Chapter

Methodological Procedures for Non-Linear Analyses of Physiological and Behavioural Data in Football

José E. Teixeira, Pedro Forte, Ricardo Ferraz, Luís Branquinho, António J. Silva, Tiago M. Barbosa and António M. Monteiro

Abstract

Complex and dynamic systems are characterised by emergent behaviour, self-similarity, self-organisation and a chaotic component. In team sports as football, complexity and non-linear dynamics includes understanding the mechanisms underlying human movement and collective behaviour. Linear systems approaches in this kind of sports may limit performance understanding due to the fact that small changes in the inputs may not represent proportional and quantifiable changes in the output. Thus, non-linear approaches have been applied to assess training and match outcomes in football. The increasing access to wearable and tracking technology provides large datasets, enabling the analyses of time-series related to different performance indicators such as physiological and positional parameters. However, it is important to frame the theoretical concepts, mathematical models and procedures to determine metrics with physiological and behavioural significance. Additionally, physiological and behavioural data should be considered to determine the complexity and non-linearity of the system in football. Thus, the current chapter summarises the main methodological procedures to extract positional data using non-linear analyses such as entropy scales, relative phase transforms, non-linear indexes, cross correlation, fractals and clustering methods.

Keywords: complex systems, positioning, physiology, performance analysis, soccer

1. Introduction

Applying non-linear theory and approaches has been a growing research interest in sports sciences fields such as performance analysis [1–4]. It is assumed that time-based and team sports display non-linear characteristics [5–8]. Football is deemed as a complex and dynamic system where players perform intermittent movements in time-space coordination [9–11]. The prominence of this research topic is due to several factors, amongst which the shift in the paradigm from linear to non-linear frameworks that has been applied to a wide variety of fields and settings, besided the ready access to technology (e.g., tracking systems) providing large datasets, time series outputs and new time-motion approaches [12–14].

Nonlinear theory and complex sciences are disruptive of linear frameworks [15, 16]. Linear systems assume a linearity on time-varying case, an input-output statistic and a linear state feedback [17]. Considering the linear system theory, an internal and external structure developing feedback control strategies for simultaneous stabilisation of the system [16]. Based on this, theoretical models quantify the relationship between human movement (input) and performance (output), considering the athlete as a linear system [18, 19]. Desynchronization between internal (such as heart rate measure, perceived exertion and biochemical procedures) and external components (i.e., movement speed, body impacts, metabolic power, accelerations and decelerations) may affect the performance [18, 20]. Small changes in the inputs determine proportional and measurable changes in the output, reporting linearity characteristics such as controllability, observability and canonical structure [21]. These assumptions determines approaches focused on the linearity of the system, reporting an fitness-fatigue binomial with a related dependence on dose-response relationships [22, 23]. However, the accuracy of these theoretical models has been challenged for being feeble and for the lack of individualised measurement [23]. Moreover, the ecological dynamisms of informational contexts, social relations and human movement variability are not considered in linear analyses [24–26]. Human movement and collective behaviour are not characterised by the linearity of the systems (as in team sports, like football) and the linear theory could be deemed as a reductionist approach to the problem [5, 6, 26]. Thus, the individual and collective performance has been reported using a complex and dynamic perspective [26–31]. Under these assumptions, biological systems are characterised by non-linearity, interaction-dominant dynamics, emergent behaviour, self-similarity, self-organisation and a chaotic component [32]. Literature reports several nomenclatures for the topic as complex adaptive systems [8, 33, 34], complex and dynamic systems [6, 26] or non-linear and dynamical self-organisation systems [27, 34, 35].

The ready access to cutting-edge technology was another reason for this field of research to increase. Such technology eventually became more affordable and user friendly. The use of tracking data started by assessing the individual players' movement, and later integrated spatial-temporal patterns based on Cartesian and Euclidean references [13, 36, 37]. Over the last two decades, positional data has been verified in football training and match-play to assess the complexity of the systems inherent to the individual movements and collective coordination [31, 38, 39]. Positional dataset can be applied to measure both physical and tactical measures [10, 40–45]. However, analysis do not always integrate different performance indicators [10, 46, 47]. Usually, studies focused only in a single performance dimension, however, football is a multifaceted sport with the physical, tactical, and technical factors amalgamating to influence performance with each factor not mutually exclusive of another [47]. Integrating performance metrics remain rarely described in current literature, concerning football environments [36]. That creates issues in the performance analysis, leading to the fact that the integrative approaches remain understudied. This research gap may be a very important topic to enhance knowledge about the theoretical concepts, mathematical models and methodological procedures of the non-linear approaches to integrate physiological and behavioural data in football.

2. Theoretical concepts of the non-linear approaches

2.1 Football as a complex and dynamic Cartesian coordinates

Football is an invasion game characterised as a complex and dynamical systems with a goal-oriented adaptation amongst teammate and opponents [9, 48].

Previously, to measure and tracking player's movements, mapping tactical actions and modelling collective behaviour were time-consuming processes [2, 4, 49]. Observational and notational analysis had scarce technological and procedural means to support the occurrence of the large number of physical, technical and tactical actions of the football game [49–51]. The wearable technologies as tracking systems allowed real-time access the players' position on the field during training and competition [31, 38, 39]. Positional dataset can be captured at different frequencies by using tracking systems as global positioning systems (GPS) tracking systems [52, 53], micro-electromechanical systems (MEMS) [36, 54], local radio-based local positioning systems (LPM) [55, 56], computerised-video systems [57, 58] and tracking system [59, 60]. This is largely due to the high cost associated with its use, which restricts its use almost exclusively to professional settings in male players [20, 61]. The validity and accuracy of these time-motion methodologies is well documented with an excellent reliability (coefficient of variation, CV: 1.02–1.04%) [52, 53]. However, the integration of the different devices still needs further studies [60, 62, 63]. Using this techniques to collect data, the players' movements are possible to be framed in a Cartesian referential (football field), represented by time series of Cartesian coordinates (xx, yy) [36]. Also, this approach allows spatiotemporal patterns to be assessed with a physical and tactical significance for coaches, performance analysts and researchers [30, 36]. Capturing collective and tactical performance also requires knowledge of the tactical-strategic variables that mediate intra- and inter-team behaviour [40, 64, 65]. According to Duarte et al. [11], inter-player and team coordination reports mutual influence of each player on the behaviour of dyadic systems shaped emergent performance outcomes. Sampaio and Vitor [64] applied positional data to calculate mean position in relation to the geometric centre of the team. Thus, longitudinal and lateral directions are established by the players' dyads and geometrical centre of the team (i.e., team centroid) [40, 45]. Movement patterns and inter-player coordination as a key issue in non-linear signal processing method [6, 64, 66]. Length, width and centroid distance as measures of team's tactical performance were also applied in youth football by Folgado et al. [66]. Synchronisation and synergy are theoretical terms recurrently used to assess intra-team and inter-team coordination and assess spatial-temporal displacement for goal orientation and team success [11, 65]. Typically, this special-temporal displacement has been assessed by the distance between player's dyads [42, 45]:

$$D\left(a_{x_{(t)},y_{(t)}}, b_{x_{(t)},y_{(t)}}\right) = \sqrt{\left(a_{x_{(t)},y_{(t)}}\right)^2 + \left(a_{x_{(t)},y_{(t)}}\right)^2} \qquad (1)$$

where D is the distance, a is the player, x and y are the coordinates, t is the time, and b is the teammate or opponent. Team dispersion can be measured using effective playing position (EPS), surface area or spatial exploratory indexes using Euclidean (planar) coordinates [67–69]. For their measurement dyads players are achieved to measure the magnitude of the variability, predictability, stability and/or regularity in the distance between players [11, 40]. However, the individual preponderance within the synergies and synchronicities of the team remains somehow challenging to measure [70]. Additionally, ball possession is a critical issue in this positional data modelling given the importance of distinguishing phases of play [44, 64]. Due to this fact, different levels of analysis must be considered such as micro, meso and macro [13, 71, 72]. Moreover, positional and behavioural data cannot be interpreted separately, because football performance is a multifactorial phenomenon [10, 46, 47].

2.2 Current performance metrics to measure physiological demands under a linear framework

The training process requires a systematic physiological and biomechanical stimulus to ensure optimal adaptations and an adequate performance [19]. Several theoretical frameworks have been developed to assess the quantity and quality of the training and competition demands [18, 19]. These training load-based consider the linear system theory, likewise dose-response relationship and fitness-fatigue binomial. Fitness-fatigue model approach was originally proposed by Bannister [73]:

$$p(k) = p^* + c_1 \sum_{i=1}^{k-1} u(i) \exp^{\frac{-(k-1)}{\tau_1}} - \sum_{i=1}^{k-1} c_2(i) u(i) \exp^{\frac{-(k-i)}{\tau_2}} \quad (2)$$

were $p(k)$ is the measured fitness (or performance) with gain term (k) and time constant (τ_1) [23]. Cumulative effect of training as a key guidance to the individual athlete's performance [20, 74]. Training load concept has been developed under the assumption that the athlete is a linear system [74, 75]. It is possible to breakdown training load into external and internal load. The external load describes the work rate (i.e., physical or biomechanical output), regardless of the biochemical and psychophysiological response [20, 76]. In training environments, it is mainstream to reports the work rate [49, 77], workload [78, 79] and training load [18, 19]. When load measures are also applied to analyse match performance, quite often is noted the physical performance [47], activity profile [80, 81] or match running performance [82–84]. Wherefore, integrating load measure with other performance factors is a current research-practice gap when determining metrics with physiological and behavioural significance [10].

