# Reducing variability of a critical dimension

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## Introduction

Sometimes statistically easy ways emerge for solving problems that at first glance seem to require more sophisticated methodologies.

We encountered the case presented here a few years ago when an important automotive company (tier 1 supplier) asked us to help in programming a recently installed adaptive control system. Specifically, they were having problems with an important dimension.

### Process and part description. Objective.

The part in question is a component of the car's braking system. The manufacturing process requires great precision because small variations in the component's critical dimensions produce important changes in its performance. One of these dimensions (Dimension A) is the distance between the pushrod and the upper part of the shell (Figure 1).

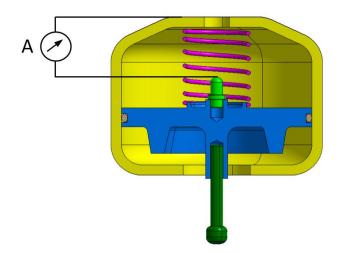
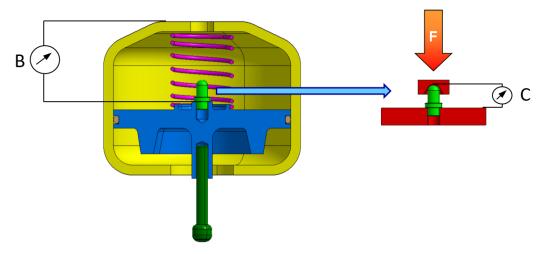


Figure 1. Scheme of the component, with the critical Dimension A indicated

Due to its form of construction and assembly, which includes creating a vacuum inside, the dimensions of the shell present variability that is not compatible with the precision required in Dimension A. To solve this problem, Dimension B (Figure 2) of each unit is measured before inserting the pushrod. This measure is used to regulate the force (F) that will be applied to insert the pushrod, in order to obtain a Dimension C such that A obtains the desired value.



*Figure 2. Scheme of the component showing dimension B and the deformation of the pushrod before its insertion* 

With the current process, the variability of Dimension A is greater than what is desired, thus causing an inacceptable level of nonconformance in the product. Since different attempts at finding the causes of variability and solving the problem had been unsuccessful, the company decided to incorporate the history of the A values into the algorithm to calculate F (as shown in Figure 3).

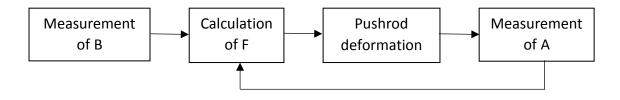


Figure 3. Scheme of the proposed new regulation method

The company made several attempts to reduce the variability by including the prediction of the value of A, based on the previous values. After all of these were unsuccessful, they decided to seek our help in programming the adaptive control system to reduce the variability.

## Analysis of the situation

The first step was to collect data on the behavior of Dimension A by calculating F based only on Dimension B of the shell. We decided to collect data from three production lots corresponding to three production days in two different weeks. We paid special attention to the collection of process variables that could be related to Dimension A and to any strange circumstances that could explain the anomalies or changes in level.

The time series plots in Figure 4 show the values obtained from Dimension A in the three lots. Note: A constant has been added to the real values, as requested by the company for reasons of confidentiality.

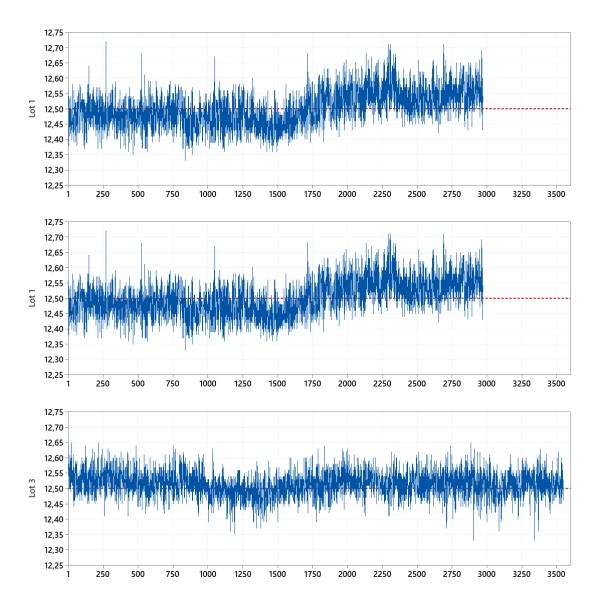


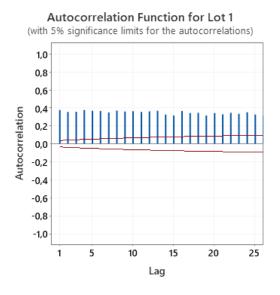
Figure 4. Time series plot of Dimension A (F based only on Dimension B)

