General Theory of Measurement with Two Copies of a Quantum State

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We analyze the results of the most general measurement on two copies of a quantum state. We show that by using two copies of a quantum state it is possible to achieve an exponential improvement with respect to known methods for quantum state tomography. We demonstrate that μ can label a set of outcomes of a measurement on two copies if and only if there is a family of maps C_μ such that the probability $\mathrm{Prob}(\mu)$ is the fidelity of each map, i.e., $\mathrm{Prob}(\mu) = \mathrm{Tr}[\rho C_\mu(\rho)]$. Here, the map C_μ must be completely positive after being composed with the transposition (these are called completely copositive, or CCP, maps) and must add up to the fully depolarizing map. This implies that a positive operator valued measure on two copies induces a measure on the set of CCP maps (i.e., a CCP map valued measure).

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One of the postulates of quantum theory tells us how to compute probabilities for the outcomes of measurements: If the system was prepared in the state ρ, for every outcome μ of a measurement there is a projector P_μ such that the probability of occurrence of μ is the expectation value of the projector P_μ in the state ρ, i.e., $\mathrm{Prob}(\mu) = \mathrm{Tr}(\rho P_\mu)$. To represent mutually exclusive outcomes the projectors must be orthogonal and they must add up to the identity to ensure that the total probability is unity (i.e., $P_\mu P_\nu = \delta_{\mu\nu} P_\mu$, $\sum_\mu P_\mu = I$). This postulate, originally formalized by von Neumann [1], was extended in the 1970s [2] when the notion of generalized measurement was introduced. In such measurement a positive operator A_μ (not necessarily a projector) is associated with every outcome μ and the probability of occurrence of μ is $\mathrm{Prob}(\mu) = \mathrm{Tr}(\rho A_\mu)$. The operators A_μ add up to the identity and define a so-called positive operator valued measure (POVM). Neumark's theorem [3] establishes that POVM measurements are equivalent to projective measurements for an extended system: Any POVM can be implemented via a projective measurement on the original system supplemented with an appropriately chosen ancillary system.

In this Letter we analyze the predictions of quantum theory concerning the results of measurements performed when two identically prepared quantum systems are simultaneously available. More precisely, we assume that a source produces the state $\rho^{(A,B)} = \rho \otimes \rho$ (A and B label two systems prepared in the same state ρ). Our goal is twofold: (a) to determine the possible distributions for the measurement outcomes and (b) to present a solution to the problem of efficient universal state tomography using copies. We divide the presentation in two parts. First we prove the following theorem [completely copositive map valued measure (CCPMVM)]: Given two systems prepared in the same state ρ, μ can label a set of possible outcomes of a measurement on $\rho^{(A,B)} = \rho \otimes \rho$ if and only if there is a family of completely copositive (CCP) maps C_μ such that the probability of occurrence $\mathrm{Prob}(\mu)$ is the fidelity of the map C_μ, i.e., $\mathrm{Prob}(\mu) = \mathrm{Tr}[\rho C_\mu(\rho)]$. The maps satisfy the condition $\sum_\mu C_\mu = \mathcal{E}$, where $\mathcal{E}$ is the map for which $\mathcal{E}(\rho) = I$ for any state ρ. Moreover C_μ must be CCP which means that the composition of C_μ with the transposition must be completely positive. The theorem establishes a connection between families of CCP maps and measurements with copies: A measurement on two copies defines a CCPMVM and vice versa. In the second part of this Letter we establish the tomographic power of this type of measurement: We show that availability of two copies gives an exponential advantage in solving the problem of quantum state tomography enabling us to construct a universal quantum state detector to efficiently estimate partial purities and other interesting quantities.

