. For the lowest value explored ($r = 0.005$), we can see a different behavior around the unstable equilibrium IV, which seems to indicate a link between the reduction of the stable attractor force and the reduction of r.

Last, in order to complete a global analysis, we test the system behavior under changes in the initial conditions. Figure 6(a) shows trajectories in the plane a_1-a_3 for $Q = 5$, obtained from the mean-field approach simulations, where we have tried different initial conditions for the numbers a_i, keeping $r = 0.5$ (the different trajectories in the plane are indicated in the plot as l_i, $i = 1, 2, 3, 4$). We can see that

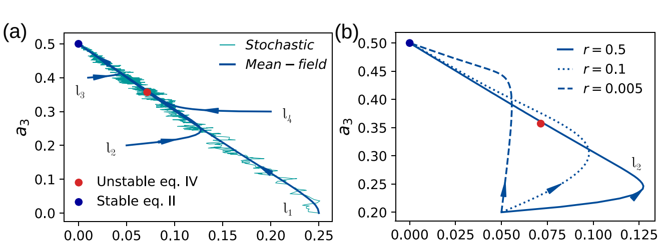

FIG. 6. Trajectories in the plane a_1-a_3: (a) changing the initial conditions, keeping $r = 0.5$, and (b) changing parameter r, for trajectory starting from the same initial condition as l_2 in (a). The blue lines correspond to the solution of the differential equations, while the cyan curve in (a) corresponds to a single realization of the stochastic simulation, starting from the initial condition of trajectory l_2.

depending on the proximity to the stable attractor II the system will explore the unstable equilibrium IV, as in the case of trajectories l_1, l_2, and l_4, or it will not as in the case of trajectory l_3. For the case of l_1 we have additionally plotted the trajectory obtained from a single stochastic realization.

The plot in Fig. 6(b), on the other hand, complements the information given in Fig. 5(d), showing the effect of parameter r on the trajectories in the plane a_1-a_3. In this regard, the parameter r regulates the velocity of the transitions $A_1 \rightleftharpoons A_2$ (and also $B_2 \rightleftharpoons B_3$). When r is strong enough, these transitions occur much faster than the rest and, thus, populations A_1 and A_2 tend to have the same number of individuals, independently of the initial conditions. This aspect explains why all the trajectories in Fig. 6(a) rapidly approximate to the line $a_3 = -2a_1$. If the system starts from an initial condition with a low value of a_3, then the trajectory will be forced to go through equilibrium IV, as can be seen in the figure. When we reduce the value of r, it is harder for the system to equal the values of a_1 and a_2 and, thus, the trajectories in general deviate from the line and equilibrium IV is avoided, as we show in Fig. 6(b).

In the frame of the proposed model, state II can be related to the change /Φ/→/h/ in Castilian, and state III to that observed in other Romance languages like Portuguese or Catalan. Therefore, for $Q > 2$, the model seems to capture very well the current pronunciations that emerged—from the real LS—in the Iberian peninsula. On the other hand, equilibrium IV describes an emergent state of mixed pronunciation, which means there are agents using different APs to pronounce the same word. This is rarely observed in the real case, but can be used to understand the existence of some regionalism or local accents in the peninsula [41].

V. CONCLUSIONS

In this work, we have proposed a model to study phonetic changes in the AP used to pronounce a single word as an evolutionary process guided by the social interaction of imitation between two groups of people with different phonological systems. Inspired by the case of the change /Φ/→/h/ in the Castilian language, we have studied a fixed population made up of two groups of interacting people, A and B, such that group A has a trend to produce frontal fricatives and conversely group B has a trend to produce back fricatives. The rules of the model were proposed based on empirical observations and were thought to link the phonetic changes with a process of social interactions inter- and intragroup. The model was mathematically formalized in Sec. II as a stochastic process where the variable $S \in \{1, 2, 3\}$, representing the AP for every agent in the population, changes according to the proposed interaction rules.