2.3 Physiological and behaviour dataset in football environments: a integration approach using non-linear procedures

Football performance, a multifactorial phenomenon, dependant on a variety of factors such as environmental, contextual, physical, technical, tactical and psychophysiological [46, 47, 83]. These factor are not mutually exclusive of one another, what makes relevant an integrated approach to provide holistic insights about performance analysis [47, 83]. On regular basis, each of these factors are analysed in isolation without taking the others into account, leading to 1-dimensional insights [83]. Bradley and Ade [47] proposed a theoretical model emphasising on high-intensity running efforts during match-play advocating a contextualization of these running-based actions amongst technical and tactical activities [47]. This becomes of utmost relevance considering the football game [9, 65]. This is what mediates the players' decision making throughout the game according a team strategy previously defined [13]. Several authors have tried to establish ecological approaches to evaluate training and match outcomes, including non-linear approaches [11, 65]. Non-linear analyses were fundamentally performed on competitive game [41, 85] and limited training tasks as small and large-side games [42–45, 67, 69]. It is therefore important to understand mathematical models and methodological procedures underpinning non-linear analyses to assess their significance in applied research and applied settings, identify possible research gaps to be explored and, be aware of potential limitations and criticisms (**Figure 1**).

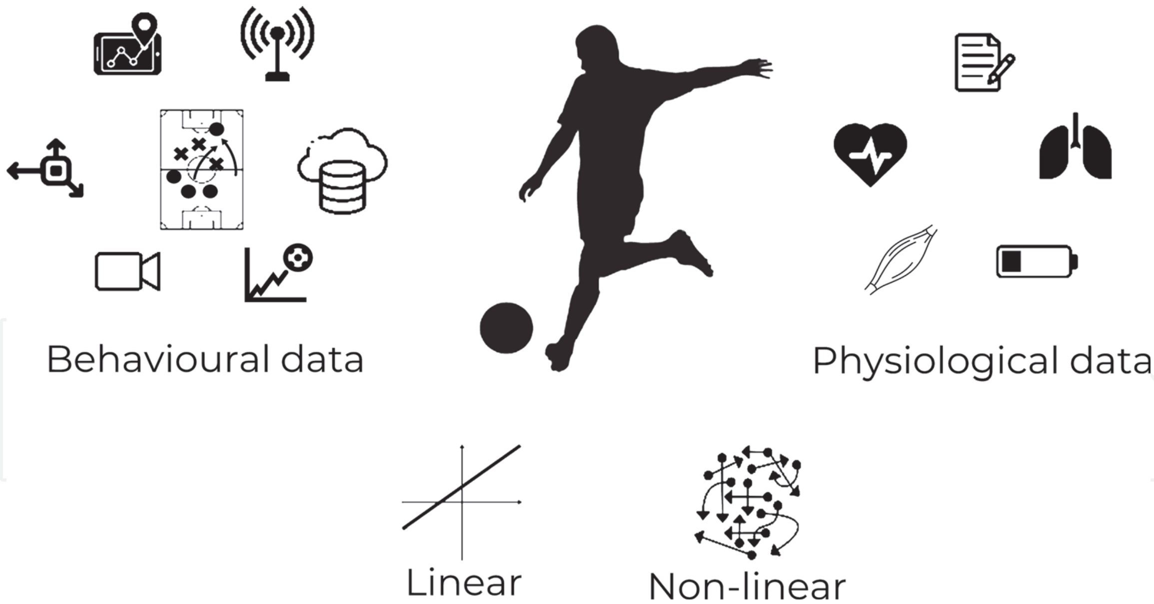

Figure 1.
Physiological and behavioural dataset in football environments.

3. Mathematical models and methodological procedures of the non-linear approaches

Non-linear approaches have been recurrently applied in football using complexity principles [14, 31, 36]. The informational context and spatiotemporal determinants that mediate players' perception and action are analysed by nonlinear and dynamic proprieties of the football game [6, 28]. It is assumed that environmental, task and organismic constraint influence individual and collective behaviour [5, 28]. This behaviour has a physiological cost over time that must be measured [18, 20, 76]. Informative content can be classified as different domains of variability, namely the frequency domain, the entropy domain and the scale-invariant domain [5, 86]. In biological systems, sequential time-series have become outstanding data analysis in multifaceted context [87–90]. According to Bravi et al. [86] the time-series data can be described through five different domains of variability: (1) statistical (i.e., statistical properties of the distribution in a stochastic process); (2) geometric (i.e., properties of the dataset shaped in a certain space); (3) energetic (i.e., energy or power of the time-series); (4) informational (i.e., degree of irregularity/complexity inherent to the order of the elements in a time-series); invariant (i.e., fractality or unchanging attributes over time or space). In football, time-series data application has been widely applied [14, 36]. Low et al. [36] organised the non-linear methods into measures of the regularity (or predictability) and synchronisation. Geometrical centre and periodic phase oscillators has been considered to analyse players' synchronisation and modelling the coordination of a team [14]. However, remains unclear the application of time-series data from an integrated approach perspective [10, 47, 83]. Thus, it is paramount to determine mathematical models and methodological procedures for non-linear time-series data analysis, bearing in mind an integrative approach. Therefore, the following subsections elaborate on the different non-linear mathematical models possible to apply in football.

3.1 Entropy

Entropy is a non-linear and informational parameter applied to describe variability, regularity or predictability of the movement/performance uncover the inter-player's interactions [86, 89, 91]. That, is entropy parameters describes the degree of irregularity/complexity inherent to the order of the elements in a time-series [86]. There are several types of entropy reported in the literature and applied in football research from the integrative perspective, amongst which Approximate Entropy (ApEn) [6, 11, 42], Sample Entropy (SampEn) [44, 92], Cross-sample Entropy (Cross-SampEn) [92] and Boltzmann-Gibbs-Shannon Entropy (ShannonEn) [43]. ApEn expresses the probability that the sequence configuration in a time-series data allows the prediction of the configuration from another sequence from a distance apart [89, 91]. ApEn was derivate from Kolmogorov-Sinai entropy and ranged amongst $0-2$ where lower values correspond to more predictable and higher values stands more unpredicted patterns within time-series ($0 \leq$ ApEn ≤ 2) [89].

$$\mathrm{ApEn}(m, r, N) = \phi^{m}(r) - \phi^{m+1}(r) \tag{3}$$

where m is the window length distance amongst comparting time-series points, r is the similarity radius, N is the time-series length, and ϕ is the probability that points m distance within a tolerance level (r) [36, 90]. $C_i^m(r)$ measures how similar are the regularity of the data points in the window length (m) having regard to the following $\phi^m(r)$ [6, 36]:

$$\phi^{m}(r) = (N - m + 1)^{-1} \sum_{i=1}^{N-m+1} \ln C_i^m(r) \tag{4}$$

From a practical point of view, the imputed ApEn values should be computed with 2 to vector length (m) and 0.2 * standard deviation to the tolerance (r) [41, 45, 93]. ApEn reliability for short time series is low, providing relative consistency during changes in input parameters (m, r and N) [1, 89, 91]. For this reason, Richman and Moorman [89] developed SampEn where their logarithm is simpler with shorter time-series records than ApEn that is heavily dependent on the length record causing lacks of relative consistency. SampleEn logarithm has been was developed on the basis Grassberger et al. [94] reporting a larger window length and a greater relative consistency than ApEn [89–91]. Likewise, SampEn values close to zero indicates a regular or near-periodic time-series sequence, while the higher values reports a more unpredictable pattern ($0 \leq$ SampEn $< \infty$) [88, 91, 92]. SampEn measures the negative logarithmic probability that two similar sequences of m points extracted within the tolerance limits (r) for a window length ($m + 1$) [36, 92] .

$$\mathrm{SampEn}(m, r, N) = -\ln \frac{\phi^{m+1}(r)}{\phi^{m}(r)} \tag{5}$$

Where, m, r, N, and ϕ retain the meaning from ApEn equation, whereby and $m + 1$ windows are compared for eliminating the self-matching bias in the ApEn [90]. $\boldsymbol{\phi^{m+1}(r)}$ reports the total number of time-series sequences in a window length $m + 1$ as far the $\boldsymbol{\phi^{m}(r)}$ expresses the total template in a length m within the aforementioned tolerance level (r) [36].

