
International Journal of Production Research

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/tprs20

# A model for designing a procurement-inventory system as a defence against a recurring epidemic

Albert Corominas

**To cite this article:** Albert Corominas (2021): A model for designing a procurement-inventory system as a defence against a recurring epidemic, International Journal of Production Research, DOI: 10.1080/00207543.2021.1919779

**To link to this article:** https://doi.org/10.1080/00207543.2021.1919779

INTERNATIONAL JOURNAL OF PRODUCTION RESEARCH
https://doi.org/10.1080/00207543.2021.1919779



# A model for designing a procurement-inventory system as a defence against a recurring epidemic

Albert Corominas

Institute of Industrial and Control Engineering, Universitat Politècnica de Catalunya, Barcelona, Spain

**ABSTRACT**
The COVID-19 pandemic has caused a general shortage of personal protection products and therapeutic devices, which has highlighted the need for each country to have its own production resources and not depend solely on imports. Given the time that elapses between the onset of an epidemic and its detection, as well as the time required to activate production and the lead time of purchasing operations, it is necessary to have a permanent reserve, which we call shield stock, in order to immediately meet the demand for equipment at the beginning and throughout the course of the epidemic. This situation is analysed in order to identify the most relevant decisions in the scenario described, formulate a cost optimisation model and develop procedures to find the most economical combination of shield stock, domestic production capacity and imports to guarantee the immediate satisfaction of demand and the restoration of the shield stock after the epidemic, as a preventative measure. The procedure is illustrated with a specific pattern of the spread of the epidemic and some numerical examples.

**ARTICLE HISTORY**
Received 21 July 2020
Accepted 6 April 2021

**KEYWORDS**
Procurement-inventory management; epidemic; COVID-19; stockpiling; supply chain design

## 1. Introduction

The COVID-19 pandemic, both because it was unexpected and because of its characteristic rapid expansion, high mortality rate and the initial lack of vaccines or treatments, has been an unprecedented challenge for the health systems of all the world's countries. Many of them have suffered from an insufficient supply of personal protective equipment and devices or instruments needed to diagnose and care for patients. In a pandemic like COVID-19, some of these items must be available without delay at the time they are needed, as their lack has very serious or even fatal consequences.

Examples of these products are personal protective equipment (PPE), ventilators, diagnostic systems or medicines for the treatment of the disease. To set out our ideas, we will now refer to PPE on the understanding that the developments we present are also applicable to other products that have a demand with characteristics similar to those of PPE. Similar requirements of items such as ammunition or specific materials can arise from situations of a different nature, such as the outbreak of conflict or the sudden increase in demand for a fashion product, but we will not refer to them in the rest of this work.

As the experience with COVID-19 has dramatically shown, when an epidemic outbreak occurs the demand for various products experiences a substantial and abrupt increase that often cannot be met with the available production capacity. Therefore, in these cases and concerning essential products, in order to meet the demand at any moment from the beginning to the end of the outbreak, there must be a sufficient permanent stock and, possibly, the country's own production has to be complemented with imports from countries with excess manufacturing capacity. In addition, once the epidemic is over, the system needs to be prepared for the next outbreak.

Hence, being prepared to adequately deal with an epidemic of this nature has a considerable cost that should be minimised. Thus, the problem arises of designing a procurement-inventory system with the best combination of permanent stock (henceforth called shield stock), production capacity available, domestic production and imports that will allow the required constraints to be fulfilled.

The goal of this paper is to identify the most important decisions in the scenario described, model the situation, and develop procedures to find the optimal values of the main elements of the procurement-inventory system corresponding to an item essential in the fight against the epidemic.

The following section is devoted to an overview of the related literature. Section 3 includes the assumptions that define the problem, the notation and the formulation of

**CONTACT** Albert Corominas albert.corominas@upc.edu

the optimisation model. Section 4 sets out the calculation and optimisation of costs which, in Section 5, are illustrated with the application to the case of a triangular evolution of the epidemic. Finally, Section 6 presents a summary of the results and indications on possible future lines of research.

## 2. Literature overview

The papers related more or less directly to the problem addressed in this study deal with epidemics, disruptions, disasters and the impact of these events on supply chains. Therefore, they are placed mainly in the fields of humanitarian, disaster and health care management.

Both disasters and epidemics require as quick a reaction as possible. However, disasters, i.e. sudden calamitous events such as earthquakes or floods, cause great damage, loss and destruction, generating immediate and important needs of resources for assistance. Epidemics, on the other hand, last for weeks or months, with both an ascending phase and a descending one; they generate a continuous consumption of resources which, in the early stages after the start of the outbreak, remains generally low. The differences between both events must be taken into account in logistics and inventory management.

Whybark (2007) pointed out that little literature was available on disaster relief inventories and called for research into their management. Nine years later, however, Balcik, Bozkir, and Kundakcioglu (2016), in a literature review on inventory management in humanitarian supply chains, included seventy-five references on papers proposing 'policies and models to determine how much to stock, where to stock and when to stock throughout the humanitarian supply chain'. Some papers (Hale and Moberg 2005; Rawls and Turnquist 2010; Jahre, Pazirandeh, and Van Wassenhove 2016; Sharifyazdi et al. 2018; Hansen, Friedrich, and Transchel 2020) focus mainly on the pre-positioning of emergency supplies.

On the other hand, with an approach closer to that adopted in the present paper, there is a predominance of recent articles that consider pre- and post-disaster decisions regarding inventory management and procurement in an integrated manner. To this end, Balcik and Ak (2014) and Hu, Han, and Meng (2017) present two-stage stochastic programming models for joint decision making concerning inventory and procurement in humanitarian relief. Torabi et al. (2018) point out that the optimum levels of pre-position relief items are affected by pre-disaster contractual agreements and post-disaster procurements; they also indicate, however, that framework agreements (Balcik and Ak 2014) frequently used by humanitarian organisations can be considered as a kind of inventory prepositioning, since they do not rely on the suppliers' post-disaster production capacities. Although it has no direct relation with disaster management, Takemoto and Arizono (2020) is interesting insofar as it analyses coordination in supply chains with capacity reservation contracts.

Other papers devoted to disaster management (Boin, Kelle, and Whybark 2010; Day 2014) focus on the resilience of the disaster relief supply chain networks and make recommendations to enhance supply chain resilience. Of course, the resilience of the supply chains is an important topic which is not always related to disasters or epidemics and reviewing the literature on this subject is beyond the scope of the present paper.

The COVID-19 outbreak, although relatively recent, has already led to a certain number of published papers, particularly about the impacts of the new virus on supply chains. Ivanov (2020) uses simulation to predict the impact of epidemic outbreaks on global supply chains. Ivanov and Dolgui (2020) discuss the viability of intertwined supply chains to improve resilience. Singh et al. (2020) deal with the disruptions in food supply chains caused by COVID-19 and they also indicate the need to adopt mathematical approaches to deal with issues related to manufacturing essential items and shortages of healthcare equipment to combat the consequences of the pandemic. Sodhi, Tang, and Willenson (2021) examine the causes of the prolonged shortages of critical products in the US, as a consequence of the COVID-19, and propose a research agenda to develop responsive supply chains to fight future pandemics. Ivanov (2021) proposes and discusses four adaptive strategies to maintain supply chain viability when facing a pandemic.

Regarding epidemics, Dasaklis, Pappis, and Rachaniotis (2012) provide an overview of the literature on their control and the required logistics operations. In particular, the authors examine the contributions regarding logistical attributes and methodologies. They show that stockpiling medical resources was common practice in order to face the consequences of influenza and similar epidemics and that, in contrast, inventory replenishment policies during an ongoing epidemic outbreak were scarce; moreover, they point out that most papers included in the review consider only the purchasing cost, leaving aside other components of inventory management (such as handling, picking, packing and preparing for shipment) that involve additional costs. Paul and Venkateswaran (2018), who deal with the dynamic replenishment of drugs during an epidemic, advocate combining resource allocation with the appropriate supply chain management. They highlight the importance of the detection threshold on the dynamics of the epidemic and state that most of the resource allocation models assume that resources are already available.

Significant examples of papers adopting this assumption are Radonovich et al. (2009), Rebmann et al. (2011) and Hashikura and Kizu (2009), who point out that even the amount of PPE that should be stored in hospital settings has been unclear. Harrington and Hsu (2010), however, deal with the so-called Manufacturer Reserve Programs, used by manufacturers to promote stockpiling of drugs by organisations such as hospitals in preparation for epidemics.

Summing up, this concise analysis of the literature shows that in order to face the consequences of an epidemic such as COVID-19 it is necessary, before the detection of the outbreak, to adopt measures with regard to essential products that include the formation of sufficient stock to provide an immediate response in the early stages of the epidemic's spread. A robust and resilient supply chain is also needed, which includes the possibility of purchasing the product from suppliers that have stocks initially and manufacturing capacity throughout the course of the epidemic.

