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Isoscaling and the high-temperature limit

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This study shows that isoscaling, usually studied in nuclear reactions, is a phenomenon common to all cases of fair sampling. Exact expressions for the yield ratio R_{21} and approximate expressions for the isoscaling parameters α and β are obtained and compared to experimental results. It is concluded that nuclear isoscaling is bound to contain a component due to sampling and, thus, a word of caution is issued to those interested in extracting information about the nuclear equation of state from isoscaling.

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I. INTRODUCTION

Recent studies of the isospin dependence of nuclear reactions at intermediate energies have been studied by comparing fragmenting collisions of similar mass and energies but different isospin. Experimentally, the ratio of isotope yields in two different systems, 1 and 2, $R_{21}(n, z) = Y_2(n, z)/Y_1(n, z)$, has been seen to follow, approximately, an exponential function of the neutron number, n, and the proton number, z, of the isotopes; dependence known as *isoscaling* [1–3]:

$$R_{21} = Y_2(n, z)/Y_1(n, z) = C \exp(\alpha n + \beta z), \qquad (1)$$

where α and β are fitting parameters and C is a normalization constant. This fit has been studied under the light of several theoretical models of nuclear reactions and, under different assumptions (see, e.g., Ref. [4]), the fitting parameters can be expected to be related to the symmetry term of the equation of state, $C_{\rm sym}$, through

$$\alpha = \frac{4C_{\rm sym}}{T}[(Z_1/A_1)^2 - (Z_2/A_2)^2], \qquad (2)$$

where T is the assumed temperature of both reactions. Thus the interest on studying the isoscaling phenomenon: R_{21} has the potential of elucidating the behavior of $C_{\rm sym}$ at varying isospin, temperature, and so on. It is relevant to note that according to the above displayed relation the coefficient α should approach 0 as T increases.

In a series of recent works, however, it has been shown that the phenomenon of isoscaling can be found very early—before thermalization—in classical molecular dynamics simulations of nuclear reactions [5], as well as in nonthermal physical phenomena, such as in percolating networks. In particular, percolation in two "colors" (i.e., protons and neutrons [6]), or in extended "polychromatic" nets [7], has demonstrated that the isoscaling behavior of the form of Eq. (1) emerges as a direct consequence of simple combinatorial problems with, e.g., $\alpha = \ln(q_2/q_1)$, where $q_i = N_i/A_i$. These results point to the fact that isoscaling, although connected to the equation of state, can also be produced by nonthermal processes and then probabilistic aspects of the problem can play a disturbing role complicating the determination of $C_{\rm sym}$ from isoscaling deermination. Thus the motive of the present work is to study the probabilistic aspects of isoscaling.

In this study we investigate the phenomenon of isoscaling in the very simplest scenario—free of the geometrical constraints imposed by bond percolation—of sampling "protons" and "neutrons" directly from an urn. In Sec. II we will obtain the correct isoscaling law using probabilistic arguments and will show that Eq. (1) is a limiting approximation to the exact expression. In Sec. III we compare our results to experimental values, after which the manuscript closes listing several conclusions.

II. ISOSCALING AND SAMPLING

Consider the problem of building clusters containing a number a of nucleons by simply grabbing these a particles from an urn in which there are A particles composed of Z "protons" and N "neutrons," i.e., $A = N + Z$. We assume the sampling to be without replacement, and with no interactions among the particles nor with the urn itself; these premises are known as simple random sampling (SRS) in the statistics circles.

To use this setup to study isoscaling we first focus on determining the yield of fragments, $Y(n, z)$, that the previous scheme would produce after a large number of samplings. It stands to reason that such yield would be directly related to the probability of drawing n neutrons and z protons, i.e., $a = n + z$. Repeating then for a second urn with a different isotopic composition, one can easily obtain the corresponding R_{21} and, thus, the scaling law.