Multiscale entropy (MSE) as Cross-ApEn and Cross-SampEn was recently introduced from the primary entropy procedures (i.e. ApEn and Cross-ApEn) [90, 95]. Therefore, cross-entropy methods quantify the degree or complexity of coupling between two cross-sequences while the primary entropy techniques evaluated the asynchronism between two time series [87, 90]. Cross-SampEn remain a greater relative consistent than Cross-ApEn, being defined as long as one template finds a time-series sequence [89]. Mostly, Cross-SampEn has been recurrently used in football settings to measure players' synchrony [36, 92]. Cross-SampEn, the templates are chosen from the series u and compared with vectors from v, while the negative logarithm accounts the ratio of two average between outputs [36, 89]:

$$\text{Cross} - \text{SampEn}(m, r, N\ ) = \left\{ -\ln \frac{\phi^{m+1}(r)}{\phi^{m}(r)} \right\} \tag{6}$$

Boltzmann-Gibbs-Shannon entropy was applied by Ric et al. [43] to measure temporal diversity and structural flexibility of the players. This entropy-based technique was originally applied by Balescu [96], reporting the configuration's probabilities as the large N in a relative frequency occurring in a stationary distributions described by Shannon [97]:

$$H(x) = \sum_{i=1}^{n} p_i \log_b p_i \tag{7}$$

where $\boldsymbol{p_i} = \boldsymbol{n_i}/N$, where $\boldsymbol{n_i}$ and N is the frequency and total number of the configuration, respectively [1, 43]. Predictable and unpredictable patterns were also reported as lower and higher entropy values, which is presented by absolute or normalised forma ($0 \leq \boldsymbol{H}(\boldsymbol{x}) \leq 1$) [36].

MSE techniques was applied in football by Canton et al. [44] to identify how positioning the goals in diagonal configurations on the pitch modifies the external training load and the tactical behaviour in youth football environments (i.e., small-sided games). The authors applied a SampleEn algorithm to compute entropy values in different timescales, calculating the area under for complexity index as reported in multiple entropy analysis for time-series [90, 98]. MSE techniques reports the point-to-point fluctuations over a time-series range [44, 90, 98] as:

$$y_j^{\tau} = \frac{1}{\tau} \sum_{i=(\ j-1)\tau+1}^{j\tau} x_i \tag{8}$$

Where, timescales is $\boldsymbol{\tau}$, $\boldsymbol{y_i}$ is the data point in the constructed time-series and $\boldsymbol{x_i}$ is a data point in the original time-series through a window length (N). Complex index (C_1) is calculated using the area under the constructed time-series and the original time scale curve [90, 98]:

$$C_1 = \sum_{i=1}^{N} \text{En}(i), 1 \leq y_j \leq \frac{N}{\tau} \tag{9}$$

Where, En is the reported entropy parameter at the time scale i and N is the total number of time scale used for the C_1 calculation [90].

3.2 Relative phase (Hilbert transform)

Relative phase was extensity reported in football within an integrative framework [40, 41, 67]. Using a Hilbert transform [$\boldsymbol{\phi}(\boldsymbol{t})$], relative phase computes the difference between two signals reporting coupled oscillators and stable patterns of synchronisation, in-phase and anti-phase, when players moved in the same or opposite directions [36, 99]. Hilbert transform was originally introduce by Gabor

[100] with aim to measure the phase and amplitude for a signal. Palut and Zanone [99] configured a phase diagram plotting the imaginary part of the Hilbert transform:

$$\phi(t) = \phi_1(r) - \phi_2(r)$$
$$\phi(t) = \arctan \frac{H_1 s_2(t) - H_2(t) s_1(t)}{s_1(t) s_2(t) + H_1(t) H_2(t)} \tag{10}$$

Where, $H_1 s_2(t)$ and $H_2(t) s_1(t)$ were the Hilbert transform from the two compared signals [36, 99]. In football, longitudinal and lateral directions within the pitch were reported using near-in-phase synchronisation of each players' dyads [40, 41, 67]. The percentage of time spent in each near-in-phase mode of coordination was computed to verify in-phase synchronisation ($-30° \le \phi \le 30°$), anti-phase synchronisation ($-180° \le \phi \le -150°$ or $150° \le \phi \le 180°$) or without pattern synchronisation (all other ϕ degree) [36, 99]. Oscillation in football environment has been a recurrent non-linear approaches to process x- and y-directions and positions covered by football players in centroid and effective playing space zone [38, 101]. Sampaio and Vitor [64] displayed the relative phase post-test value to measure movement patterns and inter-player coordination. This study reported a higher regularity of the patterns with the increasing of the expertise level. There is a gap in understanding how the physiological dataset can influence the intra- and inter-team variability that needs to be further studied [47]. It remains to be understood how this varies across the different levels of the expertise.

3.3 Complex index

Non-linear parameters are often transformed into reliable complexes indices to measure complexity in football settings [43, 69, 93]. Dynamic overlap is a complex index used to compare time-series against the average cosine auto-similarity of the overlap between configurations within time lags [102]. It is an informational non-linear parameter that expresses how timescale statured in a dynamic behaviour using the exploratory breadth at different timescales [43, 69]:

$$< q_{\mathrm{d}}(t) > \; = (1 - q_{\mathrm{stat}}) t^{\alpha} + q_{\mathrm{stat}} \tag{11}$$

Where, $\boldsymbol{< q_{\mathrm{d}}(t) >}$ is the dynamic overload value, t is the time lag, $\boldsymbol{q_{\mathrm{stat}}}$ is the horizontal asymptote and $\boldsymbol{\alpha}$ expresses the gradient. Dynamic overlap tends to infinity wherefore predictable and unpredictable reflects zero an one values, respectively ($0 \le \boldsymbol{q_{\mathrm{d}}(t)} \le 1$) [36, 43, 69]. Additionally, trapping strength is the overall behavioural flexibility performed at lower and highest values of the time scale [43].

Another complex index reported in the literature is the stretch index, which can be defined as distances amongst players and the geometrical centre of the team [45, 85]. That is, this complex index measures the spatial expansion or contraction [103] as:

$$s_{\mathrm{ind}} = \frac{\sum_{i=1}^{N} w_i d_i}{\sum_{i=1}^{N} w_i} \tag{12}$$

Here $\boldsymbol{d_i}$ is the distance between player i and their geometrical team centroid [103]. Stretch index can be expressed in meters, and is also computed by CV and entropy parameters [93, 104]. Otherwise, spatial exploration index (SEI) is obtained with the width and length displacements, computing the distance from each data point in the time-series according to geometrical centre [1, 2].

3.4 Correlation index

Windowed and cross correlation were also applied to assess collective behaviour through positional data in football training and match environments [36, 65, 92]. Cross correlation function is well-supported in the human movement research, wherein the overlapping time windows that enclosed the time-series sequence under analysis [105, 106]. Cross-Correlation function multiplied the point-to-point amongst two time-series data series, reporting the sum of the products and the respective relationship quantification [105]:

$$r_{xy} = \sum_{i=0}^{N-1} x_i y_i \tag{13}$$

Where $\boldsymbol{r_{xy}}$ is the correlation across the window length of each analysed time-series (N); $\boldsymbol{x_i y_i}$ are the data point of the calculated data series. Boker et al. [106] described cross-correlation with the pairwise dataset at two different time-series signals in accordance with:

$$r_{xy\tau} = \frac{1}{N-\tau} \sum_{i=1}^{N-\tau} \frac{(x_i - \overline{x})\left(y_{i+\tau} - \overline{y}\right)}{\sigma(x)\sigma(y)} \tag{14}$$

Where $\boldsymbol{\tau}$ is the observations across cross-correlation (r) amongst time-series data point ($\boldsymbol{x_i y_i}$); $\overline{\boldsymbol{x}}$ and $\overline{\boldsymbol{y}}$ are the means, $\boldsymbol{\sigma(x)\sigma(y)}$ are the standard deviations in the studied window length (N) reporting positives and negatives correlation ($-1 \leq r \leq 1$) [105–107]. Pearson coefficient (r) was expressed as [105]:

$$r_{xy} = \frac{N(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n \sum x^2 - (\sum x^2)][n \sum y^2 - (\sum y^2)]}} \tag{15}$$

where, r_{xy} maintains the meaning to the previous equation. Person-moment correlation was used to estimate in-phase synchronisation (i.e., r values close to -1), anti-phase synchronisation (i.e., r values close to 1) and without pattern (i.e., r = 0) [36, 92]. It remains to be seen whether matrix correlation may be applied to correlate players synchronisation with physiological insight, since it was only applied to assess dynamic collective behaviours in isolation individual physiological demands [65, 92, 107]. Complementarily, a study associated the cross-correlation with the an vector coding technique to analyse coordination patterns between teams during offensive sequences that ended in shots on goal or defensive tackles [108]. The authors based their non-linear analysis the relative motion plot proposed by Sparrow et al. [109]:

$$\theta(i) = \arctan\left|\frac{\theta_2(i+1) - \theta_2(i)}{\theta_1(i+1) - \theta_1(i)}\right|, i = 1, 2, \ldots, n-1 \tag{16}$$

where i is the time-series data point in a right horizontal and n is the total frame for each timescale [108]. As well, the $\theta(i)$ is the coupling angles between in the second, third and fourth adjusted quadrant (i.e., $\pi - \theta_i$, $\pi + \theta_i$, $2\pi - \theta_i$) [36, 109]. Near-in-phase synchronisation were computed using this non-linear technique, expressing the in-phase (22.5° $\leq \theta_i <$ 67.5° or 202.5° $\leq \theta_i <$ 247.5°) and anti-phase (22.5° $\leq \theta_i <$ 67.5° or 202.5° $\leq \theta_i <$ 247.5°). Even more, attacking and defensive team phases are also reported by Moura et al. [108]. By distinguishing the phases of the play, this non-linear method could allow to understand how physiological demands can affect the intra- and inter-synchronisation in offensive and defensive phases.

