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Demand model estimation from smartphone data. An application to assert new urbanistic development scenarios

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Abstract

The pervasive use of mobile devices has brought a valuable new source of data. The work presented here has a twofold objective: firstly, to demonstrate the capability of mobile phone records to feed traditional trip-based demand models and, secondly, to assert the possibilities of using developed models to estimate the effects of new urbanistic development scenarios. Detailed trip data for the metropolitan area of Barcelona are reconstructed from mobile phone records. This information is then employed as input for building a set of demand trip-based models and to apply these daily-based models to the appraisal of new development scenarios in a VISUM model of the city. The model calibration and validation process proves the quality of the models obtained. Our results show the way in which the generated trips are distributed into the study area and modal share is modified in the considered scenarios.

Keywords: mobile phone records; trip-based demand models.

1. Introduction

The consequences of population growth and increasing rates of urbanization are multi-faceted, but the associated pressures on constrained resources are becoming a major issue worldwide. This growing pressure induces, among other things, mobility and traffic worldwide and increases all the negative externalities associated to it. New urban developments and transformations also create mobility demand. Therefore, both variables cannot be planned independently from each other. Urban planning mechanisms need to integrate transportation evaluation tools, in

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particular modeling techniques. New ICT (Information and Communication Infrastructures) sources for modeling travel demand are potential data that appear in the modelling landscape and that have to be properly intergrated and combined into the macro-scale level modelling for strategic transportation planning analysis.

Although four-step transport demand models have been popular since the 70s and the 80s, Ortúzar and Willumsen (2011), still today their full implementation at the metropolitan and regional level represents an important challenge to public authorities. On the one hand, there exist governance issues for it, because metropolitan areas are composed by many cities with very different characteristics and resources. This fragmentation affects the accuracy of these models since available information – not only behavioral data, but also network supply data – is very heterogeneous. Additionally, higher scale (e.g. regional, or national) authorities might have implemented some of these models covering greater areas, but in many cases the level of detail is much broader leaving important gaps that are key at metropolitan level. Thus, modeling should satisfy both; firstly, the adequate level of detail and secondly, it should cover large areas (with the data availability challenges that implies).

On the other hand, there is often a lack of transdisciplinary within different branches of public administration. Thus, transport operations and infrastructure planning departments might be segregated and work with different models. But also, even in the transport management realm, different operators might also use different models and techniques. Evidently, the link with urban planning disciplines is even more diffuse and very seldom urban planning variables are connected to these transport models in practice, ending up in analyzing mobility from the supply perspective only.

2. Motivation

The metropolitan administration, the *Area Metropolitana of Barcelona* (AMB), is in the process of drafting a new master plan, PDU (Avanç PDU, 2019) which will define new urban developments and transformations for the next decades, and obviously, transport issues take a very relevant role in it. PDU needs of a quantitative approximation on how planned urban developments will affect overall mobility. That is the main motivation of developing the four-step model presented here.

The work presented here has a twofold objective: firstly, to assert several urbanistic development scenarios by estimating the effects of new urban development scenarios in a VISUM platform, PTV-AG (2020), and secondly, to demonstrate the capability of mobility data elaborated from mobile phone records to feed traditional trip-based transport models Cáceres et al. (2008 and 2012), Ros-Roca et al. (2019). Trip-based and tour-based four-stage models, Ortúzar and Willumsen (2011), still remain the dominant framework for operational transportation planning and policy analysis, Litman (2020), although there have been widely criticized by some authors. Four-step models perform reasonably well in representing and forecasting aggregate travel demand as requested by urban planners and constitute reference framework (UTP framework). Travel demand models included in the modelling stage are:

- Trip generation and attraction (the number of trips to be generated and attracted by transport analysis zones)
- Trip distribution (where those trips go)
- Mode choice (how the trips will be divided among the available modes of travel)
- Trip assignment (predicting route choice)

The results obtained through this four-step process vary widely, since they depend on the quality of the assumptions and data used, as well as the particular model's sophistication. Since the aim of the study is forecasting new strategic scenarios on a daily base, a macroscopic approach has been selected for transportation planning modelling. Demand prediction inaccuracies alone reduce efficiency of infrastructure investments. Fortunately, as sensors and localization technologies have become ubiquitous over the past decade, mobility data has increasingly grown in volume giving rise to new opportunities for high accuracy demand modelling. With the emergence of these technologies, and their rapid growth, traffic monitoring have been enabled at unprecedent scales, Chen et al (2016).

