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Discount Rates for Seed Capital Investments

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Abstract

So far, the estimation of discount rates required by entrepreneurs has remained a mystery. Mongrut and Ramirez (2006) made a contribution to this area by deriving the lower bound discount rate for a non-diversified entrepreneur in an emerging market. However, they used a quadratic utility function, which does not have desirable assumptions. In this research one extends the previous work by deriving expressions of discount rates using a Hyperbolic Absolute Risk Aversion (HARA) utility function that includes the quadratic and the logarithmic forms as special cases. Furthermore, one also assumes the entrepreneur with the lowest risk-aversion that invests almost all his capital in his project or firm and whose level of wealth approaches to zero. One finds that both expressions depend upon entrepreneur's risk-aversion and a measure of the project total risk. Maintaining constant the risk-free rate, we simulate the expressions of discount rate for the quadratic form and the logarithmic form. As expected, the entrepreneur's required returns (discount rates) are highly sensitive in both specifications and all values were lower than 50% and most of them were lower than 25%, but higher than the assumed risk-free rate.

Keywords: Seed Capital, discount rates, entrepreneurship

JEL codes: L26 and M13

1. Introduction

In a pioneering work, McInish and Kudla (1981) argued that the so called two-fund separation theorem, originally developed by Tobin (1958), does not hold in the case of closely-held firms and small firms. Hence, the appropriate discount rate for valuating these firms would be required rates of return from the owners of the business instead of a market discount rate.

The two-fund separation theorem states that the optimal portfolio of risky securities is exactly the same regardless of the investors' risk preferences[1]. This property is extremely important for asset pricing because it allows us to estimate the cost of equity capital under equilibrium conditions, such as in the case of the Capital Asset Pricing Model (CAPM). Recently, Breuer and Gürtler (2007) have shown that in order to meet the two-fund separation theorem two restricted conditions must be met: the utility function must be always defined and marginal utility must be positive. Unfortunately, these well-defined problems restrict seriously the application of the two-fund separation theorem.

This problem is even worse given the fact that not all preferences can be represented with utility functions. This is the main argument that Meyer (2007) advocates to favor marginal utility function instead of the utility function itself because the former encompasses a bigger possible set of risk preferences.

Another avenue that the literature has pursued is to impose certain restrictions to the distribution of stock returns in order to attain the two-fund separation theorem. Ross (1978) has shown the conditions that must be fulfilled to apply the theorem independently of the investor utility function. Wei et.al (1999) have shown that, although general elliptical distributions can guarantee the two-fund separation property, the identification of the "true" return distribution is rather difficult, not to say impossible. It is clear by now that using distribution properties of stock returns, is not a good avenue to guarantee the two-fund separation theorem. Furthermore, to follow the specification of utility functions that belong to the Hyperbolic Absolute Risk Aversion (HARA) family, only guarantee this separation property under restricted conditions.

The previous discussion is important if one is interested in obtaining the market value of an investment project. If this is not the case, the argument related to the distribution of stock returns becomes irrelevant as the capital market is not going to be a benchmark and the argument related to risk preference (utility function and marginal utility) becomes more relevant because it is the only way to specify the risk preferences and discount rates for seed capital projects and firms.

Unfortunately, there is a cost of using subjective discount rates because the estimated Net Present Value (NPV) turns out to be just one profitability indicator instead of being a normative investment rule (Zurita, 2005). In other words, two entrepreneurs could assess differently the same investment project being both estimations valid. This is not possible under market-derived conditions such as the ones imposed by the Capital Asset Pricing Model (CAPM).

[1] Cas and Stiglitz (1970) managed to show the conditions under which the two-fund separation theorem can be applied under hyperbolic preferences.

Another problem related with the estimation of subjective discount rates is the possible bias in the estimation of the parameters. These different biases have been explained by Fuenzalida et al. (2007), so one must use a heuristic procedure in order to avoid as much as possible the potential biases.

Despite these disadvantages, it is convenient to derive formulas for the project's discount rates under certain assumptions because these will provide magnitudes of how much we could gain or we could lose with the project in different scenarios through a prospective analysis for valuating the project. In this prospective analysis what matters are the foreseen contingent strategies for increasing the likelihood of success of the project rather than using just one indicator (i.e. NPV) to accept or reject the investment proposal.

In fact, Mongrut and Ramirez (2006) estimated a formula for project discount rate for investors with quadratic preferences in an incomplete market. They concluded that, for entrepreneurs with the lower risk aversion, the discount rate was not unique in an incomplete market. Even more, the discount rate is bounded from below and we can only estimate the maximum value for the investment project, but not its minimum value. In this situation the discount rate will depend upon three parameters: the risk-free rate, the reward-to-variability ratio (RTV) of the project, and the project total risk.

Mongrut (2015) expanded the risk analysis process proposed by Fuenzalida et al. (2007) to estimate the project's total risk. This expanded process includes a previous prospective analysis, so the project's NPV will be estimated for different scenarios and where the entrepreneurs' risk-aversion coefficient could change. In this way, one may estimate project's NPV using the estimated total risk for each year of the time-horizon with different discount rates and within each scenario.

Although the discount rate expression provided by Mongrut and Ramirez (2006) is simple, contains unrealistic assumptions because they used a quadratic utility function that implies a constant absolute risk aversion (CARA) and it is well-known that this utility function is not increasing everywhere[2]. The main goal of this research is to derive discount rates using risk-preferences according to HARA utility functions. It turns out to be that discount rate expressions depend upon a measure of the project's total risk and the entrepreneurs' risk aversion coefficient.

