Generalised trade-off model for energy-efficient WSN synchronisation

P. Briff✉, A. Lutenberg, L. Rey Vega, F. Vargas and M. Patwary

A mathematical framework to obtain a generalised energy-efficient trade-off model for generic wireless sensor networks (WSNs) while attaining sensing node synchronisation at a given network-wide estimation error threshold is presented. The model outputs both a theoretical optimal solution as well as a set of sub-optimal solutions to cater for real-world WSN designs. The robustness of the proposed framework is examined with an example in which randomly deployed sensors are affected by path-loss and uncorrelated Rayleigh fading effects.

Introduction: Time synchronisation has become a major feature within application-specific wireless sensor networks (WSNs) to meet application demands such as data fusion, energy management and collision avoidance mechanisms. The need for energy-efficient time synchronisation has attracted intense research focus of recent years. A recent survey published in [1] details the challenges in unattended WSNs and the inevitable need for energy-efficient routing and data processing algorithms, although without mentioning the existence of a network-wide energy-efficient solution with regard to estimation error. Basagni *et al.* [2] highlight the underlying trade-off between consumed energy and data latency, and propose an energy harvesting technique to alleviate the application constraints. Lenzen *et al.* [3] have updated the PulseSync Protocol with further considerations of energy efficiency, message latency and estimation error. However, the solutions proposed in [2, 3] have not considered optimal solutions for application-specific resource trade-off while attaining a given estimation error. In this Letter, we present a generalised model providing optimal solutions with a wide range of tunability for the energy-efficient synchronisation within WSN nodes, which has not been yet studied to the best our knowledge.

Model statement: Consider the WSN depicted in Fig. 1 consisting of N number of sensor nodes randomly deployed within a given area. Assume that all nodes in the WSN under consideration are identical in their properties and affected by equal noise power levels within the network, which is a widely adopted assumption in the literature [4, p. 37]. Let two nodes u_i and u_j be located at spatial positions $\{\boldsymbol{x}_i, \boldsymbol{x}_j\} \in \mathbb{R}^3$, respectively, separated by the Euclidean distance $d_{ij} \triangleq \|\boldsymbol{x}_i - \boldsymbol{x}_j\|_2$. Let D_{ij} denote node u_i's maximum coverage radius for communicating with node u_j. Furthermore, let S_{ij} denote the transmit power emitted from node u_i when communicating with node u_j, and let γ_{0_j} denote the receiver sensitivity of node u_j. The measure of D_{ij} is determined by S_{ij} and γ_{0_j}, as well as the channel condition between nodes u_i and u_j. Node u_i is said to be 'connected with' node u_j if $D_i \ge d_{ij}$. If two nodes u_i and u_j are connected, throughout the rest of this Letter they are to be known as 'neighbours'. A WSN can be represented by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with vertices $\mathcal{V} = \{1, 2, \ldots, N\}$, a set of edges $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ and weighted adjacency matrix $\boldsymbol{A} \in \mathbb{R}^{N \times N}$ with an amplitude set $\boldsymbol{A} = \{a_{ij}\}$, where the coefficient a_{ij} represents the connection weight between nodes u_i and u_j, with $\{i, j\} \in \mathcal{V}$. To account for the spatial attenuation of transmitted signals, coefficient a_{ij} is defined as follows: $a_{ij} = 1$ if $i = j$, $a_{ij} = (d_0/d_{ij})^n$ if $d_{ij} \le D_{ij}$ or $a_{ij} = 0$ otherwise. Parameter $d_0(< d_{ij})$ is a reference distance, whereas n denotes the path-loss exponent, which ranges from $n = 2$–6 [5, p. 41].