However, we have not found direct precedents in the literature that address the scenario described in this work, which indicates the need to formalise a model and develop the procedures to find optimal solutions.

## 3. Definition and formulation of the problem

In this section we establish the assumptions defining the problem to be solved, and we formalise the objective function and the constraints that configure the corresponding cost optimisation model.

We consider a recurrent epidemic, which occurs at times difficult to predict and has a first phase of ascent and a second, usually longer, of descent, with a total duration of $T$ units of time. The duration of epidemic cycles, i.e. the time between the onset of one outbreak and that of the next, is irregular, but its expected value is known.

The treatment of the epidemic requires PPE, whose demand must be met promptly at all times. The function $D(t)(0 \leq t \leq T)$ of PPE demand accumulated since the beginning of the epidemic, expressed in PPE units, is known. It is a derivable function, where $D'(t) = d(t) \geq 0$ is a continuous, unimodal function, with $d(0) = d(T) = 0$, which has a maximum at $\hat{t}$, the peak of the epidemic. Therefore, $D(t)$ is strictly convex between 0 and $\hat{t}$ and strictly concave between $\hat{t}$ and $T$, since its derivative is increasing and decreasing during the first and the second phases of the epidemic, respectively.

Although the unimodality of the demand rate does not reflect all the real patterns of an epidemic spread, since these may possibly include local oscillations, it can be a reasonable approach in most cases, according to the information relative to COVID-19 which has been published in preprints (Rocha 2020; Shayak and Sharma 2020) or journal papers (Kim, Seo, and Yeom 2020).

In order to satisfy the condition that the demand be met without delay, the health authorities have a shield stock of $s$ units of the product and an available domestic capacity of PPE manufacturing equal to $r$ PPE units per unit of time, where it is supposed that $r$ can adopt any non-negative value. The PPE units obtained by means of this available domestic production capacity will be bought by the health authorities at a price of $c$ MU. Let $p$ be the number of units produced using this available capacity. The supply of these units can start $\tau \leq \hat{t}$ units of time from the beginning of the epidemic, where the term $\tau$ includes both the time that elapses from the start of the epidemic until its detection and the lead time of the manufacturing plant. The average cost of having the capacity reserve $r$ is equal to $\varphi(r)$ MU per epidemic cycle, where $\varphi$ is a known increasing function for $r \geq 0$. Dedicated production capacity is available to meet the regular demand for PPE and to keep the level of shield stock constant under normal circumstances.

Maintaining and holding the shield stock has an average cost of $h$ MU per unit of PPE per cycle. After the outbreak of the epidemic, the level of shield stock should be restored to the same level as at the onset, not beyond the instant $\hat{T} \geq T$ (for the sake of simplicity the case with a different value of the stock at $t = 0$ and at $t = \hat{T}$, which require slight straightforward changes in the model, is not developed here). Note that the cost $h$ may have to include that of replacing units because of spoilage, since the time between successive outbreaks may be lengthy in terms of the product's life (Sodhi, Tang, and Willenson 2021, point out that managing a stockpile requires procurement, inventory rotation, audits, and inspection to ensure all items are in good working condition). It is assumed that the units that make up the shield stock are usable at the start of the pandemic (at least until produced or bought units are available) and that the product has a long enough life so that the units obtained during the planning horizon do not need to be replaced in the course of the pandemic.

The health authorities can also resort to buying from external suppliers, at a price $\tilde{c}$ MU, $q$ PPE units which will be delivered from the instant $\tilde{\tau} \leq \hat{t}$ at the appropriate pace. The number of units purchased does not have an upper bound and can be freely set by the health authorities. Note that a part of the total units bought, $q_s$, can be used to reduce the amount of the shield stock, while $q_a = q - q_s \geq 0$ will only be used, if necessary, to complete the total number of units required to restore the level of the shield stock not later than $\hat{T}$. Therefore,

$$p + q_s + q_a = D(T) \tag{1}$$

In the case of the first outbreak of COVID-19, it may be that in some countries the moments of the onset of domestic production and the arrival of imports ($\tau$ and $\tilde{\tau}$, respectively) have come after the peak of the epidemic. However, in a model for the design of the protection system it is reasonable to assume that these actions begin shortly after detection of the outbreak, because large stocks would otherwise be required. Additionally, the fact of having operational domestic production capacity and rapid access to imports provides more flexibility when dealing with fluctuations in infections or a longer duration of the outbreak than was initially forecast. In any case, relaxing the assumption $\tau, \tilde{\tau} \leq \hat{t}$ requires only basic changes that simplify both the optimisation model and the corresponding calculations.

The assumption that $q$ does not have an upper bound can be relaxed, this implying only minor changes in the model and a lower bound on $r$ to assure the feasibility of restoring the shield stock at $\hat{T}$. Given $r$, let $\Gamma(r; s, p, q_s, q_a)$ be the expected value of the cost per epidemic cycle:

$$\Gamma(r; s, p, q_s, q_a) = c \cdot p + h \cdot s + \tilde{c} \cdot (q_s + q_a) \quad (2)$$

And let $\gamma(r)$ be the cost of the optimal combination of shield stock level, domestic production and imports:

$$\gamma(r) = \min_{s,p,q_s,q_a} \Gamma(r; s, p, q_s, q_a) \quad (3)$$

Then, $\forall r \geq 0$, the optimal expected cost per epidemic cycle is

$$\Psi(r) = \varphi(r) + \gamma(r) \quad (4)$$

where $\Psi(r)$ is the objective function to be minimised.

Additionally, from now on it will be assumed that $\tau \leq \tilde{\tau}$. Certainly, the opposite case, $\tau > \tilde{\tau}$, may occur in practice. However, in order not to lengthen the paper unnecessarily, we will not deal with it, as it requires only simple changes in the developments corresponding to assumption $\tau \leq \tilde{\tau}$. These are slightly more complex because the level of domestic production between $\tau$ and $\tilde{\tau}$ must be decided, which does not apply if $\tau > \tilde{\tau}$.

To promptly satisfy the demand at any moment during the outbreak of the epidemic, the shield stock, together with any imported units, must be sufficient to cover the maximum difference between the accumulated demand and the domestic production. Let $\Delta(r)$ be this maximum difference when PPE units are produced at a rate $r$ during all the interval $[\tau, T]$:

$$\begin{aligned} \Delta(r) &= \max_{\tau \leq t \leq T} (D(t) - r \cdot (t - \tau)) \\ &= \max_{\tau \leq t \leq T} (D(t) - r \cdot t) + r \cdot \tau \quad (5) \end{aligned}$$

Let $\rho_D$ be the minimum value of $r$ so that $\Delta(r) = D(\tau)$ (Figure 1). Then, $\Delta(r)$ strictly decreases from $\Delta(0) = D(T)$ to $\Delta(r) = D(\tau)$ $r \geq \rho_D$.

Given $r \in [0, \rho_D]$, it must be taken into account that, by virtue of the previously stated assumptions concerning $D(t)$, this maximum difference (when it is not negative) corresponds to a value of $t \in [\hat{t}, T]$. Indeed, as for $t \in [\tau, \hat{t}]$ the expression $D(t) - r \cdot (t - \tau)$ is a convex function, if it has a point with null derivative in $[\tau, \hat{t}]$ it corresponds to a minimum which has a negative value (i.e. with the accumulated value of the domestic production greater than the accumulated demand). The function reaches its maximum positive value either at

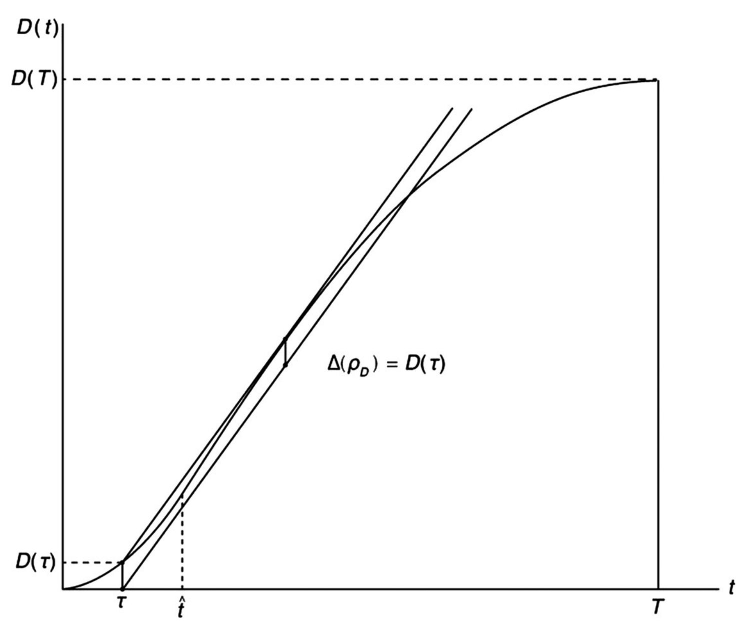



**Figure 1.** Minimum production ratio, $\rho_D$, to satisfy the demand using only the minimum shield stock and domestically produced units.