2.1. Structure of the paper

The paper is structured as follows: Section 3 describes the area of application (*Area Metropolitana de Barcelona*) while Section 4 describes the methodological approach describing data inputs and the selected four-step modelling approach; Section 5 shows the modelling results and outputs; and Section 6 summarizes the main findings.

3. Study Area Description

The application area is the Metropolitan Region of Barcelona (RMB). It is composed by 36 municipalities in the AMB subarea, with 3.239.337 inhabitants and a public transportation network with more than 200 bus lines, 4,000 stops, 10 metro lines, 15 railways lines, and two tramway networks. More than 9 million trips are produced every day. The rest of the RMB area consists of 164 municipalities and 1,848,514 inhabitants. Detailed travel demand modelling is conducted for AMB subarea (see Fig. 1).

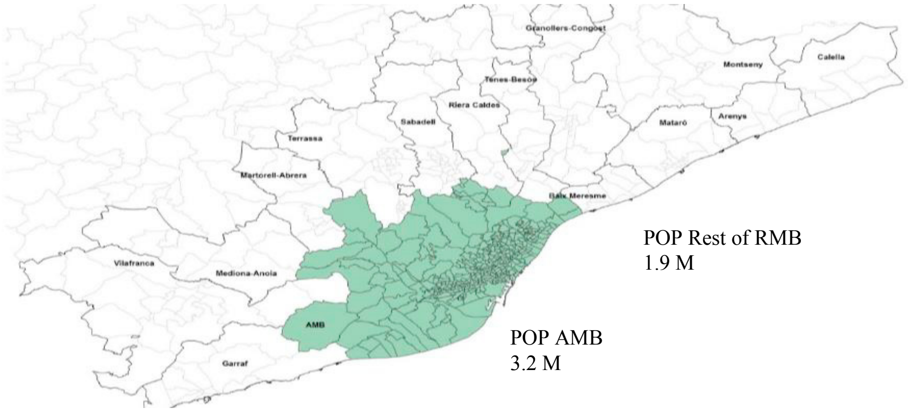


Fig. 1. RMB Study Area: Transportation Analysis Zones. AMB competence subarea in green.

3.1. PGM and PDUMAX description

Urban planners usually analyze mobility from the supply perspective. The process starts from a forecast of the population and/or economic activity (jobs) that a new urban development will concentrate, based on the existing ratios between floor m^2 and population/jobs. Then, it estimates if the existing transport networks could carry that number of users. Nevertheless, this analysis is not accurate enough and does not consider many important factors, especially the effects at the network level. Instead, when analyzing urban interactions at a broader scale, planners are familiar with population growth future trends modeled with macroeconomic factors. These forecasts are more aggregated, e.g. at a municipality level.

In this work, given the granularity of the PDU master plan, that in some cases reaches the plot detail, we can easily assign specific urban growth variables to the transport zones (see Fig. 2) without the need of interpolating variables or making additional assumptions.

The present study proposes two urban development scenarios. The first scenario could be considered as the lower bound of urban growth since it only considers feasible developments in the current metropolitan master plan (PGM-1976). This scenario involves building 39,173 new households within AMB. The second scenario, instead, is the upper bound of urban growth or the maximum feasible urban supply without compromising the metropolitan strategic values. This scenario defines 159,368 new households, 4 times more than the lower bound.

The final scenario is expected to lie within both boundaries. The aim of this study is that the results of analyzing both scenarios can influence the final choice of scenario, i.e., this is not a final check, but a dynamic evaluation within the planning process. Given the complexity of the area of analysis and all the variables involved, some KPI (key performance indicators) have been defined to state the main trade-off. In this case, the surrounding regional area (the rest of the metropolitan region, RMB) that does not belong to AMB and thus PDU does not regulate it. The incremental change of this territory has been fixed to 243,431 new households according to the current regional plan (PTMB 2010). In addition to this, both urban development scenarios consider the same new transport infrastructure level so the comparison of the generated mobility can be done properly. The choice of future transport infrastructures was done revising the different plans and programs and giving priority to those projects that are already being executed or studied.

