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CONCERNING THE CALCULATION OF TRUSSED-BEAMS

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For a single-strut trussed-beam, a comparative analysis of the internal forces' values are obtained by two methods. Initially, it is calculated as a single structure using the 'Force Method' (FM). Secondly, a separate calculation of its parts is carried out. The stiffness (main) beam is calculated as a continuous beam, whereas the struts – as a truss. It is shown that the separate calculation of the trussed-beams, can lead to errors in determining the internal forces associated with the assigned dimensions of the stiffness beam's and the pillar elements' cross-sections. The comparative analysis of internal forces' values is carried out using the software MathCAD.

Introduction. Trussed-beams are widely used in the reconstruction of industrial buildings and structures. Such structures are indistinct combined systems consisting of a stiffness beam (upper beam), reinforced from below by a hinge-rod system (strut). By the number of pillars in the truss, such structures are divided into single-strut (type with a central strut), double-strut and multi-strut systems. Depending on the structural material used in the building practice, the most common types of trussed-beams are steel, timber, steel-timber, reinforced concrete-steel.

Trussed-beams are statically indeterminate structures, and their calculation as a unified structure is set to be done by using the FM [1]. However, in the practice of engineering calculations, in order to simplify calculations, the component parts of the trussed beams are usually calculated separately: the stiffness beam is calculated as a continuous beam (two, three or multi-span), and the struts – as a truss [2] – [4].

A single-strut trussed-beam, the components of which are made of different structural materials, is studied. The beam is loaded with a uniformly distributed load along the entire span (Fig. 1,a)

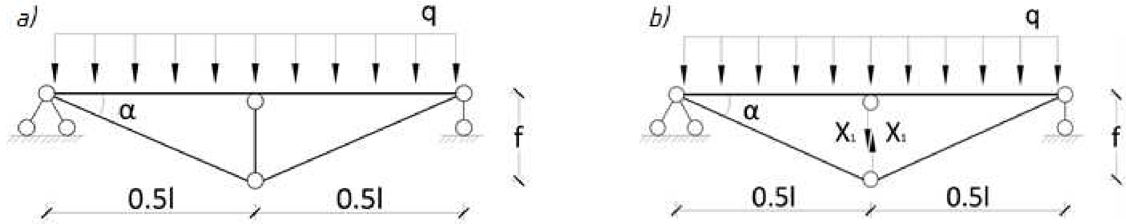

Figure 1. – A single-strut trussed-beam

A comparative analysis is conducted concerning the characteristics of the stress-strain state (SSS) of the trussed-beam, obtained by calculating by the FM as a single structure and by separately calculating its components.

When performing calculations, the following dimensionless parameters of the trussed-beam are introduced:

$\varepsilon = \frac{E_b}{E_p}$ – elasticity modulus of the stiffness beam to the pillars ratio, where E_b is the modulus of elasticity of the main beam, E_p is the modulus of elasticity of the struts;

$\omega_1 = \frac{A_b}{A_i}$, $\omega_2 = \frac{A_b}{A_v}$ – parameters of the geometric characteristics of the cross-sections, where A_b is the cross-sectional area of the stiffness beam, A_i is the cross-sectional area of the inclined struts, A_v is the cross-sectional area of the vertical strut;

$\zeta = \frac{f}{l}$ – slope parameter of the struts;

$\lambda = \frac{l}{h}$ – design flexibility parameter of the struts;

$\xi = \frac{x}{l}$ – stiffness beam's sectional abscissa.

The calculation of the trussed-beam as a single structure is carried out by the FM using the main scheme provided in Fig.1.b. The dimensionless internal forces in the main system are described by the following functional dependencies:

– unit bending moment in the stiffness beam

$$m_1'(\xi) = 0.5 \cdot \xi \quad (0 \le \xi \le 0.5)$$

$$m_1'(\xi) = 0.5 \cdot (1 - \xi) \quad (0.5 \le \xi \le 1)$$

where $m_1'(\xi) = \dfrac{m_1(x)}{l}$;

– unit axial (normal) force in the stiffness beam

$$n_{1b}(\zeta) = \frac{\zeta}{1 + 4\zeta^2}$$

– unit axial force in the inclined struts

$$n_{1i}(\zeta) = -\frac{\zeta}{\sqrt{1 + 4\zeta^2}}$$

– load bending moments in the stiffness beam

$$M_P'(\xi) = 0.5(\xi - \xi^2)$$

where $M_P'(\xi) = \dfrac{M_P(x)}{ql^2}$.