Let us prove the CCPMVM theorem. For this, we take advantage of the existence of a one-to-one correspondence between linear operators on the space $\mathsf{H} \otimes \mathsf{H}$ and linear superoperators on the space H (a superoperator on H is a map on the space of operators over H). This is the so-called Jamiołkowski isomorphism [4] which establishes that for every superoperator $\tilde{C}$ on H there is an operator $\hat{C}$ on $\mathsf{H} \otimes \mathsf{H}$ and vice versa. The correspondence is realized by the following identity: $\hat{C} = (\tilde{C} \otimes I)(|I\rangle\langle I|)$ where $|I\rangle$ is the unnormalized maximally entangled state $|I\rangle = \sum_i |ii\rangle$. The isomorphism relates positive Hermitian operators on $\mathsf{H} \otimes \mathsf{H}$ with completely positive Hermitian superoperators on H. In particular, the identity operator is associated with the completely depolarizing superoperator $\mathcal{E}$ for which the image of every trace one operator is the identity. Using this isomorphism the CCPMVM theorem can be proved as follows: An outcome μ of a generalized measurement on two copies prepared on the state $\rho^{(A,B)} = \rho \otimes \rho$ is characterized by a positive operator $\hat{C}_\mu$. The probability of such outcome is $\mathrm{Prob}(\mu) = \mathrm{Tr}(\rho \otimes \rho \hat{C}_\mu)$. Jamiołkowski isomorphism ensures that for every positive operator $\hat{C}_\mu$ there is a completely positive superoperator $\tilde{C}_\mu$ such that $\text{Prob}(\mu) = \text{Tr}[\rho \otimes \rho(\tilde{C}_\mu \otimes I)(|I\rangle\langle I|)]$. By replacing the explicit form of the state $|I\rangle$, the trace over the second copy can be computed and the probability $\text{Prob}(\mu)$ can be rewritten as $\text{Prob}(\mu) = \text{Tr}[\rho \tilde{C}_\mu(\rho^T)]$, where ρ^T denotes the usual transposition of ρ. Therefore, the probability of every outcome of a generalized measurement is $\text{Prob}(\mu) = \text{Tr}[\rho C_\mu(\rho)]$ where $C_\mu = \tilde{C}_\mu \circ \mathcal{T}$, with $\mathcal{T}$ denoting the transposition as a map. Jamiołkowski isomorphism ensures that $\tilde{C}_\mu$ is completely positive, which implies that C_μ is completely copositive. Moreover, as the POVM operators $\hat{C}_\mu$ add up to the identity, the corresponding superoperators $\tilde{C}_\mu$ must add up to the completely depolarizing map. The relation $\mathcal{E} \circ \mathcal{T} = \mathcal{T} \circ \mathcal{E} = \mathcal{E}$ completes the proof of the CCPMVM theorem. This theorem shows that the fidelity of certain families of positive superoperators has a direct physical meaning as it can be realized as the probability of a generalized measurement.

The following example is particularly significant. Consider two copies of the state ρ of an n-qubit system: $\rho^{(A,B)} = \rho \otimes \rho$ and perform a Bell measurement on all pairs formed by the jth qubit of each copy as shown in Fig. 1. In what follows we will find the CCPMVM associated with this simple measurement. We use the following notation: We expand ρ in terms of generalized Pauli operators as $\rho = \sum_{q,p} c_{q,p} T(q,p)/N$, where $N = 2^n$. Here $q = (q_1, \ldots, q_n)$ and $p = (p_1, \ldots, p_n)$ are binary n-tuples and $T(q,p)$ are n-fold tensor products of the identity and the Pauli operators on each qubit: $T(q,p) = X^{q_1} Z^{p_1} \otimes \cdots \otimes X^{q_n} Z^{p_n} (i)^{qp}$ (here $qp = \sum_k q_k p_k$). Real coefficients $c_{q,p}$ are such that $c_{q,p} = \text{Tr}[\rho T(q,p)]$.