In this frame we studied the temporal evolution of the occupation numbers, and from first principles we derived the coupled system of differential equations which defines the dynamics in the mean-field approximation. In the equilibrium, we found three nontrivial final states, which we have related to the emergence of general states of consensus in the way a word is pronounced. In this regards, we found that when the rate of interaction among agents from different groups becomes larger than the sum of the rates within each group

($q > 2p$), the system exhibits two emergent states (equilibria II and III) which capture very well both the middle-back pronunciation used in Castilian and the front-middle pronunciation observed in other Romance languages, as in the case of Portuguese. From a social point of view, we can link the condition $q > 2p$ to the situation where the relation among individuals from different groups is large enough to allow a common general consensus in spite of the cultural differences.

The model we have introduced is based on a mean-field approach since all agents interact among them; hence it does not consider the influence of the structure of social interactions in reaching a consensus for the phonetic changes. This is of course a simplification, as it is known that the structure of social interactions strongly influences human behavior and the evolution of social and cognitive processes [42]. Even more, the human mental lexicon is supposed to be assembled according to a multiplex network structure [43]. Hence, one can expect the network structure to play a key role in any particular dynamics of phonetic changes. However, Baxter *et al.* [32] proved that in several neutral interactor models inspired in Trudgill's theory for the emergence of New Zealand English, the structure of the underlying social network has a minor effect on the final state distribution of the speaker's grammar (linguemes) produced by these models. According to Trudgill's deterministic theory, frequency of use and accommodation are the only factors to be taken into account for the prevalence of a given lingueme. Our model is deterministic in terms of its parameters. The final phonetic state is correlated to the initial frequency of agents using a given phonetic system for the case $Q > 2$ (states II and III); in those states one phonetic system prevails over the other. However, the proposed imitation mechanism that rules the interaction between agents of different phonetic systems can lead to a final state in which the two phonetic systems coexist. This is particularly relevant in the analysis of Castilian phonetic when the system is considered as a whole. In this case the LS contains fricative words that keep the Latin pronunciation as well as other words that change to glottalization, which means that there is a coexistence of the articulation places in the global system.

Finally, more realistic models must consider the structures of social interactions in the dynamics. Then it would be interesting to analyze the interplay between the network topology and the dynamics of phonetic changes generated by our model. In this regard, we leave as an open problem to be faced in future works the study of the effect of the structure of social interactions in phonetic changes.

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APPENDIX: STABILITY ANALYSIS

In this Appendix we perform a stability analysis to extend the discussion in Sec. IV.

Equation system (9) can be reduced to a four-dimensional system using the constraints $a_1 + a_2 + a_3 = n_A$ and $b_1 + b_2 + b_3 = n_B$. Defining the parameters $\omega = r/p$ and $Q = q/p$ and scaling the time as $\tau = pt$, we have

$$\begin{aligned}
\dot{a_1} &= \omega(n_A - 2a_1 - a_3),\\
\dot{a_3} &= (Qb_3 - a_3)(n_A - a_1 - a_3),\\
\dot{b_1} &= (Qa_1 - b_1)(n_B - b_3 - b_1),\\
\dot{b_3} &= \omega(n_B - 2b_3 - b_1),
\end{aligned} \tag{A1}$$

where we have changed the notation $\dot{x} \equiv dx/d\tau = (1/p)dx/dt$.

The Jacobian of the reduced system is

$$J = \begin{pmatrix} -2\omega & -\omega & 0 & 0\\ a_3 - b_3Q & a_1 + 2a_3 - n_A - b_3Q & 0 & (n_A - a_1 - a_3)Q\\ (n_B - b_3 - b_1)Q & 0 & b_3 + 2b_1 - n_B - a_1Q & b_1 - a_1Q\\ 0 & 0 & -\omega & -2\omega \end{pmatrix}. \tag{A2}$$

We proceed now to analyze the stability of each equilibrium. Before we start, it is important to notice that, as we have rescaled the time as $\tau = pt$, the eigenvalues of the original system can be expressed as $\lambda = p\mu$, where μ are the eigenvalues of the reduced system.

