Optimal sensor placement methods and criteria in dynamic testing - comparison and implementation on a pedestrian bridge

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ABSTRACT: Structural health monitoring (SHM) is being widely used for the safety assessment and management of existing bridges and structures. One of the objectives related to SHM is to maximize the information gained from the structural testing, while keeping the number of sensors and consequently the cost of the sensor system to a minimum. The current work investigates four of the most influential optimal sensor placement (OSP) methods: the modal kinetic energy (MKE) method, the effective independence (EFI) method, the information entropy index (IEI) method and the MinMAC method. The methods were developed in MATLAB and used as input data the modal analysis results of a finite element model built in ANSYS of the Streicker Bridge, a pedestrian bridge located on the Princeton University Campus. The resulting sensor positions were estimated for a configuration with 14 sensors, and the four OSP methods were evaluated for different numbers of target sensors in terms of different OSP criteria: the determinant (DET) of the Fisher information matrix, the information entropy index (IEI) and the root mean square (RMS) of the off-diagonal entries of the MAC matrix. The study indicates that the EFI method should be chosen to estimate the optimal sensor positions as it provides the largest amount of information with a relatively low computation time.

KEYWORDS: Optimal sensor placement; Structural dynamics; Dynamic testing; Modal analysis; Pedestrian bridge.

1 INTRODUCTION

Bridges play a crucial role in the social and economic development of cities. The safety assessment and management of this type of structures is also particularly important and it can be carried out through different structural health monitoring (SHM) systems.

SHM systems are expected to obtain the maximum amount of information from structural testing. Generally, the higher number of sensors are placed, the more detailed information of the structure can be obtained. However, the number of sensors is in many cases constrained by high costs of data acquisition systems and accessibility limitations. The main challenge related to SHM is to optimize the trade-off between the maximal information obtained by the sensor system and the material costs for the experimental set-up.

The optimal sensor placement (OSP) has been a subject of important international research in the recent years. This paper investigates some of the most influential OSP methods and criteria and presents the implementation of the methods on a pedestrian bridge for the identification of five mode shapes.

2 OPTIMAL SENSOR PLACEMENT CRITERIA

It should be noted that the suitability of a sensor configuration depends on the evaluation criteria considered. In this paper, four influential criteria in dynamic testing (presented by Yi and Li [1]) are discussed: the measured energy per mode, the Fisher information matrix (FIM), the information entropy (IE) and modal assurance criterion (MAC).

2.1 *Measured energy per mode*

The kinetic energy is not evenly distributed into the natural modes of a structure. Therefore, the measured degrees of freedom (DOFs) of the structure are expected to capture a large part of the total kinetic energy of the structure. This criterion is based on the traditional heuristic visual inspection that consists in the visual inspection of the structure response, examination of the mode shapes of interest and selection of locations with high amplitude of responses [1].

2.2 *Fisher information matrix (FIM)*

This criterion results from minimizing the covariance matrix of the estimate error for an efficient unbiased estimator from the perspective of statistics [1]. The FIM is defined as follows:

$$\boldsymbol{Q}(\boldsymbol{L}) = (\boldsymbol{L}\boldsymbol{\phi})^{\boldsymbol{T}}(\boldsymbol{L}\boldsymbol{\phi}) \quad (1)$$

where, $\boldsymbol{\phi}$ is the matrix of mode shapes (dimensions $[n\ x\ n]$, or $[n\ x\ m]$ if m target modes are considered), $\boldsymbol{L}$ is the Boolean $[n\ x\ n]$ matrix that maps the sensor locations to the n DOFs. Thus, the Fisher information matrix $\boldsymbol{Q}$ is a $[n\ x\ n]$ matrix, or a $[m\ x\ m]$ matrix if m target modes are considered. In practice, three variants of the FIM are used: the minimum singular value, the determinant (DET) and the trace, which are maximized to increase the information acquired and to decrease the uncertainties of the estimated parameters [1].

2.3 *Information entropy (IE)*

The information entropy (IE) is the measure of uncertainty contained in the system parameters $\boldsymbol{\theta}$. A minimal information entropy corresponds to a minimal uncertainty in the system parameters and to a maximal amount of useful information contained in the measured data. Hence, the sensor set should minimize the information entropy. The IE (scalar) of a sensor configuration $\boldsymbol{L}$ is given by:

$$IE(\boldsymbol{L}, \boldsymbol{\theta_o}) = \frac{1}{2} N_\theta \ln(2\pi) - \frac{1}{2} \ln[\det\{\boldsymbol{Q}(\boldsymbol{L}, \boldsymbol{\theta_o})\}] \tag{2}$$

where, $\boldsymbol{\theta_o}$ is the optimal value of the parameter set $\boldsymbol{\theta}$ of length N_θ (number of parameters) that minimizes the IE, and $\boldsymbol{Q}(\boldsymbol{L}, \boldsymbol{\theta_o})$ is the $[N_\theta \, x \, N_\theta]$ FIM. For the case of modal identification, the parameters that are of interest are the modal coordinates. Hence, the parameter set $\boldsymbol{\theta}$ becomes a [*m x 1*] vector for *m* target modes and $\boldsymbol{Q}(\boldsymbol{L}, \boldsymbol{\theta_o})$ becomes a [*m x m*] matrix.