Let us first determine the probability of ending with a cluster composed by n neutrons and z protons, i.e., $a = n + z$. The number of ways in which a cluster of a particles can be obtained from randomly sampling an urn with A particles is $\binom{A}{a} = A!/[a!(A - a)!]$. Out of these possibilities only $\binom{N}{n} \times \binom{Z}{z}$ will correspond to clusters with n neutrons and z protons. Thus, if we now assume that the probability of getting a fragment of size a out of an urn with A particles is $P(a, A)$,

the normalized yield of such a sampling will be:

$$Y(n, N, z, Z) = P(a, A)P(n, N, z, Z, A) = P(a, A)\frac{\binom{N}{n}\binom{Z}{z}}{\binom{A}{a}}. \quad (3)$$

In particular, for the case of sampling a particles composed of n neutrons and z protons ($a = n + z$) from an urn with $A_1 = Z_1 + N_1$, the term $P(n, N_1, z, Z_1)$ becomes

$$P(n, N_1, z, Z_1, A_1) = \frac{\binom{N_1}{n}\binom{Z_1}{z}}{\binom{A_1}{a}} = \frac{N_1!}{n!(N_1 - n)!}\frac{Z_1!}{z!(Z_1 - z)!}\frac{a!(A_1 - a)!}{A_1!}.$$

And taking the ratio of this probability to the probability of obtaining n neutrons and z protons from a sampling of a nucleons from an urn with A_2 nucleons composed of N_2 neutrons and Z_2 protons, the isoscaling ratio is given exactly by

$$R_{21} = \frac{P(a, A_2)}{P(a, A_1)}\frac{P(n, N_2, z, Z_2, A_2)}{P(n, N_1, z, Z_1, A_1)} \quad (4)$$

$$= \frac{P(a, A_2)}{P(a, A_1)}\frac{\binom{N_2}{n}\binom{Z_2}{z}}{\binom{A_2}{a}}\frac{\binom{A_1}{a}}{\binom{N_1}{n}\binom{Z_1}{z}}, \quad (5)$$

which can be readily used for calculations taking $P(a, A_2)/P(a, A_1)$ as an overall normalization.

This expression, although close to the usual exponential law (1), is not a straight line in the linear-log plot of R_{21} versus N; the fact that experimental data also deviate from such a linear behavior is reassuring (see, e.g., Ref. [8]). The approximate exponential law (1) can be obtained from the exact result (4) using the binomial approximation to the hypergeometric distribution (see, e.g., Ref. [9]) which, in this case, depends on the assumption that $x \ll X_i$ for $x = a, n, z$, $X_i = A_i, N_i, Z_i$ and $i = 1, 2$. The sampling isoscaling ratio then becomes

$$R_{21} \approx \frac{P(a, A_2)}{P(a, A_1)}\exp\left[n\ln\left(\frac{q_2}{q_1}\right) + z\ln\left(\frac{p_2}{p_1}\right)\right], \quad (6)$$

where we have introduced the probabilities of extracting a neutron, $q_i = N_i/A_i$, and a proton, $p_i = Z_i/A_i$.

Comparing to Eq. (1) we identify the overall normalization constant as $C = P(a, A_2)/P(a, A_1)$ and the isoscaling parameters as $\alpha(S) = \ln(q_2/q_1) = \ln(N_2A_1/N_1A_2)$, $\beta(S) = \ln(p_2/p_1) = \ln(Z_2A_1/Z_1A_2)$, in perfect agreement with the percolation results [6,7]. Reviewing the procedure leading to Eq. (6), it is clear that exponential law is a direct result of the sampling.

[For completeness, although not relevant to the nuclear case, we note that a functionally similar result can be obtained for the case of sampling with replacement. In this case, fragments with $a = n + z$ will appear with probability $P(n, N, z, Z, A) = Cp^zq^n = Cp^z(1 - p)^{a-z}$, where C is given by the normalization $C^{-1} = (p^{A+1} - q^{A+1})/(p - q)$, with p and q defined previously. Using this for urns 1 and 2 leads to

$$R_{21} = \frac{P(a, A_2)}{P(a, A_1)}\left[\frac{p_2^zq_2^n}{p_1^zq_1^n}\right]\left[\frac{(p_2 - q_2)\left(-q_1^{a+1} + p_1^{a+1}\right)}{(p_1 - q_1)\left(-q_2^{a+1} + p_2^{a+1}\right)}\right] = C(a)\frac{P(a, A_2)}{P(a, A_1)}\left[\frac{q_2}{q_1}\right]^n\left[\frac{p_2}{p_1}\right]^z, \quad (7)$$

where $C(a) = [(2p_2 - 1)(-q_1^{a+1} + p_1^{a+1})]/[(2p_1 - 1)(-q_2^{a+1} + p_2^{a+1})]$. Because $C(a)$ is independent of n and z, Eq. (7) depends on these variables in a functionally similar manner as the isoscaling law (1).]