3.5 Fractality

Fractal dimension is an invariant non-linear parameter characterised by the unchanged proprieties of the system over time and/or space [86]. Multifractal time series expresses different local scaling exponents for a time-series dataset scaling different exponents at different times [86, 110]. Few studies applied fractal dimension to predict stability and predictability of the football players along specific training tasks [110, 111] and competitive matches [110–112]. Fractal calculus (FC) was reported using Shanon and Grünwald-Letnikov approaches [111, 112]. Grünwald–Letnikov fractional differential consideres the matrix containing the multi-player positions [111, 113]:

$$X_\delta[t] = \begin{bmatrix} x_1[t] \\ \vdots \\ x_{N_\delta[t]} \end{bmatrix}, \left(x_i[t] \in R^2\right) \tag{17}$$

where N_δ expresses the number of players in the team N_δ across a time-series analysis. The matrix $X_\delta[t]$ is the planar positioning matric of player (i) in a target team (δ) at the a concrete time (t) [111, 113]. Shannon information was expressed by [112]:

$$I[P(x)] = -\log P(x) \tag{18}$$

whereby$I$ $[P(x)]$ is the function between the cases $D^{-1}I[P(x)] = P(x)[1 - \log P(x)]$ and $D^1I[P(x)] = P(x)\left[1 - \frac{1}{P(x)}\right]$. I and D are the integral and descriptive operations [112].

A study applied multifractal dimension in football movement behaviour using Hausdorff dimension (D) [110, 112, 114]:

$$D(E) = \lim_{\varepsilon \to 0} \sup \frac{\log N_E(\varepsilon)}{-\log \varepsilon} \tag{19}$$

where N_E is the minimal diameter at the most ε needed to cover [114]. Fractional dynamics may tracking football players trajectory, which can be useful to increase the autonomy of tracking systems [113]. Additionally, fractal and multifractal analysis can be used to analyse the regularity and synchronisation of the team, as well the players' movement dynamics [110, 111, 113]. However, the use of fractal variables within an integrative approach remains little explored whereas it is important understanding the links between collective behaviour with the fractional properties of movement and its physical and physiological repercussions [47, 110]. Fractal proprieties may also increase the autonomy of the tracking systems to collect information in tracking systems such as making decisions based on it [111].

3.6 Clustering methods

Clustering methods have become popular in data mining in several research areas, including sports sciences [70, 92, 115]. Rokach and Maimon [115] was described the clustering methods in different typologies as hierarchical, partitioning, density-based, model-based, grid-based, and soft-computing methods.

Duarte et al. [92] pioneered applied a clustering method to measure overall and player team collective synchronisation in football. This method is derived from Hibert transform to calculate individual phase time-series and subsequently the cluster phase of these time-series by the natural exponential function [36, 92]. Originally, cluster phase analysis was proposed by Frank and Richardson [116] using Kuramoto's parameters for group synchronisation [117]. This clustering method calculates the mean and continuous group synchronisation $\left(\boldsymbol{\rho}_{\mathbf{group}}\ \mathbf{or}\ \boldsymbol{\rho}_{\mathbf{group},\boldsymbol{i}}\right)$ whence individual's relative phase ($\boldsymbol{\phi}_{\boldsymbol{k}}$) was measured in relation to the group measure [116]. After the Hilbert transform calculation, the continuous degree of overall team synchronisation was clustered [92]:

$$\rho_{\text{group}}(t_i) = \left| \frac{1}{n} \sum_{k=1}^{n} \exp\left\{ i\left(\phi_k(t_i) - \overline{\phi_k}\right) \right\} \right| \tag{20}$$

where overall team synchronisation $\rho_{\text{group}}(t_i) \in [0, 1]$ and mean degree to group synchronisation at every point-to-point of the time-series data (t_i) [36, 92]. $\rho_{\text{group},i}$ was computed by the inverse of the circular variance of relative phase of the cluster amplitude, $\phi_k(t_i)$, while $i = \sqrt{-1}$ as [92]:

$$\rho_{\text{group}}(t_i) = \frac{1}{N} \sum_{i=1}^{n} \rho_{\text{group},i} \tag{21}$$

Synchronisation cut-off values is zero to one representing synchronisation and unsynchronisation ($\mathbf{0} \le \boldsymbol{\rho}_{\mathbf{group}}\ \mathbf{or}\ \boldsymbol{\rho}_{\mathbf{group},\boldsymbol{i}} \le \mathbf{1}$), respectively [36, 92]. Cluster phase analysis has not yet been applied to integrate physiological and behavioural data in football [38, 101].

Furthermore, average mutual information (AMI) was also applied to measure complexity of the football patterns and expresses the amount of information one random variable contains another [118]. AMI is calculated by relative entropy between probabilities distribution and the product midst two selected variables [36]. The mathematical equation described by Cover and Thomas [118] for the calculation of mutual information is:

$$I(x, y) = \sum_{X,\, Y \in A} P(x, y) \log\left(\frac{P(x, y)}{P(x)P(y)}\right) \tag{22}$$

Where$(\boldsymbol{x})$ and $(\boldsymbol{y})$ is the team's centroid movement coordinates and A is the space discretisation [119]. $\boldsymbol{P}(\boldsymbol{x})$ and $\boldsymbol{P}(\boldsymbol{y})$ are the marginality of the probabilities distributions [36]. The AMI identify the relationships between time-series points that are not detected by linear correlation [118, 119]. Similar correlation cut-offs values were applied to predict uncertainty in less values and independence in higher values ($0 \le$ AMI <1) [119].

3.7 Frequency domain

Non-linear techniques as entropy measures can also be expressed in the frequency domain [86]. Several studies have evaluated the variability of movement comparing informational and frequency domains [10, 41, 42]. CV expresses the magnitude of the variability in the distance amongst players', expressed as percentage (%) [10, 93]:

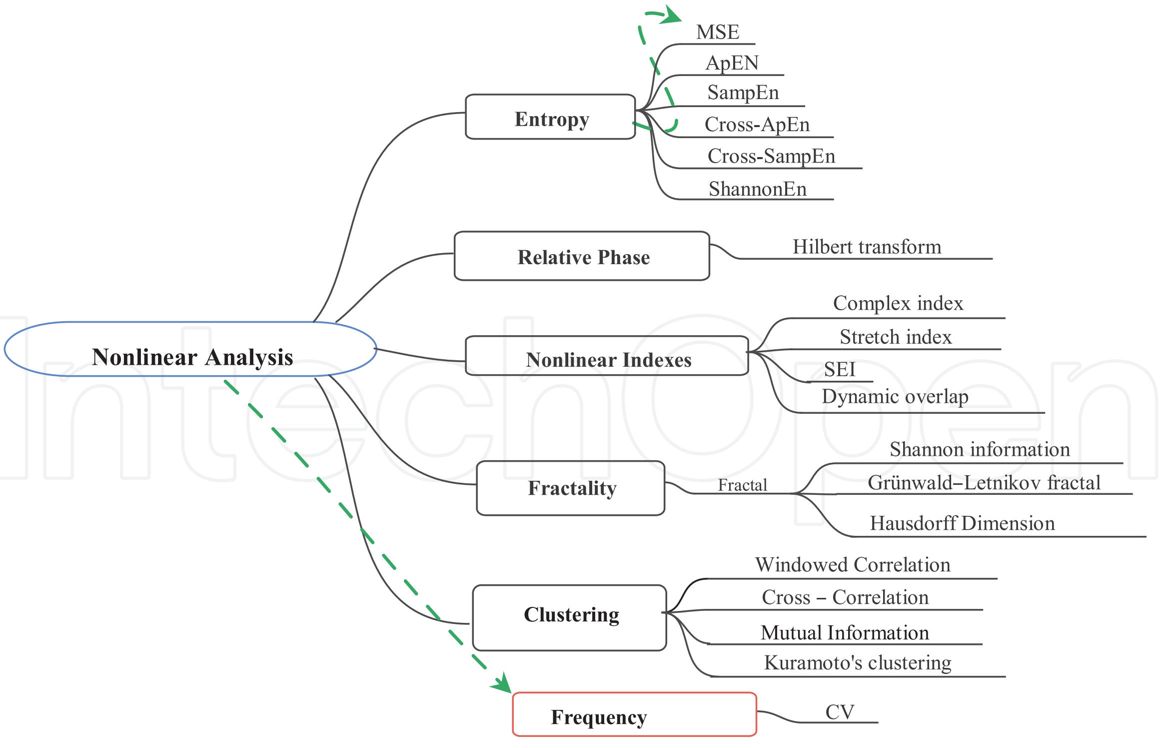

Figure 2.
Summary diagram for mathematical models and methodological procedures of the non-linear approaches.

$$CV(\%) = \frac{\sigma}{\bar{x}} 100 \tag{23}$$

Speed synchronisation has also applied into a integrative approaches in some studies [10, 41]. The near-in-phase synchronisation to players' displacements is expressed in time spent (%) of time according to speed intensity zones: 0.0–3.5 km h^{-1} (low intensity); 3.6–14.3 km h^{-1} (moderate intensity); 14.4–19.7 km h^{-1} (high intensity); and >19.8 km h^{-1} (very high intensity) [41]. Summary diagram for mathematical models and methodological procedures of the non-linear approaches are presented in **Figure 2**.