The paper is organized as follows, the next section defines the concept of a discount rate according to the features of seed capital investments, then in the third section one explains briefly all the new set of utility functions available and justifies the use of the HARA family. In the fourth section one derives an expression for the discount rate using the HARA family and assuming incomplete markets. In the fifth section, with the aid of simulation, one explores the properties of the discount rate expressions. In the last section one concludes the work.

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[2] The traditional assumptions related to utility functions are: the utility function is an increasing function everywhere of consumption or wealth (meaning that more is better), the utility function is concave and twice differentiable, where the first derivative is positive (marginal utility) and the second is negative (Gerber and Pafumi, 1999).

2. Definition of a discount rate in the context of seed capital investments

In the context of seed capital investments, the discount rate is the return that entrepreneur would require from his investment project. In this section one not only defines the discount rate, but also, one discusses the different features that a utility function must have in order to be applied in the context of seed capital investments.

Time preferences, time value of money and risk

There are two parallel strands of literature related to the estimation of the discount rate: the social choice literature and private choice literature. Although, in both cases, the discount rate is being used to discount future net benefits, what it represents is quite different.

In terms of the social choice literature what matters is the time preference problem that is the trade-off between present consumption and future consumption. Given that this trade-off is done on an individual basis, then one needs to aggregate these individual benefits into a measure of social benefit (Cameron and Gerdes, 2003). Hence, much of the literature has been devoted to estimate individual discount rates using exponential or hyperbolic utility functions, and surveys and simulations to validate them.

The main conclusion so far is that individuals' opinions about social discount rates vary substantially across samples depending on the context for making a choice and the techniques used to elicit opinions. The main problem is that most individuals do not know the meaning of a discount rate and if they know it, nothing guarantees that they are being able to exteriorize the magnitude of their individual discount rate. Hence, according to the social choice literature, there is an urgent need to gather only experts' opinions and to translate the language of a discount rate into its constituents that, hopefully, are more familiar to experts rather than non-experts.

In the case of the private choice literature, one is faced with only one choice: to invest or not to invest in a certain investment project. However, there is also the problem of social aggregation that needs to be solved. The discount rate in this context is understood as an opportunity cost that considers not only the time value of money, but that it also includes the risk to which the investor is facing. In other words, it is an opportunity cost commensurate to the project's risk level.

Here, there are two common interpretations that lead to market discount rates or to subjective discount rates. In the former case, it is being assumed that each investor holds a well-diversified portfolio of investments; hence he only needs to worry about the project's market risk, which is the project's contribution to the risk of this well-diversified portfolio. In the latter case, one assumes that investors do not hold a portfolio of investment projects and in this way they are completely exposed to the project's total risk. Naturally, these are two extremes assumptions within a range of possible degrees of diversification, but they are quite convenient because these assumptions let us to work with more tractable models.

The aggregation problem in the case of market discount rates is solved by using the two-fund separation theorem. Specifically, one postulates the existence of an investor who hold a well-diversified investment portfolio (market portfolio) that represents all individual investors in the market, so each one of the assets is priced according to its market risk. Finally, by using the market discount rate one is estimating the value of the investment project from the market's point of view (the representative investor).

In the case of individually-based discount rates the aggregation problem is of a different nature. Here we need a consensus forecast of the experts' opinion related to the critical variables of a single investment project. However, nothing guarantees that this consensus forecast is valid for the market as a whole. In fact, it is only helpful to a single investment project.

Hence, according to these two extreme points of view, the market risk is the relevant one for well-diversified investors and the total risk is the relevant one for non-diversified investors that are usually the owners of seed capital projects and firms (Erikson, 2005). Hence, it is of paramount importance the proper estimation of both risks.

The utility function for seed capital investments

In a seminal work, McMahon and Stanger (1995) discussed different factors that affect small firms' objective function. They argued that the owner-manager's utility function depends upon pecuniary returns from the business (P) (i.e. return on investment - ROI); non-pecuniary satisfactions from the business that are in the financial domain (Nf) (i.e. a good financial health, operative flexibility, and so on); non-pecuniary satisfactions from the business that are outside the financial domain (Nn) (i.e. preferred lifestyle, self-esteem, and so on); and total risk, which is the sum of systematic or market risk and the unsystematic or specific risk (ϑ). Hence, the utility function for a small firm's owner-manager will be as follows:

$$U(w) = U(P, N_f, N_n, \vartheta) \tag{1}$$

In this specification, the owner-manager's wealth is affected by all the factors already described. Furthermore, these authors went further explaining why the firms' return, firm's risk, firm's liquidity, the degree of diversification of the owners-managers, the transferability of financial and human capital, the financial flexibility and the desire to control the firm would impact this utility function through their different variables.

Although this utility function seems plausible as a generalization for small firms, there are two remarks: there are many types of small firms and hence some of them are going to be less affected by some factors, and most of these factors influence also the utility function of a corporate firm. The point here is not the difference between firms' sizes, but the different relative impact of each of these factors in the entrepreneur's utility function.

For instance, the possibility to transfer financial and human capital from one generation to the other is important for certain types of small firms such as family firms. In this case, the founder derives part of his satisfaction by looking at how his heirs take the business properly. However, for the owner-manager of a new venture it is more important to have financial flexibility because he needs funds, which are usually not provided by venture capitalists in the early stage of the business. In other words, he must be able to show to potential creditors or shareholders that he is able to cope with different scenarios using contingent strategies.

Other factors, such the firm's return, liquidity, desire of control and accountability are also important for corporate firms. In fact, there is no way in which a company can survive in the long-term if it does not have liquidity and if it does not care for the firm's stakeholders. Of course, here one must recognize that liquidity could be a matter of life and death for a small firm, but not necessarily for a corporate firm.