2.4 Modal assurance criterion (MAC)

The modal assurance criterion (MAC) index is an indicator of the degree of correlation between two mode shapes. The function of MAC index is to provide a measure of consistency (degree of linearity) between estimates of a modal vector. Usually, the MAC index is used to compare an experimental mode shape with a numerical mode shape. Nevertheless, mode shapes from two finite element (FE) models and from the same FE model (self-MAC) can be also compared.

The MAC index for a couple of [*n x 1*] column vectors $\boldsymbol{\phi_i}$ and $\boldsymbol{\phi_j}$ of the mode shape matrix of a structure is defined as a scalar constant relating the degree of consistency (linearity) between one modal vector and another reference modal vector:

$$MAC_{ij} = \frac{\left(\boldsymbol{\phi}_i^T \boldsymbol{\phi}_j\right)^2}{\left(\boldsymbol{\phi}_i^T \boldsymbol{\phi}_i\right)\left(\boldsymbol{\phi}_j^T \boldsymbol{\phi}_j\right)} \tag{3}$$

The MAC is a value that ranges between 0 and 1. A high value of MAC indicates a strong correlation between the two modes in comparison, while a low value indicates that the correlation between the two modes is weak. Usually, modes are regarded as correlated for MAC values larger than 0.8-0.9 [2].

MAC values can be represented in a MAC matrix. In the MAC matrix, diagonal elements are expected to be close to 1 whereas off-diagonal elements are expected to be close to 0. Therefore, the size of the off-diagonal elements could be an indication of optimal result [1].

Sensor configurations can be compared based on the root mean square (RMS) of the off-diagonal entries of the MAC matrix (see Equation (4)). The lower the RMS, the higher independence and distinguishability of the *m* target mode vectors.

$$RMS = \sqrt{\frac{1}{m(m-1)} \sum_{i=1}^{m} \sum_{\substack{j=1 \\ j \neq i}}^{m} MAC_{ij}^2} \qquad i = 1, \dots, m \,;\, j = 1, \dots, m \tag{4}$$

Another criterion related to the MAC is the maximum in the off-diagonal entries (MOD) of the MAC matrix. Similarly, MOD should be as low as possible. However, RMS is a more general measurement than MOD [3].

3 OPTIMAL SENSOR PLACEMENT METHODS

Many methods have been developed for the obtention of optimal sensor placement configurations in the framework of structural health monitoring. These methods range from applying constraints on the objective function (deterministic methods) to sequentially find the sensors position (sequential methods) and to applying advanced artificial analysis techniques such as the genetic algorithms (combinatorial approaches) [1].

In this section, four well-known and widely used optimal sensor placement (OSP) methods are investigated: the modal kinetic energy (MKE) method, the effective independence (EFI) method, the information entropy index (IEI) method and the MinMAC method. These four methods consider uniaxial sensors and are based on the optimization of different criteria previously defined.

3.1 Modal kinetic energy (MKE) method

The modal kinetic energy (MKE) method, adopted by Krammer [4], is related to the measured energy per mode criterion and formulates the kinetic energy distribution as follows:

$$\boldsymbol{MKE} = \mathrm{diag}(\boldsymbol{M}\boldsymbol{\phi}\boldsymbol{\phi}^{\boldsymbol{T}}) \tag{5}$$

For *m* target mode shapes and *n* DOFs, $\boldsymbol{\phi}$ is the [*n x m*] mode shape matrix and $\boldsymbol{M}$ is the [*n x n*] mass matrix. Therefore, $\boldsymbol{MKE}$ is a [*n x 1*] vector whose elements correspond to the kinetic energy associated to each DOF considering multiple mode shapes.

The DOFs are ranked according to their kinetic energy value and the N_0 DOFs with the highest values are retained as the optimal sensor locations, with N_0 being the number of target sensors. This method is not iterative.

3.2 Effective independence (EFI) method

The effective independence (EFI) method, proposed by Krammer [4], is an iterative method based on the backward sequential sensor placement (BSSP) algorithm and the Fisher Information Matrix (FIM).

The backward sequential sensor placement (BSSP) algorithm works as follows: BSSP starts with all the DOFs of the structure monitored and sensors are removed, one by one, from the position that results in the smallest increase of the objective function. This procedure is continued up to the number of target sensors N_0 is reached.