Table 1 displayed the corresponding equation, thresholds, advantages, disadvantages and practical application for each nonlinear variable.

4. Practical considerations, criticisms and future perspectives

Matlab® routines (Math-Works, Inc., Massachusetts, USA) were the most selected procedure to analyse positional dataset in football. Universal Transverse Mercator (UTM) coordinate system were used to transform latitude and longitude data points [40, 66]. Methodological procedures differ on the correction guidelines to be used and reduce tracking signal noise, advising the use of 3 Hz Butterworth low pass filter [64, 92]. Several authors ran non-linear logarithms using 20 windows of 3000 points per data collect (i.e. ranged 5–25 Hz) [40, 67]. Integrating notational analysis and video-based tracking systems has been a worthwhile strategy to provide contextual significance to positional data [67, 68, 93]. Applying new analysis techniques based on big data still lack an integrative approach, and it will be interesting to understand how future studies can do so with techniques such as machine learning, deep learning or network analysis [13, 14]. These techniques have been extensively used to analyse positional and physiological variables, but there are still few studies under an integrative perspective [13, 36, 46]. There is a

DOI: http://dx.doi.org/10.5772/intechopen.102577

Variable	Equation	Thresholds	Advantages	Disadvantages	Practical application
ApEn	$\mathbf{ApEn}(m,r,N) = \phi^{m}(r) - \phi^{m+1}(r)$	0 ≤ ApEn ≤2; close 0—predictable; close 2—unpredictable	Similar patterns will not be followed by subsequent similar observations.	Dependent on the length record causing lacks of relative consistency.	Interpersonal coordination (1-vs-1 sub-phase); Opposition and cooperation relationships on collective movement behaviour.
SampEn	$\mathbf{SampEn}(m,r,N) = -\mathbf{ln}\,\frac{\phi^{m+1}(r)}{\phi^{m}(r)}$	0 ≤ SampEn < ∞; close 0—predictable < ∞unpredictable	Shorter time-series records with a greater relative consistency	Lower complexity for a signal than white noise signal.	Diagonal positioning of the goals on SSG
ShannonEn	$H(x) = \sum_{i=1}^{n} p_i \log_b p_i$ (Eq. 7)	0 ≤$H(x)$ ≤ 1; close 0—predictable; close 1—unpredictable	Multiple optimal weights on the evaluation and self-information.	Only consider a particular event, not the meaning of the events (criteria) themselves	Dynamics of tactical behaviour emerging on different time-scale using SSG
MSE	$\mathbf{Cross} - \mathbf{SampEn}(m,r,N) = \left\{-\boldsymbol{ln}\,\frac{\phi^{m+1}(r)}{\phi^{m}(r)}\right\}$	0 ≤ Cross-SampEn < ∞; close 0—predictable; < ∞—unpredictable	Faster and allows to evaluate two time-series crossed	Loss of pattern information hidden in the time series	Assessing the dynamics of team–team and player–team synchronisation
Hilbert transform	$\phi(t) = \phi_1(r) - \phi_2(r) = \mathbf{arctan}\,\frac{H_1 s_2(t) - H_2(t) s_1(t)}{s_1(t) s_2(t) + H_1(t) H_2(t)}$	In-phase (−30° ≤ ϕ ≤ 30°) or anti-phase synchronisation (−180° ≤ ϕ ≤ −150° or 150° ≤ ϕ ≤ 180°); without synchronisation (other ϕ degree)	Require short signals than classical non-parametric methods	One dimensional processing causing phase ambiguities	Movement behaviour, speed synchronisation, inter-team distances, spatial interaction, oscillations of centroid position and surface area.
Dynamic overload	$< q_{\mathrm{d}}(t) > = (1 - q_{\mathrm{stat}}) t^{\alpha} + q_{\mathrm{stat}}$	0 ≤$q_{\mathrm{d}}(t)$ ≤ 1; close 0—predictable; close 1— unpredictable	Compare dataset using a cosine auto-similarity that increase in each time lag	Analysis allowed the slow dynamics on a long timescale	Dynamical of tactical behaviour and constrained the perceptual-motor workspace
Stretch index	$s_{\mathrm{ind}} = \frac{\sum_{i=1}^{N} w_i d_i}{\sum_{i=1}^{N} w_i}$	Near-in-phase synchronisation to players' displacements is expressed in time spent (%)	Provide the centroid position of the team and the sum of each player's dispersion on both axes	Relative stretch indexes has needed to measure two teams	Coordination and spatial interactions for opposite and team behaviours

Variable	Equation	Thresholds	Advantages	Disadvantages	Practical application
Windowed and cross correlation	$r_{xy} = \sum_{i=0}^{N-1} x_i y_i$	$-1 \le r \le 1$—phase (r close to -1), anti-phase synchronisation (r close to 1); without pattern (i.e., r = 0)	Measuring similarity, can determine time delay and the identity lagging signal	Bivariate linear association between group synchrony time-series data	Cross-correlation and peak picking for team synergies variability in tactical behaviour
Fractal Calcus	$I[P(x)] = -\log P(x)$ (Eq. (17))	0-Dimensional sets to 3-dimensional sets (D = 0–3)	Assessing fractal properties of human movement associated to sport skills and motor variability	Non-cyclicality of football movement using fractal analysis	Multifractal properties, dynamical stability and predictability of the movement.
Hausdorff dimension	$D(E) = \lim_{\varepsilon \to 0} \sup \frac{\log N_E(\varepsilon)}{-\log \varepsilon}$				
Kuramoto's Clustering	$\rho_{group}(t_i) = \frac{1}{N} \sum_{i=1}^{n} \rho_{group,i}$	$0 \le \rho_{group}$ or $\rho_{group,i} \le 1$; close 0—predictable; close 2—unpredictable	Unbiased measure of group coordination and measure to assess player–team synchrony	Achieve synchronisation modes in networks with different structures	Order, disorder and variability in spatio-temporal interactions amongst two teams
AMI (clustering)	$I(x,y) = \sum_{X,Y\ A} P(x,y) \log\left(\frac{P(x,y)}{P(x)P(y)}\right)$	$0 \le$ AMI < 1; close 0—predictable; close 1—unpredictable	Measuring the nonlinear correlation of the two centroids' movements	Disadvantages of redundancy in each class	Positional synchronisation an geometrical center modifications in team behaviour
CV%	$CV(\%) = \frac{\sigma}{x} 100$	NR	Statistical measure that is normalised and non-dimensional	Dependent on the mean values of the time-series	Speed synchronisation match-to-match variation

Abbreviations: *AMI—average mutual information; ApEn—approximate entropy; CV—coefficient of variation;* D—*dimension;* d_i—*distance between player* i; H—*Boltzmann-Gibbs-Shannon entropy;* $H_1 s_2(t)$—*Hilbert transform;* I—*Shannon information;* m—*window length; MSE—multiscale entropy;* n—*frequency;* N—*time-series length; NR—not reported;* q_d—*dynamic overload;* q_{stat}– *horizontal asymptote;* r—*correlation;* r—*similarity radius or tolerance level; SampEn—sample entropy; ShannonEn—Shannon entropy;* s_{ind}—*stretch index; SSG—small-sided games;* t—*time lag;* y_i—*data point;* τ—*timescales;* ϕ—*probability;* ρ_{group}—*group synchronisation.*

Table 1.
Summary of the non-linear variables and respective equation, thresholds, advantages, disadvantages and practical application.

lack of standardisation on non-linear measures, measurement and thresholds [20, 76]. It is even more evident in the physiological measures, therefore, the results obtained in studies that integrate positional and physiological datasets should be interpreted with caution [120]. The application of integrative approaches should also consider the boundaries between different key performance indicators such as the psychophysiological [45, 121–123], technical [44, 67, 93] and contextual factors [83, 84]. Also, acceleration outputs, metabolic power and body impacts have been poorly integrated with positional data. Behavioural data should still be better contextualised and the related-bias for physiological thresholds must be considered upon the time-dependent and transient reduction [84]. An integration approach to physiology and behavioural data must overcome some challenges on data visualisation, data processing (inherent to big data) and real-time tracking [13]. Moreover, futures researches should focus their analysis on women and sub-elite performers [20, 61].