The canonical equation of the FM has the form:

$$\delta_{11} X_1 + \Delta_{1P} = 0 \tag{1}$$

The coefficient and free term entering into equation (1) is described by the following functional dependencies obtained according to the Maxwell-Mohr formula:

$$\delta_{11}'(\varepsilon, \omega_1, \omega_2, \lambda, \zeta) = \frac{\lambda^2}{4} + 4 \cdot \varepsilon \cdot \omega_1 \frac{\zeta^2}{\sqrt{1 + 4\zeta^2}} + \omega_2 \cdot \zeta + \frac{4\zeta^2}{(1 + 4\zeta^2)^2}$$

$$\Delta_{1P}'(\lambda) = \frac{5}{32} \cdot \lambda^2$$

where $\delta_{11}' = \delta_{11} \cdot \dfrac{E_b \cdot A_b}{l}$, $\Delta_{1P}' = \Delta_{1P} \cdot \dfrac{E_b \cdot A_b}{ql^2}$.

Hence the main unknown, which is the axial force in the rack of the struts, is described by the following functional dependence

$$X_1'(\varepsilon, \omega_1, \omega_2, \lambda, \zeta) = -\frac{\Delta_{1P}'(\lambda)}{\delta_{11}'(\varepsilon, \omega_1, \omega_2, \lambda, \zeta)}, \tag{2}$$

where $X_1' = \dfrac{X_1}{ql}$.

In turn, the dimensionless internal forces in the trussed-beam are described by the following functional dependencies:

– bending moment in the stiffness beam

$$M'(\varepsilon, \omega_1, \omega_2, \lambda, \zeta, \xi) = m_1'(\xi) X_1'(\varepsilon, \omega_1, \omega_2, \lambda, \zeta) + M_P'(\xi); \tag{3}$$

– axial force in the stiffness beam

$$N_b'(\varepsilon, \omega_1, \omega_2, \lambda, \zeta) = n_{1b}(\zeta) \cdot X_1'(\varepsilon, \omega_1, \omega_2, \lambda, \zeta); \tag{4}$$

– axial force in the inclined struts

$$N_i'(\varepsilon, \omega_1, \omega_2, \lambda, \zeta) = n_{1i}(\zeta) \cdot X_1'(\varepsilon, \omega_1, \omega_2, \lambda, \zeta). \tag{5}$$

In the case of calculating the stiffness beam as a two-span continuous beam, the bending moments do not depend on the parameters of the trussed-beam and are described in dimensionless form as the following functional dependencies as a function of the stiffness beam's sectional abscissa:

$$M_{cb}'(\xi) = \frac{m_{1cb}'(\xi)}{32} + M_{Pcb}'(\xi), \tag{6}$$

where

$$m_{1cb}'(\xi) = \begin{cases} 2\xi & \text{if } 0 \le \xi \le 0.5 \\ [2\xi - 4(\xi - 0.5)] & \text{if } 0.5 \le \xi \le 1 \end{cases}$$

$$M'_{Pcb}(\xi) = \begin{vmatrix} (0.25\xi - 0.5\xi^2) & if\ 0 \le \xi \le 0.5 \\ [0.25\xi - 0.5\xi^2 + 0.5(\xi - 0.5)] & if\ 0.5 \le \xi \le 1 \end{vmatrix}$$

In turn, the dimensionless axial forces in the struts are described by the following functional dependencies as a function of the slope parameter:

$$N'_i(\zeta) = \frac{3}{16}\frac{\sqrt{1+4\zeta^2}}{\zeta},\ N'_v(\zeta) = \frac{6}{16} \quad (7)$$

Where $N'_i(\zeta) = \frac{N_i}{ql}, N'_v(\zeta) = \frac{N_v}{ql}$

The obtained functional dependencies (3), (6) for the bending moments of the trussed-beam, allow one to compare the results of their determination in two ways depending on the parameters of the trussed-beam.