The EFI method aims to maximize the linear independence between the *m* target mode shapes throughout the following [*n x 1*] vector:

$$\boldsymbol{E_D} = [\boldsymbol{L}\boldsymbol{\phi}\boldsymbol{\psi}] \circ [\boldsymbol{L}\boldsymbol{\phi}\boldsymbol{\psi}] \mathrm{diag}(\boldsymbol{\Lambda})^{-1} \tag{6}$$

where, the operator $\circ$ denotes the Hadamar product (element-wise multiplication), $\boldsymbol{L}$ is the Boolean [*n x n*] matrix that maps the sensor locations to the DOFs, $\boldsymbol{\phi}$ is the [*n x m*] matrix of mode shapes, $\boldsymbol{\Lambda}$ and $\boldsymbol{\psi}$ are the [*m x m*] matrices of eigenvalues and eigenvectors, respectively, of the following eigenvalue problem:

$$\boldsymbol{Q}(\boldsymbol{L})\boldsymbol{\psi} = \boldsymbol{\psi}\boldsymbol{\Lambda} \tag{7}$$

$\boldsymbol{Q}$ is the [m x m] Fisher Information Matrix (FIM) defined in Equation (1). According to Krammer [4], the independence distribution vector $\boldsymbol{E_D}$, defined in Equation (6), can be alternatively formulated as follows:

$$\boldsymbol{E_D} = \mathrm{diag}((\boldsymbol{L\phi})[(\boldsymbol{L\phi})^{\boldsymbol{T}}(\boldsymbol{L\phi})]^{-\boldsymbol{1}}(\boldsymbol{L\phi})^{\boldsymbol{T}}) \tag{8}$$

Before applying the method, $\boldsymbol{L}$ equals to the identity matrix because the procedure starts with all the DOFs instrumented. In every iteration, the DOF with the lowest value in vector $\boldsymbol{E_D}$ is removed from the sensor set and the mapping matrix $\boldsymbol{L}$ is updated. The procedure is continued up to the number of target sensors N_0 is reached.

3.3 Information entropy index (IEI) method

The information entropy index (IEI) method, adopted by Papadimitriou and Lombaert [5], is an iterative method which aims to find a sensor set that minimizes the information entropy (IE) defined in Equation (2). The information entropy index (IEI), defined in Equation (9), is a normalized version of the information entropy [5].

$$IEI(\boldsymbol{L}, \boldsymbol{\theta_o}) = \sqrt{\frac{\det \boldsymbol{Q}(\boldsymbol{L_{ref}}, \boldsymbol{\theta_0})}{\det \boldsymbol{Q}(\boldsymbol{L}, \boldsymbol{\theta_0})}} \tag{9}$$

where, $\boldsymbol{\theta_o}$ is the optimal value of the parameters set $\boldsymbol{\theta}$ of length N_θ (number of parameters), and $\boldsymbol{Q}(\boldsymbol{L}, \boldsymbol{\theta_o})$ is the $[N_\theta \; x \; N_\theta]$ FIM. $\boldsymbol{L_{ref}}$ is the reference sensor configuration matrix, which equals to the identity matrix if all the DOFs are monitored.
The IEI method can be used in combination with the backward sequential sensor placement (BSSP) algorithm: initially, the full configuration (n DOFs monitored) is assumed as a reference and then it is compared to all the possible configurations with one sensor less (n-1). The configuration with the lowest IEI is chosen and used in the following iteration. The procedure is continued up to the number of target sensors N_0 is reached.

3.4 MinMAC method

The MinMAC method, proposed by Carne and Dohrmann [6], is an iterative method based on the forward sequential sensor placement (FSSP) algorithm and the modal assurance criterion (MAC).
The basic steps of the forward sequential sensor placement (FSSP) algorithm are: being N_0 the number of target sensors, the position of the first sensor is chosen as the one that gives the highest reduction of the objective function. Similarly, the second sensor is located in the position that gives the highest reduction of the objective function by assuming that the first sensor was already located at its optimal position. This procedure is continued up to the number of target sensors N_0 is reached.
The MinMAC method consists in minimizing the maximum of the off-diagonal terms (MOD) of the MAC matrix (see Equation (3)) in order to determine an optimal sensor set. The basic steps of the MinMAC method are the following:

i. Choose N sensor locations (less than the required number of sensors N_0) on intuition based on a visual inspection of the structure response.
ii. The self-MAC matrix of the FEM modes is calculated for the initial sensor configuration plus one sensor ($N+1$). The diagonal elements of the self-MAC matrices are unity, in contrast to a cross-MAC matrix between FEM modes and test modes [6]. The configuration that minimizes the MOD is chosen and used in the following iteration.
iii. The procedure is continued up to the number of target sensors N_0 is reached.

As an alternative to the intuition set, an initial sensor configuration can be selected using another OSP method (e.g. the EFI method).

4 PEDESTRIAN BRIDGE

The different optimal sensor placement (OSP) methods were tested on a real scale structure: the Streicker Bridge. This pedestrian bridge, located on the Princeton University Campus (New Jersey, USA), was chosen as a test bed because it is actually instrumented with two fiber-optic-based monitoring systems: a discrete fiber Bragg-grating (FBG) monitoring system and a distributed sensing using Brillouin optical time domain analysis (BOTDA) monitoring system.
The footbridge is 104 meters long and consists of a main span and four legs (see Figure 1).