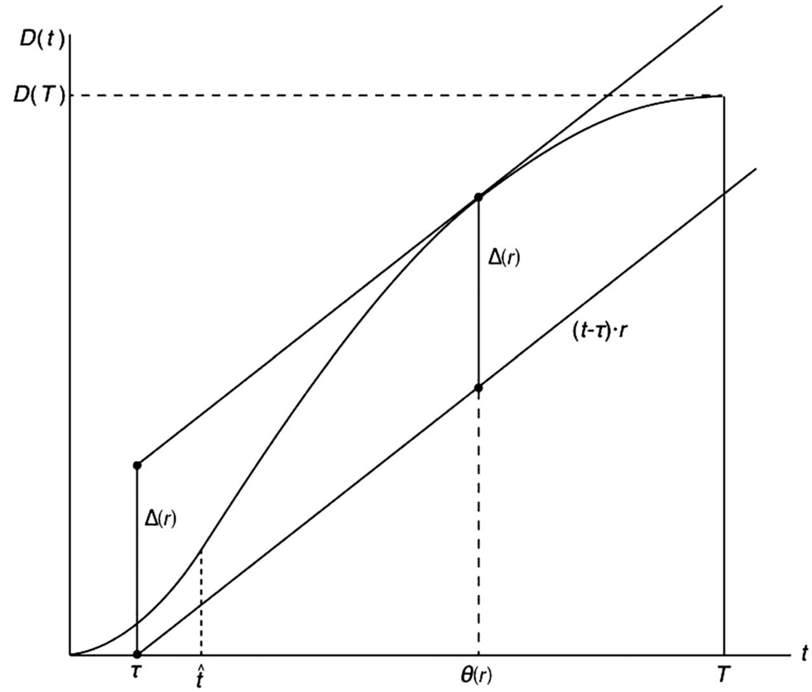



**Figure 2.** Maximum difference between accumulated demand and domestic production.

$t=\tau$ (where $D(t)-r\cdot(t-\tau)=D(\tau)\leq\Delta(r)\quad\forall r\in[0,\rho_D]$) or at $t=\hat{t}$. Hence, $\Delta(r)$ corresponds to the unique value of $t\in[\hat{t},T]$ such that the derivative of the expression $D(t)-r\cdot(t-\tau)$ with respect to $t$ is equal to 0, which leads to $d(t)=r$ (Figure 2).

Therefore, to determine $\Delta(r)$ we have to solve the equation:

$$d(t)=r \tag{6}$$

And substitute $t$ with its solution $\theta(r)$ at $D(t)-(t-\tau)\cdot r$.

From which it is:

$$\Delta(r)=D(\theta(r))-(\theta(r)-\tau)\cdot r \tag{7}$$

And, from $\frac{\mathrm{d}\Delta(r)}{\mathrm{d}r}=\frac{\mathrm{d}D(\theta(r))}{\mathrm{d}\theta}\cdot\frac{\mathrm{d}\theta}{\mathrm{d}r}-\frac{\mathrm{d}\theta}{\mathrm{d}r}\cdot r-(\theta(r)-\tau)$ and $\frac{\mathrm{d}D(\theta(r))}{\mathrm{d}\theta}=r$:

$$\mathrm{d}\Delta(r)/\mathrm{d}r=\tau-\theta(r)\leq 0 \tag{8}$$

Moreover, $\frac{d^2\Delta(r)}{\mathrm{d}r^2}=-\frac{\mathrm{d}\theta}{\mathrm{d}r}=-\left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^{-1}$ and $\frac{\mathrm{d}r}{\mathrm{d}\theta}=\frac{\mathrm{d}d(\theta)}{\mathrm{d}\theta}$, taking into account that $\theta(r)\geq\hat{t}$, giving:

$$\frac{\mathrm{d}^2\Delta(r)}{\mathrm{d}r^2}=-\left(\frac{\mathrm{d}d(t)}{\mathrm{d}t}\right)^{-1}_{|t=\theta(r)}>0 \tag{9}$$

The function $\Delta(r)$ is, therefore, decreasing and convex.

Therefore, the previously mentioned value $\rho_D$ is, taking (7) into account, the solution of the equation

$$D(\theta(r))-(\theta(r)-\tau)\cdot r=D(\tau) \tag{10}$$

And, in order to promptly satisfy the demand at any moment during the outbreak of the epidemic, the condition

$$s+q_s=\Delta(r) \tag{11}$$

must hold.

However, if $\tilde{c}<c$, it is better to import than manufacture the $(\hat{T}-\tilde{\tau})\cdot r$ units that could be obtained during the interval $[\tilde{\tau},\hat{T}]$ using the reserve of production capacity. In view of this observation, when $\tilde{c}<c$, the maximum difference between total demand and domestic production will be:

$$\hat{\Delta}(r)=D(T)-p \tag{12}$$

where $p$ is the domestic production during the interval $[\tau,\tilde{\tau}]$, so that:

$$0\leq p\leq(\tilde{\tau}-\tau)\cdot r \tag{13}$$

Moreover, there must be enough domestic production, together with any imports, to be able to restore the shield stock at $\hat{T}$. Let $P(r)=(\hat{T}-\tau)\cdot r$ be the potential domestic production between $\tau$ and $\hat{T}$ and $\rho_P$ so that $P(\rho_P)=D(T)$; i.e. the value of the available domestic production rate that would allow placing the desired level of stock at time $\hat{T}$ without resorting to imports. Then:

$$p \leq P(r) = (\hat{T} - \tau) \cdot r \tag{14}$$

$$\rho_P = \frac{D(T)}{\hat{T} - \tau} \tag{15}$$

Therefore, given that the cost $\varphi(r)$ is increasing, the values of $r$ to consider in the design of the system are those belonging to $[0, \hat{\rho}]$ where

$$\hat{\rho} = \max(\rho_D, \rho_P) \tag{16}$$

Then, the shield stock, $s$, in order to satisfy the accumulated demand at $\tau$ and $\tilde{\tau}$ (and therefore, since $D(t)$ is convex in $[0, \hat{t}]$, at any moment in $[0, \tilde{\tau}]$) must fulfil the following condition:

$$s \geq \max(D(\tau), D(\tilde{\tau}) - (\tilde{\tau} - \tau) \cdot r)$$

since any imported units cannot be available before $\tilde{\tau}$.

Now, let $\rho_{\tilde{\tau}}$ be the production rate for which $D(\tilde{\tau}) = D(\tau) + (\tilde{\tau} - \tau) \cdot r$; i.e. the minimum production rate at which the sum of a shield stock equal to $D(\tau)$ and the domestic production in $[\tau, \tilde{\tau}]$ is sufficient to meet the accumulated demand at $\tilde{\tau}$:

$$\rho_{\tilde{\tau}} = \frac{D(\tilde{\tau}) - D(\tau)}{\tilde{\tau} - \tau} \tag{17}$$

Then, $s$ has a lower bound, $\check{s}(r)$, which depends on $r$ as follows:

$$\check{s}(r) = D(\tilde{\tau}) - (\tilde{\tau} - \tau) \cdot \ \ r0 \leq r \leq \rho_{\tilde{\tau}} \tag{18}$$

$$\check{s}(r) = D(\tau) \ \rho_{\tilde{\tau}} \leq r \leq \hat{\rho} \tag{19}$$

Note that if $\tilde{\tau} < \tau$, the lower bound of $s$ is equal to $D(\tilde{\tau}) \, \forall r \in [0, \hat{\rho}]$ and the definition of $\rho_{\tilde{\tau}}$ makes no sense.

When the available production rate, $r$, is not high enough for $D(T)$ units to be manufactured from $\tau$ to $\hat{T}$, units must be imported to cover the difference $\delta(r)$, where

$$\begin{aligned}\delta(r) &= \max(D(T) - P(r), 0) \\ &= \max(D(T) - (\hat{T} - \tau) \cdot r, 0)\end{aligned} \tag{20}$$

Since $P(r)$ is the potential domestic production and not the actual one, $\delta(r)$ is a lower bound on $q$, the total number of imported PPE units. Even when the domestic production capacity can provide $D(T)$ PPE units throughout the interval $[\tau, \hat{T}]$, with the aim of minimising the total cost and depending on the values of $h, c$ and $\tilde{c}$, it may be appropriate to import a certain number of units instead of manufacturing them using domestic production capacity. As previously mentioned, part of the imported units, $q_s$, can be used to complement the shield stock in order to cover the maximum difference $\Delta(r)$ between accumulated demand and accumulated domestic production. Therefore,

$$q_s \leq \Delta(r) - \check{s}(r) \tag{21}$$

Note that $\Delta(r) - \check{s}(r) \geq 0 \, \forall r \in [0, \hat{\rho}]$ because $\Delta(r) - D(\tilde{\tau}) + (\tilde{\tau} - \tau) \cdot r$ has a negative derivative $(\tau - \theta(r) + \tilde{\tau} - \tau = \tilde{\tau} - \theta(r) < 0)$ and therefore decreases from $\Delta(0) - D(\tilde{\tau}) = D(T) - D(\tilde{\tau}) > 0$ to $\Delta(\rho_{\tilde{\tau}}) - D(\tau) \geq 0$.