The outcomes of all Bell measurements can be collected in two binary n-tuples (a,b) where (a_k, b_k) identify the state $|\beta_{a_k,b_k}\rangle$ detected at site $k = 1, \ldots, n$ (Bell state $|\beta_{a_k,b_k}\rangle$ is an eigenstate of $X_k \otimes X_k$ and $Z_k \otimes Z_k$ with eigenvalue $(-1)^{a_k}$ and $(-1)^{b_k}$, respectively). The probability of occurrence for every possible outcome $\text{Prob}(a,b)$ turns out to be $\text{Prob}(a,b) = \sum_{q,p} (-1)^{aq+bp+qp} c_{q,p}^2 / N^2$. These probabilities are, as the CCPMVM theorem ensures, the fidelities of CCP maps. Indeed, one can show that $\text{Prob}(a,b) = \text{Tr}[\rho C_{a,b}(\rho)]$, where the corresponding map is $C_{a,b}(\rho) = T(b,a)\rho^T T(b,a)/N$. The CCP character of these maps is evident since they are obtained as the composition of the transposition with a completely positive superoperator. For the simplest case of a single qubit, where the coefficients $c_{1,0}$, $c_{1,1}$, and $c_{0,1}$ are the three Cartesian components of the Bloch vector $\vec{p}$ parametrizing the state as a linear combination of the three Pauli operators: $\rho = (I + \vec{p} \cdot \vec{\sigma})/2$. Then, the maps $C_{a,b}$ are such that $C_{a,b}(\rho) = (I + \vec{p}_{a,b} \cdot \vec{\sigma})/4$. These operators are proportional to states with polarization vectors $\vec{p}_{a,b} = (-1)^a p_x \hat{x} + (-1)^{a+b+1} p_y \hat{y} + (-1)^b p_z \hat{z}$. Therefore, the map $C_{1,1}$, corresponding to the singlet $|\beta_{1,1}\rangle$, realizes a full inversion on the Bloch sphere. The other Bell states have maps corresponding to reflections about the three Cartesian planes (where one Cartesian component of $\vec{p}$ changes sign [5]). Adding these four maps we obtain the fully depolarizing one. Probabilities for the four Bell measurements are quadratic in the components of $\vec{p}$: $\text{Prob}(a,b) = (1 + \vec{p} \cdot \vec{p}_{a,b})/4$.

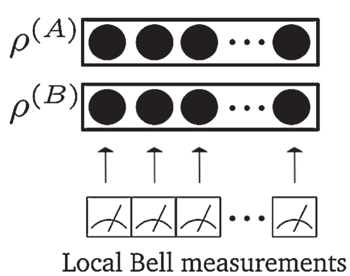

FIG. 1. Proposed scheme for full state tomography.

We can now show an important result of this Letter: The measurement described in Fig. 1 can be used to devise an efficient strategy for quantum state tomography (QST) [6,7]. The goal of QST is to extract information about the state $\rho = \sum_{q,p} c_{q,p} T(q,p)/N$. As there are N^2 unknown coefficients, full QST is always a hard task. Moreover, a naive approach (like measuring all Pauli operators on each qubit) would require the use of N^2 different experimental setups. In addition, some tomographic methods would require us to repeat the experiments an exponentially large number of times in order to estimate any coefficient $c_{q,p}$ with fixed precision. Here, we will show that if two copies are available at each time the complexity of QST is drastically reduced. Thus, we will extend previous results [8–13] showing that with two copies of a quantum state we could use just a single experimental setup for QST (a so-called "universal state detector"). More importantly, we will show that in order to gather the information to estimate every coefficient c_α^2 with fixed precision we need to repeat the experiment a number of times that is independent of the number of qubits and is only fixed by the required precision. For this purpose we can proceed as follows: After performing the Bell measurements described in Fig. 1 we can multiply the detected values of the suitable operators $X_j \otimes X_j$ and $Z_j \otimes Z_j$ to obtain the value of any Pauli operator of the form $T(q,p) \otimes T(q,p)$. In this way we can estimate $|c_{q,p}|$ by evaluating the expectation value of the above operator and using the fact that