From these three plots, we can see that the process presents three types of variability:

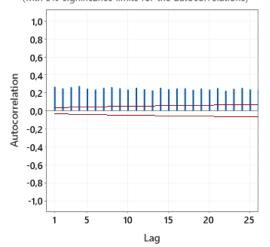
- Short-term fluctuations around the mean.
- Changes in the level of the mean.
- Occasional abnormal values occur (outliers).

A careful analysis of the possible causes of the changes gave no answers for none of the three types of variability. We were frustrated, especially with regard to the mean level changes and the outliers.

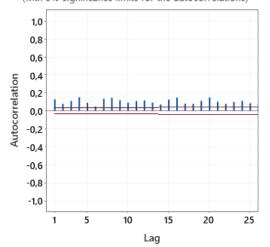
The next step was to ascertain whether Dimension A presented an autocorrelation structure that would allow for forecasting. Figure 5 shows the autocorrelation and partial autocorrelation functions of each lot.



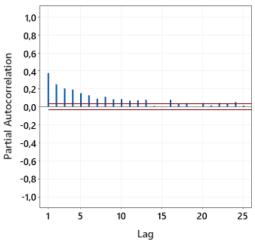
Autocorrelation Function for Lot 2 (with 5% significance limits for the autocorrelations)



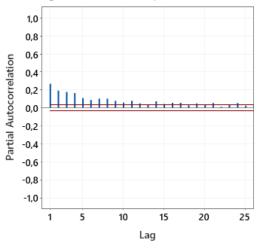
Autocorrelation Function for Lot 3 (with 5% significance limits for the autocorrelations)



Partial Autocorrelation Function for Lot 1 (with 5% significance limits for the partial autocorrelations)



Partial Autocorrelation Function for Lot 2 (with 5% significance limits for the partial autocorrelations)



Partial Autocorrelation Function for Lot 3 (with 5% significance limits for the partial autocorrelations)

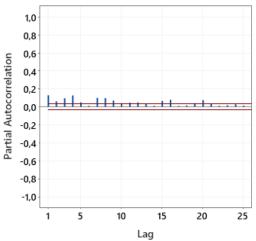


Figure 5. Autocorrelation and partial autocorrelation functions of the three lots.

The plots show similar behavior in all three lots: the ACF decreases very slowly and the PACF shows a certain relationship between close observations, especially in the first lot. Attempting to model them leads to identifying very unstable ARMA (1,1) and ARIMA (0,1,1) models. Although the series could be considered stationary in terms of constant mean and variance, everything seems to indicate that in reality they are not. Their autocorrelation structure changes over time.

Therefore, it seems that in this case very little can be done to reduce the variability of short-term fluctuations, and nothing to reduce the occurrence of outliers. Thus, we deemed it sensible to focus on reducing variability due to changes in the mean level.

## **Control strategy**

The control strategy employed must aim to eliminate the effect of level changes and, if at all possible, take advantage of the information provided by the previous observations to mitigate short-term fluctuations. Furthermore, this strategy must meet two conditions: it must not be affected by abnormal values and not cause overadjustment.

The available data allowed establishing a reference for how much variability could be reduced when eliminating level changes. Furthermore, they could also be used to test and assess the effect of different strategies.

To estimate the minimum variability that could be attained if there were no mean level changes, we considered that all the variability is random in a group of 50 consecutive units, since the change in the mean has inertia and lacks the time to move in the period needed to manufacture 50 units. Thus, we have estimated the variance without mean level changes by the average of the variances of groups of 50 consecutive observations. We did the same for other group sizes, and the differences in the estimated variance are not relevant.

Table 1 shows the standard deviation for the initial situation of each lot and the one obtained by eliminating the level changes. Obviously, the latter value that we use as a reference must be close to the variability if the process mean were stable.