Suppose that to minimise the total cost $q_s$ has to take its feasible maximum value, i.e. $q_s = \Delta(r) - \check{s}(r)$ (this happens, as will be made clearer below, when $\tilde{c} < c + h$) and $q_a$ just the value necessary to restore, along with $q_s$ and the domestic production, the shield stock, $s$, at $\hat{T}$ (this happens when $\tilde{c} \geq c$), i.e. $q_a = \max(\delta(r) - \Delta(r) + \check{s}(r), 0)$. Let $y(r) = \delta(r) - \Delta(r) - \check{s}(r)$. Then, $y(0) = \delta(0) - \Delta(0) + \check{s}(0) = D(T) - D(T) + D(\tilde{\tau}) = D(\tilde{\tau}) > 0$, while $y(\rho_P) = \delta(\rho_P) - \Delta(\rho_P) + \check{s}(\rho_P) = 0 - \Delta(\rho_P) + D(\tau) \leq 0$. Therefore, there is at least one value $r \in [0, \rho_P]$ for which $y(r) = 0$. Moreover, $y'(r) = -\hat{T} + \theta(r) + \tau - \tilde{\tau} < 0$ for $r \in [0, \rho_{\tilde{\tau}})$ and $y'(r) = -\hat{T} + \theta(r) < 0$ for $r > \rho_{\tilde{\tau}}$, i.e. $y(r)$ is a strictly decreasing function in $[0, \rho_P]$, which implies that $y(r) = 0$ has a unique solution. Let $\rho_a$ be this solution, i.e.

$$\rho_a = r | \delta(r) - \Delta(r) + \check{s}(r) = 0 \tag{22}$$

Then, when $q_s = \Delta(r) - \check{s}(r)$, $q_a = \delta(r) - \Delta(r) + \check{s}(r)$ $(r \in [0, \rho_a])$ and $q_a = 0$ $(r \geq \rho_a)$. Therefore, $r > \rho_a$ implies $q_s = q > \delta(r)$. On the other hand, since $\delta(\rho_P) = 0$, $\rho_a \leq \rho_P$.

Now, we have all the necessary elements to formulate the model for minimising the total average cost per epidemic cycle. The proposed notation, previously introduced together with the adopted assumptions, is summarised in the following list:

| | |
|---|---|
| $T$ | Duration of the epidemic outbreak |
| $D(t) 0 \leq t \leq T$ | Accumulated demand of PPE at $t$ |
| $d(t) = D'(t) 0 \leq t \leq T$ | Instantaneous rate of PPE demand |
| $h$ | Average holding cost per unit of PPE in the shield stock during an epidemic cycle |
| $s$ | Units of PPE in the shield stock |
| $r$ | Available domestic production rate |
| $\varphi(r)$ | Average cost, per epidemic cycle, of having available a production rate equal to $r$ |
| $\tau$ | Time, from the onset of the outbreak, at which domestic production can begin |
| $c$ | Price of domestically produced PPE |
| $p$ | Number of units of domestically produced PPE |

(*continued*).

| | |
|---|---|
| $\hat{T}$ | Time, from the onset of the outbreak, at which the shield stock level should be restored |
| $P(r) = (\hat{T} - \tau) \cdot r$ | Potential capacity of domestic production through $[\tau, \hat{T}]$ |
| $\delta(r) = \max(D(T) - P(r), 0)$ | Difference between the total demand of PPE during the outbreak and the potential capacity of domestic production through $[\tau, \hat{T}]$ |
| $\rho_P = D(T)/(\hat{T} - \tau)$ | Minimum production rate to manufacture $D(T)$ units of PPE through $[\tau, \hat{T}]$ |
| $\check{s}(r)$ | Lower bound of the shield stock, which depends on $r$. |
| $\tilde{\tau}$ | Time, from the onset of the outbreak, at which the arrival of imported units can begin |
| $\tilde{c}$ | Price of an imported unit of PPE |
| $\rho_{\tilde{\tau}} = (D(\tilde{\tau}) - D(\tau))/(\tilde{\tau} - \tau)$ | Minimum production rate to meet the demand in $[\tau, \tilde{\tau}]$ with domestic units when $s = D(\tau)$ |
| $q$ | Total number of imported units |
| $\Delta(r) = \max_{\tau \le t \le T} (D(t) - r \cdot (t - \tau))$ | Maximum difference between the accumulated values of demand and domestic production |
| $\hat{\Delta}(r) = D(T) - p$ | Maximum difference between the total demand and domestic production when $r \le \rho_{\tilde{\tau}}$ and nothing is produced throughout $[\tilde{\tau}, \hat{T}]$ |
| $\rho_D$ | Production rate necessary to have $\Delta(r) = D(\tau)$ |
| $\hat{\rho} = \max(\rho_D, \rho_P)$ | Upper bound on $r$ |
| $q_s$ | Imported units used, together with those of the shield stock, to cover the difference $\Delta(r)$ |
| $q_a$ | Imported units used, together with $q_s$, to cover the difference $\delta(r)$ |
| $\rho_a$ | Production rate for which, when $q_s$ takes its maximum feasible value, $q_a = 0$ |
| $\Gamma(r; s, p, q_s, q_a)$ | Average cost, per epidemic cycle, of producing domestic units, importing and maintaining the shield stock |
| $\gamma(r) = \min_{s,p,q_s,q_a} \Gamma(r; s, p, q_s, q_a)$ | Minimum of $\Gamma$ function for a given value of $r$ |
| $\Psi(r) = \varphi(r) + \gamma(r)$ | Total average cost per epidemic cycle |

Additionally, the notation for an optimal value of a variable, $x$, is $x^*$.

Therefore, the optimisation model, bearing (1), (11)–(14) and (16)–(18) in mind, can be formulated as follows (in fact, this model can be separated into two: one for $\tilde{c} \ge c$ and another for $\tilde{c} < c$):

$$\begin{aligned}
&\text{OPTM-1 minimise } \Psi(r) = \varphi(r) + \Gamma(r; s, p, q_s, q_a) = \\
&\quad \varphi(r) + c \cdot p + h \cdot s + \tilde{c} \cdot (q_s + q_a) \\
&\quad \text{s.t.} \\
&\quad p + q_s + q_a = D(T) \\
&\quad s + q_s = \Delta(r) \quad \tilde{c} \ge c \\
&\quad s + q_s = \hat{\Delta}(r) \quad \tilde{c} < c \\
&\quad s \ge \check{s}(r) \\
&\quad p \le (\hat{T} - \tau) \cdot r \quad \tilde{c} \ge c \\
&\quad p \le (\tilde{\tau} - \tau) \cdot r \quad \tilde{c} < c \\
&\quad r \le \hat{\rho} \\
&\quad r, s, p, q_s, q_a \ge 0
\end{aligned}$$

Although the model involves few variables, it is difficult to solve directly because of the non-linearity of the objective function and the constraints $s + q_s = \Delta(r)$ and $s + q_s = \hat{\Delta}(r)$. Moreover, the constraint $s \ge \check{s}(r)$ must be developed, since its formulation is different ((17) or (18)), depending on the value of $r$. Note, however, that OPTM-1 is equivalent to

$$\begin{aligned}
&\text{OPTM-2 minimise } \Psi(r) = \varphi(r) + \gamma(r) \\
&\text{s.t.} \quad 0 \le r \le \hat{\rho}
\end{aligned}$$

where $\gamma(r) = \min_{s,p,q_s,q_a} \Gamma(r; s, p, q_s, q_a)$, subject to the appropiate constraints. This way, the problem basically involves finding the relevant expressions for $\gamma(r)$, as then the objective function depends on a single variable and its optimisation does not present any special difficulty.