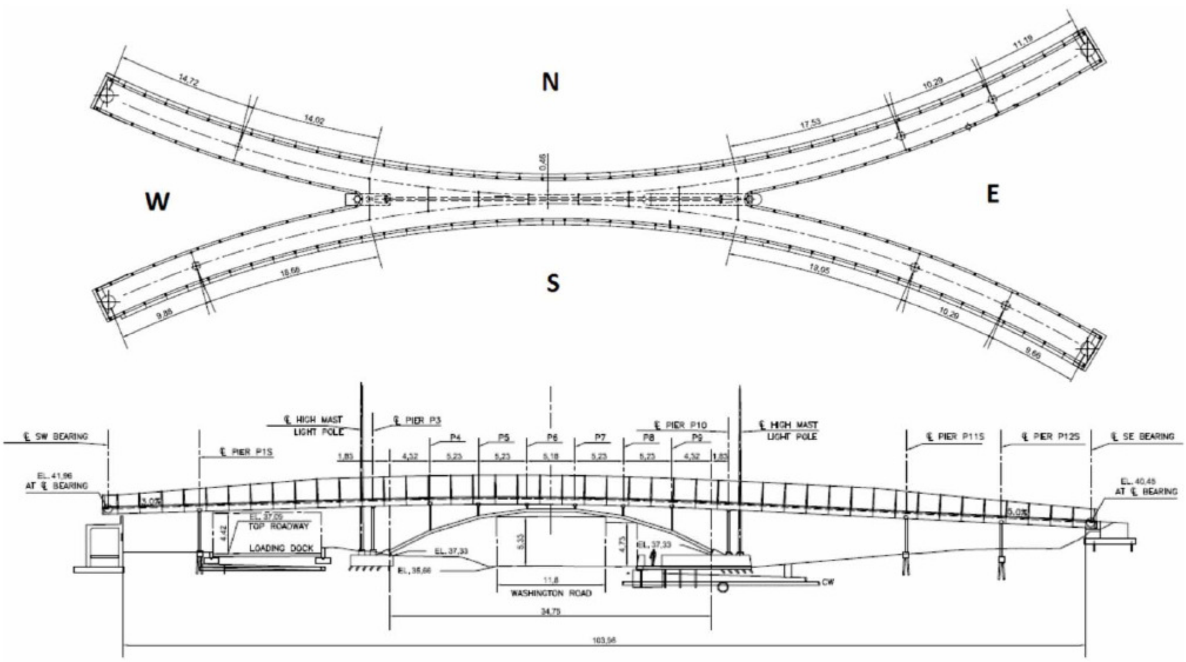


Figure 1. Plan and elevation drawings of the Streicker Bridge (PU & HNTB).

The deck, made of post-tensioned high-performance concrete, is connected through six spandrels to a steel arch in the main span (deck-stiffened arch) and is supported by eight Y-shaped piers in the lateral legs (see Figure 2). Arch, spandrels and piers are made of weathering steel tubes filled with self-consolidating concrete.

At the four abutments, the deck rests on elastomeric neoprene bearings. Both piers and arch are supported on concrete footings. The deck is connected to piers and spandrels through fixed connections.

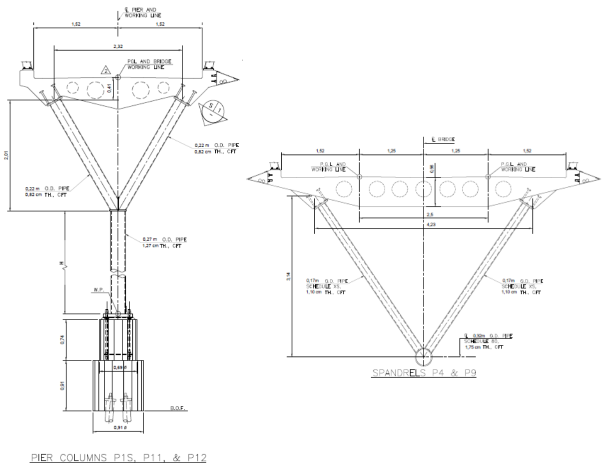


Figure 2. Detail of one pier, left, and one spandrel, right (PU & HNTB).

In order to provide input data for the OSP methods a three-dimensional finite element (FE) model of the footbridge built by Lizana [7] was used (see Figure 3). The FE model, built in ANSYS Mechanical APDL, contains 97 nodes and 83 Timoshenko beam elements with 6 DOFs at each node named BEAM188 in ANSYS.

