Effects of an imposed axial flow on a Ferrofluidic Taylor-Couette flow

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Summary We investigate the effects of an externally imposed axial mass flux [1, 2] (axial pressure gradient, axial through flow) on ferrofluidic Taylor-Couette flow under the influence of either an axial or a transverse magnetic field [3]. Without imposed axial through flow, due to the symmetry-conserving axial field and the symmetry-breaking transverse field, it gives rise to various vortex flows in ferrofluidic Taylor-Couette flow such as wavy Taylor vortex flow (wTVF), wavy spiral vortex flow (wSPI) and wavy vortex flows (wTVFHx and wSPIHx), which are typically produced by a non-linear interaction of *rotational, shear* and *magnetic instabilities*. In addition, when an axial through flow is imposed to a ferrofluidic Taylor-Couette flow in the presence of such a magnetic field, new helical vortex structures are born. In particular, we uncover *'modulated Mixed-Cross-Spirals'* with a combination of at least *three different* dominant azimuthal wavenumbers.

FIXED TRANSVERSAL MAGNETIC FIELD.

To investigate the effects of an externally imposed axial through flow Re, for a fixed transversal magnetic field ($s_x = 0.6$), examine dynamics of flow states by varying Re, specially focusing on bifurcation phenomenon for two fixed inner Reynold numbers, $Re_i = 110$ and 270. Due to a transversal magnetic field, all flow states are *inherently* $3D$ and wavy-like modulated flow [4] containing specific higher modes $m \pm 2$.

BIFURCATION SCENARIO

For $Re_i = 110$ and $Re = 0$ the system shows *multistability* having three stable states, $L1-wSPI_{H_x}$, $R1-wSPI_{H_x}$ and $wTVF_{H_x}$. By increasing Re, both flow states $R1 - wSPI_{H_x}$ and $wTVF_{H_x}$ move to $L1 - wSPI_{H_x}$, and then follow the destiny of $L1 - wSPI_{H_x}$ (Fig. $1(a)$).

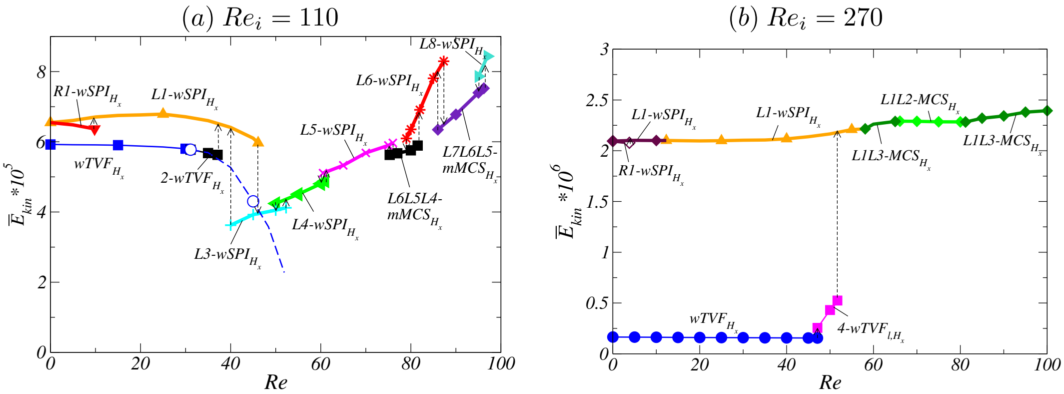

Figure 1: Bifurcation scenarios vs. imposed axial through flow Re. Bifurcation scenarios for (a) $Re_i = 110$ and (b) $Re_i = 270$, respectively. Shown is the total (time-averaged) modal kinetic energy E_{kin}. Different flow structures are labeled. Vertical arrows illustrate the transition direction to another stable state, when a flow state loses its stability. Thin [thick] lines correspond to toroidally closed [helical] flow states.