Figure 2 shows the bending moment diagrams, obtained in two ways, for the trussed-beam, made of the same material ($\varepsilon = 1$), with the following values of the remaining parameters $\omega_1 = 1,\ \omega_2 = 1,\ \lambda=5,\ \zeta = 0.25$.

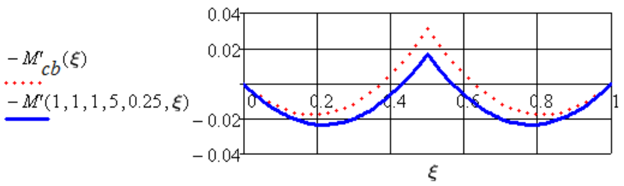

Figure 2. – Bending moment diagrams

From the above diagrams, it can be seen that the bending moments in the stiffness beam obtained in the calculation by the FM are quantitatively different from the moments, if it is calculated as a continuous beam. In the support's section, they decrease by 46%, and in the middle of the spans they increase by 33%. The difference in the values of the bending moments obtained in two ways, substantially depends on the other parameters of the trussed-beam.

Figure 3 shows the bending moment diagrams of the stiffness beam as a function of the slope parameter ζ.

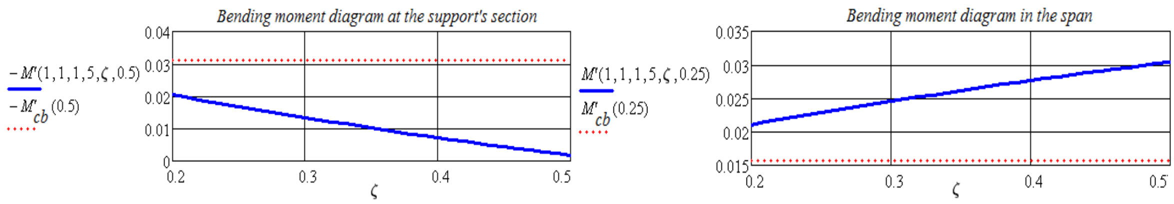

Figure 3. – Bending moment diagrams as a function of the slope parameter

From the above diagrams, it can be seen that with an increase in the slope parameter, the difference in the magnitudes of the bending moments increases. When the value $\zeta = 0.5$ in the reference section, it is 94.5%, and in the span – 67.5%.

Figure 4 shows the bending moment diagrams of the stiffness beam as a function of the flexibility parameter λ.

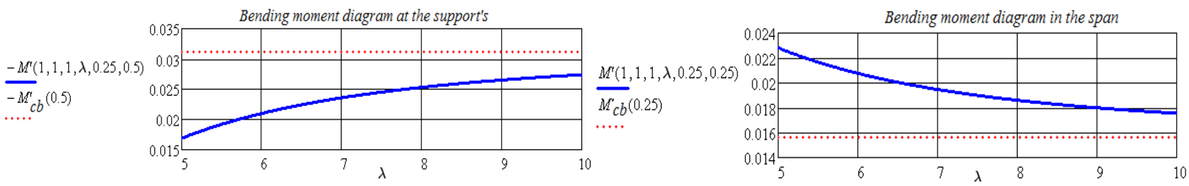

Figure 4. – Bending moment diagrams as a function of the flexibility parameter

From the above diagrams, it can be seen that when the flexibility parameter increases, the difference in the values of bending moments decreases. With the value $\lambda = 10$ in the reference section, it is 12.4%, and in the span – 8.8%.