	Number of units	Mean	Variability (Standard deviation)	
			Initial situation	Reference (without mean level changes)
Lot 1	2969	12.501	0.0601	0.0482
Lot 2	3318	12.528	0.0419	0.0361
Lot 3	3546	12.514	0.0429	0.0402

Table 1. Initial situation and target values for the variability of reference lots

Using the data from the three lots, we tested the two control strategies discussed below.

#### Strategy 1: Correction based on the moving average

This consists of introducing a correction equal to the difference between the average of the last *n* observations and the target value. The following parameters were considered.

- Number of previous values considered in the adjustment (*n*): If *n* is set to 1, this means that only the previous value is considered.
- Exponential weighting factor of the values used for the adjustment (p): The influence of the values considered in the adjustment can be decreased exponentially, such that the most recent ones have more influence than those that are further away. The value of these weights can range from p = 0 (a situation in which only the last value obtained has influence) to p = 1 in which all values have the same influence (the moving average case).
- Outlier limit (k): In the graphic that shows the evolution of the values obtained (Figure 1), some peaks correspond to observations that are very far from the mean. However, these values do not mean the process has been disrupted, since they appear sporadically and correct themselves. For calculating the adjustment, we did not include the values that are outside the interval: target value ±k.
- Correction threshold (u): Introducing very small corrections can mean introducing corrections into an already well-balanced process, thus causing an increase in variability due to overadjustment. We have established a minimum threshold from which the correction will be made, that is, it is corrected only when the necessary correction is greater than u. In the particular case of u = 0, this means that it is always corrected.

We developed a small routine to determine the combination of parameter values that minimize the variability of the data of each lot. These values are: p = 1, n = 50, k = 0.15, and u = 0.

#### Strategy 2: Correction based on the difference

This strategy consists of introducing a correction that is proportional to the difference between the last value obtained and the target value. This correction is updated with the information provided by each new observation. The parameters used to apply this strategy are:

- Outlier limit (k): Same as in the previous strategy.
- Correction factor (*f*): This is the factor by which the difference between the last observation and the target value is multiplied in order to update the correction that will be applied to the next observation. The values of *f* range from 0 to 1.

As was done for the previous strategy, a routine has been developed to sweep all combinations of parameter values and select the combination that gives the best results.

The value chosen for f is 0.04, which is the value that is common to all three lots. Regarding k, the optimal value is different for each lot, but the curve that relates the standard deviation to the value of k is very flat in the area of minimum values (kbetween 0.10 and 0.20), so we chose the median value k = 0.15.

A detailed explanation of the two tested strategies can be found in Grima, Puig, and Tort-Martorell (2013), along with the routines used to identify the optimal values of the considered parameters.

### Comparison of the two strategies

Both strategies achieve not only a significant reduction in variability, but they also obtain a value that is close to the reference and therefore to the minimum possible value without making relevant changes to the manufacturing process.

Figure 6 shows the standard deviations obtained in each lot with each strategy, comparing them with the initial and reference values.

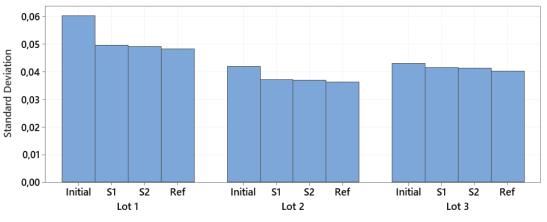


Figure 6. Comparison of the standard deviations

The second strategy was chosen because it provides slightly better results than the moving average strategy and is easier to implement.

## **Results and learning points**

Once the control algorithm was implemented, the results were, essentially, as expected. The standard deviation of Dimension A was reduced, and consequently so was the number of defects, which dropped from close to 3% to around 0.8%. It is worth noting

that it does not seem possible, without changes in the process, to greatly improve the level achieved.

Nowadays, it is normal to find industrial processes with the technology to collect large amounts of data on their behavior. Unfortunately, it is also quite common to get little or no benefit from these data. This case shows how a process was improved by employing simple techniques to understand its behavior, using the available data to simulate it, and then finding a good control algorithm.

In this case, the strategy used is justified by the autocorrelation structure of the process data. However, from a practical point of view, this strategy is often a good option even when the behavior of the process can be modeled.

## References

Grima, P. Puig, X and Tort-Martorell, X. 2013 Reducing Variability by Monitoring and Feedback Adjustment. *DYNA* 88(6):659-665. <u>https://doi.org/10.6036/5781</u>