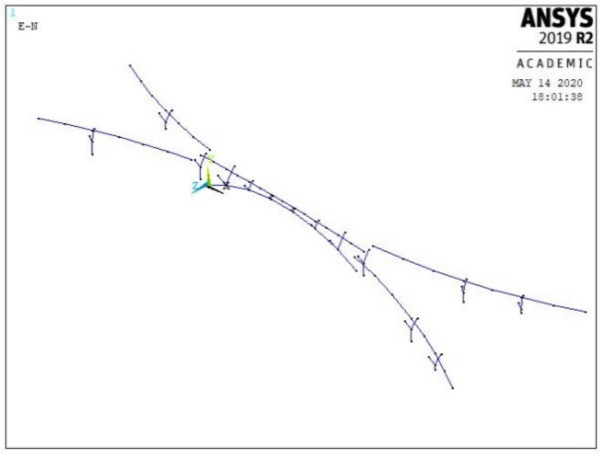


Figure 3. View of Model B in ANSYS [7].

The deck is connected to piers and spandrels through rigid body constraints, named CERIG in ANSYS, which link the centroid of the deck to the two upper nodes of the of every Y-shaped pier and spandrel and constrain the 6 DOFs.

The boundary conditions are an idealization of the drawings' representations. The bases of the piers and of the arch are fixed, while the supports at the four abutments are simple supports where all translations are constrained, whereas all rotations are allowed.

The first five eigenmodes, reported in Table 1, were used as input to the OSP analysis (m=5). The mode shapes related to the first five eigenmodes can be considered flexural (vertical displacements are dominant) according to the modal deformations in the FE model and the modal participation mass.

Table 1. Modal frequencies calculated using FE model [7].

Mode N.	Mode shape	f_{FEM} (Hz)
1	Flexural	**3.12**
2	Flexural	**3.22**
3	Flexural	**3.60**
4	Flexural	**3.76**
5	Flexural	**4.19**

The eigenfrequencies from modal analysis were compared to two natural frequencies experimentally determined by Sigurdardottir and Glisic [8] with the SHM systems installed in the Streicker Bridge. The experimental mode shapes were not obtained. Table 2 shows that there is an excellent match between the first numerical and experimental frequencies. However, the second experimental frequency matches the fourth numerical frequency. Relative errors ε are below 2%.

Table 2. Comparison between experimental and numerical frequencies [7].

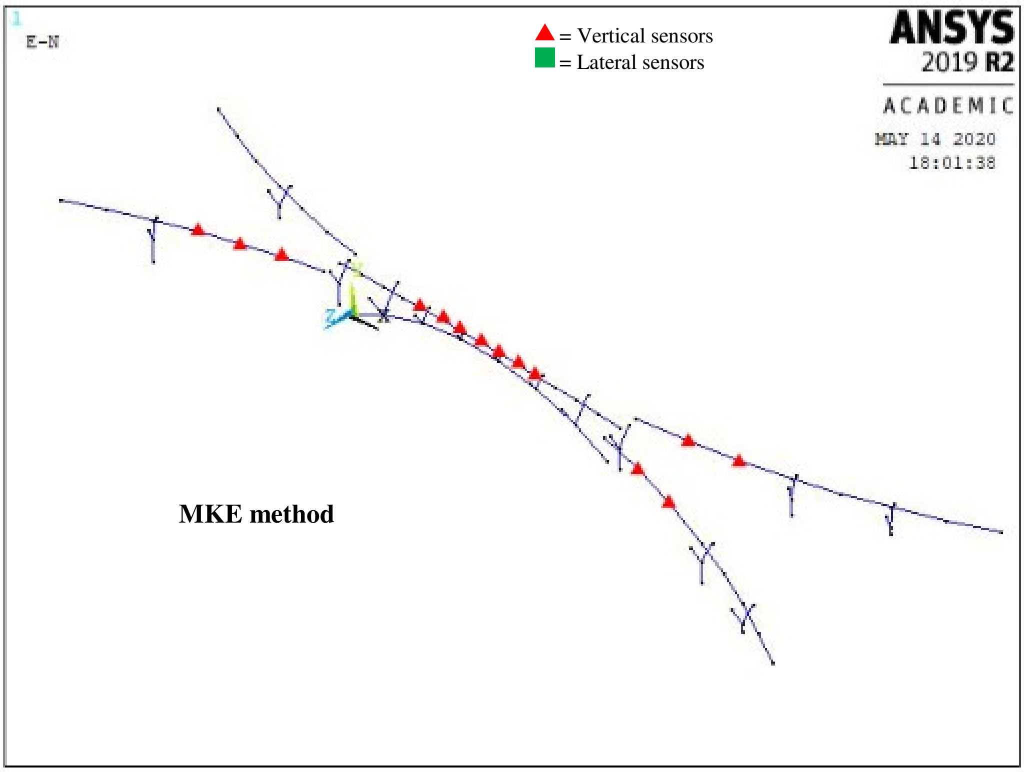
f_{EXP} (Hz) Sigurdardottir and Glisic [8]	f_{FEM} (Hz)	ε (%)
3.11	3.12	0.32
3.72	3.76	1.08