Figure 5 shows the bending moment diagrams of the trussed-beam as a function of the structural material when $\varepsilon < 1$, with the following values of the remaining parameters $\omega_1 = 1$, $\omega_2 = 1$, $\lambda=5$, $\zeta = 0.25$

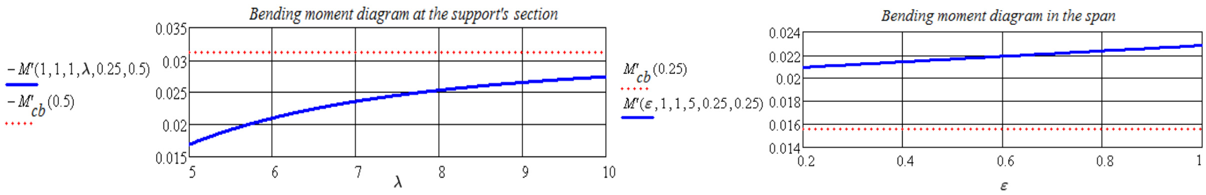

Figure 5. – Bending moment diagrams as a function of the structural material

From the above diagrams, it can be seen that the difference in bending moments obtained by two methods for a trussed-beam made of different structural materials decreases and, if the value $\varepsilon = 0.2$ in the reference section is 33.9%, and in the span – 24.2%.

The obtained functional dependences (4), (5), (7) for the axial forces of the trussed-beam allow us to compare the results of their determination in two ways depending on the parameters of the trussed-beam. Comparison of the obtained results allows us to draw the following conclusions.

First, it should be noted that if by using the first method, the axial forces in the struts depend on the parameters of the trussed-beam, in the second method, the axial forces in the pillars depend only on the slope parameter, and the axial force in the rack of the truss does not depend at all from the parameters of the trussed-beam.

Figure 6 shows the axial forces diagrams of the vertical rack strut, calculated by the two methods, as a function of the slope ζ and flexibility λ parameters

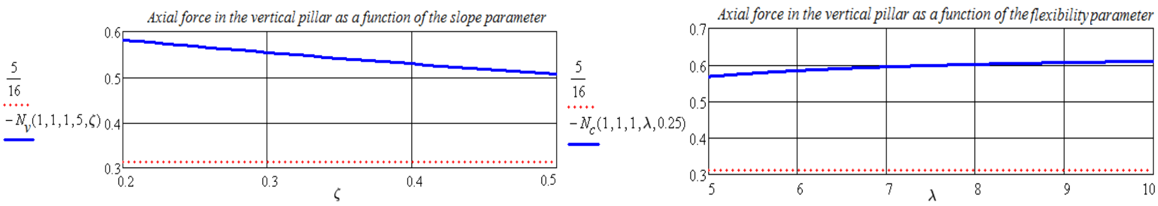

Figure 6. – Axial forces diagrams of the vertical strut as a function of the slope and flexibility parameters

From the above diagrams, it can be seen that with an increase in the slope parameter, the difference in the axial force in the vertical strut of the truss, obtained in two ways, decreases, and with increasing the flexibility parameter, it increases. In the considered intervals, with the change of the parameters, the decrease in the first case is 5.3%, and the increase in the second is 3.8%. These differences are reduced for the trussed-beam made up of different structural materials ($\varepsilon < 1$).

Figure 7 shows the axial forces diagrams of the inclined struts, calculated by the two methods, as a function of the slope ζ and flexibility λ parameters

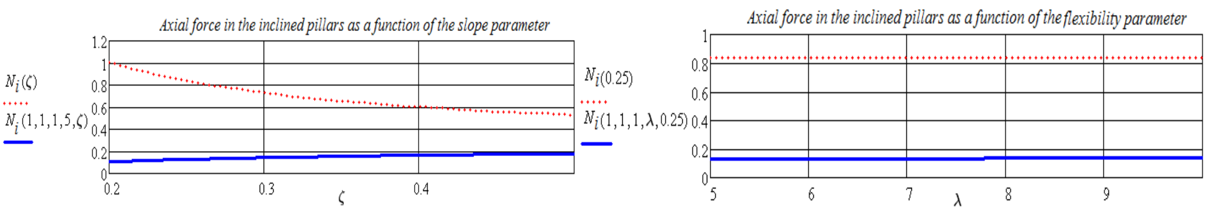

Figure 7. – Axial forces diagrams of the inclined struts as a function of the slope and flexibility parameters

From the above diagrams, it can be seen that with an increase in the slope parameter, the difference in the axial forces in the inclined struts of the truss, obtained in two ways, decreases by a factor of 1.5, and does not change with the increase of the flexibility parameter. These differences are practically the same for the trussed-beam, made up of different structural materials ($\varepsilon < 1$).