The degree of diversification of the owner-manager and the firm's risk are crucial because they change the valuation tools for small firms and in particular for seed capital investments. A seed capital project is usually run by a single non-diversified entrepreneur that faces the project's total risk. A market valuation does not apply here because there is no a representative investor, only matter the entrepreneur and the experts' opinions concerning a particular investment proposal. Besides, given that the entrepreneur puts all his money in the project or firm then he will face the project's total risk. Furthermore, in this case the traditional value-additivity principle breaks up, because the entrepreneur could derive some positive synergies by investing in different projects[3].

Another issue worth to discuss is whether to include or not in the utility function the non-pecuniary satisfactions that do not lie within the financial domain (Nn). Although it is important to have a preferred lifestyle, security by being self-employed, employment to family members, and self-esteem; one must recognize that most of these satisfactions do not apply for all small firms and some of them are beyond the firm's fate, so it is debatable whether to include them or not in the entrepreneur's utility function.

For instance, having a good self-esteem is a personal attribute that a person must cultivate regardless whether he becomes an entrepreneur or not. Of course, if he is interested to become an entrepreneur, as a preferred lifestyle, he will get an important satisfaction by being an entrepreneur independently if his business becomes successful or not.

In other words, if self-esteem and become an entrepreneur by conviction are two attributes beyond the firm's fate, is it important to value the marginal satisfaction that the entrepreneur may obtain through his entrepreneurial adventure? Due to the fact that the entrepreneur will obtain satisfaction anyway by running his business, one may think about these non-pecuniary satisfactions as belonging to a superior indifference curve, but all with the same slope. In other words, it is like having a constant that moves the utility function upward, but these non-pecuniary satisfactions do not restrict the shape and the optimum solution. Hence one may decide not to include this constant in the entrepreneur's utility function[4].

Concerning the non-pecuniary satisfactions related to the financial domain (Nf), they could be included through different scenarios as contingent strategies that improve the project financial health and its likelihood of success. Of course one must also include them in the entrepreneur's utility function and obtain a discount rate accordingly, but one may also include them through a prospective analysis of the project[5].

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3 The value-additivity principle states that the value of a firm, for a well-diversified investor, is equal to the value of the firm without the project plus the NPV of the project. This is possible because only matters the firm's contribution to the well-diversified investor's portfolio, so any potential synergy between the project and the firm does not have a market value because the investor could diversify better and quicker than the firm. In the case of a non-diversified entrepreneur, this principle breaks-up.

4 This decision amounts to picture a self-confident entrepreneur who decided to be an entrepreneur by conviction and who runs his business in a strictly professional way..

5 For example, James (1999) has included the transferability issue into the owner-manager's utility function of a family firm.

In brief, one believes that the utility function for entrepreneurs who are facing a seed capital investment may be a subset of the utility function described by McMahon and Stanger (1995):

$$U(w) = U(P, \vartheta) \tag{2}$$

Note that "P" is related to pecuniary returns from the business, especially the return on investment (ROI) that helps to provide a good salary for owner-manager. Given that most entrepreneurs with seed capital investment opportunities are not diversified, the second parameter (ϑ) is related to the project's total risk. The non-pecuniary satisfaction related to the financial domain (Nf) are addressed directly in the prospective analysis and the pecuniary satisfactions outside of the financial domain (Nn) are not considered in the entrepreneur or owner-manager's utility function.

3. The choice of the utility function for discount rates

The field of risk preferences (utility functions) was fostered by the work of Arrow (1965, 1971) and Pratt (1964). In particular, they both proposed the following specification of utility functions[6]:

$$U(C_t) = -e^{-\alpha C_t} \tag{3}$$

This function is well-known as the negative exponential and it has a constant absolute risk-aversion coefficient ($A(c_t) = \alpha$). They also proposed the following specification[7]:

$$U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma} \tag{4}$$

This utility function is known as the power utility function where the parameter (γ) represents the level of relative risk-aversion. Unfortunately, both specifications only let us vary the magnitude of the risk-aversion (absolute or relative), but they do not allow us to change the slope of these risk-aversion measures (Meyer, 2007).

The slope represent the behaviour of the individual's risk- aversion because it may depict decreasing (or increase) of the absolute risk-aversion (DARA or IARA) .In one hand, a DARA behaviour implies that richer people are less risk adverse than poorer people, so they require a smaller payment. In the other hand IARA implies that richer people require a larger payment than poorer people in order to enter in a lottery game. Naturally, the DARA case is more realistic than the IARA case.

6 One can also express the utility function in terms of wealth (W) instead of consumption (C) without loss of generality.

7 The coefficient of relative risk-aversion (CRRA) is equal to $R(c_t) = c_t A(c_t)$ and it implies that risk-aversion also depends upon the individual's level of consumption, which in turn depends on his wealth.

In 1971, Merton proposed the Hyperbolic utility function with Absolute Risk-Aversion (HARA) that has two big advantages: it includes the previous specifications as particular cases and it includes a wider range of risk preference specifications. Furthermore a particular choice of function parameters can change the slope of the risk aversion coefficients (CARA and CRRA):

$$U(C_t) = \frac{\gamma}{1-\gamma}\left[\mu + \frac{\alpha C_t}{\gamma}\right]^{1-\gamma} \tag{5}$$

Where:

$\alpha > 0$; $\mu + \frac{\alpha C_t}{\gamma} > 0$; and $C_t > 0$

For instance, if $\gamma > 0$ then absolute risk-aversion is decreasing, while if $\gamma < 0$ then absolute risk-aversion is increasing. If $\alpha = \infty$ then the coefficient of relative risk-aversion will be equal to γ.