Consider the situation in which node u_j estimates u_i's clock offset, denoted by θ_{ij}, by means of one-way message exchange, as in the Flooding Time Synchronization Protocol [6]. At a given time instant, node u_i transmits a message modulated in signal $q_{ij}(t)$ over a flat-fading channel with gain $g_{ij}(t)$, and node u_j receives $y_{ij}(t) = g_{ij}(t)q_{ij}(t) + w(t)$, where $w(t)$ is an additive white Gaussian noise process with power σ^2. Signal q_{ij} is received by node u_j with a probability of success determined by the channel's outage probability. More precisely, node u_i sends m_{ij} number of messages to u_j, which successfully receives M_{ij} messages, where $M_{ij} \le m_{ij}$. The expectation of M_{ij}, denoted as $\tilde{m}_{ij}$, can be obtained as $\tilde{m}_{ij} = m_{ij} \cdot (1 - P_{\text{out}_{ij}})$ [7], where $P_{\text{out}_{ij}}$ denotes the outage probability of the wireless link. In general, the outage probability is a function S_{ij}, g_{ij}, γ_{0_j}, σ_j^2 at the receiver, a_{ij} and the relative velocity between sensors v_{ij}. For Cramer-Rao efficient estimators of θ_{ij}, the estimation error incurred by node u_j when estimating u_i's clock offset, denoted as ϵ_{ij}, is given by $\epsilon_{ij} = \sigma_V^2/\tilde{m}_{ij}$ [7], where σ_V^2 is the variance of the measured θ_{ij}. Thus, the total local estimation error on node u_i when estimating its neighbours' clock offsets, denoted as ϵ_i, is

$$\epsilon_i \triangleq \sum_j \epsilon_{ij} = \sum_j \frac{\sigma_V^2}{m_{ij}\left(1 - P_{\text{out}_{ij}}\right)} \quad \forall j \in \mathcal{V}, \quad j \ne i \tag{1}$$

Energy optimisation against network-wide estimation error: Node u_i's pairwise synchronisation energy, defined as the local average energy function of node u_i when synchronising with u_j, is dictated by $E_{ij} = S_{ij} m_{ij} \delta_{ij}$ [7], where $\delta_{ij} = T_m/\left(1 - P_{\text{out}_{ij}}\right)$ represents each message's average delivery time and T_m is each message's time duration. In general, node u_i shall synchronise with $(N-1)$ nodes, consuming a total synchronisation energy, denoted as E_i, equal to $E_i \triangleq \sum_j E_{ij}\ \forall j \in \mathcal{V},\ j \ne i$. Energy being a scarce resource, E_i is to be minimised. Moreover, it is required to guarantee that network-wide synchronisation is achieved within a maximum tolerable threshold, denoted by $\epsilon_{\max}$. Hence, the following problem can be stated:

$$\text{Minimise } E_i \text{ s.t.} \quad \sum_i \epsilon_i \le \epsilon_{\max} \quad \forall i \in \mathcal{V} \tag{2}$$

The objective is then to find the optimal pairs $\{S_{ij}, \epsilon_{ij}\}\ \forall i, j \in \mathcal{V}$ that solve for the optimisation problem expressed in (2).

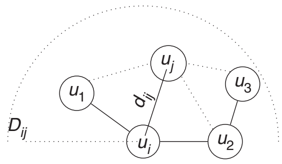

Fig. 1 *Node u_i is connected to node u_j since $d_{ij} < D_{ij}$*

Tight, optimal solution: An optimal solution for equality in (2) can be found using the Lagrange multipliers method, by stating

$$\nabla E_i = \lambda \nabla \left(\sum_i \epsilon_i \right) \quad \text{s.t.} \quad \sum_i \epsilon_i = \epsilon_{\max} \quad \forall i \in \mathcal{V} \tag{3}$$