5. Conclusion

Physiological assessment to monitoring training and match load has been carried out mainly under a linear perspective. Positional data to assess tactical behaviour considers fundamentally the theory of the complex systems and non-linear dynamics. Thus, an integrative approach allows a more holistic and extensive evaluation of the performance as a multifactorial phenomenon. This chapter summarises the theoretical concepts, mathematical models and methodological procedures to be applied by researchers and practitioners in training and match settings in football. The non-linear techniques reported more often in the literature were entropy, relative phase, complex indexes, correlation matrixes, clustering methods, frequency-based measures, fractals and multifractals. Correlation matrixes, clustering methods and fractality have not yet been applied in an integrative perspective in football. Finally, using non-linear approaches to integrate physiological and behavioural data remains a research-practice gap to be explored in the next years.

Acknowledgements

This research was supported by Portuguese Foundation for Science and Technology, I.P. (project UIDB04045/2021).

Conflict of interest

The authors declare no conflict of interest.

Author details

José E. Teixeira[1,2,3*], Pedro Forte[1,3,4], Ricardo Ferraz[1,5], Luís Branquinho[1,4,5], António J. Silva[1,2], Tiago M. Barbosa[1,3] and António M. Monteiro[1,3]

1 Research Centre in Sports, Health and Human Development, Covilhã, Portugal

2 Department of Sports, Exercise and Health Sciences, University of Trás-os-Montes e Alto Douro, Vila Real, Portugal

3 Department of Sport Science, Instituto Politécnico de Bragança, Bragança, Portugal

4 Department of Sports, Higher Institute of Educational Sciences of the Douro, Penafiel, Portugal

5 Department of Sports Sciences, University of Beira Interior, Covilhã, Portugal

*Address all correspondence to: jose.eduardo@ipb.pt

References

[1] Silva P, Duarte R, Esteves P, Travassos B, Vilar L. Application of entropy measures to analysis of performance in team sports. International Journal of Performance Analysis in Sport. 2016;**16**(2):753-768

[2] O'Donoghue P. Research Methods for Sports Performance Analysis. London: Routledge; 2009

[3] Passos P, Araújo D, Volossovitch A. Performance Analysis in Team Sports. London: Routledge, Taylor & Francis; 2016

[4] Sarmento H, Clemente FM, Araújo D, Davids K, McRobert A, Figueiredo A. What performance analysts need to know about research trends in association football (2012–2016): A systematic review. Sports Medicine. 2018;**48**(4):799-836

[5] Barbosa TM, Goh WX, Morais JE, Costa MJ, Pendergast D. Comparison of classical kinematics, entropy, and fractal properties as measures of complexity of the motor system in swimming. Frontiers in Psychology. 2016;**7**:1566

[6] Duarte R, Araújo D, Folgado H, Esteves P, Marques P, Davids K. Capturing complex, non-linear team behaviours during competitive football performance. Journal of Systems Science and Complexity. 2013;**26**(1):62-72

[7] McGarry T, Anderson DI, Wallace SA, Hughes MD, Franks IM. Sport competition as a dynamical self-organizing system. Journal of Sports Sciences. 2002;**20**(10):771-781

[8] Kelso JAS, Schöner G. Self-organization of coordinative movement patterns. Human Movement Science. 1988;**7**(1):27-46

[9] Gréhaigne JF, Bouthier D, David B. Dynamic-system analysis of opponent relationships in collective actions in soccer. Journal of Sports Sciences. 1997;**15**(2):137-149

[10] Gonçalves B, Coutinho D, Travassos B, Folgado H, Caixinha P, Sampaio J. Speed synchronization, physical workload and match-to-match performance variation of elite football players. PLoS One. 2018;**13**(7):e0200019

[11] Duarte R, Araújo D, Davids K, Travassos B, Gazimba V, Sampaio J. Interpersonal coordination tendencies shape 1-vs-1 sub-phase performance outcomes in youth soccer. Journal of Sports Sciences. 2012;**30**(9):871-877

[12] Goes FR, Meerhoff LA, Bueno MJO, Rodrigues DM, Moura FA, Brink MS, et al. Unlocking the potential of big data to support tactical performance analysis in professional soccer: A systematic review. European Journal of Sport Science. 2021;**21**(4):481-496

[13] Rein R, Memmert D. Big data and tactical analysis in elite soccer: Future challenges and opportunities for sports science. Springerplus. 2016;**5**(1):1410

[14] Memmert D, Lemmink KAPM, Sampaio J. Current approaches to tactical performance analyses in soccer using position data. Sports Medicine. 2017;**47**(1):1-10

[15] Anderson BDO. Internal and external stability of linear time-varying systems. SIAM Journal on Control and Optimization. 1982;**20**(3):408-413

[16] Saberi A, Stoorvogel A, Sannuti P. Internal and External Stabilization of Linear Systems with Constraints. In Systems & Control: Foundations & Applications. Birkhäuser, Boston: Springer Science & Business Media; 2012

[17] Callier FM, Desoer CA. Linear System Theory. Springer Science & Business Media; 2012

[18] Vanrenterghem J, Nedergaard NJ, Robinson MA, Drust B. Training load monitoring in team sports: A novel framework separating physiological and biomechanical load-adaptation pathways. Sports Medicine. 2017;**47**(11): 2135-2142

[19] Impellizzeri FM, Rampinini E, Marcora SM. Physiological assessment of aerobic training in soccer. Journal of Sports Sciences. 2005;**23**(6):583-592

[20] Teixeira JE, Forte P, Ferraz R, Leal M, Ribeiro J, Silva AJ, et al. Monitoring accumulated training and match load in football: A systematic review. International Journal of Environmental Research and Public Health. 2021;**18**(8):3906

[21] Weiss L, Kalman RE. Contributions to linear system theory. International Journal of Engineering Science. 1965; **3**(2):141-171

[22] Peterson MD, Rhea MR, Alvar BA. Maximizing strength development in athletes: A meta-analysis to determine the dose-response relationship. Journal of Strength and Conditioning Research. 2004;**18**(2):377

[23] Busso T. Variable dose-response relationship between exercise training and performance. Medicine & Science in Sports & Exercise. 2003;**35**(7):1188-1195

[24] Goldstein EB. The ecology of J. J. Gibson's perception. Leonardo. 1981; **14**(3):191

[25] Davids K, Araujo D, Shuttleworth R, Button C. Acquiring skill in sport: A constraints-led perspective. International Journal of Computer Science in Sport. 2003;**2**:31-39

[26] Seifert L, Button C, Davids K. Key properties of expert movement systems in sport: An ecological dynamics perspective. Sports Medicine. 2013; **43**(3):167-178

[27] Stergiou N, Buzzi U, Kurz M, Heidel J. Nonlinear tools in human movement. In: Stergiou N, editor. Innovative Analyses of Human Movement. Champaign, IL: Human Kinetics; 2004

[28] Davids K, Button C, Araújo D, Renshaw I, Hristovski R. Movement models from sports provide representative task constraints for studying adaptive behavior in human movement systems. Adaptive Behavior. 2006;**14**(1):73-95

[29] Araújo D, Silva P, Davids K. Capturing group tactical behaviors in expert team players. In: Baker J, Farrow D, editors. Routledge Handbook of Sport Expertise. London: Routledge; 2015

[30] Travassos B, Davids K, Araujo D, Esteves P. Performance analysis in team sports: Advances from an ecological dynamics approach. International Journal of Performance Analysis in Sport. 2013;**13**:89-95

[31] Balague N, Torrents C, Hristovski R, Davids K, Araújo D. Overview of complex systems in sport. Journal of Systems Science and Complexity. 2013; **26**(1):4-13

[32] Kellert SH. Borrowed Knowledge: Chaos Theory and the Challenge of Learning across Disciplines. Chicago: University of Chicago Press; 2009. p. 303

[33] Ramos-Villagrasa PJ, Marques-Quinteiro P, Navarro J, Rico R. Teams as complex adaptive systems: Reviewing 17 years of research. Small Group Research. 2018;**49**(2):135-176

[34] Banerjee S. Chaos and complexity theory for management: Nonlinear dynamics. Hershey, PA: IGI Global; 2012

[35] Pikovsky A, Rosenblum M, Kurths J. Synchronization: A universal concept in nonlinear sciences. 2002;**70**:655-655

[36] Low B, Coutinho D, Gonçalves B, Rein R, Memmert D, Sampaio J. A systematic review of collective tactical behaviours in football using positional data. Sports Medicine. 2020;**50**(2):343-385

[37] Clemente FM, Martins FML, Couceiro MS, Mendes RS, Figueiredo AJ. Developing a football tactical metric to estimate the sectorial lines: A case study. In: Murgante B, Misra S, Rocha AMAC, Torre C, Rocha JG, Falcão MI, et al, editors. Computational Science and its Applications – ICCSA 2014. Cham: Springer International Publishing; 2014

[38] Lames M, Erdmann J, Walter F. Oscillations in football - order and disorder in spatial interactions between the two teams. International Journal of Sport Psychology. 2010;**41**:85-86