If $\gamma \to \infty$ then $\mu = 1$ and the so called generalized power utility function appears:

$$U(C_t) = \frac{\gamma}{1-\gamma}\left[1 + \frac{\alpha C_t}{\gamma}\right]^{1-\gamma} \tag{6}$$

From this specification, one could obtain the negative exponential function, the logarithmic utility function and the quadratic utility function:

If $\gamma \to 0$ and $\alpha > 0$, then the generalized power utility function (expression 6) converges to the negative exponential function given by expression 3 and it has an absolute risk-aversion coefficient equal to α.

If $\gamma \to 1$ then an affine transformation of expression 6 converges to the logarithmic utility function:

$$U(C_t) = Ln\left(C_t + \frac{1}{\alpha}\right) \qquad \text{Where: } C_t + \frac{1}{\alpha} > 0$$

This specification has a decreasing absolute risk-aversion coefficient equal to $A(C_t) = \frac{1}{C_t + \frac{1}{\alpha}}$. If $\alpha = \infty$ then the relative risk-aversion coefficient is equal to 1.

If $\gamma \to -1$ then the affine transformation of expression 6 will converge to the quadratic utility function:

$$U(C_t) = -\frac{1}{2}\left(\frac{1}{\alpha} - C_t\right)^2$$ Where: $\frac{1}{\alpha} - C_t > 0$

This specification has an increasing absolute risk-aversion coefficient equal to: $\frac{1}{\frac{1}{\alpha} - C_t}$.

Recently, some new utility functions have been put forward in the literature. For instance Saha (1993) introduced the expo-power (EP) utility function. The functional form is as follows:

$$U(C_t) = \vartheta - e^{-\beta C_t^{\alpha}}$$

With the following restrictions: $\vartheta > 1$ and $\alpha\beta > 0$

Given these restrictions, the absolute risk-aversion coefficient is equal to the following expression: $A(C_t) = \frac{1 - \alpha + \alpha\beta C_t^{\alpha}}{C_t}$. As Meyer (2007) pointed out, this is a two parameter model because the constant "ϑ" does not enter the risk-aversion coefficient.

The main advantage of the EP functional form over the HARA is that the EP form reduces to the CARA form with finite parameter values. Remember that in the HARA form (expression 6) one needs to assume that $\gamma \to 0$ in order to have the CARA form. Xie (2000) tried to improve also a functional form over the HARA function, so he proposed the Power Risk Aversion (PRA) functional form. The advantage of this new functional form over the HARA is that it remains well-defined over a longer range for parameter values (Meyer, 2007):

$$U(C_t) = \frac{1}{\gamma}\left\{1 - \exp\left[-\gamma\left(\frac{c_t^{1-\sigma} - 1}{1-\sigma}\right)\right]\right\}$$ With the following restrictions: $\sigma \geq 0$ and $\gamma \geq 0$

In the previous expression the exponential operator (exp) refers to the transcendental number "e". The absolute risk-aversion coefficient is given by the following expression:

$$A(C_t) = \frac{\sigma}{C_t} + \gamma C_t^{-\sigma}$$

Finally, Connife (2006) has provided the Flexible Three Parameter (FTP) functional form that encompasses the EP and PRA forms. The functional form is given by the following expression:

$$U(C_t) = \frac{1}{\gamma}\left\{1 - \left[1 - k\gamma\left(\frac{C_t^{1-\sigma} - 1}{1-\sigma}\right)\right]^{\frac{1}{k}}\right\}$$ When k=0 it reduces to the PRA (Meyer, 2007)

The absolute risk aversion coefficient is given by the following expression:

$$A(C_t) = \frac{\sigma}{C_t} + \frac{(1-k)\gamma C_t^{-\sigma}}{1 - k\gamma\left(\frac{C_t^{1-\sigma} - 1}{1-\sigma}\right)}$$

Despite the fact that using an HARA utility function just captures a more restrictive range of investors' preferences than the alternatives utility functiuons (EP, PRA and FTP forms), it is the most used closed-form specification in financial economics. For instance, the HARA utility functions have been applied for estimating discount rates in the case of bankruptcy (Barthelemy et. al., 2006), for portfolio performance (Breuer and Gürtler, 2006), for portfolio allocation with hedge funds (Popova, 2006), and for expanding conditions for pricing in incomplete markets (Menoncin (1998), Luenberger (2002), and Guu and Wang (2008)).

If one aims to capture other types of risk preferences, it would be better to follow the approach of Meyer (2007). The approach of Meyer is based upon defining a closed-form function for the marginal utility function instead of the utility function itself. This is because, the marginal utility also represents a risk preference, however not all persons have a closed-form marginal utility.

In this work, one believes that it is possible to use a broad (closed-form) utility function to estimate the lower bound of a discount rate and then, through a prospective risk-analysis, one may estimate the resulting expression's parameters. In fact, Mongrut and Ramirez (2006) have shown that with incomplete markets there could be many discount rates so the possibilities to specify different values for the parameters are enormous.

4. Discount rates for seed capital investments under incomplete markets

In this section, one uses an affine transformation of expression 6 to solve the problem stated by Mongrut and Ramirez (2006): how much to consume today and tomorrow provided that each individual has an initial wealth and each one can invest in one investment project or make a deposit in a savings account. However, in order to put the problem more realistic, one analyses the case when the entrepreneur's initial wealth level approaches to zero, when he is the entrepreneur with the lowest risk-aversion and invests all his money in the project.