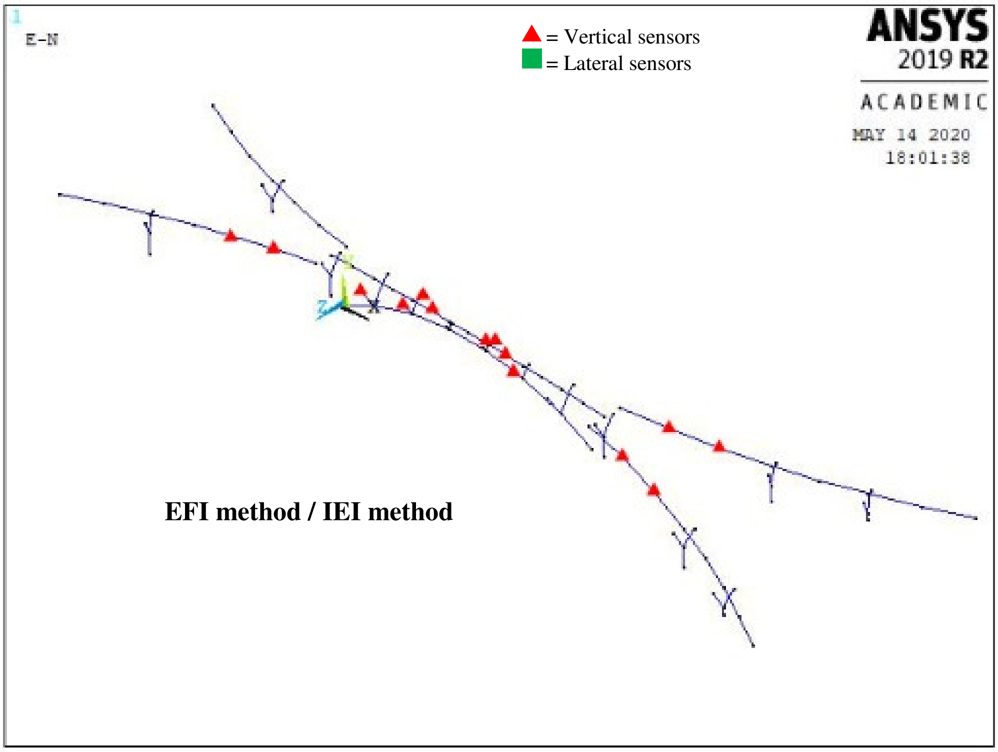
5 OPTIMAL SENSOR PLACEMENT RESULTS AND EVALUATION

The goal of the analysis is to determine the best OSP method for the identification of the first five modes of the structure. The methods, developed in MATLAB, use the modal analysis results of the FE model of the Streicker Bridge as input data. The resulting sensor positions are estimated for a configuration with 14 sensors, and the four methods are evaluated in terms of different OSP criteria for different numbers of target sensors (ranging from $N_0 = 5$ to 166).
For the identification of the first five modes, the minimum number of sensors is 5 since it is known that the number of sensors should not be less than the number of mode shapes to be identified [1].

5.1 Resulting sensor positions (14 sensors)

Considering that translations in the longitudinal direction (Y-direction) of the footbridge are almost null for the first five mode shapes, only two DOFs per node, corresponding to the vertical translation (Z-direction) and the lateral translation (X-direction), are considered in the analysis. Therefore, assuming that translations are constrained in 14 nodes due to the boundary conditions, 166 DOFs are candidate to be monitored with the 14 available sensors.
Figure 4 shows the sensor distribution estimated by the different OSP methods. The vertical sensors are represented by red triangles and the lateral sensors are represented by green squares.