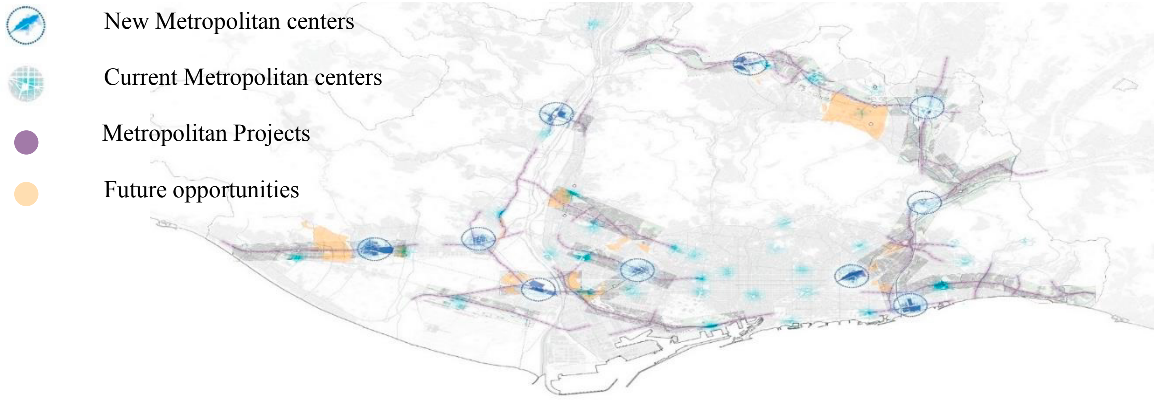


Fig. 2. New urban developments proposed by the PDU (Avanç PDU, 2019).

4. Methodological Approach

In the context of this research, we focus on daily OD destination matrices generated from smartphone data by NOMMON (https://www.nommon.es/). Modelling travel behavior trip purpose-based has been conducted. Technical details on sources and model development at each step are described. Model estimation, validation and prediction rely on RStudio (2020) and VISUM model for the study area, PTV-AG (2020). Final demand models are being integrated within the VISUM platform to support scenario evaluation by urban planners.

Below we outline the main types of data sources usually from traditional travel surveys to new sources:

- **EMO Transportation Analysis Zones (EMO-TAZ)** defined by transportation authorities. Metropolitan Area of Barcelona (AMB) study area is split into 372 TAZ and 210 TAZ for the rest of the study area.
- **OD matrices produced by NOMMON (2019).** Expanded number of trips segmented by daily periods, gender, age group (in four groups defined as 16-29, 30-44, 45-64 and >64), purpose on origin and purpose on destination (H-home, W-work, NF-casual and O-others) and EMO-TAZ origin, EMO-TAZ destination and EMO-TAZ residential. No trip travel times or lengths are included.
- **EMEF 2018.** Traditional mobility survey that analyzes the working day mobility of residents in the RMB area aged 16 and over. Spatial granularity at municipality level, but Barcelona is divided into districts (10) leading to a total of 296 macrozones, only 45 of them in the AMB area. Data collected for each journey refers to: origin and destination macrozones, purpose, mode, travel start time and duration (min), vehicle use, parking use, etc.
- **Land use data**. Elaborated by urban planning authorities for the current situation and future scenarios referred to EMO-TAZ spatial granularity. It consists on population segmented by gender, age group (5 groups), education, scholarship opportunities, services, land use, number of stops in public network, average income surface, etc.
- **VISUM model**. Developed by ATM (Metropolitan Transport Authority). It contains private transport network and public transport lines for an extended area that includes RMB and OD daily matrices for public transport, light vehicles and heavy vehicles (2019 data). It is the source for OD skim matrices for the base scenario. The model is being updated to include future scenario supply and OD demand matrices.

4.1. Modelling approach

Travel demand models from household surveys is a common topic already covered by literature and practitioners. Nevertheless, smarphone-based demand models are a novelty for practitioners and one of the goals of this work. Data fusion from data sources has been undertaken to take benefit from all available sources. Some addressed topics:

- Transport Analysis Zone (TAZ) systems mismatch. EMEF 2018 addressed macrozones. ATM model considers a TAZ system based on EMO-TAZ composed by 582 zones for the extended metropolitan region of Barcelona (RMB). NOMMON data consists of 372 EMO-TAZ plus 14 macrozones representing the rest of the RMB.
- Trip Purposes. A very detailed list of purposes is considered in EMEF 2018 household travel survey, ATM model contains no information about trip purposes and NOMMON data infers a restricted set of purposes. A unified set of six purposes was defined to orchestrate EMEF and NOMMON sources (HBW-Home Based Work, HBC-Home Based Casual, HBO-Home Based Other, NHBW-Non-Home Based Work, NHBO-Non-Home Based Other and NHBH-Return Home). The modelling approach framework is described in Fig. 3.