III. COMPARISON TO EXPERIMENTAL ISOSCALING

The energy dependence of the isoscaling parameters has already been explored in collisions [5]; here we compare the isoscaling parameter α obtained from experiments to those from samplings and study their variation as a function of beam energy.

In view of the previous result, namely the fact that isoscaling can be expected from the mere act of fragmenting a system, the question to answer now is: what fraction of the nuclear isoscaling is due to sampling?

In what follows we will show to which extent combinatorial effects (i.e., symentropy effects; see Ref. [12]), are relevant by comparing the value of α obtained from experimental calculations with the one resulting from the simple combinaorial analysis. It is worth realizing at this point that in the case of the combinatorial analysis we are disregarding all correlations, in particular those associated with energy terms and then correspond to the very high temperature limit (see, e.g., Ref. [7]). We then expect that the contribution of combinatorial terms to be apparent in collisions at high energies. Here we present a direct comparison of our findings to two sets of experimental data—not to try to reproduce them but to assess the relative magnitude of the sampling contribution to isoscaling and to attempt to draw a baseline from which future experimental studies will be able to extract the nuclear contribution to this effect. The data used were obtained by Yennello *et al.* [11] at the Cyclotron Institute of Texas A&M University.

The values of $\alpha(E)$ used in this comparison are shown in the inset of Fig. 1 and correspond to the ratios of the yields $Y(^{40}\mathrm{Ar} + {}^{58}\mathrm{Fe})/Y(^{40}\mathrm{Ca} + {}^{58}\mathrm{Ni})$, $Y(^{58}\mathrm{Fe} + {}^{58}\mathrm{Fe})/Y(^{58}\mathrm{Ni} + {}^{58}\mathrm{Ni})$, $Y(^{40}\mathrm{Ar} + {}^{58}\mathrm{Ni})/Y(^{40}\mathrm{Ca} + {}^{58}\mathrm{Ni})$, and $Y(^{58}\mathrm{Fe} + {}^{58}\mathrm{Ni})/Y(^{58}\mathrm{Ni} + {}^{58}\mathrm{Ni})$ at the energies shown. The main panel of Fig. 1 shows the ratio of $\alpha(E)$ to the corresponding parameter obtained from the sampling, $\alpha(S) = \ln(N_2A_1/N_1A_2)$; cf. Eq. (6).

It is easy to see that at all energies $\alpha(E) > \alpha(S)$ and that for large energies $\alpha(E) \to \alpha(S)$. A second point of interest is the fact that on division by $\alpha(S)$, the previously scattered four curves of $\alpha(E)$ values collapse into a single group.