Increasing Re the state $wTVF_{H_x}$ becomes unstable at $Re \approx 35$, where a wavy flow state $2 - wTVF_{H_x}$ with dominant mode amplitudes $(2, \pm 1)$ bifurcates. At $Re \approx 38$, $2 - wTVF_{H_x}$ finally loses its stability, and moves towards a modulated helical spiral state $L1 - wSPI_{H_x}$. The helical downward propagating state $R1 - wSPI_{H_x}$ becomes unstable at $Re \approx 10$, and then moves toward a stable helical upward propagating state $L1 - wSPI_{H_x}$. Starting with $L1 - wSPI_{H_x}$, one obtains the following sequence of flow states (Fig. $1(a)$):

$$L1 - wSPI_{H_x} \to L3 - wSPI_{H_x} \to L4 - wSPI_{H_x} \quad \to \quad L5 - wSPI_{H_x} \to$$
$$L6L5L4 - mMCS_{H_x} \to L6 - wSPI_{H_x} \quad \to \quad L7L6L5 - mMCS_{H_x} \to L8 - wSPI_{H_x}.$$

While most steps in this scenario can be anticipated, i.e. increasing the helicity - larger azimuthal modes m with increasing Re one eventually finds a *new* type of *mixed mode states* as non-linear interaction. We will call this states *modulated Mixed-Cross-Spirals* ($mMCS_{H_x}$)[5] as they appear with *three* dominant azimuthal wavenumbers. For instance, $L6L5L4 - mMCS_{H_x}$ and $L7L6L5 - mMCS_{H_x}$.

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At larger $Re_i = 270$ (Fig. 1(b)) the sequence of flow state appearing out of $wTVF_{H_x}$ with increasing Re is quite different: $wTVF_{H_x} \rightarrow 4 - wTVF_{l,H_x} \rightarrow L1 - wSPI_{H_x}$.

At $Re \approx 47.6$, $wTVF_{H_x}$ bifurcates to $4 - wTVF_{l,H_x}$ with dominant azimuthal wavenumber $m = 4$. However flow states with lower azimuthal wavenumber m, as $2 - wTVF_{l,H_x}$ and $3 - wTVF_{l,H_x}$ were temporarily observed as *unstable* and only *transient* states. Further, at $Re \approx 52$, $4 - wTVF_{l,H_x}$ loses its stability, and then moves towards the helical $L1 - wSPI_{H_x}$. As before, within this transition process, $5 - wTVF_{l,H_x}$ and $6 - wTVF_{l,H_x}$ appear as *unstable transient* states. Interestingly we could not observe any other stable $wTVF_{l,H_x}$ with larger azimuthal wavenumber $m \geq 7$.

MODULATED MIXED-CROSS-SPIRALS

The spatial structure of a *stable* $L6L5L4 - mMCS_{H_x}$ (Fig. 2), which exists between $L5 - wSPI_{H_x}$ and $L6 - wSPI_{H_x}$ clearly illustrates the dominance of the largest mode $m = 6$ while at the same time the influence and modulation of *both* other dominant modes $m = 5$ and $m = 4$ are visible. The dominant azimuthal wavenumber m is decreasing from the inner towards the outer cylinder (Fig. 2(4)). Due to the symmetry-breaking effect of a transversal magnetic field,