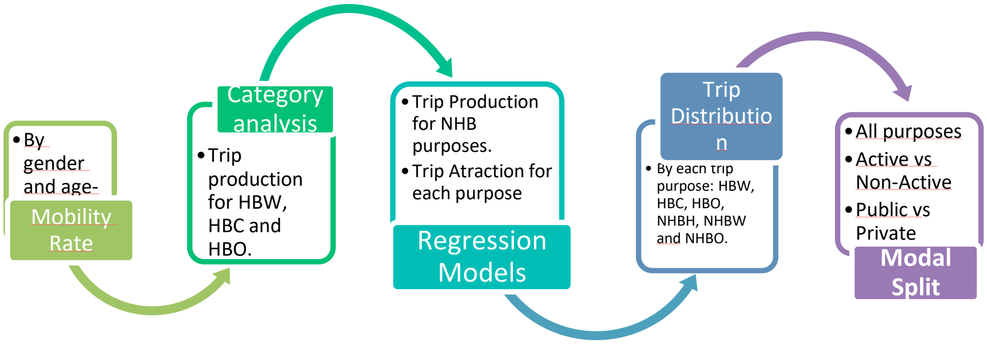


Fig. 3. Modelling approach

4.1.1. Mobility rate

Mobility rate modelling has been considered as a binary logit model depending on gender and age-group based on EMEF 2018 household survey valid for the whole study area. Several models with better properties from the statistical point of view were found, but explanatory variables related to car availability and activity group were not available at the EMO-TAZ spatial granularity. Smartphone-based data does not contain any data to infer inmobility rates.

4.2. Trip Production and Attraction by purpose

Category analysis for HB purpose trips was initially conducted from smarphone-based data. HBW, HBC and HBO purposes for AMB subarea. Initially, generalized linear models for a counting process of number of trips for the selected purpose per trip-maker available to travel modelled as Poisson targets were developed. Explanatory qualitative variables related to per-capita rent (5 groups), gender and age-group (4 groups) were included. These models were not satisfactory since residual deviance was too high, so an alternative approach focused on EMO-TAZ variables using group defined by gender and age-group and generalized linear modelling for purpose trip rate and covariates as per-capita rent and squared meters of roof percentage used for each economic sector (agriculture, industrial, residential, civil equipment, shopping, service sector, other and unclassified land-use). Poisson model were checked for over-dispersion and statistical tests were not rejected; thus negative binomial targets were proposed. Models excluding non-significative variables, collinearity and influent data were addressed. Gamma models were also assessed and rejected, although improving Poisson modeling, they do not enhance negative binomial models for the selected targets.

Non-home-based purpose generation models have been developed by traditional linear regression modeling for total number of trips per purpose at EMO-TAZ level and explanatory variables related to the population per gender and group age, percentage of dwelling units according to surface groups (<45m^2, 46-60 m^2, 61-80 m^2, 81-100 m^2, 101-120 m^2 and >120 m^2), education group (5 levels), number of civil equipment (school places, health services, train stations, etc. in 32 categories), edified floor surface in economic sectors (7 groups), percentage of morphological residential use (4 groups), number of public transport stops (train, metro, bus, tram, etc), average per-capita yearly rent, special spots (university, airport, etc) and zone area. A normalized principal component analysis (PCA) was addressed to select for each block of variables (population, dwelling surface, edified floor surface in economic sectors, level of education, civil equipment, and transport facilities) the most representative variables according to Kaiser criteria. From more than 200 variables a reduced set of 30 was obtained and defined the initial models that are reduced by removing collinear variables (belonging to different blocks) and non-significant variables according to Fisher test. Final models selected based on BIC (Bayesian Information Criteria) once influential observations were re-weighted to minimize their impact in the estimation of parameters. EMO-TAZ generated trips by NHBW, NHBO and NHBH purposes for internal trips to AMB subarea were estimated. HB and NHB purposes were also addressed for inter AMB-RMB subarea trips. Home-based and non-home-based purpose attraction models have been developed by traditional linear regression modeling for total number of trips per purpose at EMO-TAZ level. A logarithm transformation for the total number of trips according to explanatory variables by blocks was considered.