Secondly, it should be noted that in the case of a separate calculation of the trussed-beam, additional axial forces arising in the stiffness beam are considered independent of the parameters of the trussed-beam.

Figure 8 shows the axial forces diagrams of the stiffness beam as a function of the slope ζ and flexibility λ parameters

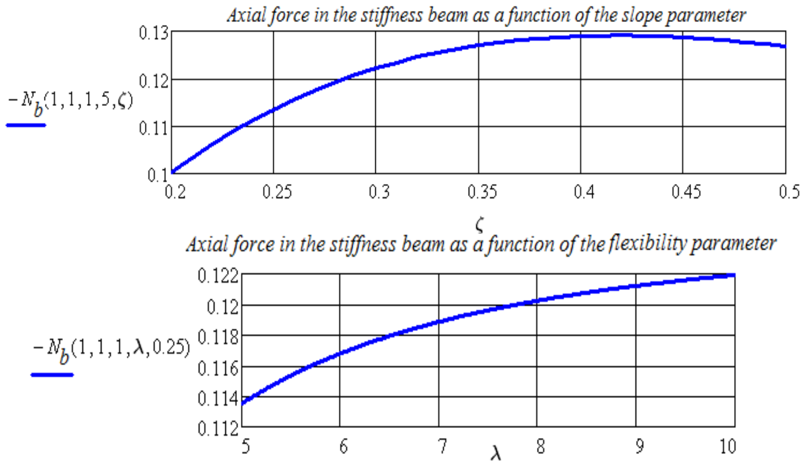

Figure 8. – Axial forces diagrams of the stiffness beam as a function of the slope and flexibility parameters

From the above diagrams, it can be seen that the axial forces in the stiffness beam increase, both with the increase in the slope parameter and with the increase in the flexibility parameter. In the considered intervals, with the change of the parameters, in the first case, the increase is 39.5%, in the second is 16.5%.

Figure 9 shows the axial forces diagrams of the stiffness beam as a function of the structural material when $\varepsilon < 1$, with the following values of the remaining parameters $\omega_1 = 1, \ \omega_2 = 1, \ \lambda=5, \ \zeta = 0.25$

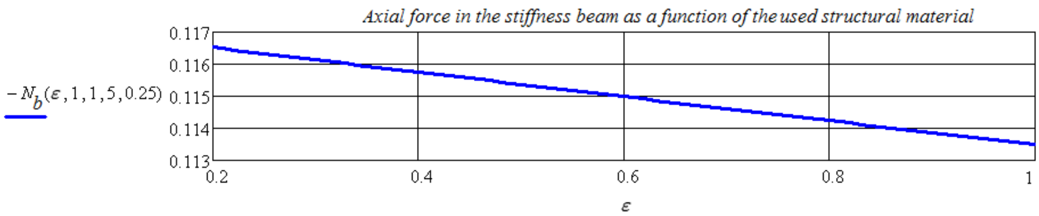

Figure 9. – Axial forces diagrams of the stiffness beam as a function of the structural material

From the above diagram, it can be seen that the axial forces in the stiffness beam of the trussed-beam, made of different structural materials, additionally increase, if the value $\varepsilon = 0.2$ in the reference section increases by 3%.

Thus, the separate calculation of trussed-beams used in the practice of engineering calculations, can lead to errors in determining the internal forces, with the assigned dimensions of the stiffness beam's and the strut elements' cross-sections and the assessment of their bearing capacity.

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