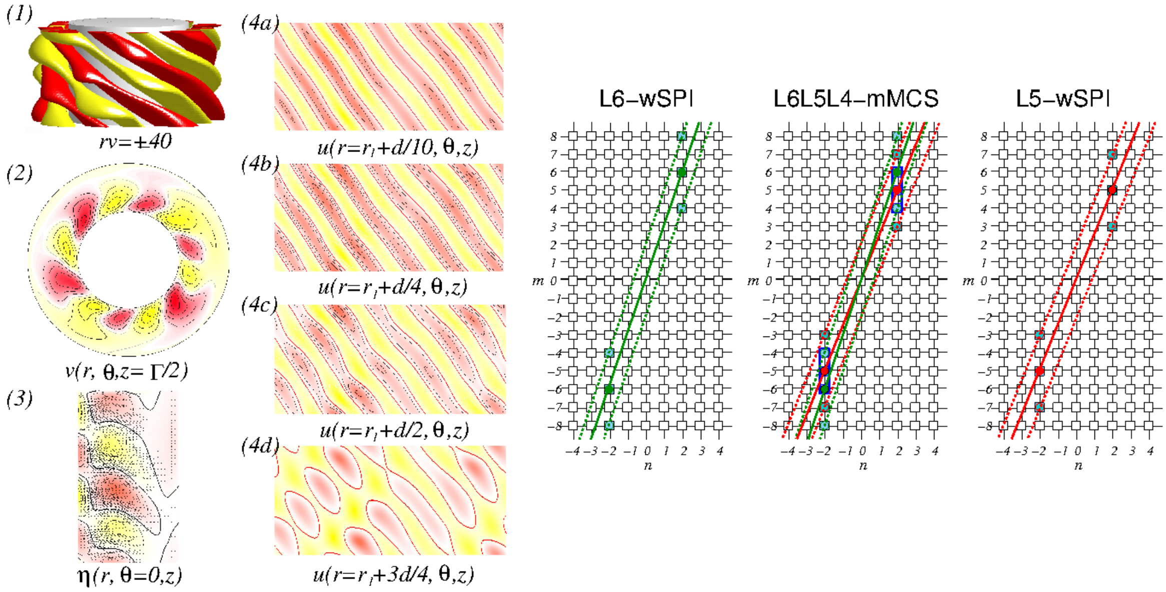

Figure 2: *Left:* Flow visualization of flow states L6L5L4-mMCS$_{H_x}$ at $s_x = 0.6$ & $s_z = 0.0$, $Re_i = 110$ and fixed axial through flow $Re = 82$. (1) Isosurface of rv; (2) The azimuthal velocity $v(r, \theta)$ at mid-plane; (3) Vector plot $[u(r, z), w(r, z)]$ of the radial and axial velocity components including color-coded azimuthal vorticity $\eta(r, \theta = 0, z)$). $(4a - d)$ The radial velocity $u(r, \theta, z)$ at different radial positions as indicated. *Right:* Dominant excited modes (colored squares) of the different solutions in the two-dimensional Fourier mode space (m, n). Filled circles denote linearly driven modes and linear Fourier mode subspaces are indicated by thick lines. They represent $L5 - wSPI_{H_x}(5n \pm 2, n)$, $L6 - wSPI_{H_x}(6n \pm 2, n)$, and a combination of both in $L6L5L4 - mMCS_{H_x}$.

the helical state $L5 - wSPI_{H_x}$ [$L6 - wSPI_{H_x}$] has its dominant mode $m = 5$ [$m = 6$] and an additionally stimulated modes $m \pm 2 = \{3, 7\}$ [$m \pm 2 = \{4, 8\}$] [4], respectively. When the states $L5 - wSPI_{H_x}$ and $L6 - wSPI_{H_x}$ come *close* in Re, they stimulate one of these additional modes (here $m = 4$) as a *nonlinear interaction* of dominant modes and thus a new state with *three* dominant modes is born. The correspondingmode space of $L6L5L4 - mMCS_{H_x}$ can be (linearly) constituted as a combination of two mode spaces of $L5 - wSPI_{H_x}$ and $L6 - wSPI_{H_x}$ [$(5n \pm 2, n)$ and $(6n \pm 2, n)$].

CONCLUSIONS

When an axial mass flux is applied to a ferrofludic system, the dynamics of a system can be described by results of competition of the *three* different instabilities; *centrifugal instability* due to rotation, *shear instability* due to axial mass flux and *magnetic instability* due to applied magnetic fields. Due to competition of these instabilities, previously unknown *new* flow states appear. These are *localized wavy Taylor vortices* ($wTVF_l$ and $wTVF_{l,H_x}$) and *modulated Mixed-Cross-Spirals* ($mMCS_{H_x}$). $mMCS_{H_x}$ appear as a byproduct of an interaction of an axial through flow and a transversal magnetic field. In fact the symmetry breaking nature of a transverse magnetic field is responsible for the appearance of $mMCS_{H_x}$ due to its additionally stimulated modes $m \pm 2$.

References

[1] Chossat P., and Iooss G., The Couette-Taylor Problem. (Springer, Berlin, 1994).
[2] Hoffmann C., Lücke M., and Pinter A., Spiral vortices and Taylor vortices in the annulus between rotating cylinders and the effect of an axial flow. *Phys. Rev. E* **69**, 056309, 2004.
[3] Altmeyer S., and Do Y., Effects of an imposed axial flow on a Ferrofluidic Taylor-Couette flow. *Scientific Reports* **9**, 15438, 2019.
[4] Altmeyer S., Hoffmann C., Leschhorn A., and Lücke M., Influence of homogeneous magnetic fields on the flow of a ferrofluid in the Taylor-Couette system. *Phys. Rev. E* **82**, 016321, 2010.
[5] Altmeyer S., and Hoffmann C., Secondary bifurcation of mixed-cross-spirals connecting travelling wave solutions. *New J. Phys.* **12**, 113035, 2010.