$$\langle T(q,p) \otimes T(q,p) \rangle_{\rho\otimes\rho} = c_{q,p}^2. \tag{1}$$

Moreover, the number of experimental runs M_E required to estimate any $|c_{q,p}|$ with fixed precision is independent of the number of qubits and is only fixed by the precision. This can be seen as follows: First, we notice that every measurement yields binary values for $T(q,p) \otimes T(q,p)$. The central limit theorem implies that after $\tilde{M}_E$ repetitions, the average $\tilde{c}^2_{q,p}$ has a standard deviation $\sigma_{q,p}$ satisfying $\sigma_{q,p} \leq 1/\sqrt{\tilde{M}_E}$. Thus, there is a number k such that $c^2_{q,p} \in [\tilde{c}^2_{q,p} - k\sigma_{q,p}; \tilde{c}^2_{q,p} + k\sigma_{q,p}]$ with probability p. Therefore, with the same probability p, $|c_{q,p}|$ will be found in an interval centered at $|\tilde{c}_{q,p}|$ with a width $\frac{k\sigma_{q,p}}{2|\tilde{c}_{q,p}|}$. On the other hand, if one wants to estimate each $|c_{q,p}|$ larger than a fixed δ with an uncertainty ϵ, and obtain a correct value with probability p, the number of required repetitions is $\tilde{M}_E \geq k^2/4\delta^2\epsilon^2$ where k is chosen to satisfy $p = \mathrm{erf}(\frac{k}{\sqrt{2}})$. Thus, the number of repetitions does not depend on n but only on the precision ϵ, the minimum measurable value δ and the probability of success p. This implies that the method is "quantum efficient" as the number of quantum resources (i.e., copies of the state, measurements, etc.) is constant given a required precision. However, classical resources to determine every $c_{q,p}$ are still exponential in n due to the fact that there are N^2 such coefficients. This strategy determines $c_{q,p}$ up to a sign. Once the coefficients satisfying $|c_{q,p}| \geq \delta$ are known, it is possible to directly determine each sign using ordinary tomographic methods (of course, it is not possible to do that efficiently if the coefficients are exponentially small or the number of such coefficients is exponentially large).

It is important to notice that the setup of Fig. 1 is a universal quantum state detector that, as opposed to previously proposed ones, is efficient. Indeed, the detector is universal and efficient since it is a single experimental apparatus that can be used to estimate any set of coefficient c_{mn} with fixed precision investing resources that scale as a polynomial of n. Universal state detectors that use ancillary systems prepared in a given state ρ_0 (which is not related with the state ρ) were introduced before [14]. However, as we will show now, these detectors are inefficient. The way such detectors work is as follows: We prepare the ancillary system in the state $\rho_0 = \sum_{q,p} c^{(0)}_{q,p} T(q,p)/N$ and perform joint Bell measurements on every pair of qubits (as in Fig. 1). As the system-ancilla ensemble is in state $\rho \otimes \rho_0$, we have

$$c_{q,p} c^{(0)}_{q,p} = \langle T(q,p) \otimes T(q,p) \rangle_{\rho\otimes\rho_0}. \tag{2}$$