[39] Pol R, Balagué N, Ric A, Torrents C, Kiely J, Hristovski R. Training or synergizing? Complex systems principles change the understanding of sport processes. Sports Medicine - Open. 2020;**6**(1):28

[40] Folgado H, Duarte R, Fernandes O, Sampaio J. Competing with lower level opponents decreases intra-team movement synchronization and time-motion demands during pre-season soccer matches. PLoS One. 2014;**9**(5):e97145

[41] Folgado H, Gonçalves B, Sampaio J. Positional synchronization affects physical and physiological responses to preseason in professional football (soccer). Research in Sports Medicine. 2018;**26**(1):51-63

[42] Sampaio JE, Lago C, Gonçalves B, Maçãs VM, Leite N. Effects of pacing, status and unbalance in time motion variables, heart rate and tactical behaviour when playing 5-a-side football small-sided games. Journal of Science and Medicine in Sport. 2014;**17**(2):229-233

[43] Ric A, Hristovski R, Gonçalves B, Torres L, Sampaio J, Torrents C. Timescales for exploratory tactical behaviour in football small-sided games. Journal of Sports Sciences. 2016;**34**(18):1723-1730

[44] Canton A, Torrents C, Gonçalves B, Ric A, Salvioni F, Exel J, et al. The diagonal positioning of the goals modifies the external training load and the tactical behaviour of young football players. Biology of Sport. 2021;**39**(1):135-144

[45] Ferraz R, Gonçalves B, Coutinho D, Oliveira R, Travassos B, Sampaio J, et al. Effects of knowing the task's duration on soccer players' positioning and pacing behaviour during small-sided games. International Journal of Environmental Research and Public Health. 2020;**17**(11):3843

[46] Carling C. Interpreting physical performance in professional soccer match-play: Should we be more pragmatic in our approach? Sports Medicine. 2013;**43**(8):655-663

[47] Bradley P, Ade J. Are current physical match performance metrics in elite soccer fit for purpose or is the adoption of an integrated approach needed? International Journal of Sports Physiology and Performance. 2018;**13**(5):656-664

[48] Baca A, Perl J. Modelling and Simulation in Sport and Exercise. London: Routledge; 2018

[49] O'Donoghue P. Sources of variability in time-motion data; measurement error and within player variability in work-rate. International Journal of Performance Analysis in Sport. 2004;**4**(2):42-49

[50] Anguera MT, Camerino O, Castañer M, Sánchez-Algarra P, Onwuegbuzie AJ. The specificity of observational studies in physical activity

and sports sciences: Moving forward in mixed methods research and proposals for achieving quantitative and qualitative symmetry. Frontiers in Psychology. 2017;**8**:2196

[51] Sánchez-Algarra P, Anguera MT. Qualitative/quantitative integration in the inductive observational study of interactive behaviour: Impact of recording and coding among predominating perspectives. Quality and Quantity. 2013;**47**(2):1237-1257

[52] Beato M, Coratella G, Stiff A, Iacono AD. The validity and between-unit variability of GNSS units (STATSports apex 10 and 18 Hz) for measuring distance and peak speed in team sports. Frontiers in Physiology. 2018;**9**:1288

[53] Rago V, Brito J, Figueiredo P, Costa J, Barreira D, Krustrup P, et al. Methods to collect and interpret external training load using microtechnology incorporating GPS in professional football: A systematic review. Res Sports Medicine. 2019;**28**(3):437-458

[54] Gómez-Carmona CD, Bastida-Castillo A, Ibáñez SJ, Pino-Ortega J. Accelerometry as a method for external workload monitoring in invasion team sports. A systematic review. PLoS One. 2020;**15**(8):e0236643

[55] Leser R, Baca A, Ogris G. Local positioning systems in (game) sports. Sensors. 2011;**11**(10):9778-9797

[56] Ogris G, Leser R, Horsak B, Kornfeind P, Heller M, Baca A. Accuracy of the LPM tracking system considering dynamic position changes. Journal of Sports Sciences. 2012;**30**(14):1503-1511

[57] Di Salvo C, Marco C. Validation of Prozone®: A new video-based performance analysis system. International Journal of Performance Analysis in Sport. 2006;**6**(1):108-119

[58] Castellano J, Alvarez-Pastor D, Bradley PS. Evaluation of research using computerised tracking systems (Amisco and Prozone) to analyse physical performance in elite soccer: A systematic review. Sports Medicine. 2014;**44**(5):701-712

[59] Rico-González M, Los Arcos A, Nakamura FY, Gantois P, Pino-Ortega J. A comparison between UWB and GPS devices in the measurement of external load and collective tactical behaviour variables during a professional official match. International Journal of Performance Analysis in Sport. 2020;**20**(6):994-1002

[60] Bastida-Castillo A, Gómez-Carmona CD, Sánchez EDLC, Pino-Ortega J. Comparing accuracy between global positioning systems and ultra-wideband-based position tracking systems used for tactical analyses in soccer. European Journal of Sport Science. 2019;**19**(9):1157-1165

[61] Teixeira JE, Forte P, Ferraz R, Leal M, Ribeiro J, Silva AJ, et al. Quantifying sub-elite youth football weekly training load and recovery variation. Applied Sciences. 2021;**11**(11):4871

[62] Buchheit M, Allen A, Poon TK, Modonutti M, Gregson W, Salvo VD. Integrating different tracking systems in football: Multiple camera semi-automatic system, local position measurement and GPS technologies. Journal of Sports Sciences. 2014;**32**(20):1844-1857

[63] Jackson BM, Polglaze T, Dawson B, King T, Peeling P. Comparing global positioning system and global navigation satellite system measures of team-sport movements. International Journal of Sports Physiology and Performance. 2018;**13**(8):1005-1010

[64] Sampaio J, Vitor M. Measuring tactical behaviour in football. International Journal of Sports Medicine. 2012;**33**:395-401

[65] Silva P, Chung D, Carvalho T, Cardoso T, Davids K, Araújo D, et al. Practice effects on intra-team synergies in football teams. Human Movement Science. 2016;**46**:39-51

[66] Folgado H, Lemmink KAPM, Frencken W, Sampaio J. Length, width and centroid distance as measures of teams tactical performance in youth football. European Journal of Sport Science. 2014;**14**(Suppl. 1):S487-S492

[67] Folgado H, Bravo J, Pereira P, Sampaio J. Towards the use of multidimensional performance indicators in football small-sided games: The effects of pitch orientation. Journal of Sports Sciences. 2019;**37**(9):1064-1071

[68] Olthof SBH, Frencken WGP, Lemmink KAPM. Match-derived relative pitch area changes the physical and team tactical performance of elite soccer players in small-sided soccer games. Journal of Sports Sciences. 2018; **36**(14):1557-1563

[69] Ric A, Torrents C, Gonçalves B, Torres-Ronda L, Sampaio J, Hristovski R. Dynamics of tactical behaviour in association football when manipulating players' space of interaction. PLoS One. 2017;**12**(7): e0180773

[70] Carrilho D, Santos Couceiro M, Brito J, Figueiredo P, Lopes RJ, Araújo D. Using optical tracking system data to measure team synergic behavior: Synchronization of player-ball-goal angles in a football match. Sensors. 2020;**20**(17):4990

[71] Davids K, Hristovski R, Araújo D, Serre NB, Button C, Passos P. Complex Systems in Sport. London: Routledge; 2013

[72] Ric A, Torrents C, Gonçalves B, Sampaio J, Hristovski R. Soft-assembled multilevel dynamics of tactical behaviors in soccer. Frontiers in Psychology. 2016;**7**:1513

[73] Bannister EW, Clavert TW, Savage MV, Bach T. A systems model of training for athletic performance. Australian Journal of Sports Medicine. 1975;**7**:57-61

[74] Foster C, Florhaug JA, Franklin J, Gottschall L, Hrovatin LA, Parker S, et al. A new approach to monitoring exercise training. Journal of Strength and Conditioning Research. 2001;**15**(1): 109-115

[75] Coutts A, Kempton T, Crowcroft S, Coutts AJ, Crowcroft S, Kempton T. Developing athlete monitoring systems: Theoretical basis and practical applications. In: Kellmann M, Beckmann J, editors. Sport, Recovery and Performance: Interdisciplinary Insights. Abingdon, UK: Routledge; 2018

[76] Miguel M, Oliveira R, Loureiro N, García-Rubio J, Ibáñez SJ. Load measures in training/match monitoring in soccer: A systematic review. International Journal of Environmental Research and Public Health. 2021;**18**(5): 2721

[77] Carling C, Bloomfield J, Nelsen L, Reilly T. The role of motion analysis in elite soccer: Contemporary performance measurement techniques and work rate data. Sports Medicine. 2008;**38**(10): 839-862

[78] Bowen L, Gross AS, Gimpel M, Li F-X. Accumulated workloads and the acute: Chronic workload ratio relate to injury risk in elite youth football players. British Journal of Sports Medicine. 2017;**51**(5):452-459

[79] Gabbett TJ, Nassis GP, Oetter E, Pretorius J, Johnston N, Medina D, et al.