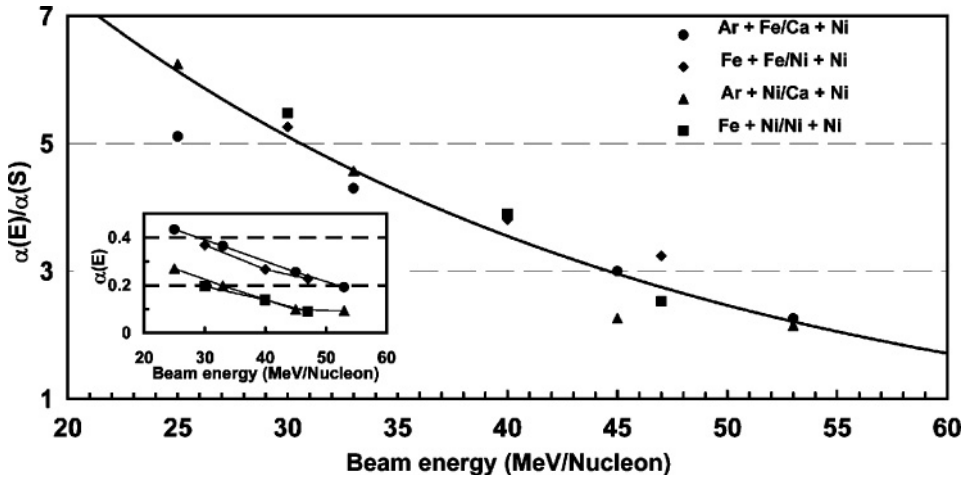

FIG. 1. Ratio of experimental $\alpha(E)$ to the sampling $\alpha(S)$ as a function of energy.

IV. CONCLUSIONS

The conclusions of this work are quite simple. First we note that isoscaling can be expected to appear in any system undergoing a *fair* sampling; nuclear fragmentation, being a type of sampling, is bound to exhibit this phenomenon.

Second, the inherent correlations of nuclear systems are expected to have an effect on the *fairness* of the sampling modifying the yield ratio R_{21} and the isoscaling parameters α and β, as was demonstrated by a direct comparison to experimental results.

Finally, a word of caution is needed if one attempts to extract information about the nuclear equation of state from isoscaling. Given that it is now known that a sampling-related isoscaling is ever present, obtaining quantities such as C_{sym} from Eq. (2) is not straightforward. In principle, the experimental results should contain the isoscaling produced by sampling, and—in some regime—both $R_{21}(E)$ and $\alpha(E)$ should tend to $R_{21}(S)$ and $\alpha(S)$.

The indication that $\alpha(E) \to \alpha(S)$ at high energies indicates (perhaps) that at those energies nuclear binding is less important and the "sampling" is closer to that of a noninteracting urn; at lower energies, however, the reaction has longer interaction times and the phase space available for the *sampling* becomes a complex function of the energy distancing itself from the fair sampling case of Eqs. (4) and (6). This is consistent with the implications of Eq. (2) that, through its inverse dependence on the temperature, indicates that in the limit of high energies the expected contribution from the equation of state to the isoscaling coefficients should vanish: $\alpha \to 0$. Likewise, studies of bond percolation on polychromatic substrates [7,12] (generated through the nuclear lattice model at temperature T) have shown that in the limit of high temperatures only the probabilistic term survives.

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[1] M. B. Tsang , C. K. Gelbke, X. D. Lui, W. G. Lynch *et al.*, Phys. Rev. C **64**, 054615 (2001).
[2] M. B. Tsang, W. A. Friedman, C. K. Gelbke, W. G. Lynch, G. Verde, and H. Xu, Phys. Rev. Lett. **86**, 5023 (2001).
[3] A. R. Raduta, Eur. Phys. J. **A24**, 85 (2005).
[4] A. S. Botvina, O. V. Lozhkin, and W. Trautmann, Phys. Rev. C **65**, 044610 (2002).
[5] C. O. Dorso, C. R. Escudero, M. Ison, and J. A. López, Phys. Rev. C **73**, 044601 (2006).
[6] A. Dávila, C. Escudero, J. A. López, and C. O. Dorso, Physica **A374**, 663 (2007).
[7] C. O. Dorso, Phys. Rev. C **73**, 034605 (2006).
[8] M. Mocko, Ph. D. thesis, Michigan State University (2006).
[9] S. Ross, *First Course in Probability* (Prentice Hall, Upper Saddle River, NJ, 2005).
[10] G. Chaudhuri, S. Das Gupta, and M. Mocko, arXiv:0711.4992v1 [nucl-th] (2007).
[11] D. V. Shetty, S. J. Yennello, and G. A. Souliotis, Phys. Rev. C **75**, 034602 (2007).
[12] L. G. Moretto, C. O. Dorso, J. B. Elliott, and L. Phair, Phys. Rev. C **77**, 037603 (2008).

QA 2,QC 3,QA 4,QA