Therefore, knowing $c^{(0)}_{q,p}$ and measuring the expectation value appearing in (2) we can determine $c_{q,p}$. Clearly, the method is such that we can determine $c_{q,p}$ only if the corresponding $c^{(0)}_{q,p}$ is nonvanishing. Moreover, the smaller the value of $c^{(0)}_{q,p}$, the higher the precision required in the estimation of $\langle T(q,p) \otimes T(q,p) \rangle_{\rho\otimes\rho_0}$. Thus, a truly universal detector would require all coefficients to be nonvanishing, but in such a case they would all be very small. This is the origin of the inefficiency. More precisely, consider first a state for which $|c^{(0)}_{q,p}|$ are maximal. This is the case for the so-called stabilizer states [defined as common eigenstates of a commuting set of N Pauli operators $T(q,p)$'s]. For such a state there are N nonvanishing coefficients $c^{(0)}_{q,p}$'s taking values equal to ± 1. For such ρ_0 the detector can only be used to estimate N $c_{q,p}$'s providing no information about the $N^2 - N$ remaining ones, denying its universality. On the other hand, all the coefficients could be estimated using a state ρ_0 with nonvanishing $c^{(0)}_{q,p}$ for all (q,p)'s. The problem for such unbiased ρ_0 is that all $c^{(0)}_{q,p}$'s are exponentially small. The reason for this is that $\sum_{q,p} c^{(0)2}_{q,p}/N \leq 1$. Therefore, each coefficient $c^{(0)}_{q,p}$ is $O(1/\sqrt{N})$. Then, if we use (2) to estimate them with fixed precision, we need exponentially high precision in the estimation of the expectation value $\langle T(q,p) \otimes T(q,p) \rangle$. For this reason the method is inefficient (the universal detector would have to be used an exponentially large number of times to achieve a fixed precision). Clearly, the use of a copy instead of an ancilla provides a simple way out of this problem.

Full quantum state tomography is always exponentially hard as the number of unknown parameters scales as 4^n. Therefore, it is crucial to conceive efficient methods for partial characterization of quantum states. Remarkably, the strategy described above is also a solution for this problem. To see this we consider "coarse-grained" Bell measurements: For any Bell state $|\beta_{m,n}\rangle$ we can estimate the probability to detect an even (odd) number of them in the measurement of all pairs. Then, we can compute $\Delta\mathrm{Prob}_{m,n} = \mathrm{Prob}(\text{even } \#|\beta_{m,n}\rangle) - \mathrm{Prob}(\text{odd } \#|\beta_{m,n}\rangle)$. It is simple to show that $\Delta\mathrm{Prob}_{m,n} = \mathrm{Tr}[O_{m,n}(\rho)\rho]$ where the (not necessarily positive) map $O_{m,n}$ is such that

$$\Delta\mathrm{Prob}_{m,n} = \frac{1}{N}\sum_{q,p} s_{q,p} c^2_{q,p}. \tag{3}$$

Here, the N^2 components of the vector $s_{q,p}$ are $s_{q,p} = (-1)^{(m+1)(\alpha_x+\alpha_y)}(-1)^{(n+1)(\alpha_z+\alpha_y)}$, where α_x (respectively, α_y, α_z) denotes the number of qubits for which the Pauli operator $T(q,p)$ contains an X (respectively, Y, Z) operator. For example, for the singlet $|\beta_{11}\rangle$, $s_{q,p} = 1$ for every α. For any other Bell state half of the components of $s_{q,p}$ are equal to $+1$ and the other half are equal to -1. For the singlet the above formula reduces to

$$\Delta\mathrm{Prob}_{1,1} = \frac{1}{N}\sum_{q,p} c^2_{q,p} = \mathrm{Tr}(\rho^2). \tag{4}$$

Thus, this measurement reveals the purity of the state. Partial purities can be detected in the same way: Consider the state ρ_J, obtained after tracing out the qubits for which the binary n-tuple J is zero. Purity of such state is the sum of $c^2_{q,p}$ for the coefficients associated with Pauli operators containing the identity in the qubits for which the corresponding component of J is zero. To obtain it we must use (4) counting singlets only in the qubits where the corresponding bit of J is equal to unity. The above method for estimating purity is equivalent to the one proposed by

Ekert *et al.* who used the fact that purity is equal to the expectation value of the swap operator in the state $\rho^{(A)} \otimes \rho^{(B)}$ [15]. But our results also show that, by making more general coarse-grained Bell measurements, we are not only able to efficiently detect partial purities. In fact, with the same effort we reveal other quantities that partially characterize the quantum state and have the form $\Delta P_S = (\sum_{(q,p)\in S} c_{q,p}^2 - \sum_{(q,p)\in \bar{S}} c_{q,p}^2)/N$, where $\{S, \bar{S}\}$ is a partition of the N^2 coefficients $c_{q,p}$ in two halves. It is possible to generalize this even further by grouping Bell states in each pair of qubits (in this case one can attain linear combinations with vectors $s_{q,p}$ that have a different number of ± 1 components). Other weighted sums of squares of c_α over certain sets of (q, p)'s can also be obtained in this way.