It is important to state beforehand that the solution this problem can be considered a generalized solution only the in the sense that it accounts for a broader set of risk preferences rather than the quadratic utility function used by Mongrut and Ramirez (2006). However, it should be clear by now that using expression 6 is only a subset of all possible risk preferences.

All the remaining assumptions made by Mongrut and Ramirez (2006) are also made in this derivation namely, it is an individual optimization meaning that each individual must define the parameters of the resulting discount rate expression according to his risk aversion and the specifics of the project. However, one can also define the lower bound of the discount rate with the entrepreneur with the lowest risk-aversion, which implies an absolute risk aversion coefficient equal to 1.

One starts by optimizing the following two-period objective function (one also assumes that the utility function is time separable):

$$U(C_t, C_{t+1}) = \frac{\gamma}{1-\gamma}\left[\left(1+\frac{\alpha C_t}{\gamma}\right)^{1-\gamma} - 1\right] + \delta E_t\left\{\frac{\gamma}{1-\gamma}\left[\left(1+\frac{\alpha C_{t+1}}{\gamma}\right)^{1-\gamma} - 1\right]\right\}$$

$$U(C_t) = \frac{\gamma}{1-\gamma}\left[\left(\frac{1}{\alpha}+\frac{C_t}{\gamma}\right)^{1-\gamma} - 1\right]$$

Subject to the following constraint: $E(R_{t+1}) = \dfrac{C_{t+1}}{W_t - C_t}$

Where:

$E(R_{t+1})$: Represents the project expected return in period "t+1"
W_t: The entrepreneur initial wealth level.
C_t: The entrepreneur's consumption of today
C_{t+1}: The entrepreneur's consumption of tomorrow

The stochastic discount factor (SDF) of the previous problem is equal to:

$$m_{t+1} = \delta\left(1+\frac{\alpha C_t}{\gamma}\right)^{\gamma}\left(1+\frac{\alpha C_{t+1}}{\gamma}\right)^{-\gamma} \tag{7}$$

In order to estimate the parameters of the SDF for this individual, one may state the following system of equations (Mongrut and Ramirez, 2006):

$$1 = E_t(m_{t+1}R_{t+1})$$
$$\frac{1}{R_f} = E_t(m_{t+1})$$

Using the SDF from equation 7, this system of equation translates into the following system:

$$1=\delta\left(1+\frac{\alpha C_t}{\gamma}\right)^{\gamma}E_t\left[\left(1+\frac{\alpha R_{t+1}(W_t-C_t)}{\gamma}\right)^{-\gamma}R_{t+1}\right] \tag{8}$$

$$\frac{1}{R_f}=\delta\left(1+\frac{\alpha C_t}{\gamma}\right)^{\gamma}E_t\left[\left(1+\frac{\alpha R_{t+1}(W_t-C_t)}{\gamma}\right)^{-\gamma}\right] \tag{9}$$

From equations 8 and 9, it follows the following condition:

$$0=E_t\left[(-R_f+R_{t+1})\left(1+\frac{\alpha R_{t+1}(W_t-C_t)}{\gamma}\right)^{-\gamma}\right] \tag{10}$$

$$0\approx E_t\left[\sum_{k=0}^{\infty}-Binomial(-\gamma,k)(R_f-R_{t+1})R_{t+1}^{k}\left(\frac{\alpha(W_t-C_t)}{\gamma}\right)^{k}\right] \tag{11}$$

Simplifying even more expression 11 it yields:

$$E_t(R_{t+1})-R_f\approx E_t\left[\sum_{k=1}^{n}Binomial(-\gamma,k)(R_f-R_{t+1})R_{t+1}^{k}\left(\frac{\alpha R_{t+1}(W_t-C_t)}{\gamma}\right)^{k}\right] \tag{12}$$

Expression 12 is very important because it allow us to derive the expressions of the discount rates for two risk preferences: quadratic and logarithmic. It is not possible to obtain the discount rate corresponding to the case of a negative exponential utility function because $\alpha\neq 0$ and 1. One must remember that the coefficient of absolute risk-aversion is equal to α .

The case of a quadratic utility function appears when the following conditions are met: $\gamma\rightarrow -1$, k = -1 and $\alpha>0$. By replacing these conditions in equation 12 yields:

$$E_t(R_{t+1})-R_f\approx E_t\left[\sum_{k=-1}^{-1}Binomial(1,-1)(R_f-R_{t+1})R_{t+1}^{-1}\left(\frac{\alpha R_{t+1}(W_t-C_t)}{-1}\right)^{-1}\right] \tag{13}$$

Simplifying the previous expression yields:

$$E_t(R_{t+1})-R_f\approx E_t\left[\frac{R_{t+1}-R_f}{R_{t+1}(W_t-C_t)\alpha}\right] \tag{14}$$

Now, one must consider the following two conditions. The first condition is related to the entrepreneur with the lowest risk-aversion coefficient and the second condition refers to the low initial level of wealth of the entrepreneur

$$A = \frac{1}{\frac{1}{\alpha} - C_t} = 1 \Rightarrow C_t = \frac{1-\alpha}{\alpha}$$

$$W_t \to 0$$

Given these two conditions, equation 14 provides the lower bound for the discount rate for entrepreneurs with quadratic preferences:

$$E_t(R_{t+1}) \geq R_f + \left(\frac{1}{\alpha - 1}\right) RTV(CV_t) \tag{15}$$

Where:

$RTV = \dfrac{E_t(R_{t+1}) - R_f}{\sigma(R_{t+1})}$ is the reward-to-variability ratio and

$CV_{t+1} = \dfrac{\sigma(R_{t+1})}{E_t(R_{t+1})}$ is the coefficient of variation

However, for the entrepreneur with the lowest risk-aversion coefficient, one may assume that $RTV \approx 1$, hence equation 15 boils down to:

$$E_t(R_{t+1}) \geq R_f + \left(\frac{1}{\alpha - 1}\right)(CV_{t+1}) \tag{16}$$

Note that in this case, the discount rate has three parameters: the risk-free rate (Rf), the coefficient that measure the magnitude of the entrepreneur risk-aversion ($1/_{\alpha - 1}$) and the coefficient of variation that comes from the seed capital project. Besides, the absolute coefficient of risk-aversion is increasing because $\gamma < 0$ and $\alpha > 1$.