The athlete monitoring cycle: A practical guide to interpreting and applying training monitoring data. British Journal of Sports Medicine. 2017;**51**(20):1451-1452

[80] Nobari H, Fani M, Clemente FM, Carlos-Vivas J, Pérez-Gómez J, Ardigò LP. Intra- and inter-week variations of well-being across a season: A cohort study in elite youth male soccer players. Frontiers in Psychology. 2021;**12**:671072

[81] Aquino R, Carling C, Maia J, Vieira LHP, Wilson RS, Smith N, et al. Relationships between running demands in soccer match-play, anthropometric, and physical fitness characteristics: A systematic review. International Journal of Performance Analysis in Sport. 2020;**20**(3):534-555

[82] Vieira LHP, Carling C, Barbieri FA, Aquino R, Santiago PRP. Match running performance in young soccer players: A systematic review. Sports Medicine. 2019;**49**(2):289-318

[83] Paul DJ, Bradley PS, Nassis GP. Factors affecting match running performance of elite soccer players: Shedding some light on the complexity. International Journal of Sports Physiology and Performance. 2015;**10**(4):516-519

[84] Teixeira JE, Leal M, Ferraz R, Ribeiro J, Cachada JM, Barbosa TM, et al. Effects of match location, quality of opposition and match outcome on match running performance in a Portuguese professional football team. Entropy. 2021;**23**(8):973

[85] Gonçalves BV, Figueira BE, Maçãs V, Sampaio J. Effect of player position on movement behaviour, physical and physiological performances during an 11-a-side football game. Journal of Sports Sciences. 2014;**32**(2):191-199

[86] Bravi A, Longtin A, Seely AJ. Review and classification of variability analysis techniques with clinical applications. BioMedical Engineering OnLine. 2011;**10**:90

[87] Pincus SM. Approximate entropy as a measure of system complexity. Proceedings of the National Academy of Sciences of the United States of America. 1991;**88**(6):2297-2301

[88] Preatoni E, Ferrario M, Donà G, Hamill J, Rodano R. Motor variability in sports: A non-linear analysis of race walking. Journal of Sports Sciences. 2010;**28**(12):1327-1336

[89] Richman JS, Moorman JR. Physiological time-series analysis using approximate entropy and sample entropy. American Journal of Physiology-Heart and Circulatory Physiology. 2000;**278**(6):H2039-H2049

[90] Busa MA, van Emmerik REA. Multiscale entropy: A tool for understanding the complexity of postural control. Journal of Sport and Health Science. 2016;**5**(1):44-51

[91] Yentes JM, Hunt N, Schmid KK, Kaipust JP, McGrath D, Stergiou N. The appropriate use of approximate entropy and sample entropy with short data sets. Annals of Biomedical Engineering. 2013;**41**(2):349-365

[92] Duarte R, Araújo D, Correia V, Davids K, Marques P, Richardson MJ. Competing together: Assessing the dynamics of team–team and player–team synchrony in professional association football. Human Movement Science. 2013;**32**(4):555-566

[93] Coutinho D, Gonçalves B, Travassos B, Folgado H, Figueira B, Sampaio J. Different marks in the pitch constraint youth players' performances during football small-sided games. Research Quarterly for Exercise and Sport. 2020;**91**(1):15-23

[94] Grassberger P, Schreiber T, Schaffrath C. Nonlinear time sequence analysis. International Journal of Bifurcation and Chaos. 1991;**01**(03): 521-547

[95] Jamin A, Humeau-Heurtier A. (Multiscale) cross-entropy methods: A review. Entropy. 2020;**22**(1):45

[96] Balescu R. Equilibrium and nonequilibrium statistical mechanics. NASA STI/Recon Technical Report A. 1975;**76**:32809

[97] Shannon C. A mathematical theory of communication. Bell System Technical Journal. 1948;**27**(3):379-423

[98] Costa M, Goldberger AL, Peng C-K. Multiscale entropy analysis of biological signals. Physical Review E. 2005;**71**(2): 021906

[99] Palut Y, Zanone P-G. A dynamical analysis of tennis: Concepts and data. Journal of Sports Sciences. 2005;**23**(10): 1021-1032

[100] Gabor D. Theory of communication. Journal of the Institution of Electrical Engineers. 1943; **93**:429-457

[101] Frencken W, Lemmink K, Delleman N, Visscher C. Oscillations of centroid position and surface area of soccer teams in small-sided games. European Journal of Sport Science. 2011; **11**(4):215-223

[102] Hristovski R, Davids K, Araujo D, Passos P, Torrents C, Aceski A, et al. Creativity in sport and dance: Ecological dynamics on a hierarchically soft-assembled perception-action landscape. In: Davids K, Hristovski R, Araújo D, Balagué N, Button C and P. Passos, editors. Complex systems in sport. London: Routledge; 2013. pp. 259-271

[103] Clemente FM, Couceiro MS, Martins FML, Mendes R, Figueiredo AJ. Measuring tactical behaviour using technological metrics: Case study of a football game. International Journal of Sports Science & Coaching. 2013;**8**(4): 723-739

[104] Travassos B, Gonçalves B, Marcelino R, Monteiro R, Sampaio J. How perceiving additional targets modifies teams' tactical behavior during football small-sided games. Human Movement Science. 2014;**38**: 241-250

[105] Derrick TR. Thomas JM. Time Series Analysis: The Cross-Correlation Function; 2004. p. 189

[106] Boker SM, Xu M, Rotondo JL, King K. Windowed cross-correlation and peak picking for the analysis of variability in the association between behavioral time series. Psychological Methods. 2002;**7**(3):338-355

[107] Silva P, Vilar L, Davids K, Araújo D, Garganta J. Sports teams as complex adaptive systems: Manipulating player numbers shapes behaviours during football small-sided games. Springerplus. 2016;**5**(1):191

[108] Moura FA, van Emmerik REA, Santana JE, Martins LEB, Barros RML de, Cunha SA. Coordination analysis of players' distribution in football using cross-correlation and vector coding techniques. Journal of Sports Sciences. 2016;**34**(24):2224–2232

[109] Sparrow WA, Donovan E, van Emmerik R, Barry EB. Using relative motion plots to measure changes in intra-limb and inter-limb coordination. Journal of Motor Behavior. 1987;**19**(1):115-129

[110] Freitas Cruz I, Sampaio J. Multifractal analysis of movement behavior in association football. Symmetry. 2020;**12**(8):1287

[111] Couceiro MS, Clemente FM, Martins FML, Machado JAT. Dynamical

stability and predictability of football players: The study of one match. Entropy. 2014;**16**(2):645-674

[112] Lopes AM, Tenreiro Machado JA. Entropy analysis of soccer dynamics. Entropy. 2019;**21**(2):187

[113] Couceiro MS, Clemente FM, Martins FML. Analysis of football player's motion in view of fractional calculus. Central European Journal of Physics. 2013;**11**(6):714-723

[114] Ramirez-Arellano A, Bermúdez-Gómez S, Hernández-Simón LM, Bory-Reyes J. D-summable fractal dimensions of complex networks. Chaos, Solitons & Fractals. 2019;**119**:210-214

[115] Rokach L, Maimon O. Clustering methods. In: Maimon O, Rokach L, editors. Data Mining and Knowledge Discovery Handbook. Boston, MA: Springer US; 2005

[116] Frank TD, Richardson MJ. On a test statistic for the Kuramoto order parameter of synchronization: An illustration for group synchronization during rocking chairs. Physica D: Nonlinear Phenomena. 2010;**239**(23): 2084-2092

[117] Kuramoto Y, Nishikawa I. Statistical macrodynamics of large dynamical systems. Case of a phase transition in oscillator communities. Journal of Statistical Physics. 1987; **49**(3):569-605

[118] Cover TM, Thomas JA. Elements of Information Theory. New York: Wiley; 1991. p. 542

[119] Silva P, Duarte R, Sampaio J, Aguiar P, Davids K, Araújo D, et al. Field dimension and skill level constrain team tactical behaviours in small-sided and conditioned games in football. Journal of Sports Sciences. 2014;**32**(20): 1888-1896

[120] Gonçalves B, Coutinho D, Exel J, Travassos B, Peñas C, Sampaio J. Extracting spatial-temporal features that describe a team match demands when considering the effects of the quality of opposition in elite football. PLoS One. 2019;**14**:e0221368

[121] Ferraz R, Gonçalves B, Coutinho D, Marinho DA, Sampaio J, Marques MC. Pacing behaviour of players in team sports: Influence of match status manipulation and task duration knowledge. PLoS One. 2018;**13**(2): e0192399

[122] Branquinho L, Ferraz R, Travassos B, Marques MC. Comparison between continuous and fractionated game format on internal and external load in small-sided games in soccer. International Journal of Environmental Research and Public Health. 2020;**17**(2): 405

[123] Branquinho L, Ferraz R, Travassos B, Marinho DA, Marques MC. Effects of different recovery times on internal and external load during small-sided games in soccer. Sports Health. 2021;**13**(4):324-331