where $\lambda \in \mathbb{R}$ is the Lagrange multiplier and $\nabla(\cdot)$ is calculated with respect to all the variables composing the energy function and for the $(N-1)$ neighbours of node u_i. Parameters a_{ij}, γ_{0_j}, σ_i^2 and v_{ij} are assumed constant and known. Therefore, it holds that $\epsilon_{ij} \triangleq \epsilon_{ij}(S_{ij})$, $P_{\text{out}_{ij}} \triangleq P_{\text{out}_{ij}}(S_{ij})$, $P'_{\text{out}_{ij}} \triangleq \mathrm{d}P_{\text{out}_{ij}}/\mathrm{d}S_{ij}$ and $E'_{ij} \triangleq \mathrm{d}E_{ij}/\mathrm{d}S_{ij}$. Thus, (3) implies minimisation of each component of E_i, leading to

$$E'_{ij} = \lambda \epsilon'_{ij} \tag{4}$$

$$E'_{ij} = \frac{\sigma_V^2 T_M}{\epsilon_{ij}\left(1 - P_{\text{out}_{ij}}\right)^2} \left(1 - \frac{\epsilon'_{ij} S_{ij}}{\epsilon_{ij}} + \frac{2 S_{ij} P'_{\text{out}_{ij}}}{1 - P_{\text{out}_{ij}}} \right) \tag{5}$$

Equation (4) implies terms in (5) not containing ϵ'_{ij} vanish, hence

$$2 S_{ij} P'_{\text{out}_{ij}} = 1 - P_{\text{out}_{ij}} \tag{6}$$

Let $S_{ij_{\text{opt}}}$ denote the optimal transmit power that solves (6). Hence, from (4) and (5) it follows that the optimal pairwise estimation error for the link between nodes u_i and u_j, denoted by $\epsilon_{ij_{\text{opt}}}$, is given by:

$$\epsilon_{ij_{\text{opt}}} = \sqrt{- \frac{S_{ij_{\text{opt}}}}{\lambda\left(1 - P_{\text{out}_{ij}}(S_{ij_{\text{opt}}})\right)^2 \sigma_V^2 T_M}} \tag{7}$$

The value of λ in (7) is obtained by plugging each $\epsilon_{ij_{\text{opt}}}$ into the summation constraint in (3) and solving for λ, namely

$$\lambda = -\frac{1}{\epsilon_{\max}^2} \left(\sum_i \sum_{j, j \ne i} \sqrt{\frac{S_{ij_{\text{opt}}}}{\left(1 - P_{\text{out}_{ij}}(S_{ij_{\text{opt}}})\right)^2 \sigma_V^2 T_M}} \right)^2 \tag{8}$$

Note in (8) that $\lambda < 0$ for all non-zero $S_{ij_{\text{opt}}}$, which indicates that the optimal solutions of (3) can only be energy minima.

Flexible, sub-optimal solution: In practice, WSN sensors may be unable to tune their transceivers to the exact value of $S_{ij_{\text{opt}}}$. Therefore, it is useful to find flexible solutions suitable for different realistic scenarios. Namely, inequality constraints in (2) can be attained at a predefined constant E_{ij}, which implies $\nabla E_i = 0$ over the set of solutions $\{S_{ij}, \epsilon_{ij}(S_{ij})\}$ for all $i, j \in \mathcal{V}$, or

$$E'_{ij} = \frac{\partial E_{ij}}{\partial S_{ij}} + \frac{\partial E_{ij}}{\partial \epsilon_{ij}} \epsilon'_{ij} = 0 \qquad (9)$$

After simple mathematical operations, (9) leads to

$$\epsilon'_{ij} = \frac{1}{S_{ij}} + \frac{2P'_{\text{out}_{ij}}}{1 - P_{\text{out}_{ij}}} \epsilon_{ij}, \quad \epsilon_{ij}(S_{ij_{\text{opt}}}) = \xi_{ij} \epsilon_{ij_{\text{opt}}} \qquad (10)$$

where $S_{ij_{\text{opt}}}$ is a reference transmit power that sets the initial condition of the estimation error, given by $\epsilon_{ij_{\text{opt}}} \xi_{ij}$. Using separation of variables, system (10) solves to