Alternatively, one may study the case of the logarithmic risk preference. This situation appears when the following conditions are met: $\gamma \to 1$, k = 1 and $\alpha > 0$. By replacing these conditions in equation 12 yields:

$$E_t(R_{t+1}) - R_f \approx E_t\left[\sum_{k=1}^{1} Binomial(-1,1)(R_f - R_{t+1}) R_{t+1}^{1} \left(\frac{\alpha R_{t+1}(W_t - C_t)}{1}\right)^{1}\right] \tag{17}$$

Simplifying this expression taking into account the following property yields expression 18:

$$E_t\left[R_{t+1}^2\right] = \sigma^2(R_{t+1}) + E_t\left[R_{t+1}\right]^2$$

$$E_t[R_{t+1}]-R_f \approx \frac{\alpha(W_t - C_t)CV_t\sigma(R_{t+1})}{CV_t - \alpha(W_t - C_t)\sigma(R_{t+1})}\sigma(R_{t+1}) \qquad (18)$$

Again in expression 18 one needs to consider the aforementioned two additional restrictions:

$$A = \frac{1}{\frac{1}{\alpha}+C_t} = 1 \Rightarrow C_t = \frac{\alpha - 1}{\alpha}$$

$$W_t \to 0$$

Replacing these two restrictions in equation 18, yields the lower bound discount rate for an entrepreneur with a logarithmic risk preference:

$$E_t[R_{t+1}] \geq R_f + \left[\frac{(1-\alpha)\sigma^2(R_{t+1})}{CV_{t+1} - (1-\alpha)\sigma(R_{t+1})}\right]CV_{t+1} \qquad (19)$$

Note that in this case the discount rate has also three parameters: the risk-free rate (Rf), the risk-aversion "α", and the project's total risk. However, the project total risk is being represented with two estimators: the standard deviation (and variance) of the project returns and the coefficient of variation of the project returns. This specification implies a decreasing risk-aversion coefficient and $0 < \alpha < 1$.

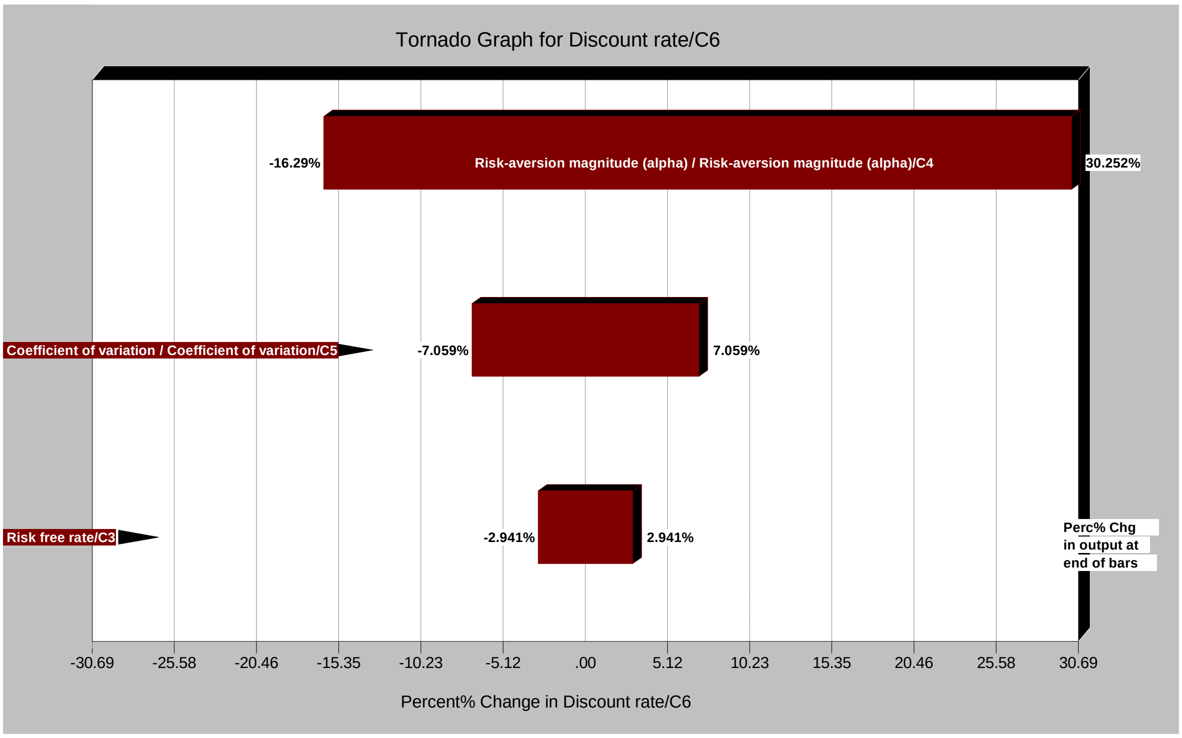
5. Properties of the discount rate expressions

In this section one identifies the critical parameters in both expressions for the lower bound of the discount rate (expressions 16 and 19) and then one performs a simulation analysis

5.1 Analysis of the discount rate with quadratic preferences

Figure No 1 shows the results of the sensitivity analysis using the expression 16. As one may observe, the critical variable is the entrepreneur's risk-aversion coefficient and it has an asymmetric impact over the estimated discount rate because a decrease of 10% in the alpha parameter increases the project's discount rate in more than 30%..