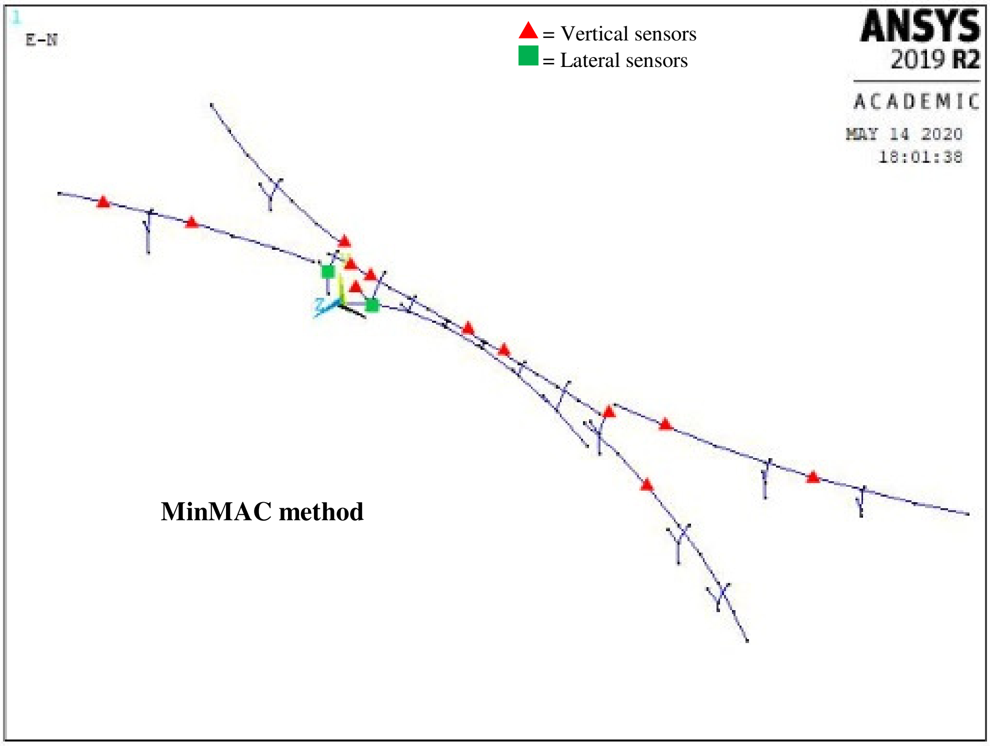


Figure 4. Positions of the DOFs to be monitored for the MKE, EFI, IEI and MinMAC methods (14 sensors).

Since the first five modes are flexural, all the sensors were expected to be placed measuring in the vertical direction. The MinMAC method, whose initial sensor set is formed by a single sensor located in the midspan of the longest span of the bridge, leads to a sensor set less likely to capture the five flexural modes as it places two sensors in the lateral direction. Conversely, the MKE, EFI and IEI methods place all the sensors measuring in the vertical direction. The EFI and IEI methods give the same sensor positions. None of the four methods place sensors on the northwest leg, which is the leg with the shortest longest span (abutment to pier span) compared to the other legs.

It must be remarked that the flexural deflection in the bridge deck induces some torsional deformation due to the complex geometry of the structure. The EFI and IEI methods would be capable of identifying torsion in the main span deck since they place vertical sensors on 5 upper nodes of main span spandrels. The MKE method, on the other hand, places all the sensors on deck nodes and then it would not be able to capture torsion in any part of the deck.

5.2 Evaluation of the OSP methods in terms of different OSP criteria

Table 3 summarizes the values of the OSP criteria for the sensor configurations with 14 sensors. The EFI and IEI methods are the finest methods in terms of the determinant (DET) of the Fisher information matrix and the information entropy index (IEI), whereas the MinMAC method gives the best results in terms of the root mean square (RMS) of the off-diagonal entries of the MAC matrix.

Table 3. Values of the OSP criteria for 14-sensors configurations.

Method	DET	IEI	RMS
MKE	8.21E-23	9.25	0.062
EFI	1.14E-22	8.02	0.026
IEI	1.14E-22	8.02	0.026
MinMAC	4.57E-24	40.08	0.006
All DOFs	7.34E-21	1.00	0.075

For a range of 5 to 166 sensors, the EFI and IEI methods obtain the highest DET and the lowest IEI (see Figure 5 and Figure 6, respectively). Consequently, the optimization of these two methods leads to the largest volume of information and the minimal uncertainty in the system parameters for any number of sensors. Their equivalence in terms of the resulting sensors positions for any number of sensors implies that their optimization criteria are analogous.

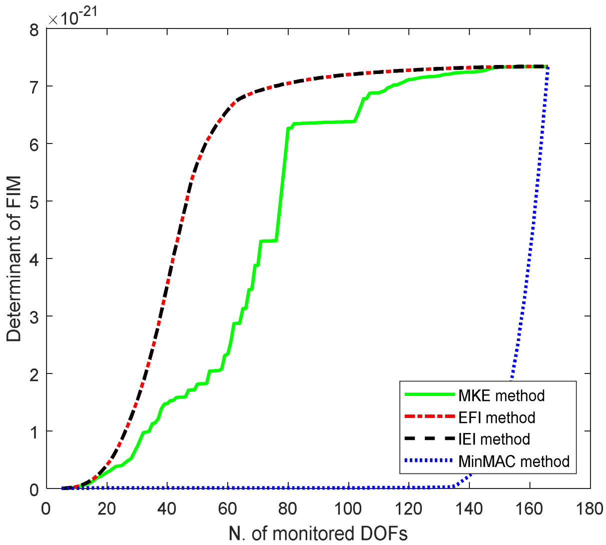


Figure 5. Determinant of the FIM (DET) as a function of the number of monitored DOFs.