It is interesting to consider another related coarse-grained Bell measurement: we can estimate the probability $p_{m,n}^{(\mathrm{all})}$ to find all pairs of qubits in the subspace orthogonal to $|\beta_{m,n}\rangle$. For the case of the singlet this is related to the multipartite concurrence that for a pure n-qubit state is [12]

$$\mathcal{C}(\rho) = 2^{1-n/2}\sqrt{(2^n - 2) - \sum_l \mathrm{Tr}(\rho_l^2)}. \quad (5)$$

Here the n-tuple l labels every nontrivial subset of the n qubits. As noticed in [13], this can be rewritten as

$$\mathcal{C}(\rho) = 2\sqrt{1 - p_{1,1}^{(\mathrm{all})}}; \quad (6)$$

therefore, concurrence of a pure state can be estimated efficiently using the type of coarse-grained Bell measurements we described above. The probability $p_{1,1}^{(\mathrm{all})}$ is also a quadratic form of the coefficients $c_{q,p}$: $p_{1,1}^{(\mathrm{all})} = \sum_{q,p} c_{q,p}^2 3^{\alpha_0}/N^2$, where α_0 is the number of qubits for which $T(q, p)$ contains a factor equal to the identity. More generally, if we measure $p_{m,n}^{(\mathrm{all})}$, we obtain a quantity that is not a concurrence but provides different tomographic information. It can be expressed as $p_{m,n}^{(\mathrm{all})} = \sum_{q,p} c_{q,p}^2 f_{q,p}/N^2$, where $f_{q,p} = 3^{\alpha_0}(-1)^{(m+1)(\alpha_x+\alpha_y)+(n+1)(\alpha_z+\alpha_y)}$.

As we mentioned, for an efficient partial QST it is necessary to estimate differences between probabilities such as in (3). Such probabilities, as the CCPMVM theorem states, are fidelities of CCP maps. Thus, the right-hand side of Eq. (3) can be expressed as the fidelity of a map which is the difference between two CCP maps: $\Delta P_{m,n} = \mathrm{Tr}[\rho O_{m,n}(\rho)]$ ($O_{m,n}$ is not CCP). One of such maps is the identity, which is not CCP since it is not completely positive after being composed with the transposition. For that case, one finds $\Delta P_{1,1} = \mathrm{Tr}(\rho^2)$. Thus, purity can never be evaluated as the probability of a generalized measurement but only as the difference between them. In turn, fidelities of positive maps that are not CCP can be related via the Jamiołkowski isomorphism with expectation values of Hermitian operators (not necessarily positive). Such Hermitian operator can be written as the difference between two positive operators. For the above example, where the positive map is the identity, it turns out that the nonpositive operator is the one implementing the swap between the two copies as the well-known identity $\Delta P_{1,1} = \mathrm{Tr}(\rho^2) = \mathrm{Tr}(\mathrm{SWAP}\rho \otimes \rho)$ shows.

Summarizing, we showed that by performing general measurements on two copies of a quantum state one always detects probabilities that are fidelities of completely copositive maps. Interestingly, this tells us that certain quantities (like purity) can never be directly obtained as the probability of a given outcome of such measurements whereas other quantities could. Moreover, we showed that the use of two copies of a quantum state gives an exponential advantage for quantum state tomography by enabling the construction of a universal state detector.

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