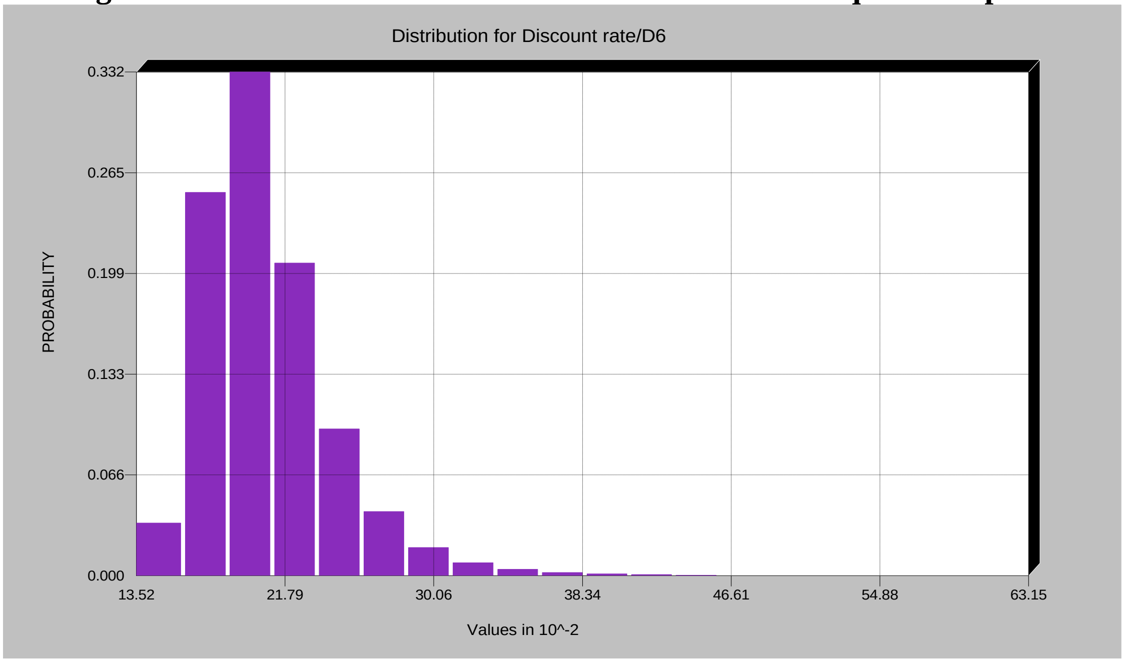
Figure No 1: Critical parameters for the estimation of discount rates with quadratic preferences



Source: Own elaboration

Figure No 2 shows the results of simulating discount rates assuming a constant risk-free rate of 5%, a constant coefficient of variation equal to 0.15, and an alpha that is normally distributed with a mean of 2 and a standard deviation of 20%. The resulting discount rates tend to cluster around 20%. Although this is the lower bound, one may assume a different scenario and by getting different values for the parameters (especially for the coefficient of variation)

Figure No 2: Simulated values for the discount rate with quadratic preferences

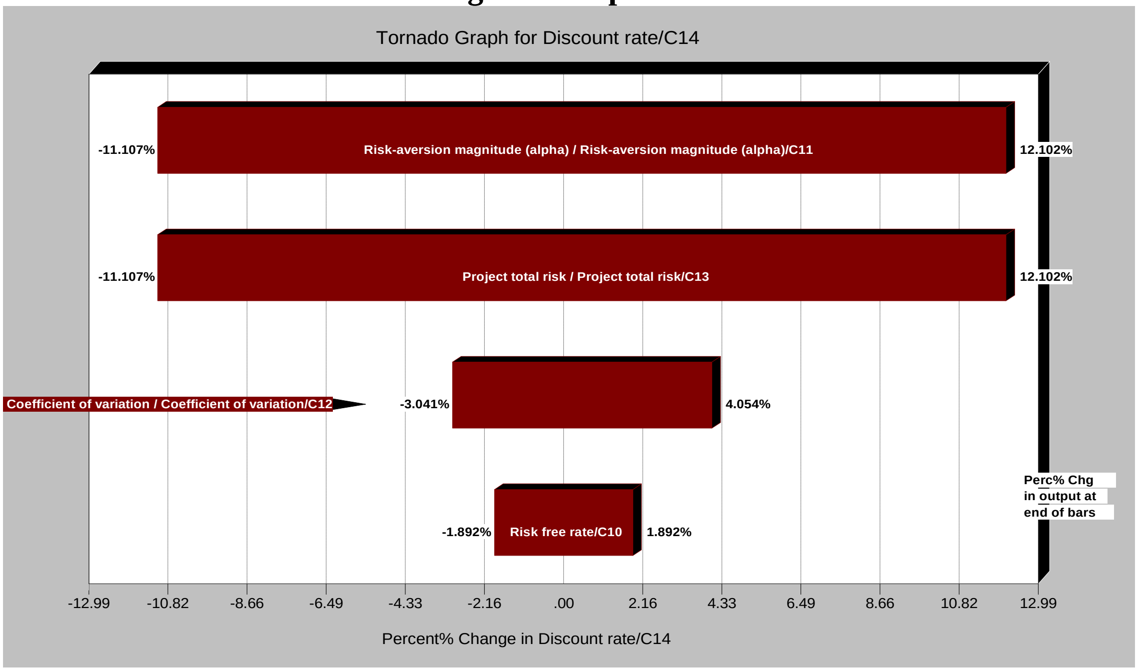


Source: Own elaboration

5.2 Analysis of the discount rate with logarithmic preferences

Figure No 3 shows the critical variables for this type of discount rate. The main conclusion is that the alpha and the project total risk are crucial parameters to estimate the project's discount rates. An increase of 10% in the risk aversion parameter alpha will decrease the project discount rate in 11%. It is interesting to note that although the alpha parameter is still important, its relative impact has been diminished.