4.3. Trip Distribution by purpose

Once trip generation and attraction models for purpose and subarea are completed, spatial distribution for purpose and subarea has been addressed. Intra-zonal trips were removed for trip distribution modelling. Six distribution models were calibrated for internal trips to AMB subarea, one for each purpose (HBW, HBC, HBO, NHBH, NHBW and NHBO) and two models, HB and NHB purposes, for inter AMB-RMB trips. All of them taking smartphone-based data. While household surveys usually contain some trip duration information, as it is the case in EMEF 2018, smartphone-based data does not include any data related to trip duration. The authors used 2019 ATM model containing private and public transport network models and matrices to calculate skim matrices after user equilibrium assignment and transit assignment (both under VISUM platform) and develop ATM model-based weighted OD trip travel times, to be combined to smartphone-based data.

Gravity models with general deterrence function were estimated once friction factors per purpose were calculated using a tri-proportional balancing method. Total travel times were discretized into 23 intervals with a maximum value of 150 min and convergence was achieved with 10-12 iterations to get an error less than 0.1%. Intra-trips modelling is omitted. Several deterrence function fits have been proposed leading to OD subarea and purpose specific models obtained from linear adjustment to the friction factors obtained after triproportional balancing estimation.

4.4. Modal Split

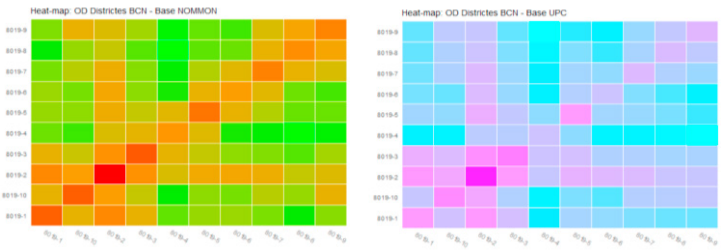
Modal use is not available in smartphone-based data, thus it was inferred from external sources using an aggregated zone-based approach that constitutes a weak modeling step, but actually, the only realistic possibility. ATM model contains private (light plus heavy vehicles) OD matrices and public transport (all modes) OD matrices. The first step has to split all modes OD data into active OD trips (walking, cycle and micromobility) and motorized OD trips. EMEF 2018 data has been used to estimate a binary logit model to split active versus non-active (motorized) trips, explanatory variable is OD direct distance (from ATM model) in a 4th order function.

Modal split of non-active mobility into private and public transport modes has been addressed with a binary logit based on ATM model OD matrices for the RMB subarea. The final model considers as characteristics variables related to origin and destination zones, the absolute difference of *per capita* rent, number of metro stops and number of suburban train stops and alternative variables as the difference between transit and private for trip distance and journey perceived time and access walking time and egress walking time for public transport. The most important variable is the difference between OD perceived journey time and private travel time.

5. Modeling results

Datasets are processed with RStudio 1.3.959 (2020) and R version 4.0.2 (2020-06-22) (R Core Team, 2020) in a Windows 10 x86_64-w64-mingw32/x64 (64-bit) platform and 30 GB RAM requirements for data exploitation. Estimated trip-based demand models are available and implemented in RStudio. They are being adapted to be included as python-based procedures in ATM VISUM model. A set of twenty production and attraction models have been developed. Eight distribution models by purpose and subarea pair-based and two aggregated bimodal split models have been finally selected. Production and attraction models offer validated forecasts for the base scenario over 85% (R2 comparison). Distribution modelling is the well-known weak step in the UTP framework and validation results at EMO-TAZ level provide 60% R2 goodness of fit. Aggregated modal split for motorized private-public transport has required manual intervention to sort satisfactory vs weak goodness of fit OD pairs (classified into good-regular-bad fit) to be accounted for in the forecasting demand of future scenarios.