To determine $\gamma(r)$, the following optimisation problem has to be solved:

$$\begin{aligned}
&\text{OPTM-3 minimise } \Gamma(r; s, p, q_s, q_a) \\
&\quad = c \cdot p + h \cdot s + \tilde{c} \cdot (q_s + q_a) \\
&\quad \text{s.t.} \\
&\quad p + q_s + q_a = D(T) \\
&\quad s + q_s = \Delta(r) \quad \tilde{c} \ge c \\
&\quad s + q_s = \hat{\Delta}(r) \quad \tilde{c} < c \\
&\quad s \ge \check{s}(r) \\
&\quad p \le (\hat{T} - \tau) \cdot r \quad \tilde{c} \ge c \\
&\quad p \le (\tilde{\tau} - \tau) \cdot r \quad \tilde{c} < c \\
&\quad s, p, q_s, q_a \ge 0
\end{aligned}$$

where $\gamma$ is not a variable, but a parameter. Using $p + q_s + q_a = D(T)$, $p$ can be replaced with $D(T) - q_s - q_a$ in the objective function; on the other hand, using $s + q_s = \Delta(r)$ or $s + q_s = \hat{\Delta}(r)$, $s$ can be replaced, respectively, with $\hat{\Delta}(r) - q_s$ or $\Delta(r) - q_s$. After these replacements, the following optimisation model is obtained:

$$\begin{aligned}
&\text{OPTM-4 minimise } c \cdot D(T) + h \cdot \Delta(r) \\
&\quad + (\tilde{c} - c - h) \cdot q_s + (\tilde{c} - c) \cdot q_a \\
&\quad \text{s.t.} \\
&\quad q_s \le \Delta(r) - \check{s}(r) \\
&\quad p \le (\hat{T} - \tau) \cdot r \quad \tilde{c} \ge c \\
&\quad p \le (\tilde{\tau} - \tau) \cdot r \quad \tilde{c} < c
\end{aligned}$$

$$p + q_s + q_a = D(T)$$
$$p, q_s, q_a \geq 0$$

where the term $h \cdot \Delta(r)$ in the objective function should be replaced with $h \cdot \hat{\Delta}(r)$ when $\tilde{c} < c$.

## 4. Cost optimisation for given values of *r*

As indicated in the previous section, to solve the problem the $\gamma(r)$ function must be specified. This will come from solving OPTM-4, whose objective function shows that the structure of the optimal solution (i.e. the one that minimises $\Gamma(r)$ for a given value of $r$) depends essentially on the relationship between the values of $c, \tilde{c}$ and $h$.

There are two main cases concerning the costs relationship, namely: $\tilde{c} < c$ and $c \leq \tilde{c}$. Moreover, the characteristics of the optimal solutions also depend on the relationship of the value of $r$ with those of $\rho_{\tilde{\tau}}, \rho_{q_a}, \rho_P$ and $\rho_P$.

As only the two variables concerning imports, $q_s$ and $q_a$, are involved in the objective function of OPTM-4, in order to find the optimal solutions each one of these variables must take their maximum or minimum value compatible with the constraints, whether its coefficient in the objective function is negative or positive. The values of the other decision variables, $s$ and $p$, are easily deduced from those of $q_s$ and $q_a$.

### 4.1. Case $\tilde{c} < c$

As pointed out above, production can be ruled out when importing is cheaper than producing, except perhaps in the interval $[\tau, \tilde{\tau}]$, since domestic production in this interval may contribute to reducing the level of the shield stock and, therefore, the corresponding holding cost. Moreover, since the maximum reduction of the shield due to produced units is $D(\tilde{\tau}) - D(\tau)$ and $\varphi(r)$ is increasing, it is not necessary to consider values of $r$ greater than $\rho_{\tilde{\tau}}$. Then, we have:

$$\text{OPTM-5 minimise } c \cdot D(T) + h \cdot \hat{\Delta}(r)$$
$$+ (\tilde{c} - c - h) \cdot q_s + (\tilde{c} - c) \cdot q_a$$
$$\text{s.t.}$$
$$q_s \leq \hat{\Delta}(r) - D(\tilde{\tau}) + p$$
$$0 \leq p \leq (\tilde{\tau} - \tau) \cdot r$$
$$p + q_s + q_a = D(T)$$
$$p, q_s, q_a \geq 0$$

where according to (12) $\hat{\Delta}(r) = D(T) - p$, giving $q_s \leq D(T) - D(\tilde{\tau})$. Since the coefficients of $q_s$ and $q_a$ in the objective function are negative, these variables must take their maximum feasible values, i.e. $q_s = D(T) - D(\tilde{\tau}), q_a = D(\tilde{\tau}) - p$, giving

$$\Gamma(r) = \tilde{c} \cdot D(T) + h \cdot D(\tilde{\tau}) + (c - h - \tilde{c}) \cdot p \quad (23)$$

To minimise this expression we must make $p = 0$ or $p = (\tilde{\tau} - \tau) \cdot r$, depending on whether the sign of the coefficient $(c - h - \tilde{c})$ is positive or negative, respectively (if $c - h - \tilde{c} = 0, p$ can take any value between 0 and $(\tilde{\tau} - \tau) \cdot r$). Hence:

For $\tilde{c} \leq c - h$:

$$q_s = D(T) - D(\tilde{\tau}) \quad (24)$$
$$q_a = D(\tilde{\tau}) \quad (25)$$
$$\gamma(r) = \tilde{c} \cdot D(T) + h \cdot D(\tilde{\tau}) \quad (26)$$

Which is independent from $r$ and, with respect to $D(t)$, depends only on the total demand, $D(T)$, and on the accumulated demand up to the instant $\tilde{\tau}$, $D(\tilde{\tau})$.

For $c - h \leq \tilde{c} < c$:

$$q_s = D(T) - D(\tilde{\tau}) \quad (27)$$
$$q_a = D(\tilde{\tau}) - (\tilde{\tau} - \tau) \cdot r \quad (28)$$
$$\gamma(r) = \tilde{c} \cdot D(T) + h \cdot D(\tilde{\tau}) - (\tilde{c} + h - c) \cdot (\tilde{\tau} - \tau) \cdot r \quad (29)$$

which, with respect to $D(t)$, as in the previous equation, depends only on $D(T)$ and $D(\tilde{\tau})$.

### 4.2. Case $c \leq \tilde{c}$

In this case OPTM-4 comes to

$$\text{OPTM-6 minimise } c \cdot D(T) + h \cdot \Delta(r)$$
$$+ (\tilde{c} - c - h) \cdot q_s + (\tilde{c} - c) \cdot q_a$$
$$\text{s.t.}$$
$$q_s \leq \Delta(r) - \check{s}(r)$$
$$p \leq (\hat{T} - \tau) \cdot r$$
$$p + q_s + q_a = D(T)$$
$$p, q_s, q_a \geq 0$$

The coefficient of $q_a$ is non-negative. However, that of $q_s$ may be negative or non-negative, depending on whether $\tilde{c} < c + h$ or $c + h \leq \tilde{c}$.

*Subcase* $c \leq \tilde{c} < c + h$

Since the coefficients of $q_s$ and $q_a$ in the objective function are negative and non-negative, respectively, $q_s$ must take the highest possible value, $q_s = \Delta(r) - \check{s}(r)$, and $q_a$ the lowest.

For $0 \leq r \leq \rho_{\tilde{\tau}}$ ($\check{s}(r) = D(\tilde{\tau}) - (\tilde{\tau} - \tau) \cdot r$):

$0 \le r \le \rho_{\tilde{\tau}}$ and $r \le \rho_a$:

$$q_s = \Delta(r) - D(\tilde{\tau}) + (\tilde{\tau} - \tau) \cdot r \tag{30}$$

$$q_a = D(T) - (\hat{T} - \tau) \cdot r - q_s \tag{31}$$

$$\gamma(r) = \tilde{c} \cdot D(T) + h \cdot D(\tilde{\tau}) - (h \cdot (\tilde{\tau} - \tau) + (\tilde{c} - c)(\hat{T} - \tau)) \cdot r \tag{32}$$

$0 \le r \le \rho_{\tilde{\tau}}$ and $r \ge \rho_a$:

$$q_s = \Delta(r) - D(\tilde{\tau}) + (\tilde{\tau} - \tau) \cdot r \tag{33}$$

$$q_a = 0 \tag{34}$$

$$\gamma(r) = c \cdot D(T) + (h + c - \tilde{c}) \cdot D(\tilde{\tau}) + (c - \tilde{c}) \cdot \Delta(r) - (h + c - \tilde{c}) \cdot (\tilde{\tau} - \tau) \cdot r \tag{35}$$

For $\rho_{\tilde{\tau}} \le r \le \hat{\rho}$ ($\check{s}(r) = D(\tau)$):

$\rho_{\tilde{\tau}} \le r \le \hat{\rho}$ and $r \le \rho_a$

$$q_s = \Delta(r) - D(\tau) \tag{36}$$

$$q_a = D(T) - (\hat{T} - \tau) \cdot r - q_s \tag{37}$$

$$\gamma(r) = \tilde{c} \cdot D(T) + h \cdot D(\tau) + (c - \tilde{c}) \cdot (\hat{T} - \tau) \cdot r \tag{38}$$

$\rho_{\tilde{\tau}} \le r \le \hat{\rho}$ and $r \ge \rho_a$

$$q_s = \Delta(r) - D(\tau) \tag{39}$$

$$q_a = 0 \tag{40}$$

$$\gamma(r) = c \cdot D(T) + (h + c - \tilde{c}) \cdot D(\tau) + (\tilde{c} - c) \cdot \Delta(r) \tag{41}$$

*Subcase $c + h \le \tilde{c}$*

With the coefficients of $q_s$ and $q_a$ being non-negative in the objective function, the sum of both variables must have its minimum feasible value, i.e. $q_s + q_a = \delta(r)$; if $\delta(r) > 0$, as the coefficient of $q_s$ is less than that of $q_a$, the former can take the maximum value compatible with the constraints.