$$\epsilon_{ij}(S_{ij}) = \epsilon_{ij_{\text{opt}}} \xi_{ij} \frac{S_{ij}}{S_{ij_{\text{opt}}}} \left[\frac{1 - P_{\text{out}_{ij}}(S_{ij_{\text{opt}}})}{1 - P_{\text{out}_{ij}}(S_{ij})} \right]^2 \qquad (11)$$

where ξ_{ij} is a degree of freedom of the energy-efficient pairwise estimation error, in the range $\xi \in (0, 1]$. To ensure the fulfilment of constraint (2), $\epsilon_{ij}(S_{ij})$ must be capped to $\epsilon_{ij_{\text{opt}}}$. Note that $\xi_{ij} = 1$ corresponds to the optimal solution found by the Lagrange method, whereas decreasing ξ_{ij} allows variation of S_{ij} and ϵ_{ij} in an amplitude range determined by ξ_{ij}, at fixed synchronisation energy.

Application example: Consider the WSN in Fig. 1 with $N = 5$ nodes designated u_i, with $i \in \mathcal{V} = \{1, 2, \ldots, N\}$, randomly deployed in a sphere of radius $r = 6$ m. Nodes' position tolerances are modelled by a zero-mean normal distribution with standard deviation $\sigma_d = 0.03$ m. Assume that nodes communicate over a channel that experiences combined path-loss and uncorrelated Rayleigh fading effects. Using the simplified path-loss model, the power received by sensor u_j from transmitter sensor u_i, denoted as S_{ij_R}, is equal to $S_{ij_R} = K a_{ij} S_{ij}$, where K is a unitless constant, and the outage probability is given by [5, p. 169]

$$P_{\text{out}_{ij}}(S_{ij}) = 1 - \exp\left(-\frac{\gamma_{0_j}}{\bar{\gamma}_{S_{ij}}}\right) \approx 1 - \exp\left(-\frac{\gamma_{0_j}\sigma^2}{K a_{ij} S_{ij}}\right) \qquad (12)$$

where γ_{0_j} represents the minimum acceptable signal-to-noise ratio. The transmit power S_{ij} and the noise power σ^2 in (12) are expressed in watts. Power from interferer sources is within parameter σ^2. Plugging (12) into (6) leads to $S_{ij_{\text{opt}}} = 2\sigma^2 \gamma_{0_j}/(K a_{ij})$. Hence, the value of each $\epsilon_{ij_{\text{opt}}}$ is obtained by plugging $S_{ij_{\text{opt}}}$ into (7) and (8).

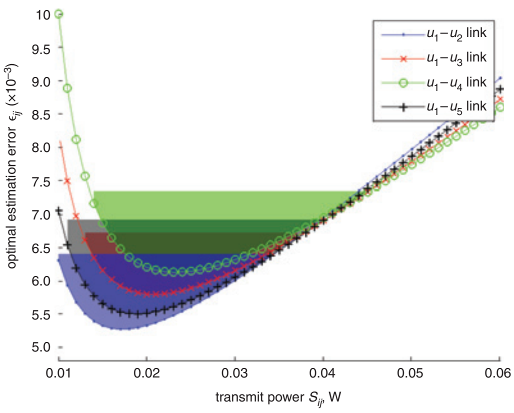

Fig. 2 *Optimal estimation error between node u_1 and its neighbours*

Fig. 2 shows the Monte Carlo simulation outputs of pairwise estimation errors for node u_1 when synchronising nodes u_2 to u_5 using MATLAB©. The shaded areas show the region in which each pairwise estimation error can be freely varied, through parameter ξ_{ij}, without compromising the network-wide estimation error. The region of freedom is upper bounded by $\epsilon_{ij_{\text{opt}}}$. Also, note that the pairwise estimation error reaches a minimum at $S_{ij_{\text{opt}}}$. For constant S_{ij}, the required synchronisation energy can be further reduced at the expense of ϵ_{ij}. Fig. 3 depicts the pairwise synchronisation energy against transmit power and estimation error. For the sake of clarity, and without loss of generality, only the energy for pair $u_1 - u_3$ is drawn. Note that, for a given ξ_{ij}, the synchronisation energy remains constant along the solutions of (11), which is observed as level curves of the synchronisation energy.