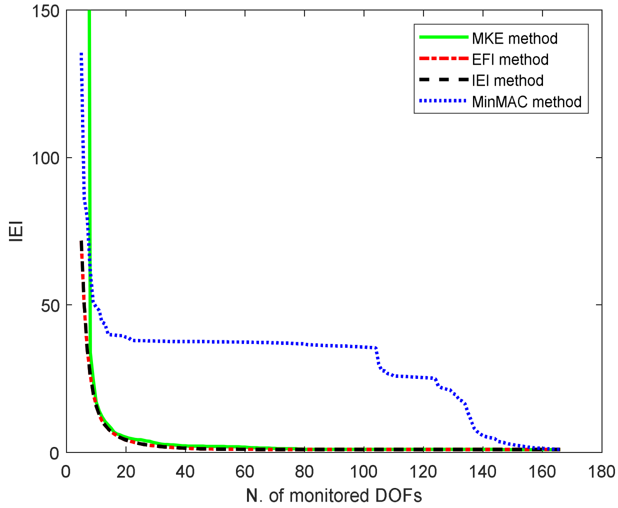


Figure 6. Information entropy index (IEI) as a function of the number of monitored DOFs.

Figure 7 illustrates the evolution of the RMS for a range of 5 to 166 sensors. The MinMAC method performs best for any number of monitored DOFs. This was expected since the MinMAC method minimizes the maximum in the off-diagonal entries (MOD) of the MAC matrix during the optimization process. In the range of 19 to 79 sensors, the MKE method obtains lower values of the RMS than the EFI and IEI methods.

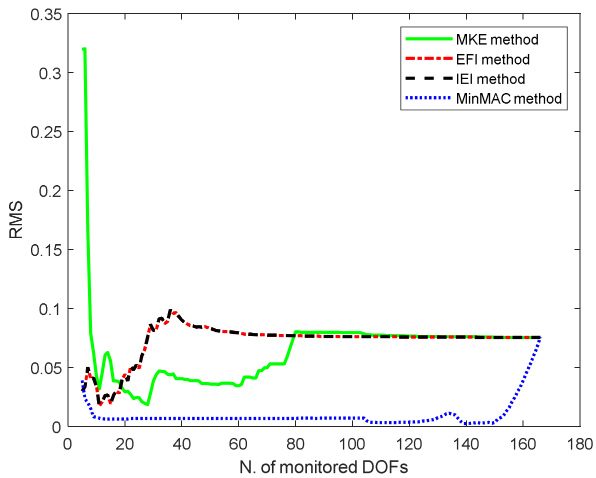


Figure 7. Root mean square (RMS) of the off-diagonal entries of the MAC matrix as a function of the number of monitored DOFs.

The computation time was not defined as an OSP criterion. However, it can be evaluated with the aim of comparing the computational efficiency of the methods. Figure 8 shows the evolution of the computation time for a range of 5 to 166 sensors. The MKE method presents the lowest computation time for any total number of sensors, closely followed by the EFI method. It must be remarked that even though the EFI and IEI methods give the same sensor positions, the computation time of the EFI method is significantly lower.

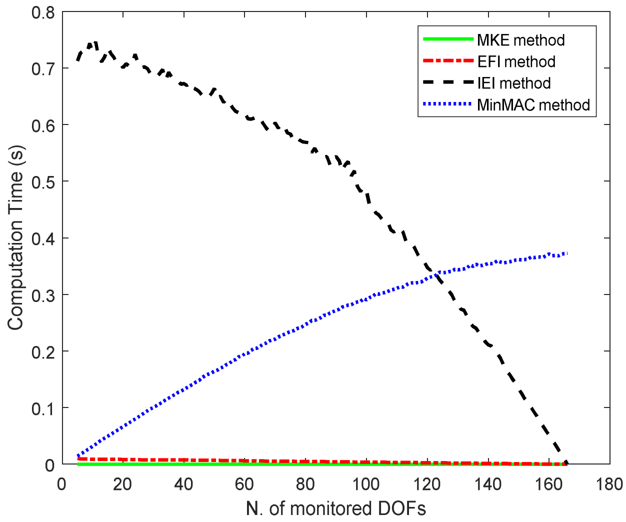


Figure 8. Computation time as a function of the number of monitored DOFs.

6 CONCLUSIONS

In this paper, four of the most influential OSP methods were evaluated in terms of different OSP criteria for the identification of the first five mode shapes of a pedestrian bridge.

The EFI and IEI methods are equivalent methods in terms of the resulting sensor positions and obtain the highest DET (largest volume of information) and the lowest IEI (minimal uncertainty in the system parameters). However, the computation time of the EFI method is significantly lower.

The MinMAC method procures the lowest RMS values (highest independence and distinguishability of the target mode vectors), but its performance in terms of the DET and the IEI is not satisfactory.

The MKE method presents the lowest computation time for any total number of sensors. In the range of 19 to 79 sensors, the MKE method obtains lower values of the RMS than the EFI and IEI methods.

According to the results, in a dynamic testing of a pedestrian bridge, the EFI method should be chosen to estimate the optimal sensor positions as it provides the largest amount of information with a relatively low computation time.

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