For $0 \le r \le \rho_{\tilde{\tau}}$ and $r \le \rho_a$:

$$q_s = \Delta(r) - D(\tilde{\tau}) + (\tilde{\tau} - \tau) \cdot r \tag{42}$$

$$q_a = D(T) - (\hat{T} - \tau) \cdot r - q_s \tag{43}$$

$$\gamma(r) = \tilde{c} \cdot D(T) + h \cdot D(\tilde{\tau}) - (h \cdot (\tilde{\tau} - \tau) + (\tilde{c} - c) \cdot (\hat{T} - \tau)) \cdot r \tag{44}$$

For $\rho_{\tilde{\tau}} \le r \le \hat{\rho}$ and $r \le \rho_a$:

$$q_s = \Delta(r) - D(\tau) \tag{45}$$

$$q_a = D(T) - (\hat{T} - \tau) \cdot r - q_s \tag{46}$$

$$\gamma(r) = \tilde{c} \cdot D(T) + h \cdot D(\tau) - (\tilde{c} - c) \cdot (\hat{T} - \tau) \cdot r \tag{47}$$

$\rho_{q_a} \le r \le \rho_P$:

$$q_s = D(T) - (\hat{T} - \tau) \cdot r \tag{48}$$

$$q_a = 0 \tag{49}$$

$$\gamma(r) = (\tilde{c} - h) \cdot D(T) + h \cdot \Delta(r) - (\tilde{c} - c - h) \cdot (\hat{T} - \tau) \cdot r \tag{50}$$

$r \ge \rho_P$:

$$q_s = q_a = 0 \tag{51}$$

$$\gamma(r) = c \cdot D(T) + h \cdot \Delta(r) \tag{52}$$

### 4.3. Summary of the cost optimisation procedure

In all the cases and subcases studied in 4.1 and 4.2, we have deduced formulas giving, for each value of $r$, the optimum values of the decision variables and of $\gamma(r)$. Note that the data and the functions involved in the formulas are different, depending on the relationships between the unit costs.

Therefore, given $\varphi(r)$, the objective function $\Psi(r) = \varphi(r) + \gamma(r)$ is also defined for any value of the parameters (note that it is reasonable to suppose that the function $\varphi$, corresponding to having a production rate available in $[\tau, \hat{T}]$,is different from that for availability in $[\tau, \tilde{\tau}]$).

The optimisation of $\Psi(r)$ does not present any major difficulty, because it depends on a single variable and can be computed for any of its values. Given the unit costs and under the adopted assumptions $\gamma(r)$, which is decreasing, is derivable in each of the relevant subintervals of $[0, \hat{\rho}]$. If $\varphi(r)$, which is increasing, is also derivable then $\Psi(r)$ is derivable in each subinterval; the necessary condition for a point to be optimal is when the derivative is zero, or is a point with a discontinuity in the derivative going from negative (left) to positive (right). However, it is generally not possible to find a closed form expression for the optimal value, $r^*$, of the available rate of domestic production.

Once the optimal value of the available production rate is obtained, finding the optimal values of the other variables is straightforward, using the appropriate formulas from those we have seen in 4.1 and 4.2.

The optimisation procedure can be schematised as follows:

Given the $h, c, \tilde{c}, \tau, \tilde{\tau}, T$ and the function $d(t)$:

- $\tilde{c} \le c - h$: Find $D(\tau)$and $D(T)$, apply (24)-(26).
- $c - h \le \tilde{c} < c$: Given $\varphi(r)$, find $D(\tilde{\tau})$and $D(T)$, apply (29) and (4), find $r^*$and apply (27)–(28) with $r = r^*$.
- $c \le \tilde{c} < c + h$: Given $\varphi(r)$, find $D(\tau), D(\tilde{\tau})$, $D(T)$, $\Delta(r)$, $\rho_P$, $\rho_D$, $\hat{\rho}$, $\rho_{\tilde{\tau}}$, and $\rho_a$.
  $0 \le r \le \rho_{\tilde{\tau}}$ and $r \le \rho_a$ (CON1): apply (32) and (4).

$0 \leq r \leq \rho_{\tilde{\tau}}$ and $r \geq \rho_a$ (CON2): apply (35) and (4).
$\rho_{\tilde{\tau}} \leq r \leq \hat{\rho}$ and $r \leq \rho_a$ (CON3): apply (38) and (4).
$\rho_{\tilde{\tau}} \leq r \leq \hat{\rho}$ and $r \geq \rho_a$ (CON4): apply (41) and (4).
Find $r^*$ and apply (30)–(31), (33)–(34), (36)–(37) or (39)–(40), according to whether $r^*$ fulfils, respectively, CON1, CON2, CON3 or CON 4.

- $c + h \leq \tilde{c}$: Given $\varphi(r)$, find $D(\tau), D(\tilde{\tau}),\ D(T),\ \Delta(r)$, $\rho_P, \rho_D, \hat{\rho}, \rho_{\tilde{\tau}}$, and $\rho_a$.
  $0 \leq r \leq \rho_{\tilde{\tau}}$ and $r \leq \rho_a$ (CON1): apply (44) and (4).
  $\rho_{\tilde{\tau}} \leq r \leq \hat{\rho}$ and $r \leq \rho_a$ (CON3): apply (47) and (4).
  $\rho_a \leq r \leq \rho_P$ (CON5) apply (50) and (4).
  $r \geq \rho_P$ (CON6) apply (52) and (4).
  Find $r^*$ and apply (41)–(42), (45)–(46), (48)–(49) or (51), according to whether $r^*$ fulfils, respectively, CON1, CON3, CON5 or CON 6.

Once the optimal solution has been found, the corresponding objective function can be used to analyse the sensitivity of the optimal values of the cost and variables with respect to variations of the data. Take, for instance, Equation (35), and suppose that there is a delay, $\varepsilon$, in detecting the onset of the outbreak, small enough not to change the structure of the optimal solution. This means an increase of $\Delta(r)$ equal to $D(\tau + \varepsilon) - D(\tau)$ and that the values $\tau$ and $\tilde{\tau}$ have to be replaced, respectively, with $\tau + \varepsilon$ and $\tilde{\tau} + \varepsilon$. From this observation it can be deduced that the increase in the total cost due to delay is equal to $(h + c - \tilde{c}) \cdot (D(\tilde{\tau} + \varepsilon) - D(\tilde{\tau})) + (c - \tilde{c}) \cdot (D(\tau + \varepsilon) - D(\tau))$, with, therefore, a local rate of increase equal to $(h + c - \tilde{c}) \cdot d(\tilde{\tau}) + (c - \tilde{c}) \cdot d(t)$. Similarly, a sensitivity analysis can be performed in relation to other parameters, such as $\hat{T}$ or to variations in the ordinates of the function $d(t)$.

## 5. Application to the case of triangular propagation

In this section we use a triangular pattern of epidemic's spread (Figure 3) to illustrate the application of the above results. It is not claimed, however, that the expansion of a pandemic like COVID-19 fits this pattern. The mere visual examination of the infection curves as a function of time shows that a triangle that is never below the observed curve can differ significantly from it especially at the beginning and end of the epidemic, since initially its growth is relatively slow and at the end, when the infection rate is already very low, the relative decrease also is. However, the calculations to be made if a different pattern is adopted are similar, though perhaps a little more laborious, to those presented in this section.

Regarding the triangular spreading pattern, the notation and numeric values used are as follows:

$$d(t) = \alpha \cdot \ t(0 \leq t \leq \hat{t}),$$

$$d(t) = \beta \cdot (T - t)\ (\hat{t} \leq t \leq T) \tag{53}$$

$$\alpha \cdot \hat{t} = \beta \cdot (T - \hat{t}\ ) \tag{54}$$

and therefore:

$$\hat{t} = \beta / (\alpha + \beta) \cdot T \tag{55}$$

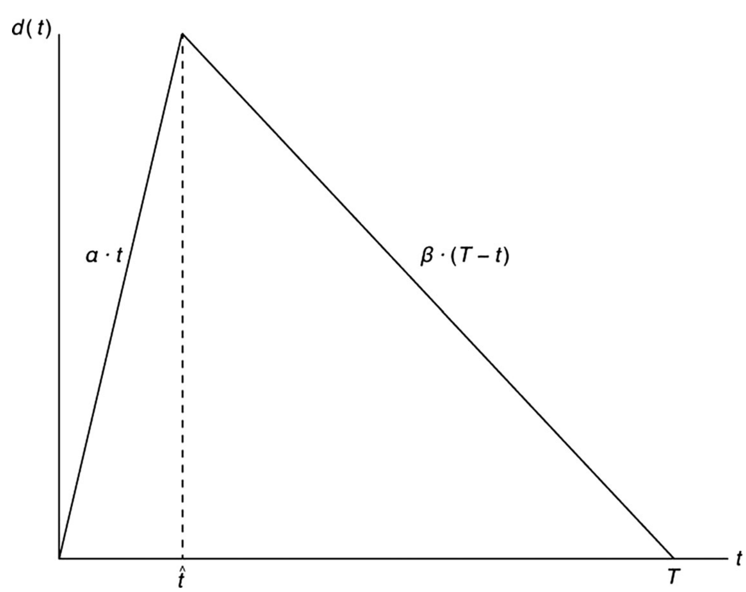



**Figure 3.** Triangular pattern of the epidemic spread.

If, as usual, the descent of the epidemic lasts longer than the rise, we will have $\beta < \alpha$.