Figure No 3: Critical parameters for the estimation of discount rates with logarithmic preferences

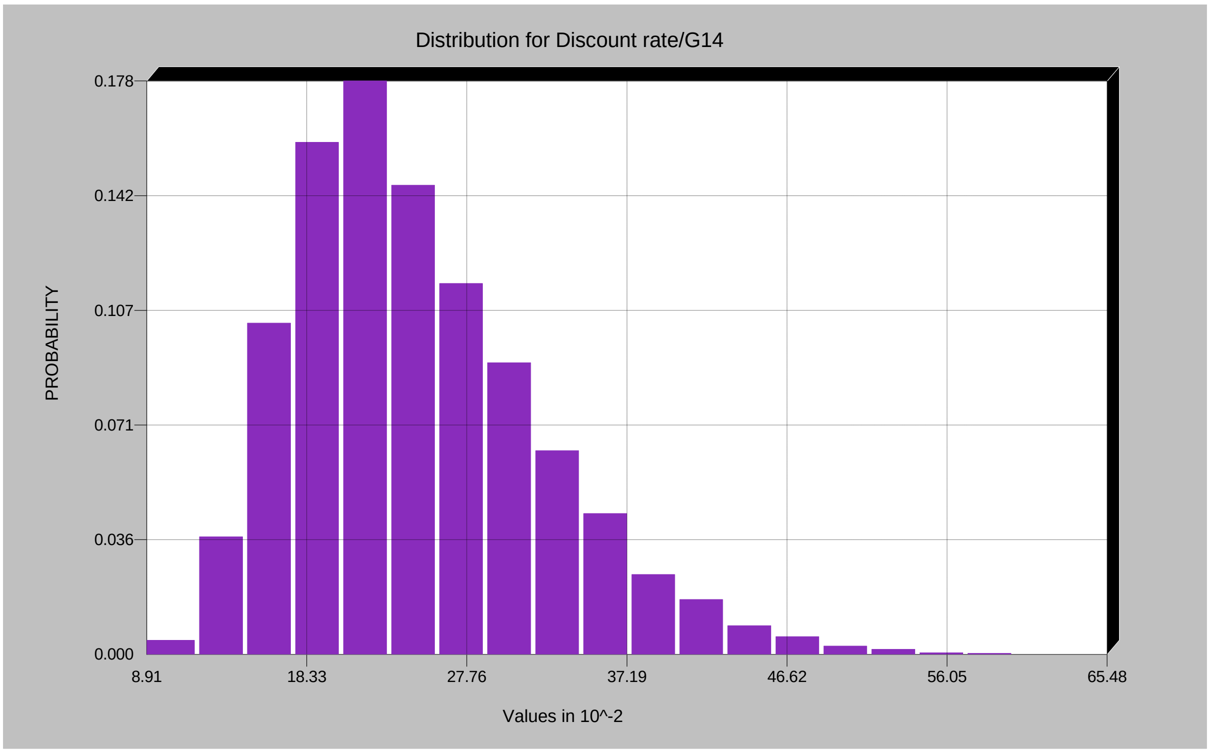


Source: Own elaboration

Figure No 4 shows the simulation results for the discount rates with logarithmic preferences. Here, one uses a risk free rate of 5%, a coefficient of variation of 0.5, an alpha parameter that is normally distributed with a mean of 0.5 and a standard deviation of 10%. Besides the project total risk is distributed according to a uniform distribution with a lower value of 0.2 and an upper value of 0.4.

From this analysis one may conclude that the joint effect of both critical variables tend to increase the simulated values of the discount rates. However, the net effect is still asymmetric and it is skewed to the right as it should be because they represent lower values.

Figure No 4: Simulates values for the discount rate with logarithmic preferences



Source: Own elaboration

6. Conclusion

In this work one has obtained two expressions for estimating subjective discount rates using the HARA utility function. In this way, one has obtained more generalized results than those of Mongrut and Ramirez (2006). Furthermore, these two expressions have been obtained assuming an entrepreneur with the lowest risk-aversion coefficient, who almost lacks initial wealth, and who puts all his money in the investment opportunity (non-diversified entrepreneur). One believes that this entrepreneur's profile is very common in emerging markets.

The utility specification applied in this work only includes pecuniary returns (such as the project return on investment - ROI and total risk). Depending on the specification, total risk is measured using the standard deviation or the coefficient of variation. From the results, it seems that depending on the entrepreneur's risk preferences, the risk-aversion parameter could be more or equally important as the measure of the project's total risk.

However, given the assumptions, it seems more realistic to use the logarithmic specification instead of the quadratic one because the former assumes DARA.

Given the importance of the entrepreneur's risk aversion, it is important that future work is directed towards improve the measurement of it. At the end, what matters in the valuation of seed capital investments is consistency, not accuracy in the values.

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