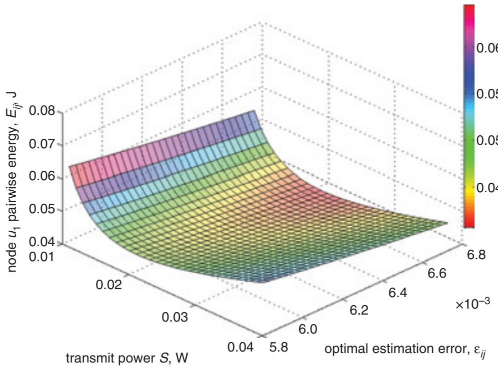

Fig. 3 *Synchronisation energy level curves for link $u_1 - u_3$*

Conclusion: The generalised model presented in this Letter contemplates the main parameters that drive energy-efficient synchronisation within a WSN. The proposed solutions, supported by simulation results, show how to fine tune both pairwise transmit power and local estimation error for each node in the network, in order to meet the expected network-wide estimation error requirement in an energy-efficient manner. More importantly, the system model is based on a generic WSN, without being constrained by network size or topology, which makes it a powerful tool for energy-aware WSN design.

30 July 2014
doi: 10.1049/el.2014.2753
One or more of the Figures in this Letter are available in colour online.

P. Briff (*LSE, FIUBA, University of Buenos Aires, Buenos Aires, Argentina*)

✉ E-mail: pbriff@fi.uba.ar

A. Lutenberg (*LSE, FIUBA and CONICET, Buenos Aires, Argentina*)

L. Rey Vega (*LPSC, FIUBA, and CSC-CONICET, Buenos Aires, Argentina*)

F. Vargas (*Catholic University – PUCRS, Porto Alegre, Brazil*)

M. Patwary (*SPCR Laboratory, FCES, Staffordshire University, Stafford, United Kingdom*)

References

1 Celandroni, N., *et al.*: 'A survey of architectures and scenarios in satellite-based wireless sensor networks: system design aspects', *Int. J. Satell. Commun. Netw.*, 2013, **31**, (1), pp. 1–38
2 Basagni, S., Naderi, M., Petrioli, C., and Spenza, D.: 'Wireless sensor networks with energy harvesting', *Mob. Ad Hoc Netw., Cutt. Edge Dir.*, 2013, pp. 701–736
3 Lenzen, C., Sommer, P., and Wattenhofer, R.: 'PulseSync: an efficient and scalable clock synchronization protocol'. Available at http://www.groups.csail.mit.edu/tds/papers/Lenzen/LSW14pulsesync.pdf, accessed April 2014
4 Serpedin, E., and Chaudhari, Q.: 'Synchronization in wireless sensor networks: parameter estimation, performance benchmarks, and protocols' (Cambridge University Press, Cambridge, UK, 2009)
5 Goldsmith, A.: 'Wireless communications' (Cambridge University Press, Cambridge, UK, 2005), pp. 24–179
6 Maroti, M., Kusy, B., Simon, G., and Ledeczi, A.: 'The flooding time synchronization protocol'. Proc. of 2nd Int. Conf. on Embedded Networked Sensor Systems, 2004, pp. 39–49
7 Briff, P., Lutenberg, A., Rey Vega, L., Vargas, F., and Patwary, M.: 'A primer on energy-efficient synchronization of WSN nodes over correlated Rayleigh fading channels', *IEEE Wirel. Commun. Lett.*, 2013, **3**, (1), pp. 38–41