Evaluating (A2) at equilibrium I and computing its eigenvalues, we have

$$\begin{aligned}
\mu^B_\pm &= \frac{n_B - 2\omega \pm \sqrt{(n_B - 2\omega)^2 + 4n_B\omega}}{2},\\
\mu^A_\pm &= \frac{n_A - 2\omega \pm \sqrt{(n_A - 2\omega)^2 + 4n_A\omega}}{2}.
\end{aligned} \tag{A3}$$

The four eigenvalues are always real, and two of them (the ones with the plus sign) are always positive. This means that equilibrium I is always unstable, and thus it lacks physical interest.

For equilibrium II, the corresponding eigenvalues are

$$\begin{aligned}
\mu_\pm &= \frac{2n_A - Qn_B - 4\omega \pm \sqrt{(2n_A - Qn_B)^2 + 16\omega^2}}{4},\\
\mu_3 &= -\frac{n_B}{2},\\
\mu_4 &= -2\omega.
\end{aligned} \tag{A4}$$

The eigenvalues μ_-, μ_3, and μ_4 are always negative, but μ_+ can be either positive or negative. The region of the parameter space where it is negative (and thus where the equilibrium is stable) is given by

$$Q > \frac{2n_A}{n_B}. \tag{A5}$$

Considering $n_B = 1 - n_A$, the previous inequality can be solved for n_A as

$$n_A < \frac{Q}{2+Q}. \tag{A6}$$

Equilibrium III is symmetric with respect to equilibrium II and its eigenvalues are

$$\begin{aligned}
\mu_{\pm} &= \frac{2n_B - Qn_A - 4\omega \pm \sqrt{(2n_B - Qn_A)^2 + 16\omega^2}}{4},\\
\mu_3 &= -\frac{n_A}{2},\\
\mu_4 &= -2\omega.
\end{aligned} \tag{A7}$$

Thus, the equilibrium is stable when

$$Q > \frac{2n_B}{n_A}, \tag{A8}$$

which can be also expressed as

$$n_A > \frac{2}{2+Q}. \tag{A9}$$

Equilibrium IV exists only if α and β are simultaneously greater than zero. It can be shown that the condition for this to happen is

$$\left(n_A - \frac{2}{2+Q}\right)\left(n_A - \frac{Q}{2+Q}\right) < 0. \tag{A10}$$

To analyze the stability, let us first consider the particular case $n_A = n_B$. In this case, the corresponding eigenvalues are

$$\begin{aligned}
\mu_{1,\pm} &= \frac{-[n_A + 2\omega(2+Q)]}{2(2+Q)}\\
&\quad \pm \frac{\sqrt{[n_A + 2\omega(2+Q)]^2 - 4\omega n_A(4 - Q^2)}}{2(2+Q)},\\
\mu_{2,\pm} &= \frac{-[n_A + 2\omega(2+Q)]}{2(2+Q)}\\
&\quad \pm \frac{\sqrt{[n_A + 2\omega(2+Q)]^2 - 4\omega n_A(2+Q)^2}}{2(2+Q)}.
\end{aligned} \tag{A11}$$

From these eigenvalues, $\mu_{1,\pm}$ and $\mu_{1,-}$ are always negative while $\mu_{1,+}$ is negative if and only if $Q < 2$. For the general case $n_A \neq n_B$, we could not get explicit expressions for the stability analysis of this equilibrium. Instead, we performed simulations starting from different combinations of parameters and we found that, as long as condition (A10) is satisfied, equilibrium IV is stable for $Q < 2$ and unstable for $Q > 2$.

Using Eqs. (A6), (A9), and (A10), and taking into account the discussion in the previous paragraph, we can draw a phase diagram for our model in the plane (n_A, Q), as we show in Fig. 2.

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