Trip distribution detailed result analysis has been addressed by comparing travel times, OD tables between functional subareas, heatmaps and numerical indicators as MAPE (Mean Absolute Percentage Error), MaxMAPE (Max Absolute Percentage Error), R2 and MSSIM. MSSIM was proposed by some authors Wang et al. (2004) and have been modified in Ros-Roca et al (2020), to properly address OD matrix comparison (see Fig. 4). MISSIM is imported from image recognition and it is composed by luminance (it corresponds to the intensity of illumination), which is, indeed, the mean of the different pixels in a sub-matrix. Contrast is the root of the squared average between pixels once the luminance is removed from the sub-matrix, making it the standard deviation. Structure is the comparison between the two sub-matrices using the covariance. It is normalized to [-1,1].



$$\mathrm{SSIM}(\mathbf{a},\mathbf{b}) = L(\mathbf{a},\mathbf{b})^{\alpha} C(\mathbf{a},\mathbf{b})^{\beta} S(\mathbf{a},\mathbf{b})^{\gamma}$$

$$\begin{cases} L(\mathbf{a},\mathbf{b}) = \dfrac{2\mu_a\mu_b + C_1}{\mu_a^2 + \mu_b^2 + C_1} \\ C(\mathbf{a},\mathbf{b}) = \dfrac{2\sigma_a\sigma_b + C_2}{\sigma_a^2 + \sigma_b^2 + C_2} \\ S(\mathbf{a},\mathbf{b}) = \dfrac{\sigma_{ab} + C_3}{\sigma_a\sigma_b + C_3} \end{cases}$$

Fig. 4. Smartphone-based data and UPC model. OD trips between Barcelona districts. Left-reference data and right-model. SSIM definition.

5.1. Forecasting travel demand for future scenarios

After validating the obtained models in the first three steps, forecasting of PGU and PDUMAX modal OD matrices has been addressed. PGM scenario (lower bound for current planning) percentage comparison with respect the base-scenario for all modes and public transport is presented according to OD functional macrozones (Table 1).

Table 1. PGM Forecasting. OD matrices for all modes and public transport according to Macrozones. Relative difference to base scenario.

	Destination															
	BCN		BCNNord		DeltaLlobregat		LlobregatContinu		RMB		VallBaixaOrdal		Vallès		Total All Modes	Total Public Transport
Origin	All Modes	Public Transport	All Modes	Public Transport	All Modes	Public Transport	All Modes	Public Transport	All Modes	Public Transport	All Modes	Public Transport	All Modes	Public Transport		
BCN	-1%	2%	0%	5%	10%	22%	2%	5%	-6%	12%	0%	8%	9%	23%	2%	11%
BCNNord	-1%	4%	1%	7%	2%	19%	3%	7%	-5%	31%	2%	12%	5%	12%	1%	13%
DeltaLlobregat	3%	16%	-1%	14%	6%	15%	6%	23%	7%	12%	0%	16%	8%	23%	4%	17%
LlobregatContinu	3%	5%	3%	8%	11%	28%	3%	8%	2%	23%	3%	11%	11%	15%	5%	14%
RMB	-8%	22%	-9%	40%	2%	21%	-3%	28%	0%	3%	-9%	38%	3%	58%	-3%	30%
ValBaixaOrdal	-1%	4%	0%	11%	3%	24%	1%	10%	-6%	47%	-1%	10%	7%	18%	0%	18%
Vallès	12%	26%	8%	9%	21%	38%	16%	19%	5%	87%	13%	16%	4%	17%	11%	30%
Grand Total	**1%**	**11%**	**0%**	**13%**	**8%**	**24%**	**4%**	**14%**	**0%**	**31%**	**1%**	**16%**	**7%**	**24%**	**3%**	**19%**

6. Conclusions and Further Discussion

This paper presents the implementation of a four-step model in the Area Metropolitana de Barcelona (3.2 mio hab and 632 km^2). This methodology is a novel approach as it has been developed to serve urban planning purposes. Its main objective is to evaluate different scenarios considered in the new metropolitan master plan (PDU). In this work, two scenarios are considered: a lower and an upper bound related to the amount of urban development.

Additionally, four-step models are fed not only with diverse urban and socioeconomic data, but also with mobility data coming from surveys and mobile phone records. This paper demonstrates how not only is feasible to combine these data but that this combination can yield to more accurate results as it takes advantage of the capabilities from both worlds. Purpose and OD subarea based demand models have been successfully estimated and linked to conduct travel demand forecasting in PDU scenarios.

The final network assignment is undergoing and will shed some light in how the different urban scenarios are contributing to the generation of mobility. Therefore, the next steps will be to evaluate these results, the upper and lower bounds, to support the decision making for the ultimate PDU urban development scenario. That scenario, of course, will be eventually evaluated with the same model. At the same time, using the input from these upper and lower bound matrices, a new scenario related to transport supply (new infrastructures and services) will be proposed.

Acknowledgements

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