So:

$$D(t) = \frac{\alpha}{2} \cdot t^2 \quad 0 \le t \le \hat{t} \tag{56}$$

$$D(t) = -\frac{\beta}{2} \cdot t^2 + \beta \cdot T \cdot t - \frac{\beta}{2} \cdot T \cdot \hat{t} \quad \hat{t} \le t \le T \tag{57}$$

$$D(T) = \frac{\alpha \cdot \beta}{2 \cdot (\alpha + \beta)} \cdot T^2 = \frac{\alpha}{2} \cdot T \cdot \hat{t} \tag{58}$$

We also present numerical results for four values of $\tilde{c}$, corresponding to the values:

$$h = 2,\ c = 5,\ \alpha = 60,\ \beta = 15,\ T = 100,$$
$$\tau = 5,\ \tilde{\tau} = 10,\ \hat{T} = 150$$

And, therefore:

$$\hat{t} = 20, D(\tau) = 750, D(\tilde{\tau}) = 3,000,$$
$$D(T) = 60,000, \rho_{\tilde{\tau}} = 450, \rho_P = 413.79$$

It is also assumed that $\varphi(r) = 60 \cdot r$.

Given that in this section we consider the four possible relevant relationships between $h, c$ and $\tilde{c}$, there are elements (values and functions) involved in more than one of these cases; we will determine all of these elements before presenting the solution for the four values of $\tilde{c}$ considered.

Equations (6) and (7) must be used to determine $\Delta(r)$. The maximum difference between demand and domestic production, $\forall r \in [0, \rho_D]$, is reached at $t | d(t) = r = \beta \cdot (T - t)$, i.e. $\theta(r) = T - r/\beta$ (for $r > \rho_D, \Delta(r) = D(\tau)$). Then:

$$\Delta(r) = \frac{r^2}{2 \cdot \beta} - (T - \tau) \cdot r + D(T) \quad \forall r \in [0, \rho_D];$$
$$\Delta(r) = D(\tau) \quad \forall r > \rho_D \tag{59}$$

From (45) we can determine $\rho_D$ with the Equation (10):

$$\Delta(r) = \frac{r^2}{2 \cdot \beta} - (T - \tau) \cdot r + D(T) = D(\tau) \tag{60}$$

Which in the triangular case is, therefore, a second degree equation.

$$\frac{d\Delta(r)}{dr} = \frac{r}{\beta} - (T - \tau) \quad (\le 0 \forall r \in [0, \beta \cdot (T - \tau)]) \tag{61}$$

$$\frac{d^2\Delta(r)}{dr^2} = \frac{1}{\beta} > 0 \tag{62}$$

Therefore $\Delta(r)$, as shown in Section 3, is convex and non-increasing in the whole set of valid values of $r$ (since $\rho_D \le \beta \cdot (T - \hat{t}) \le \beta \cdot (T - \tau)$).

For the numerical example,

$$\Delta(r) = \frac{r^2}{30} - 95 \cdot r + 60.000 \quad \forall r \in [0, \rho_D] \tag{63}$$

And the solution of (46) $\Delta(r) = D(\tau) = 750$ gives $\rho_D = 921.88$ and $\hat{\rho} = \max(\rho_P, \rho_D) = 921.88$.

On the other hand, solving (22), i.e. $D(T) - (\hat{T} - \tau) \cdot r - \Delta(r) + \check{s}(r) = 0$, which is also a second degree equation, gives for $\check{s}(r) = D(\tilde{\tau}) - (\tilde{\tau} - \tau) \cdot r = 3{,}000 - 5 \cdot r$, $\rho_a = 52.85$.

We now have all the elements to find the optimal solution in any case for the value of $\tilde{c}$.

$\tilde{c} < c - h$; example: $\tilde{c} = 2$.

$$\Psi^* = 2 \cdot 60,000 + 2 \cdot 3,000 = 126,000\text{MU}$$
$$r^* = 0; s^* = 3,000; q_s^* = 57,000; q_a^* = 3,000; p^* = 0$$

$c - h \le \tilde{c} < c$; example: $\tilde{c} = 4$.

$$\Psi(r) = 60 \cdot r + 246,000 - 5 \cdot r = 246,000 + 55 \cdot r (0 \le r \le 450)$$
$$r^* = 0; s^* = 3,000; q_s^* = 57,000; q_a^* = 3,000;$$
$$p^* = 0; \Psi^* = 246,000$$

$c \le \tilde{c} \le c + h$; example: $\tilde{c} = 6$.

$$0 \le r \le \rho_a = 52.85 \quad \gamma(r) = 366,000 - 155 \cdot r$$
$$52.85 = \rho_a \le r \le \rho_{\tilde{\tau}} = 450$$
$$\gamma(r) = 363,000 - 100 \cdot r + \frac{r^2}{30}$$
$$450 = \rho_{\tilde{\tau}} \le r \le \hat{\rho} = 921.88$$
$$\gamma(r) = 360,750 - 95 \cdot r + \frac{r^2}{30}$$
$$\Psi(r) = \varphi(r) + \gamma(r) = 60 \cdot r + \gamma(r)$$

There are no sign changes of the derivative at the points separating the intervals and $\Psi(r)$ has zero derivative at $r = 525$:

$$r^* = 525; \Delta(r^*) = 19,312.50; s^* = 750;$$
$$q_s^* = 18,562.50; q_a^* = 0; p^* = 41,437.50;$$
$$\Psi^* = 351,562.50$$

$c + h \le \tilde{c}$; example: $\tilde{c} = 8$.

$$0 \le r \le \rho_a = 52.85 \quad \gamma(r) = 486,000 - 445 \cdot r$$
$$52.85 = \rho_a \le r \le \rho_P = 413.79$$

$$\gamma(r) = 480,000 - 335 \cdot r + \frac{r^2}{15}$$

$$413.79 = \rho_P \leq r \leq \hat{\rho} = 921.88$$

$$\gamma(r) = 420,000 - 190 \cdot r + \frac{r^2}{15}$$

$$\Psi(r) = \varphi(r) + \gamma(r) = 60 \cdot r + \gamma(r)$$

There are no sign changes of the derivative at the points separating the intervals and the derivative of $\Psi(r)$ is negative $\forall r \in [0, \hat{\rho}]$, so:

$$r^* = 921.88; \Delta(r^*) = 750; s^* = 750; q_s^* = q_a^* = 0;$$
$$p^* = 60,000; \Psi^* = 356,812.80$$

In the case that domestic production capacity has a fixed cost, $K$, it is necessary to add it to the optimal cost obtained by considering only the variable costs and to compare the total cost with that corresponding to $r = 0$. For example, for $\tilde{c} = 6$ the cost would be $K + 351,562.50$, which should be compared to the cost of having a shield stock equal to 3,000 and buying $D(T)$ units, i.e. $2 \cdot 3,000 + 6 \cdot 60,000 = 366,000$.

## 6. Conclusions and possible extensions

The recent intense universal epidemic of COVID-19 has emphasised the need to have robust systems for supplying essential protection, diagnostic and treatment products, which experience a sudden and intense increase in demand for a limited time when an epidemic or similar incident occurs.

This paper describes the relevant decisions for determining the basic dimensions of a procurement and inventory system for these products. It presents a model to optimise the aforementioned decisions, in which the key variable is the available rate of domestic production, from which the level of the permanently available shield stock and the quantity that must be imported are derived. The analysis of the model shows that there are four basic situations, depending on the relationship between the purchase price of the product in the market and the production and holding costs. Equations are shown for each of the possible situations, under very general assumptions; when adapted to the specific propagation curve in each case, they allow the total cost (which includes holding, production and purchasing costs) to be determined in terms of the available rate of production. The objective function results from adding this function (which obviously decreases strictly in the range of permitted values for the available rate of production) to the increasing function, which gives the cost corresponding to the availability of capacity. The properties of the objective function depend on those of the two functions that compose it, but ultimately it is a function of a single variable defined in a finite interval, so its optimisation does not present any special difficulty.

The use of the model is illustrated by its application to a triangular-type epidemic spread and some numerical examples.

Summing up, concerning the quantitative aspects of the decisions involved, the model shows that the main dimensions of the system can be specified, under very general assumptions, with very little computation effort.

From the point of view of public health management, the proposed model highlights some of the most relevant decisions to protect citizens against an epidemic (the logistics systems for the supply and distribution of the product are outside the frame of the model). On the one hand, it is necessary to size and set up a system of storage and permanent conservation of shield stock. On the other hand, there must be current contracts with domestic industries that will guarantee the rapid availability of productive capacity. Finally, it is necessary to establish the commercial relations that will allow essential products to be imported in the short term.

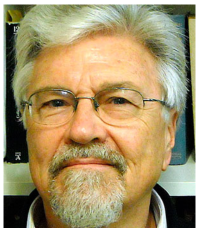
The model also illustrates the fact that the imported units, in addition to being necessary in many cases to complement domestic production in order to restore the shield stock before the next outbreak of the epidemic, can be useful for reducing costs. Even though they might cost more than domestically produced units, they allow the size of the shield stock to be reduced and therefore save the corresponding holding costs.

The limitations of this work derive from the assumptions adopted for the optimisation model approach. As indicated in Section 3, these assumptions are reasonably realistic but might not reflect all the scenarios that may actually arise. In some cases (unlimited imports, positions in the outbreak of $\tau$ and $\tilde{\tau}$ and the relationship between both moments) it has already been indicated that if these assumptions are modified the changes required in the model are slight and may even simplify the formulation and calculations. In others, they require substantial modifications and, therefore, may be proposed as future lines of research, such as: the consideration of risk (in demand forecasts, the start, volume and price of domestic production and imports), prices dependent on quantity, the possible nonlinearity of the shield stock holding costs, the existence of several suppliers with limited capacities and different prices and, in the case of items having a short life relative to the duration of the outbreaks, the repercussions of product deterioration with the passing of time.

## Disclosure statement

No potential conflict of interest was reported by the author.

## Notes on contributor



*Albert Corominas* is an emeritus professor at the Universitat Politècnica de Catalunya (UPC), Spain. He holds degrees in computer science from the Universidad Politécnica de Madrid and in engineering from the Universidad del País Vasco (UPV) and a PhD that is also from the UPV. After having worked as a consultant, he taught operations research and industrial engineering, for more than forty years, at the UPC and other universities. His current research interests include supply chain design, operations management, scheduling and combinatorial optimisation. He has been the director of several research projects and is author or co-author of books and papers that have been published in journals such as IJPR, Omega, IJPE, EJOR and JORS.

## References

Balcik, B., and D. Ak. 2014. "Supplier Selection for Framework Agreements in Humanitarian Relief." *Production and Operations Management* 23 (6): 1028–1041.

Balcik, B., C. D. C. Bozkir, and O. E. Kundakcioglu. 2016. "A Literature Review on Inventory Management in Humanitarian Supply Chains." *Surveys in Operation Research and Management Science* 21 (2): 101–116.

Boin, A., P. Kelle, and D. C. Whybark. 2010. "Resilient Supply Chains for Extreme Situations: Outlining a New Field of Study." *International Journal of Production Economics* 126 (1): 1–6.

Dasaklis, T. K., C. P. Pappis, and N. P. Rachaniotis. 2012. "Epidemics Control and Logistics Operations: A Review." *International Journal of Production Economics* 139: 393–410.

Day, J. M. 2014. "Fostering Emergent Resilience: The Complex Adaptive Supply Network of Disaster Relief." *International Journal of Production Research* 52 (7): 1970–1988. doi:10.1080/00207543.2013.787496.

Hale, T., and C. R. Moberg. 2005. "Improving Supply Chain Disaster Preparedness: A Decision Process for Secure Site Location." *International Journal of Physical Distribution & Logistics Management* 35 (3): 195–207. doi:10.1108/09600030510594576.

Hansen, O., H. Friedrich, and S. Transchel. 2020. "An Inventory Management Approximation for Estimating Aggregated Regional Food Stock Levels." *International Journal of Production Research* 58 (19): 5769–5785. doi:10.1080/00207543.2019.1657248.

Harrington, J. E., and E. B. Hsu. 2010. "Stockpiling Anti-Viral Drugs for a Pandemic: The Role of Manufacturer Reserve Programs." *Journal of Health Economics* 29 (3): 438–444. doi:10.1016/j.jhealeco.2010.02.004.

Hashikura, M., and J. Kizu. 2009. "Stockpile of Personal Protective Equipment in Hospital Settings: Preparedness for Influenza Pandemics." *American Journal of Infection Control* 37: 703–707. doi:10.1016/j.ajic.2009.05.002.

Hu, S. L., C. F. Han, and L. P. Meng. 2017. "Stochastic Optimization for Joint Decision Making of Inventory and Procurement in Humanitarian Relief." *Computers & Industrial Engineering* 111: 39–49.

Ivanov, D. 2020. "Predicting the Impacts of Epidemic Outbreaks on Global Supply Chains: A Simulation-Based Analysis on the Coronavirus Outbreak (COVID-19/SARS-CoV-2) Case." *Transportation Research Part E: Logistics and Transportation Review* 136: 1–14.

Ivanov, D. 2021. "Supply Chain Viability and the COVID-19 Pandemic: A Conceptual and Formal Generalisation of Four Major Adaptation Strategies." *International Journal of Production Research*, doi:10.1080/00207543.2021.1890852.

Ivanov, D., and A. Dolgui. 2020. "Viability of Intertwined Supply Networks: Extending the Supply Chain Resilience Angles Towards Survivability. A Position Paper Motivated by COVID-19 Outbreak." *International Journal of Production Research* 58 (10): 2904–2915. doi:10.1080/00207543.2020.1750727.

Jahre, M., A. Pazirandeh, and L. Van Wassenhove. 2016. "Defining Logistics Preparedness: A Framework and Research Agenda." *Journal of Humanitarian Logistics and Supply Chain Management* 6 (3): 372–398. doi:10.1108/JHLSCM-04-2016-0012.

Kim, Y.-J., M. H. Seo, and H.-E. Yeom. 2020. "Estimating a Breakpoint in the Pattern of Spread of COVID-19 in South Korea." *International Journal of Infectious Diseases* 97: 360–364.

Paul, S., and J. Venkateswaran. 2018. "Inventory Management Strategies for Mitigating Unfolding Epidemics." *IISE Transactions on Healthcare Systems Engineering* 8 (2018): 167–189.

Radonovich, L. J., P. D. Magalian, M. K. Hollingworth, and G. Baracco. 2009. "Stockpiling Supplies for the Next Influenza Pandemic." *Emerging Infectious Diseases* 15, doi:10.3201/eid1506.081196.

Rawls, C. G., and M. A. Turnquist. 2010. "Pre-positioning of Emergency Supplies for Disaster Response." *Transportation Research Part B: Methodological* 44 (4): 521–534.

Rebmann, T., B. Citarella, S. Alexander, B. Russell, and J. C. Volkmann. 2011. "Personal Protective Equipment Use and Allocation in Home Health During Disasters." *American Journal of Infection Control* 39: 823–831.

Rocha, A. F. 2020. Regression Polynomial Analysis of the Covid 19 Epidemics: Regional Dynamics in Spain and Germany. doi:10.13140/RG.2.2.33177.65121

Sharifyazdi, M., K. A. Navangul, A. Gharehgozli, and M. Jahre. 2018. "On- and Offshore Prepositioning and Delivery Mechanism for Humanitarian Relief Operations." *International Journal of Production Research* 56 (18): 6164–6182. doi:10.1080/00207543.2018.1477260.

Shayak, B., and M. M. Sharma. 2020. Retarded Logistic Equation as a Universal Dynamic Model for the Spread of COVID-19. doi:10.1101/2020.06.09.20126573

Singh, S., R. Kumar, R. Panchal, and M. K. Tiwari. 2020. "Impact of COVID-19 on Logistics Systems and Disruptions in Food Supply Chain." *International Journal of Production Research*, doi:10.1080/00207543.2020.1792000.

Sodhi, M. S., C. S. Tang, and E. T. Willenson. 2021. "Research Opportunities in Preparing Supply Chains of Essential

Goods for Future Pandemics." *International Journal of Production Research*, doi:10.1080/00207543.2021.1884310.

Takemoto, Y., and I. Arizono. 2020. "Moral Hazard Problem and Collaborative Coordination in Supply Chain with Capacity Reservation Contract." *International Journal of Production Research* 58 (8): 2510–2526. doi:10.1080/00207543.2019.1633027.

Torabi, S. A., I. Shokr, S. Tofighi, and J. Heydari. 2018. "Integrated Relief pre-Positioning and Procurement Planning in Humanitarian Supply Chains." *Transportation Research Part E: Logistics and Transportation Review* 113: 123–146.

Whybark, D. C. 2007. "Issues in Managing Disaster Relief Inventories." *International Journal of Production Economics* 108 (1–2): 228–235.